Data Management and Probability

General Curriculum Outcome G:

Students will represent and solve problems involving uncertainty.

GCO (G): Students will represent and solve problems involving uncertainty.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) make predictions regarding, and design and carry out, probability experiments and simulations in relation to a variety of real-world situations

SCO: By the end of grade 9, students will be expected to

G1 make predictions of probabilities involving dependent and independent events by designing and conducting experiments and simulations

Elaboration – Instructional Strategies/Suggestions

G1 In grade 7, students explored independent events, using tree diagrams and area models. These may need to be revisited briefly with students.

It is important to give the students a clear understanding of the difference between events which are dependent and those which are independent. If the probability of the second event is affected by the outcome of the first event, then the two events are dependent. However, if the probability of the second event is not influenced by the outcome of the first event, then the two events are independent. Most situations that students have encountered in the curriculum that involve two events are independent. One of the most common experiments to illustrate dependence versus independence is simulated through drawing objects from a container. If you replace the first object before drawing the second, then the second event becomes independent of the first, whereas, if the first object is not replaced, then the second event is dependent on the first. Students should draw tree diagrams or area diagrams for both situations to help them understand the difference.

❐ Put two red and three whites cubes in a bag.
   a) What is the probability that two red cubes will be drawn from the bag if there is no replacement?
   b) What is the probability that two red cubes will be drawn from the bag if the first is replaced before drawing the second?

These questions can be answered experimentally by placing the cubes in a bag and conducting a series of trials. Such experiments may be done for their own sake, or to simulate some other event which cannot be modelled directly. The situation above could simulate a situation where a family has two boys and three girls and we want to find the probability that the first two children born are boys. Naturally, replacement would be required in this situation, since the events are independent of each other.

A simulation can be generated using a graphing calculator or computer software. A spreadsheet can be useful to record data. Whenever technology is available, it can be used as a support in achieving this outcome, but it should not totally replace hands-on activities. Simulation was addressed in grade 7, and more sophisticated situations involving complementary events were studied in grade 8. See the instructional implications for grade 7 for a detailed explanation of how to conduct a simulation.

GCO (G): Students will represent and solve problems involving uncertainty.

Worthwhile Tasks for Instruction and/or Assessment

Performance

G1.1 Ask students to work in groups to design a simulation for determining the probability that, in a family of three, all children will be girls. The use of a spreadsheet or graphing calculator is encouraged, but this activity can also be achieved using three coins.

a) Ask students to select an appropriate model for the simulation, and justify their choice. [The model can be two-colour counters, where one colour represents girls and the other represents boys.]
b) Ask students what the probability is that there will be at least one boy in this family of three children.
c) Ask them what the probability is that there will be all boys in this family of three children.

G1.2 Prior to class, put two counters of one colour (black) and four of another colour (red) in a bag. Ask students to work with a partner, where one person acts as recorder and the other draws objects from a bag.

a) Ask them to draw a counter from the bag, and then replace the counter and draw again, and record the results of the two draws. Ask them to repeat this activity 50 times and record the results.
    i) Have students use the results to estimate the probability of getting two blacks, two reds, and a black and a red.
    ii) Have students use the results to estimate the probability of not getting two that are the same colour.
b) Ask students to draw a counter from the bag, and then draw a second counter. Ask them to record the results of the two draws, and then return the two counters to the bag. Have them repeat this process for 50 trials.
    i) Ask students to use the results to estimate the probability of getting two blacks, two reds, and a red and a black.
    ii) Ask students to use the results to estimate the probability of not getting two that are the same colour.
c) Ask students which of part a) and b) would they describe as dependent, and which would they describe as independent. Ask them to explain their choice.
d) Ask students to pool their results and discuss any differences they notice between the pooled data and the data collected by each pair.

G1.3 Sue placed two green and two red cubes in a bag. She wanted to find the probability of drawing two green if the first one is not returned before drawing the second. Ask students to design an experiment to solve this problem and conduct it. [Have students save the data collected in each of G1.1, G1.2, and G1.3 since it will be useful for outcome G3.]

Suggested Resources

Interactions 9

G1 Ch. 8, pp. 174-183

Mathpower 9

G1 xxii,
Ch. 9, pp. 348-355

GCO (G): Students will represent and solve problems involving uncertainty.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) make predictions regarding, and design and carry out, probability experiments and simulations in relation to a variety of real-world situations

ii) derive theoretical probabilities, using a range of formal and informal techniques

SCO: By the end of grade 9, students will be expected to

G2 determine theoretical probabilities of independent and dependent events

Elaboration – Instructional Strategies/Suggestions

G2 It is not always directly obvious when events are dependent or independent. Sometimes students will need to discuss whether two events are truly independent of each other. In general, event A is independent of event B if the probability of A is not affected by the occurrence or nonoccurrence of B. Likewise, event B is dependent on event A when the result in event B is directly affected by the occurrence or nonoccurrence of A.

- ❒ Three red and one white counter are placed in a bag. The probability of drawing two reds when the first one is replaced before drawing the second is $\frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$. Ask students to create a tree diagram and use it to explain why this is true.
- ❒ The probability of drawing two reds wheg the first one is not replaced before drawing the second is $\frac{3}{4} \cdot \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$. Once the first red is drawn, the sample size for the second event changes to two red and one white. Naturally, in this situation, the probability of getting two reds if a white is drawn on the first try is 0. Ask students to explain why this is so.

In general, for any two independent events, A and B, the probability of A and B is equal to P(A) · P(B). Students can also use tree diagrams to solve compound-event problems. This would be considered an informal method. For the examples above, probability tree diagrams are shown. The various branches represent different probabilities; therefore, the branches are labelled in order to help clarify what each branch represents.

$\frac{3}{4}$	$\frac{3}{4}$	$\frac{9}{16}$
	$\frac{1}{4}$	$\frac{3}{16}$
$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{16}$
	$\frac{1}{4}$	$\frac{1}{16}$

$\frac{3}{4}$	$\frac{2}{3}$	$\frac{6}{12}$	$\frac{1}{2}$
	$\frac{1}{3}$	$\frac{3}{12}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{3}{3}$	$\frac{3}{12}$	$\frac{1}{4}$
	$\frac{0}{3}$	$\frac{0}{12}$	0

These tree diagrams are quite different from the ones studied in grade 7. In grade 7, tree diagrams were used to identify all the possible outcomes.

GCO (G): Students will represent and solve problems involving uncertainty.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

G2.1 A box contains 3 red balls and 2 blue balls.

a) You remove two balls from the box; the second one is removed without replacing the first.
   i) Draw a tree diagram to show all the possible outcomes for this situation.
   ii) What is the probability of drawing two blue balls?
b) Suppose you draw two balls from the box, and the first one is replaced before drawing the second.
   i) Draw a tree diagram to show all the possible outcomes for this situation.
   ii) What is the probability of drawing two blue balls?

G2.2 The following describes events A and B. Decide whether the events are dependent or independent and explain your thinking.

a) A. Mrs. Brown's first child was a boy.
   B. Mrs. Brown's second child will be a boy.
b) A. It snowed last night.
   B. Jon will be late for school this morning.
c) A. Leif swam 2 hours every day for the last ten months.
   B. Leif's swimming times have improved.
d) A. Allison got an A in her last math test.
   B. Allison will get an A in her next math test.
e) A. Matthew got a head in his last coin toss.
   B. Matthew will get a head in his next coin toss.

G2.3 This problem is based on the same situation as that described in G1/2.1. In this situation, students are expected to solve the problem, using theoretical probability. Prior to class, put two black counters and four red counters in a bag.

a) Ask students to find the probability of drawing two black counters, two red counters, and a red and a black counter from the bag when the first one is replaced before drawing the second.
b) Ask students to find the probability of drawing two black counters, two red counters, and a red and a black counter from the bag when the first one is not replaced before drawing the second.

G2.4 Sue placed two green and two red cubes in a bag. Find the probability of drawing two green cubes if the first one is not returned before drawing the second.

Suggested Resources

Interactions 9

G2 Ch. 8, pp. 174-183

Mathpower 9

G2 Ch. 9, pp. 350-355

GCO (G): Students will represent and solve problems involving uncertainty.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) derive theoretical probabilities, using a range of formal and informal techniques

iii) determine and compare experimental and theoretical results

iv) relate a variety of numerical expressions to the corresponding experimental or simulation situation

SCO: By the end of grade 9, students will be expected to

G3 demonstrate an understanding of how experimental and theoretical probabilities are related

G4 recognize and explain why decisions based on probabilities may be combinations of theoretical calculations, experimental results, and subjective judgments

Elaboration – Instructional Strategies/Suggestions

G3 Once students have worked with probability experiments and derived theoretical probability, they should be able to compare the results obtained from each method. Students should be able to relate the experimental probability with results achieved, using the definition of theoretical probability. Discuss with the students when they can be comfortable that the experimental probability is a close approximation of the theoretical probability, and what can be done to increase their confidence in experimental results. Discussion here should focus on the influence of increasing sample size. For example, students can look at the results when they have collected a sample of size 50, pool their data with another student's to have data for sample size 100, and then pool the data for the whole class to see what effect a very large sample size may have on the results.

G4 Students should relate to how decision making is affected by the combination of probability and subjective judgments. For example, consider the variety of strategies people use when choosing their lottery numbers. Some use the same numbers for repeated lotteries, others use past frequencies to select their numbers, and others allow their numbers to be randomly selected.

Another example to consider is the impact that the probability of rainfall has on decisions made about whether to engage in some outdoor sports, replace a window in a house, or put the laundry out on a clothes line.

Students might engage in evaluating situations that are amenable to reasonably accurate predictions, those that are questionable, and those for which the unknowns are not quantifiable. Road accidents with/without seatbelts is a good example for safe prediction, while the use of airbags involves a more questionable situation. There are many situations where the unknowns are so great that probabilistic arguments only appear authoritative, such as life on other planets, dangers of transgenic animals, and the threat of global warming. Discuss with students: What are the reasons for the uncertainty? What are the important questions to ask regarding a situation in order to reduce it to probabilistic form?

❐ Discuss why knowing that one party is favoured to win an election by 65% of voters may or may not influence voting of individuals on election day.

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Worthwhile Tasks for Instruction and/or Assessment

Performance

G3.1 Ask students to

a) roll two dice 50 times to determine the experimental probability of rolling a total of 7
b) use a rectangular array to determine the theoretical probability of getting a seven
c) discuss the difference in theoretical and experimental probability in this case, and why there are differences

Pencil and Paper

G3.2 Compare and discuss the results in G1.2 and G2.3.

G3.3 Find the theoretical probability associated with G1.1, and compare it to the experimental results.

Interview

G4.1 Tell students that they have been told that they tested positive on a medical test that is 90% accurate. The disease for which they were tested is very rare – only one person in a million suffers from it. Ask students if they should assume they have the disease, and to explain their answer. [Students should discuss the fact that the sample size for positive must be very small because the disease is so rare. On the other hand, the sample size for negative is probably very large for exactly the same reason. We can, therefore, probably have greater confidence in negative results then positive results.]

Presentation

G4.2 Odette knows that theoretically she has a 1 in 2 chance of getting a head when she flips a coin. Claude had a particular coin that, when flipped 50 times, came up heads 40 of the 50 times. Ingrid feels that, even if there is an equal chance of getting heads, heads will appear more often because she feels it is her lucky choice. Ask students to categorize the three situations as subjective (based on opinion), experimental, or theoretical, and present to the class how each can play a part in decision making.

Portfolio

G4.3 A notorious individual was found guilty of murder, partly because of forensic testing which pointed to him. It was later discovered that the lab which did the testing knew that the sample came from a suspect, and were being asked for a judgment by the prosecutor. Ask students what biases may affect the judgment of the lab. Ask them if these biases could have changed the probability estimates of the lab. Ask students if there is any way that these biases might be reduced or eliminated. Ask if their solutions would be very difficult or costly to implement.

Suggested Resources

Interactions 9

G3 Ch. 8, pp. 188

G4 Ch. 8, pp. 182-183

Mathpower 9

G3 Ch. 9, pp. 350-355