Prince Edward Island
CANADA

Education and Early Childhood Development
English Programs

Prince Edward Island Mathematics Curriculum

CURRICULUM

Mathematics

Grade 1

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Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base. Developers have examined the curriculum proposed throughout Canada and secured the latest research in the teaching of mathematics, and the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for K-9 Mathematics* (2006) has been adopted as the basis for a revised mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories, and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP, and on the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➢ Essential Graduation Learnings

Essential graduation learnings (EGLs) are statements describing the knowledge, skills, and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills, and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work, and study today and in the future. Essential graduation learnings are cross curricular, and curriculum in all subject areas is focussed to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will demonstrate knowledge, skills, and attitudes expressed as essential graduation learnings, and will be expected to

- respond with critical awareness to various forms of the arts, and be able to express themselves through the arts;
- assess social, cultural, economic, and environmental interdependence in a local and global context;
- use the listening, viewing, speaking, and writing modes of language(s), and mathematical and scientific concepts and symbols, to think, learn, and communicate effectively;
- continue to learn and to pursue an active, healthy lifestyle;
- use the strategies and processes needed to solve a wide variety of problems, including those requiring language and mathematical and scientific concepts;
- use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

➢ Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate to all educational partners high expectations for students in mathematics education. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to

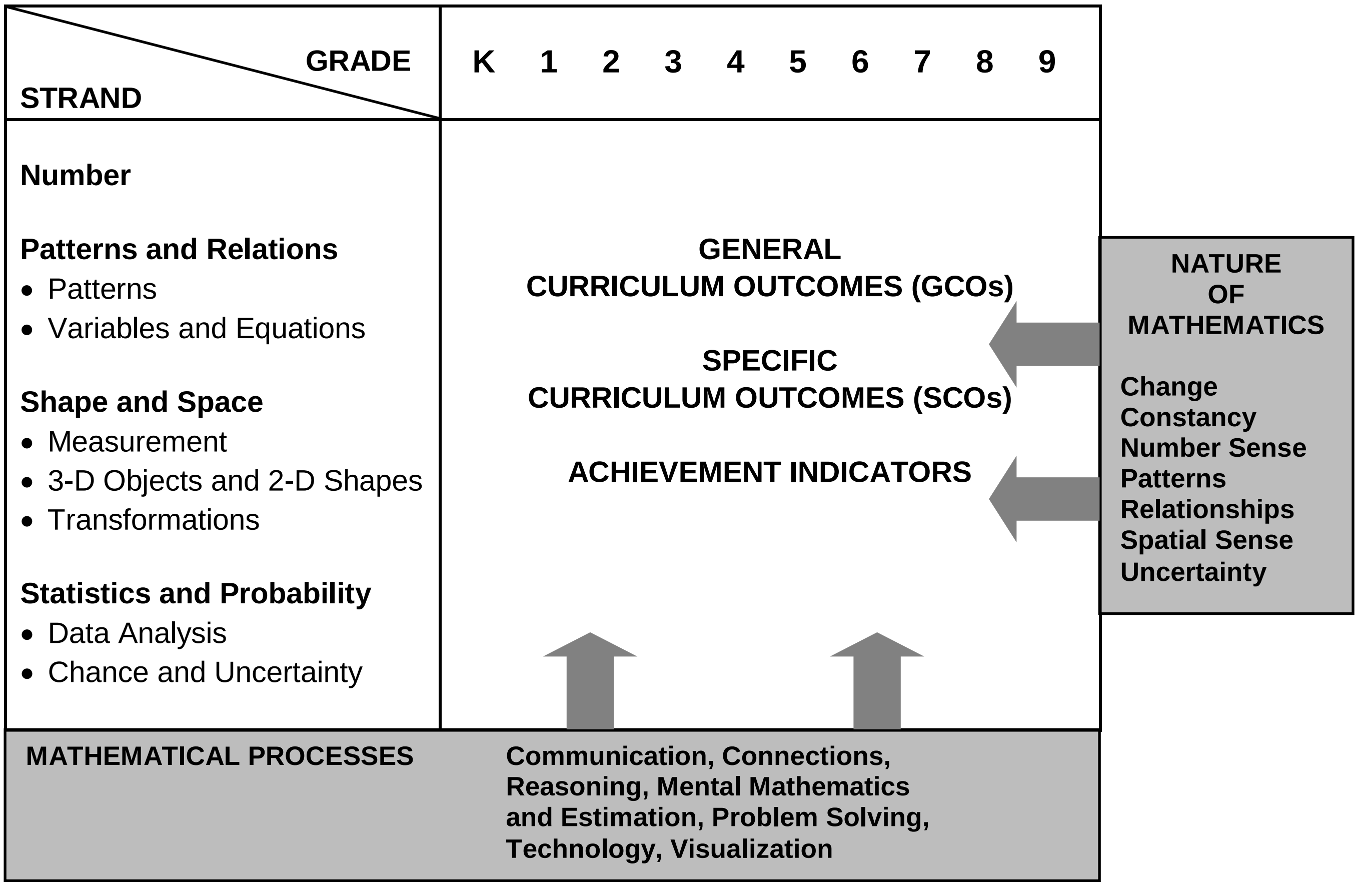
- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning;
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks;
- exhibit curiosity.

Conceptual Framework for K-9 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into four strands, namely Number, Patterns and Relations, Shape and Space, and Statistics and Probability. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connections among concepts both within and across strands. Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- Decreasing emphasis on rote calculation, drill, and practice, and the size of numbers used in paper and pencil calculations makes more time available for concept development.
- Problem solving, reasoning, and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➢ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]**
- develop visualization skills to assist in processing information, making connections, and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

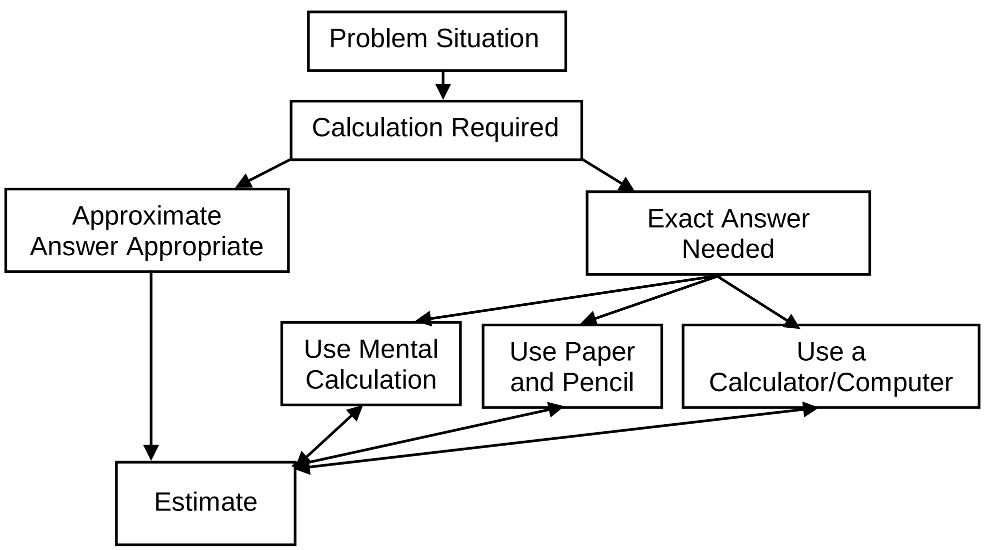
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves calculation without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need - more than ever before - with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein, 2001). Mental mathematics “provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers” (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know when to estimate, what strategy to use, and how to use it. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you. . . ?" or “How could you. . . ?” the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- using estimation
- guessing and checking
- looking for a pattern
- making an organized list or table
- using a model
- working backwards
- using a formula
- using a graph, diagram, or flow chart
- solving a simpler problem
- using algebra.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations;
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K-3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform, and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and knowledge of several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

➢ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics which are woven throughout this document. These components include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180^0.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (*The Primary Program*, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, and flexible and intuitive with numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collecting and analysing of data, and describing relationships visually, symbolically, orally, or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to interpret representations of 2-D shapes and 3-D objects, and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 2-D shapes and 3-D objects.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to use dimensions and make predictions about the results of changing dimensions.

- Knowing the dimensions of an object enables students to communicate about the object and create representations.
- The volume of a rectangular solid can be calculated from given dimensions.
- Doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Young children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Their mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Initial problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➢ Connections across the Curriculum

There are many possibilities for connecting Grade 1 mathematical learning with the learning occurring in other subject areas. Making connections between subject areas gives students experiences with transferring knowledge and provides rich contexts in which students are able to initiate, make sense of, and extend their learnings. When connections between subject areas are made, the possibilities for transdisciplinary inquiries and deeper understanding arise. When making such connections, however, teachers must be cautious not to lose the integrity of the learning in any of the subjects.

➢ Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding, and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should replace some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill-oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child’s learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a teacher can give a parent clearer understanding of the mathematics curriculum and of the child’s progress in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent’s window to the classroom.

➢ Diversity in Student Needs

Every class has students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters, but should be designed to help all students (whether strong, weak or average) to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson from which all students come away with a better understanding of what the solution to an equation really means.

➢ Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean not only that enrolments of students of both genders and various cultural backgrounds in public school mathematics courses should reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➢ Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English deficiencies and cultural differences must not be barriers to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and co-ordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication "as an essential part of mathematics and mathematics education (p.60)." The *Standards* elaborate that all students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading, and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers and will facilitate "communicating to learn mathematics and learning to communicate mathematically" (NCTM, p.60).

To this end,

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counsellors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not on other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated with appropriate language support to both students and parents;
- to verify that barriers have been removed, educators should monitor enrolment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➢ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development - such as poverty alleviation, human rights, health, environmental protection, and climate change - into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental, and economic perspective, and exploring how those factors are interrelated and interdependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database

Resources for Rethinking, found at **http://r4r.ca/en**. It provides teachers with access to materials that integrate ecological, social, and economic spheres through active, relevant, interdisciplinary learning.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, or teaching has been effective, or how best to address student learning needs. The quality of the assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others.

➢ Assessment

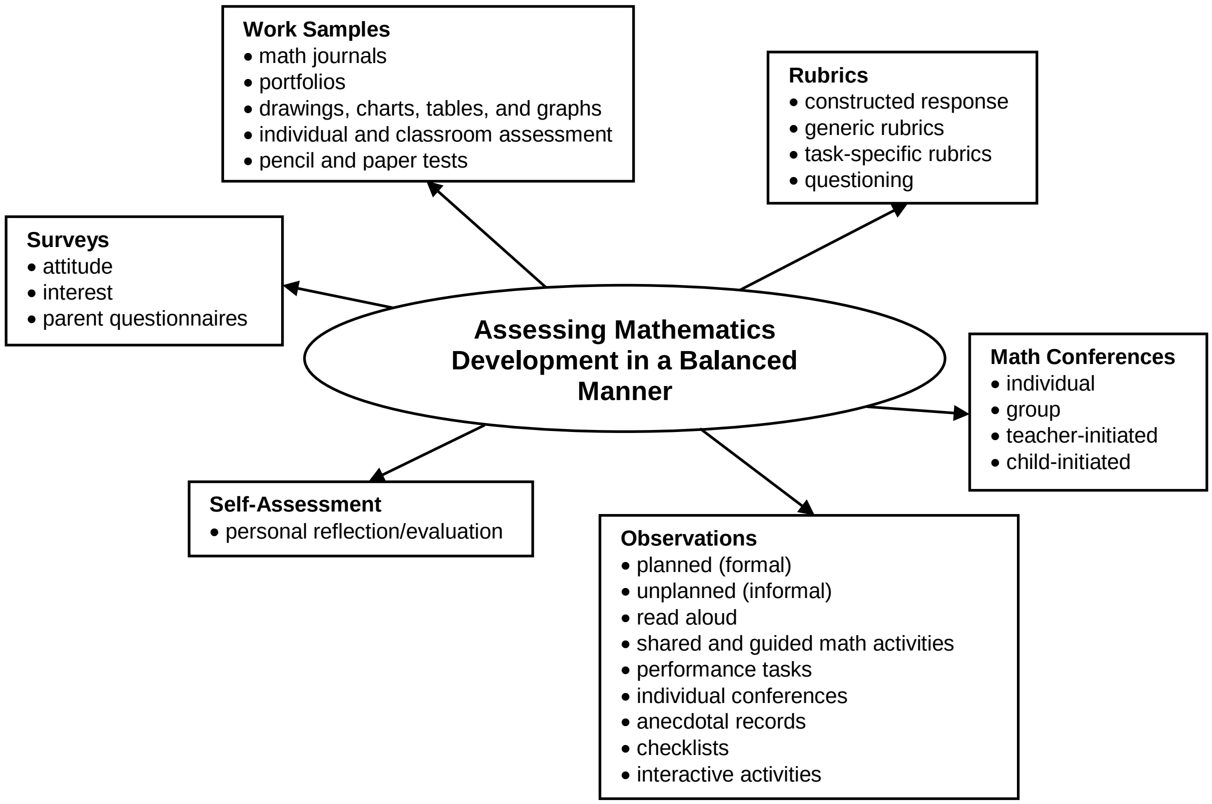
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children’s learning;
- providing information to teachers on the effectiveness of their teaching, the program, and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources, including

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment.

This balanced approach for assessing mathematics development is illustrated in the diagram below.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning, and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used

- to engage students in their own learning and self-assessment;
- to help students understand what is important in the mathematical concepts and particular tasks they encounter;
- to develop effective habits of metacognition and self-coaching;
- to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used

- to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
- to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
- to provide feedback to students about how they are doing and how they might improve;

- to provide feedback to other professionals and to parents about how to support students' learning.

Assessment *of* learning is used

- to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
- to facilitate reporting;
- to provide the basis for sound decision-making about next steps in a student's learning.

➢ Evaluation

Evaluation is the process of analysing, reflecting upon, and summarizing assessment information, and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires

- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information;
- using a high level of professional judgment in making decisions based upon that information.

➢ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes, and phone calls.

➢ Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration, and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles, as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes, or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.

- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.
- Assessment reports should be clear, accurate, and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that

- the best interests of the student are paramount;
- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes;
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he/she knows and can do.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island mathematics curriculum are organized into four strands across the grades K-9. They are Number, Patterns and Relations, Shape and Space, and Statistics and Probability. These strands are further subdivided into sub-strands, which are the general curriculum outcomes (GCOs). They are overarching statements about what students are expected to learn in each strand or sub-strand from grades K-9.

Strand	General Curriculum Outcome (GCO)
Number (N)	**Number:** Develop number sense.
Patterns and Relations (PR)	**Patterns**: Use patterns to describe the world and solve problems.
	Variables and Equations: Represent algebraic expressions in multiple ways.
Shape and Space (SS)	**Measurement**: Use direct and indirect measure to solve problems.
	3-D Objects and 2-D Shapes: Describe the characteristics of 3-D objects and 2-D shapes, and analyse the relationships among them.
	Transformations: Describe and analyse position and motion of objects and shapes.
Statistics and Probability (SP)	**Data Analysis**: Collect, display, and analyse data to solve problems.
	Chance and Uncertainty: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific outcome.

The first two pages for each outcome contain the following information:

- the corresponding **strand** and **general curriculum outcome**
- the **Specific Curriculum Outcome**(s) and the mathematical **processes** which link this content to instructional methodology
- the **scope and sequence** of concept development related to this outcome(s) from K - 2
- an **elaboration** of the outcome
- a list of **achievement indicators**

  Students who have achieved a particular outcome should be able to demonstrate their understanding in the manner specified by the achievement indicators. It is important to remember, however, that these indicators are not intended to be an exhaustive list for each outcome. Teachers may choose to use additional indicators as evidence that the desired learning has been achieved.

The last two pages for each outcome contain lists of **instructional strategies** and **strategies for assessment**.

The primary use of this section of the guide is as an ***assessment for learning*** (formative assessment) tool to assist teachers in planning instruction to improve learning. However, teachers may also find the ideas and suggestions useful in gathering ***assessment of learning*** (summative assessment) data to provide information on student achievement.

NUMBER

SCO: **N1: Say the number sequence, 0 to 100, by:** **• 1s forward between any two given numbers** **• 1s forward between any two given numbers** **•2s to 20, forward starting at 0** **• 5s and 10s to 100, forward starting at 0.** **[C, CN, V, ME]**

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math/Estimation

Scope and Sequence

Kindergarten	Grade One	Grade Two
N1 Say the number sequence by 1s starting anywhere from 1 to 10 and from 10 to 1.	**N1** Say the number sequence, 0 to 100, by: • *1s forward and backward between any two given numbers* • *2s to 20, forward starting at 0* • *5s and 10s to 100, forward starting at 0.*	**N1** Say the number sequence from 0 to 100 by: • *2s, 5s and 10s, forward and backward, using starting points that are multiples of 2, 5 and 10 respectively* • *10s using starting points from 1 to 9* • *2s starting from 1.*

Elaboration

Students are developing an understanding of **number** and **counting**. They are able to count forwards and backwards and count on to 10. They should continue to practise rote counting and begin to skip count various number sequences**.** Include situations which require:

- counting forwards and backwards
- counting on by ones from a given number
- skip counting (e.g., 2, 4, 6, 8,...)

Although it is unlikely that children at this age will understand place value, students should experience a wide variety of situations which require counting beyond 10. (Students will be expected, however, to deal only with 2-digit numbers at this grade level.). Students should become familiar with **counting patterns** to 100. Include:

- skip counting by 2s, 5s, 10s (starting from 0, as well as from other numbers)
- counting using coins (pennies, nickels, dimes)
- counting on from a given number
- counting back from a given number

SCO: **N1: Say the number sequence, 0 to 100, by:**
• 1s forward and backward between any two given numbers
• 2s to 20, forward starting at 0
• 5s and 10s to 100, forward starting at 0.
[C, CN, V, ME]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Recite forward by 1s the number sequence between two given numbers (0 to 100).
- Recite backward by 1s the number sequence between two given numbers.
- Record a given numeral (0 to 100) symbolically when it is presented orally.
- Read a given numeral (0 to 100) when it is presented symbolically.
- Skip count by 2s to 20 starting at 0.
- Skip count by 5s to 100 starting at 0.
- Skip count forward by 10s to 100 starting at 0.
- Identify and correct errors and omissions in a given number sequence.

SCO: **N1: Say the number sequence, 0 to 100, by:**
• 1s forward and backward between any two given numbers
• 2s to 20, forward starting at 0
• 5s and 10s to 100, forward starting at 0.
[C, CN, V, ME]

Instructional Strategies

Consider the following strategies when planning lessons:

- The hundred chart is an excellent tool to explore counting patterns. For example, when skip counting by 5s, students might put a counter on every 5th number, reading the number as the counter is placed on it.
- A walk-on number line (on the floor) or an open number line can also be used to experience skip counting.
- Create a number line in the classroom by adding another number for each day the students are in school. This can be used to reinforce counting sequences (by 1s, 2s, 5s, and 10s).
- Attendance charts can be organized in pocket charts in rows of 5 or 10 using different colours for each group. This chart can then be used to count on and skip count.
- Calendars can also be an effective model to support counting.

Suggested Activities

- Ask students to use the repeat function on the calculator to skip count to a target number. For example: If you start at 0 and want to end on 40, by which number(s) could you skip count? What if you started at a different point? What if you wanted to end at a different point?
- Ask students to count items which occur naturally in twos (e.g., shoes, hands, eyes).
- Invite students to sing songs and recite poems which involve counting backwards and forwards and skip counting; for example:
  "Ten In A Bed", "One, Two, Buckle My Shoe", "This Old Man"
- Invite students to use calculators to count. For example, as some students place cookies into a bag and count aloud, others may repeatedly add one on calculators to keep track electronically.
- Ask the student to count aloud to 50 by 5s while using the constant function of the calculator.
- Have students work with a partner to create a number sequence with a missing number. Exchange sequence with another pair and identify the missing number.
- Ask students how many ways they can count to 30. Have students talk about their findings.

SCO: **N1: Say the number sequence, 0 to 100, by:**
• 1s forward and backward between any two given numbers
• 2s to 20, forward starting at 0
• 5s and 10s to 100, forward starting at 0.
[C, CN, V, ME]

Assessment Strategies

- Ask students to count by 2s (5s or 10s) as you clap. Have students to tell you/record the final number when you finish clapping.
- Show students a number sequence with an error or a missing number. Have students to correct the sequence.
- Have students “count-off” by 1s, 2s, 5s, and 10s. Observe whether students are able to follow the number sequence.
- Give each student in class a card with a number symbol on it. Have students put themselves in order using the number cards. The cards can be by ones, twos, or fives.
- Ask the student to count backwards starting at 18.
- Ask the student: If you count by twos, starting at 0, will you say 7? Why or why not?
- Ask the student to begin counting at 13 and stop at 25.
- Provide a hundred chart. Tell the student: I counted from 10 to 50 and only said 5 numbers. What do you think I said?
- Tell the student: I said, "10, 20, 25" when I was counting some coins. What coins do you think I had?

SCO: **N2: Recognize, at a glance, and name familiar arrangements of 1 to 10 objects or dots (*subitize*).**
[C, CN, ME, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

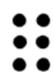
Scope and Sequence

Kindergarten	Grade One	Grade Two
N2 Recognize, at a glance, and name familiar arrangements of 1 to 5 objects or dots.	**N2** Recognize, at a glance, and name familiar arrangements of 1 to 10 objects or dots.	

Elaboration

Children need to be able to recognize, without counting, configurations or spatial patterns for small numbers of items (up to 10). This is called **subitizing** and will encourage reflective thinking while deepening their number sense.
It will be useful with respect to:

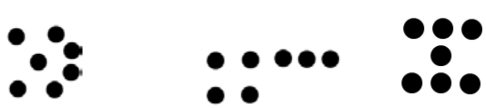
- addition; for example, 5 = 4 + 1 (or 2 + 1 + 2) is apparent from 

  and 6 = 3 + 3 or 2 + 2 + 2 is apparent from: 

- place value**;** for example, groups of 10 can be easily observed in

(Note: Dice and other games strengthen recognition of many configurations of numbers.)

Provide opportunities for students to discover which configurations are easier to recognize. For example, ask students to show 7 in several ways, and then decide which configuration(s) is (are) easiest to identify. Possible configurations might include:



SCO: **N2: Recognize, at a glance, and name familiar arrangements of 1 to 10 objects or dots.**
[C, CN, ME, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Look briefly at a given familiar arrangement of objects or dots and identify the number represented without counting.
- Look briefly at a given familiar arrangement and identify how many objects there are without counting.
- Identify the number represented by a given arrangement of objects or dots on a ten frame.

SCO: **N2: Recognize, at a glance, and name familiar arrangements of 1 to 10 objects or dots.**
[C, CN, ME, V]

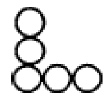
Instructional Strategies

Consider the following strategies when planning lessons:

- Subitizing should initially focus on arrangements of numbers from 1 to 5 and gradually increase. Use dice, “dot cards”, ten-frames and other models with easily recognizable configurations of numbers so that students become familiar with them. The level of difficulty can be adjusted by the arrangements used and the length of time the image is displayed. The constraint of time forces children to gradually move away from one-by-one counting and develop more efficient strategies such as counting on.

Suggested Activities

- Show students 5 counters arranged in an L-shape with equal sides.
  Ask what other numbers of counters can be arranged to form "Ls"?

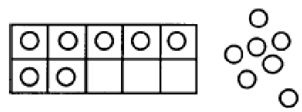


- Arrange counters on an overhead projector. Switch the light on for a few moments, but not long enough for students to count the counters. Ask: What number was represented? (Repeat several times, using different configurations of the same number.) Ask: Which configuration was easiest to recognize? Why?
- Hold up a dot card for a few seconds. Ask: How many? What did you see first?
  Include dot cards with both familiar and unfamiliar arrangements. Consider also using cards with two colors of dots.
- State a number or hold up a numeral card and students find the corresponding dot card.
- Concentration Game: (Materials: 2 sets of dot cards that show the same number) Place a number of dot cards face down in a 5 x 4 array. Students take turns turning over two cards trying to find a match.
- Play a favourite board game with your children using dot cards instead of a number cube.
- Have children sort dot card arrangements into groups that display the same number.

SCO: **N2: Recognize, at a glance, and name familiar arrangements of 1 to 10 objects or dots.** [C, CN, ME, V]

Assessment Strategies

- Show the class a dot card or ten-frame arrangement. Ask them to respond by writing the numeral on their individual board or paper.
- Ask students to draw an arrangement of numbers that makes it easy to recognize 6.
- Ask the student to arrange 8 (or other numbers) counters in a way that will make it easy to tell that there are 8.
- Ask the student to make (a) sketch/sketches showing how he/she "sees" 9. (or other numbers)
- Have a set of cards or objects in your pocket. At any time during the day, show a student one of the cards or group of objects and ask them to tell you how many.
- Explain why it might be easier to count the number of counters on the left than the number on the right.



SCO: **N3: Demonstrate an understanding of counting by:**
- **indicating that the last number said identifies "how many"**
- **showing that any set has only one count**
- **using the counting on strategy**
- **using parts or equal groups to count sets.**

[C, CN, ME, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence

Kindergarten	Grade One	Grade Two
N3 Relate a numeral, 1 to 10, to its respective quantity.	**N3** Demonstrate an understanding of counting by: • *indicating that the last number said identifies "how many"* • *showing that any set has only one count* • *using the counting on strategy* • *using parts or equal groups to count sets.*	

Elaboration

Counting is the most fundamental skill in the development of number sense and must not be confused with rote-counting. Number is an *idea,* the concept of "how much", and is represented in various quantities of objects that must be counted by children. Meaningful counting leads to the development of other critical, central ideas in mathematics. For example:

- **Cardinality**
  - The number the child ends on when counting is the number of objects in the set.
- **One-to-one Correspondence**
  - One number is said for each item in the group and is counted once and only once.
- **Inclusion**
  - Amounts *nest inside* each other: six includes five plus one; five includes four plus one, etc.
- **Unitizing**
  - Children use numbers to count not only individual objects but also groups. Unitizing is a shift in perspective for children.
- **Compensation and Equivalence**
  - If you lose 1 from one number, but gain it on the other, the total remains the same.

These ideas are *constructed* by individual children and characterize leaps in the development of each child's reasoning ability

SCO: **N3: Demonstrate an understanding of counting by:**
- **indicating that the last number said identifies "how many"**
- **showing that any set has only one count**
- **using the counting on strategy**
- **using parts or equal groups to count sets.**

[C, CN, ME, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Answer the question, "How many are in the set?" using the last number counted in a given set.
- Identify and correct counting errors in a given counting sequence.
- Show that the count of the number of objects in a given set does not change regardless of the order in which the objects are counted.
- Count the number of objects in a given set, rearrange the objects, predict the new count and recount to verify the prediction.
- Determine the total number of objects in a given set, starting from a known quantity and counting on.
- Count quantity using groups of 2s, 5s or 10s and counting on.

SCO: **N3: Demonstrate an understanding of counting by:**
• indicating that the last number said identifies "how many"
• showing that any set has only one count
• using the counting on strategy
• using parts or equal groups to count sets.
[C, CN, ME, R, V]

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Children who are successful counters, have strategies to keep track of their count, such as touching and moving each object as it is counted.
- Children should be encouraged to count items in natural situations that arise in the classroom.
- Count objects in the classroom – with large collections have students group/bundle objects in 5's or 10's
- Children might play a variety of games which require counting, for example:
  - bowling (counting both the pins knocked down and the pins left standing)
  - board games (counting the number of spaces to be moved based on a spin)
  - throwing bean bags (counting how many land in the target box)

Suggested Activities

- Request that a student draw a picture of his/her favourite toys. Then ask him/her to count the number of toys in the picture.
- Allow students to count the number of napkins, cups, plates, etc. that are on the table or are needed for snack time or a special party.
- Say: I am thinking of something in the classroom of which there are exactly 5. What do you think it could be?
- Ask students to count items which occur naturally in twos (e.g., shoes, hands, eyes). (This can be extend to fives and tens)
- Place 5 counters under a cup and tell the students that they are there. Show 3 more beside the cup. Ask: How many counters are there altogether?
- Use a walk-on number line. A student rolls 2 number cubes. He/she chooses the value on one of the cubes to stand on the number line and then moves along the number line by counting on the amount from other number cube.

SCO: **N3: Demonstrate an understanding of counting by:**
• indicating that the last number said identifies "how many"
• showing that any set has only one count
• using the counting on strategy
• using parts or equal groups to count sets.
[C, CN, ME, R, V]

Assessment Strategies

- Provide students with a number of objects. Ask them to count them. After they have counted them once, rearrange the objects and get them to tell you how many. Observe children to determine their understanding of each of the principles underlying meaningful counting. Note the way in which students count:
    - *Do they touch each object as they count?*
    - *Do they set items aside as they count them?*
    - *Do they show confidence in their count or feel the need to check?*
    - *Do they check their counting in the same order as the first count or in a different order?*
- Ask students to count a large number of items in a photo. Observe how they count.
- Ask the student to count out sixteen blocks/counters onto the table. Rearrange them by moving them around the table and then display them in two groups to display a "16" combination, (e.g., 9 in one group, 7 in the other). Ask the student how many you have altogether. Repeat using different combinations. Observe the student's method of determining how many.
- Show two groups of objects. Hide one under a piece of paper labeled with its amount. Leave the other group showing. Ask the student how many objects there are in all.
- Ask the student to count out six counters. Once they have six counters, ask them to show you a total of 14 counters. Observe whether the student is able to count on from six or recounts starting at one.

SCO: **N4: Represent and describe numbers to 20 concretely, pictorially and symbolically.** [C, CN, V] **N5: Compare sets containing up to 20 elements to solve problems using:** • **referents** • **one-to-one correspondence.** [C, CN, ME, PS, R, V] **N6: Estimate quantities to 20 by using referents.** [C, ME, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence

Kindergarten	Grade One	Grade Two
N4 Represent and describe numbers 2 to 10, concretely and pictorially. **N5** Compare quantities 1 to 10, using one-to-one correspondence.	**N4** Represent and describe numbers to 20, concretely, pictorially and symbolically. **N5** Compare sets containing up to 20 elements, using • *referents* • *one-to-one correspondence to solve problems.* **N6** Estimate quantities to 20 by using referents.	**N4** Represent and describe numbers to 100, concretely, pictorially and symbolically. **N5** Compare and order numbers up to 100. **N6** Estimate quantities to100, using referents.

Elaboration

By the end of Grade 1, students will have had many opportunities to explore relationships among the ”teen” numbers 11-19. It is critical that they come to understand that a number such as 17 is “ten and seven more” and that ten plus five or five plus ten is 15 without counting. Considerable work with a two-part mat or two ten-frames will be necessary to help children develop these important relationships.

The ability to **estimate**, a key reasoning skill in mathematics, should develop with regular practice over the course of the year. Estimation helps to develop useful benchmarks for thinking about numbers. Include situations in which sets have the same number of items but differ in the amount of physical space they cover. For small groups, ask: *Is it closer to 5 or 10?* For large collections, one might be asking whether the group is closer to 20 or 50. Teachers need to listen to students while at the same time challenging them to share their ideas about numbers.

Using numerals is society’s way of communicating about number size. It is important, therefore, that students become familiar with **standard symbols** at this time. Students need to create or collect sets, given a **numeral** and assign numerals to sets. Some students will need additional practice recording numerals. Tactile experiences such as tracing numerals and copying them are useful.

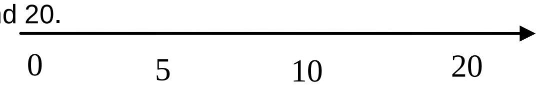
SCO: **N4: Represent and describe numbers to 20 concretely, pictorially and symbolically.**
[C, CN, V]
N5: Compare sets containing up to 20 elements to solve problems using:
- **referents**
- **one-to-one correspondence.**

[C, CN, ME, PS, R, V]
N6: Estimate quantities to 20 by using referents.
[C, ME, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

N4
- Represent a given number up to 20, using a variety of manipulatives, including ten frames and base ten materials.
- Read given number words to 20.
- Partition any given quantity up to 20 into two parts and identify the number of objects in each part.
- Model a given number, using two different objects; e.g., 10 desks represents the same number as 10 pencils.
- Place given numerals on a number line with benchmarks 0, 5, 10 and 20.



N5
- Build a set equal to a given set that contains up to 20 elements.
- Build a set that has more, fewer or as many elements as a given set.
- Build several sets of different objects that have the same given number of elements in the set.
- Compare two given sets, using one-to-one correspondence, and describe them, using comparative words such as *more, fewer*, *as many*, or *same as*.
- Compare a set to a given referent, using comparative language.
- Solve a given problem (pictures and words) that involves the comparison of two quantities.

N6
- Estimate a given quantity by comparing it to a given referent (known quantity).
- Select an estimate for a given quantity from at least two possible choices and explain the choice.

SCO: **N4: Represent and describe numbers to 20 concretely, pictorially and symbolically.**
[C, CN, V]
N5: Compare sets containing up to 20 elements to solve problems using:
- **referents**
- **one-to-one correspondence.**

[C, CN, ME, PS, R, V]
N6: Estimate quantities to 20 by using referents.
[C, ME, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students with many opportunities to represent numbers concretely.
- Allow students to make purposeful links between pictorial, concrete and symbolic representations of numbers.
- Provide the students with a number of counting activities in which sets of items numbering 11 through 19 are counted. Students will be developing number sense and recognizing that certain groupings, such as a group of ten and 7 more, make it easier to determine the size of the set (a pre-place value concept).
- Use objects that are familiar to students whenever possible when representing numbers.
- Expect students to explain their answers about numbers verbally.
- Develop an understanding of the concept of "about" as it relates to estimation activities.
  Use language like:
  More or less than ___? Are there more or less than 15 counters on the overhead projector?
  Closer to ___ or to ___? Do I have closer to 10 cubes or closer to 15 cubes in the clear glass?
  Less than ___, between ___ and ___ or more than ___? If I use this ruler to measure my desk, will it be less than 10 rulers, between 10 and 20 rulers or more than 20 rulers?
  About ___. Use one of the numbers 5, 10, 15, 20. About how many triangles are on the overhead?

Suggested Activities

- For each of the numbers from 1 to 20, have students find objects in the classroom that represent the number; e.g., twelve —there are twelve windows in the classroom.
- Tell students, "There are 16 monkeys at the zoo. Where they live, there is one big tree and one small tree. When it rains, the monkeys like to climb up a tree. One day when I visited the zoo, all the monkeys were in the trees. How many monkeys could be in the big tree and in the small tree? Are there other answers?" Draw two trees on the board and have construction paper monkeys to place in the trees. Change the position of the monkeys as students offer alternative answers.
- Using a hundred chart, ask the students, “What can you tell me about the number 17?”

SCO: **N4: Represent and describe numbers to 20 concretely, pictorially and symbolically.**
[C, CN, V]
N5: Compare sets containing up to 20 elements to solve problems using:
- **referents**
- **one-to-one correspondence.**

[C, CN, ME, PS, R, V]
N6: Estimate quantities to 20 by using referents.
[C, ME, PS, R, V]

Assessment Strategies

- Ask children to show the number 15 in as many different ways as they can.
- Tell students, "In my bowl, I have apples and bananas. There are 14 pieces of fruit altogether. How many apples are there? Draw a picture of the fruit. Are there other possibilities?"
- In groups of four, give students interlocking cubes. Give a variety of directions that use terms such as *more* or *less* and have students build towers; e.g., *Build a tower that is one more than 11. Build a tower that is two less than nine. Build a tower that is two more than 18*. Show the student a set of 10 cubes (or 15 or 20 etc.). Tell them how many there are and then hold up a larger collection of cubes. Ask, *"How many cubes do you think there are in this group?"* You might then show students what a set of 20 cubes looks like and ask them if they would like to adjust their estimates. Continue in this manner for other estimation activities.
- Ask individual students, "I was counting objects in our classroom. I counted exactly 18 of the same thing. What could I have been counting? Tell me why. What are some things I could not have been counting? Why could they not be the objects I was counting?"
- Fill a container with cubes. The container should hold almost 20 cubes. Show it to the student and then ask, "How many cubes do you think are in the container?" Have the student count the cubes. Then ask, "Are there more cubes or fewer cubes than you predicted?"
- Provide individual students or groups with sets of small ten-frames or dot cards. The teacher holds up a single ten-frame, say 6, and asks students to hold up one of their ten-frame that has fewer or more dots (or one more, two more etc.). This type of activity can also be extended to the symbolic stage involving numeral cards or a combination of numeral cards and ten-frames.

SCO: **N7: Demonstrate, concretely and pictorially, how a given number can be represented by a variety of equal groups with and without singles.** [C,R,V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence

Kindergarten	Grade One	Grade Two
	N7 Demonstrate, concretely and pictorially, how a given number can be represented by a variety of equal groups with and without singles.	**N7** Illustrate, concretely and pictorially, the meaning of place value for numerals to 100.

Elaboration

As students' counting experiences become more varied to include increasingly larger sets, they will begin to construct more efficient strategies such as *skip counting* of equal groups or *counting on* because it enables them to count larger sets of objects more quickly and efficiently. One of the *big ideas* that students need to construct in order to be able to think in this way is that of ***part-whole relationships***.

Focusing on a quantity in terms of its parts has important implications for the development of number sense. Recognizing that numbers can be *composed* from parts, or *decomposed* into parts enables students to consider the relative size of numbers and leads to an understanding that the structure of the benchmark numbers *five* and *ten* can be used to enhance fact learning and computation.

SCO: **N7: Demonstrate, concretely and pictorially, how a given number can be represented by a variety of equal groups with and without singles.**
[C,R,V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Represent a given number in a variety of equal groups with and without singles, e.g., 17 can be represented by 8 groups of 2 and one single, 5 groups of 3 and two singles, 4 groups of 4 and one single, and 3 groups of 5 and two singles.
- Recognize that for a given number of counters, no matter how they are grouped, the total number of counters does not change.
- Group a set of given counters into equal groups in more than one way.

SCO: **N7: Demonstrate, concretely and pictorially, how a given number can be represented by a variety of equal groups with and without singles.**
[C,R,V]

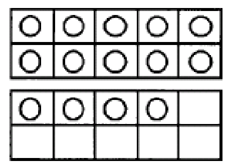
Instructional Strategies

Consider the following strategies when planning lessons:

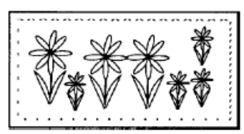
- Provide students with one type of material, such as connecting cubes or squares of coloured paper. Explore how many different combinations for a particular number can be made using two parts or more parts.

Suggested Activities

- Provide 2 ten-frames and counters for each student. Ask the students to model numbers with the counters. (Note: for numbers greater than 10, one ten-frame must be completely filled; for 5 and under, use the top row only.) Have them say the total and explain their reasoning. Practise with other numbers. Observe the students as they model additional numbers.
  - Do they remove all the counters?
  - Do they remove all the counters on the bottom frame?
  - Do they add to/remove counters on the bottom frame?
  - Are they able to verbalize appropriately?



- Give the students a picture card and ask them to model or tell a number story about a part/whole relationship.

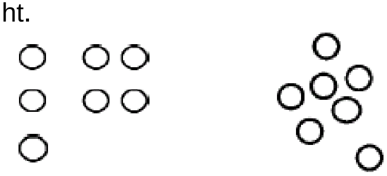


- Provide students with pattern blocks (one or two shapes at a time). The task is to create a two part design for a particular number. This can be extended to multiple shapes.
- Hold out a bar of connecting cubes, a dot strip, a two-column strip, or a dot plate showing 6 or less. Say, “I wish I had six.” The children respond with the part that is needed to make 6. Counting on can be used to check. The game can focus on a single whole, or the “I wish I had” number can change each time.
- Have students count out 11 counters on to a 3-part mat. Have them place 5 counters on one side, 5 in the middle and 1on the other side. Together count all the counters by ones. Then say, *"Five and five and one is eleven.”* Turn the mat around -"One and five and five is eleven.” Repeat with other numbers without changing the eleven side of the mat.

SCO: **N7: Demonstrate, concretely and pictorially, how a given number can be represented by a variety of equal groups with and without singles.**
[C,R,V]

Assessment Strategies

- Provide a selection of buttons, snap cubes, or bread tags to represent any number less than 20 and ask students to sort them into two or more equal groups, with or without “leftovers”. Have the students draw their groupings on paper. Students describe their thinking to their group members or to the class as a whole.
- Give the children a number and ask them to represent that number with drawings in more than one way using equal groups and “leftovers”.
- Explain why it might be easier to count the number of counters on the left than the number of counters on the right.



- Explain why it is possible to have a number such as 13 described using two or more parts in more than one way.
- Ask a student to model a number in two or more parts.

SCO: **N8: Identify the number, up to 20, that is one more, two more, one less and two less than a given number.**
[C, CN, ME, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Kindergarten	Grade One	Grade Two
	N8 Identify the number, up to 20, that is one more, two more, one less and two less than a given number.	**N8** Demonstrate and explain the effect of adding zero to or subtracting zero from any number.

Elaboration

Teachers must ensure that children develop a strong understanding of *one and two more than,* and *one and two less than* relationships for numbers 0-20. Students do not necessarily reflect on the connection between two numbers when they are counting but work with models such as ten-frames, dot-plates or cards, number lines and charts, and numeral cards will help to develop these fundamental concepts. Strategies such as *counting on and back* and *next number* can also be developed within this context of *more* and *less*.

"Though the concept of less is logically related to the concept of more (selecting the set with more is the same as *not* selecting the set with less), the word *less* proves to be more difficult for children than *more*. A possible explanation is that children have many opportunities to use the word *more* but have limited exposure to the word *less*. To help children with the concept of *less*, frequently pair it with the word *more* and make a conscious effort to ask, *"Which is less?"* questions as well as *"Which is more?"* questions. In this way, the less familiar idea can be connected with the one that is better-known."

John Van D Walle, Elementary and Middle School Mathematics
Canadian Edition, Pearson Education, 2005

SCO: **N8: Identify the number, up to 20, that is one more, two more, one less and two less than a given number.**
[C, CN, ME, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Name the number that is one more, two more, one less or two less than a given number, up to 20.
- Represent a number on a ten frame that is one more, two more, one less or two less than a given number.

SCO: **N8: Identify the number, up to 20, that is one more, two more, one less and two less than a given number.**
[C, CN, ME, R, V]

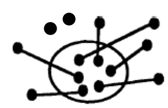
Instructional Strategies

Consider the following strategies when planning lessons:

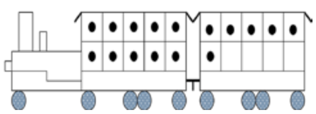
- Ensure that students' initial exploration of numbers that are "one more than," "one less than," "two more than," and "two less than" is done concretely using sets of objects.
- Give students opportunities to transfer their thinking from one representation to another. For example, showing 6 and 7 with linking cubes and then displaying the same numbers with counters on a ten-frame.

Suggested Activities

- Ask children to sort a collection of buttons by various criteria into two sets so that the sets are one more or one less than each other. Compare the size of the sets.
- Students should be able to create a set equal in number to a given set. Ask students to change their set to equal a number that is two more (less) than their current set. For example, change your set of 8 counters to show 10.
- Invite students to make up story problems to solve. For example, if the tooth fairy gives me a quarter for each tooth, and I have 4 quarters so far, how many teeth have I lost? How many quarters will I have when I lose one more tooth? Two more teeth?
- Invite students to create their own "dot" stories. For example, if the dots inside the circle are seats on a bus and the dots outside the circle are children, the story might be that there are just enough children to fill the seats. What would happen if two more children appeared? If there were two more seats, how many children could travel on the bus?



- *"If two more children want to get on, how many children will be on the train?"* Or, "*One child wants to get off the train. How many children will be on the train if there's one less?"*



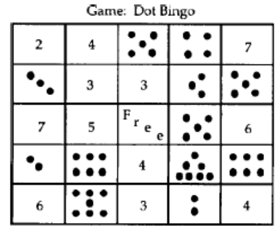
- Show students on an overhead projector a number of counters. Have the students count them. Get them to close their eyes while you change the amount by one or two. Ask the students to open their eyes and tell you how the group of counters has changed.

SCO: **N8: Identify the number, up to 20, that is one more, two more, one less and two less than a given number.**
[C, CN, ME, R, V]

Assessment Strategies

- Line up 7 boys and 5 girls. Ask: What must be changed to make the number of girls equal to the number of boys? The number of boys equal to girls?
- Have students, working in groups of 4, write down their favourite names (not their own). Ask them to sort the names into groups with names having one more letter than the other, another group with two less than the other. Have them share their findings with another group.
- Have the students play "Dot Bingo".
  Rules:
  Take turns rolling a die
  Cover any one square that is one more than the top number on the die.
  The player who first covers three in a row is the winner.
  (Ideas – Arithmetic Teacher)

Game: Dot Bingo



- Place 3 red counters and 3 blue counters in one group and 3 blue and 2 red in another, as shown:
  RRR BBB   BBB RR
  Ask: How do you know that there are more blue counters than red? How many more?
- Give students dot cards and some counters. Ask them to create sets that are "one more than," "one less than," "two more than," and "two less than" the dot cards.

SCO: **N9: Demonstrate an understanding of addition of numbers with answers to 20 and their corresponding subtraction facts, concretely, pictorially and symbolically by:**

- **using familiar and mathematical language to describe additive and subtractive actions from their experience**
- **creating and solving problems in context that involve addition and subtraction**
- **modeling addition and subtraction using a variety of concrete and visual representations, and recording the process symbolically.**

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Kindergarten	Grade One	Grade Two
	N9 Demonstrate an understanding of addition of numbers with answers to 20 and their corresponding subtraction facts, concretely, pictorially and symbolically by: • *using familiar and mathematical language to describe additive and subtractive actions from their experience* • *creating and solving problems in context that involve addition and subtraction* • *modeling addition and subtraction using a variety of concrete and visual representations, and recording the process symbolically.*	**N9** Demonstrate an understanding of addition (limited to 1 and 2-digit numerals) with answers to100 and the corresponding subtraction by: • *using personal strategies for adding and subtracting with and without the support of manipulatives* • *creating and solving problems that involve addition and subtraction* • *explaining that the order in which numbers are added does not affect the sum* • *explaining that the order in which numbers are subtracted may affect the difference.*

Elaboration

As with many early concepts, the development of the meaning of addition and subtraction cannot be rushed. It is desirable to explore adding and separating situations in a context. Students should have extensive investigative experiences in which they use a variety of concrete materials to model both operations and investigate the relationship between the operations, before moving to recording the process symbolically. It is important that problems be personalized, but students also need experience interpreting how addition and subtraction situations are portrayed in print. Include examples of:

- **active situations** which involve the physical joining/separating of sets
- **static situations** involving the implied joining/separating of sets

It is important that **all** of the following **4 structures of problems** be presented and that these are derived from students' experiences.

These structures include:

- **Join Problems**: result unknown, change unknown, initial unknown
- **Separate Problems**: result unknown, change unknown, initial unknown
- **Part-Part-Whole Problems:** whole unknown, part unknown
- **Compare Problems:** difference unknown, larger unknown, smaller unknown

(Van de Walle and Lovin, 2006, p. 67-69)

SCO: **N9: Demonstrate an understanding of addition of numbers with answers to 20 and their corresponding subtraction facts, concretely, pictorially and symbolically by:**
- **using familiar and mathematical language to describe additive and subtractive actions from their experience**
- **creating and solving problems in context that involve addition and subtraction**
- **modeling addition and subtraction using a variety of concrete and visual representations, and recording the process symbolically.**

[C, CN, ME, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Act out a given story problem presented orally or through shared reading.
- Indicate if the scenario in a given story problem represents additive and/or subtractive action.
- Represent the numbers and actions presented in a given story problem by using manipulatives, and record them using sketches and/or number sentences.
- Create a story problem for addition that connects to student experience and simulate the action with counters.
- Create a story problem for subtraction that connects to student experience and simulate the action with counters.
- Create a word problem for a given number sentence.
- Represent a given story problem pictorially or symbolically to show the additive and/or subtractive action and solve the problem.

SCO: **N9: Demonstrate an understanding of addition of numbers with answers to 20 and their corresponding subtraction facts, concretely, pictorially and symbolically by:**

- **using familiar and mathematical language to describe additive and subtractive actions from their experience**
- **creating and solving problems in context that involve addition and subtraction**
- **modeling addition and subtraction using a variety of concrete and visual representations, and recording the process symbolically.**

[C, CN, ME, PS, R, V]

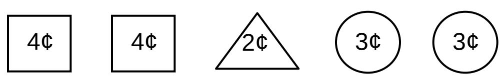
Instructional Strategies

Consider the following strategies when planning lessons:

- Personalize word problems for children. Encourage students to create a variety of meaningful problems.
- Manipulate concrete materials to model as you or students relate a word problem. Verbalize as you manipulate.
- Provide a wide variety of problem types and structures (see Elaboration).

Suggested Activities

- Choose a book, or make up a story, which tells about an addition (subtraction) situation and ask the student to model the situation with counters as you read the book.
- Present a number of shapes worth various amounts; for example,



  Have the children create various designs, using the shapes, in each case describing how much the design would be worth.
- Pose story problems such as "Janet has 6 baseball cards. Mario gives her some hockey cards. She now has 13 sports cards. How many sports cards did Mario give her?" Provide the cards and observe how students solve the problem. Students should be encouraged to share strategies with their classmates.
- Ask the student to make a drawing to model this and other structures of problems: Robert had some baseball cards. His brother convinced him to give him 2 of the cards. He now has 8 cards. How many cards did Robert have to start?
- Ask the student to think of a situation in a restaurant when someone might add. Ask them to think of a situation when they might subtract.

SCO: **N9: Demonstrate an understanding of addition of numbers with answers to 20 and their corresponding subtraction facts, concretely, pictorially and symbolically by:**
- **using familiar and mathematical language to describe additive and subtractive actions from their experience**
- **creating and solving problems in context that involve addition and subtraction**
- **modeling addition and subtraction using a variety of concrete and visual representations, and recording the process symbolically.**

[C, CN, ME, PS, R, V]

Assessment Strategies

- Model this problem for a pair of students: I had 5 pennies and now I have 9. How many pennies did I earn? Ask the pair to make up a similar problem, using objects of their choice, and to model and describe it.
- Ask students to tell an addition/subtraction story involving 8 and 5 while manipulating a model.
- Tell the students that you have a nickel and 4 pennies. You want to buy a candy that costs 3¢. Ask: How much money will be left? Tell how you know.
- Place a large number line on the floor, positioning a child on the 8 and facing the higher numbers. Ask: Where would you be if you jumped 4 spaces forward?
- Provide the children with a given number of counters. Ask them to add/remove 3 or another number of counters and tell how many are now there. Ask them to represent this symbolically.
- Tell the student that Jane had 9 pencils and lost 3, while Martha had 7 pencils and lost 2. Ask: Who has more pencils left? Explain how you know.
- Tell the student that you had 9 marbles, but lost some. There are only 4 marbles left. Ask: How many did I lose? Show how you know.

SCO: **N10: Describe and use mental mathematics strategies (memorization not intended), such as:**
• counting on and counting back
• making 10
• doubles
• using addition to subtract
to determine the basic addition facts to 18 and related subtraction facts.
[C, CN, ME, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence

Kindergarten	Grade One	Grade Two
	N10 Describe and use mental mathematics strategies (memorization not intended), such as: • *counting on and counting back* • *making 10* • *doubles* • *using addition to subtract* for the basic addition and subtraction facts to 18.	**N10** Apply mental mathematics strategies, such as: • *using doubles* • *making 10* • *one more, one less* • *two more, two less* • *addition for subtraction* to determine basic addition facts to 18 and the related subtraction facts.

Elaboration

When students' thinking has developed at least to the point where they are counting on from the large number, **strategy** learning should begin. Children should be encouraged to use the relationships between facts to learn new facts, rather than using counting to find **sums** or **differences**. Children will construct number relationships by making connections with prior knowledge. These relationships will lead to the development of a network of patterns that children will be able to access to recall number facts.

It is not intended that students recall the basic facts but become familiar with strategies to mentally determine sums and differences. Students need many rich experiences to explore strategies **concretely** and **pictorially** as this will lead them to the understanding that all of the facts are conceptually related. It is important that opportunities for student discussion and sharing of a wide variety of strategies, including their own, are provided.

SCO: **N10: Describe and use mental mathematics strategies (memorization not intended), such as:**
• counting on and counting back
• making 10
• doubles
• using addition to subtract
to determine the basic addition facts to 18 and related subtraction facts.
[C, CN, ME, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Use and describe a personal strategy for determining a given sum.
- Use and describe a personal strategy for determining a given difference.
- Write the related subtraction fact for a given addition fact.
- Write the related addition fact for a given subtraction fact.

(It is not intended that students recall the basic facts but become familiar with strategies to mentally *determine* sums and differences.)

SCO: **N10: Describe and use mental mathematics strategies (memorization not intended), such as:**
• counting on and counting back
• making 10
• doubles
• using addition to subtract
to determine the basic addition facts to 18 and related subtraction facts.
[C, CN, ME, PS, R, V]

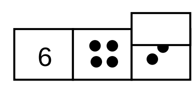
Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students with opportunities to develop their own strategies for determining a given sum or difference.
- Encourage students to invent strategies for solving problems that include making doubles, making 10, using compensation and using known facts.
- Ask students to employ as many representations as possible for determining sums and differences, including physically acting out, drawing pictures, verbally explaining their ideas, using concrete materials and writing number sentences.
- Provide students with time to learn basic facts, so they understand the operation and can invent their strategies rather than memorizing.
- Facilitate the learning addition and subtraction facts by having students solve word problems with familiar contexts.
- Encourage students to create their own word problems. They can write these down or dictate them to a scribe.

Suggested Activities

- Ask students to choose any number, add 10 and then take away 1. Get students to model this activity using ten frames. Have them repeat this activity with other starting numbers and discuss what they observe.
- Give students a bag containing 8 counters. Have students reach in the bag and remove some of the counters. Ask how many are still in the bag.
- Make missing part cards: Each card has a numeral for the whole and two dot sets with one set covered by a flap. Ask students how many are covered and write the number sentence.



- Ask students to build a linking cube train of 9 cubes with two colours in different ways.
- Have a group of approximately eight students stand in front of the room. Ask the class how many students are at the front. Divide the group into two smaller groups and ask the class how many students are at the front now and how do they know. Explore the different ways that we could partition the larger group.

SCO: **N10: Describe and use mental mathematics strategies (memorization not intended), such as:**
• counting on and counting back
• making 10
• doubles
• using addition to subtract to determine the basic addition facts to 18 and related subtraction facts.
[C, CN, ME, PS, R, V]

Assessment Strategies

- Provide students with concrete materials and present them with the following number problems. Ask them to solve the problem and the number sentence.
  - Charles has eight dimes. Danielle has four more dimes than Charles. How many dimes does Danielle have?
  - Brodie has 18 coins. Eight of his coins are dimes and the rest are quarters. How many quarters does Brodie have?
  - Sophie had 12 nickels. She gave some to her mother and now she has eight nickels. How many did she give to her mother?
  - Shona had 15 quarters. Her dad gave her some more. Now she has 18 quarters. How many did dad give her?
- Have students create their own word problems for the number family 7, 9 and 16 (numbers related by addition and subtraction). Ask them to write a problem that uses these numbers in addition and another problem that uses these numbers in subtraction.
- Have students explain how they obtain the answer for each of the following computations.
  • 8 + 9 • 6 + 4 • 7 + 8 • 4 + 7 • 9 + 6
- Ask students to write a related subtraction/addition fact for the following facts.
  • 12 + 6 = 18 • 14 + 3 = 17 • 16 – 9 = 7 • 12 – 8 = 4
- Ask the student how he/she could use 6 - 4 = 2 to figure out 6 - 3. Students may use materials to model this.

PATTERNS AND RELATIONS

SCO: **PR1: Demonstrate an understanding of repeating patterns (two to four elements) by:**
- **describing**
- **reproducing**
- **extending**
- **creating**

patterns using manipulatives, diagrams, sounds and actions.
[C, PS, R, V]

PR2: Translate repeating patterns from one representation to another.
[C, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Scope and Sequence

Kindergarten	Grade One	Grade Two
PR1 Demonstrate an understanding of repeating patterns (two or three elements) by: • *identifying* • *reproducing* • *extending* • *creating* *patterns using manipulatives, sounds and actions.*	**PR1** Demonstrate an understanding of repeating patterns (two to four elements) by: • *describing* • *reproducing* • *extending* • *creating* *patterns using manipulatives, diagrams, sounds and actions.* **PR2** Translate repeating patterns from one representation to another.	**PR1** Demonstrate an understanding of repeating patterns (three to five elements) by: • *describing* • *extending* • *comparing* • *creating* *patterns using manipulatives, diagrams, sounds and actions.* **PR2** Demonstrate an understanding of repeating patterns (three to five elements) by: • *describing* • *reproducing* • *extending* • *creating* *patterns using manipulatives, diagrams, sounds and actions (numbers to 100)*

Elaboration

The foundation of algebraic thinking is investigating patterns and their representations. We are always looking for ways to generalize and formalize regularity in mathematics. These outcomes focus on patterns and regularity and ways of representing these patterns. Not only do students need to recognize the pattern, but they must also be able to **extend** and generalize in both **words** and **symbols**. They should be able to recognize many different forms of the same pattern. They will identify **similarities** and **differences** between and among patterns.

Patterns occur regularly in students' everyday life and they will be able to identify patterns in their daily living, including physical and geometric situations as well as numbers. The patterns being explored in Grade 1 are all **repeating patterns**. Teachers should be aware that some students may extend this concept and create growing patterns. The focus for these outcomes is on the core of a repeating pattern; patterns should be written having the core repeat at least three times so the pattern is clear to students.

SCO: **PR1: Demonstrate an understanding of repeating patterns (two to four elements) by:**
- **describing**
- **reproducing**
- **extending**
- **creating**

patterns using manipulatives, diagrams, sounds and actions.
[C, PS, R, V]

PR2: Translate repeating patterns from one representation to another.
[C, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

PR1
- Describe a given repeating pattern containing two to four elements in its core.
- Identify and describe errors in a given repeating pattern.
- Identify and describe the missing element(s) in a given repeating pattern.
- Create and describe a repeating pattern, using a variety of manipulatives, diagrams, sounds and actions.
- Reproduce and extend a given repeating pattern, using manipulatives, diagrams, sounds and actions.
- Identify and describe a repeating pattern in the environment, e.g., classroom, outdoors, using everyday language.
- Identify repeating events; e.g., days of the week, birthdays, seasons.

PR2
- Represent a given repeating pattern, using another mode; e.g., actions to sound, colour to shape, ABCABC to bear eagle fish bear eagle fish.
- Describe a given repeating pattern, using a letter code; e.g., ABCABC.

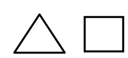
SCO: **PR1: Demonstrate an understanding of repeating patterns (two to four elements) by:**
• describing
• reproducing
• extending
• creating
patterns using manipulatives, diagrams, sounds and actions.
[C, PS, R, V]
PR2: Translate repeating patterns from one representation to another.
[C, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students with many opportunities to represent patterns concretely.
- Allow students to identify patterns in their daily lives. This can include repetitive songs and rhythmic chants that are based on repeating and growing patterns.
- Patterning activities form the basis for algebraic reasoning. Using concrete materials, students can examine how patterns can be created with things such as cubes or pattern blocks.
- Students need opportunities to create patterns and to identify the core of a pattern visually.
- Songs and poems can be used to explore patterns, such as “The Hokey Pokey”.
- Integrate patterns in physical education, music, art and other subject areas to provide a context.
- Expect students to explain, verbally, their answers about patterns.

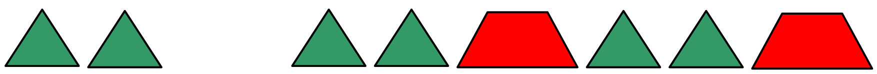
Suggested Activities

- Request that the student use pattern blocks or attribute blocks to construct a simple pattern. Ask her/him to explain the pattern to another student.
- Provide a calendar. Present the problem: Your mom, your sister and you take turns tidying up the living room floor at the end of the day. If your next turn is on Friday, on which days will your following two turns be?
- Have students create clapping patterns, or use stickers or coloured counters, to make their favourite visual patterns.
- Show students a pattern with an error or missing part in the pattern. Ask students to identify and correct the error or add the missing piece.
- Ask the student to continue the pattern begun (at right) in two different ways. 
- Give students pattern blocks and ask them to create an ABBABB pattern. Then ask students, "What would this pattern sound like?" Continue asking for other patterns, such as ABCABC, AABAAB or ABBCABBC.
- Give students a series of patterns. These may be on a page or on an overhead. Ask students to translate these patterns and others into letter representations.

SCO: **PR1: Demonstrate an understanding of repeating patterns (two to four elements) by:**
• describing
• reproducing
• extending
• creating
patterns using manipulatives, diagrams, sounds and actions.
[C, PS, R, V]
PR2: Translate repeating patterns from one representation to another.
[C, R, V]

Assessment Strategies

- Show a pattern of cubes, e.g., RGGRGGRGG, and ask students to tell you what the pattern is. Then show a different pattern of cubes; e.g., YYBYYBYYB. Have students identify the new pattern. Then ask students to tell you how the patterns are different and how they are the same.
- Tell students, "Mary has six green triangles and three orange squares." Show students the pieces on the overhead. Ask, "Can she make two different patterns?" Ask students to draw two possible patterns that Mary could make and explain the patterns. Ask, "What comes next in the pattern?"
- Ask the student to make a pattern so that a triangle is the third item.
- Tell the student that you think there is a pattern to the days (Monday, Tuesday...) in a week. Ask the student to explain the pattern.
- Show the student patterns like the one below and have them draw the missing element.



SCO: **PR3: Describe equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20).**
[C, CN, R, V]
PR4: Record equalities using the equal symbol.
[C, CN, PS, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence

Kindergarten	Grade One	Grade Two
	PR3 Describe equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20). **PR4** Record equalities, using the equal symbol.	**PR3** Demonstrate and explain the meaning of equality and inequality, concretely and pictorially. **PR4** Record equalities and inequalities symbolically, using the equal symbol or the not equal symbol.

Elaboration

When students begin the study of **equality**, it is important for them to see that the **equal sign** represents a **relation**, not an operation. It tells us that the quantity on the left is the same as the **quantity** on the right. Students should see that the expression which may include an operation really is an **equivalent** form that represents a single quantity. For example, 10 + 8 and 7 + 11 are both equivalent representations for 18.
Students could work with the pattern formed by the 3 numbers in a **number sentence** to demonstrate the equal sign shows that both sides are the same. For example:

$5 + 3 = 8 \qquad 8 = 3 + 5$

$5 + 3 = 3 + 5$

Using a balance scale, students begin to understand the concept of equating two quantities (you start with 2 different quantities and adjust them to make them equal). Working with balance scale problems, students build the foundation for further study in the area of algebra and solving equations.

In everyday life, we sort things by comparison relationships. For example, we might note that Ron is taller than Mary or that Monica takes more time than Valerie to complete her homework. Relationships also apply to number, as we might note that five is two less than seven or 12 is three more than nine. Students need to explore the concept of **inequalities** by recognizing and creating symbolic representations for "less than" and "greater than." They should recognize the relationship between these inequalities. Given two expressions, students should be able to identify if the quantities they represent are equal or not equal and how they can sort the quantities using inequalities.

SCO: **PR3: Describe equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20).**
[C, CN, R, V]
PR4: Record equalities using the equal symbol.
[C, CN, PS, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

PR3
- Construct two equal sets, using the same objects (same shape and mass), and demonstrate their equality of number, using a balance limited to 20 elements.
- Construct two unequal sets, using the same objects (same shape and mass), and demonstrate their inequality of number, using a balance limited to 20 elements.
- Determine if two given concrete sets are equal or unequal and explain the process used.

PR4
- Represent a given equality, using manipulatives or pictures.
- Represent a given pictorial or concrete equality in symbolic form.
- Provide examples of equalities where the given sum or difference is on either the left or right side of the equal symbol (=).
- Record different representations of the same quantity (0 to 20) as equalities.

SCO: **PR3: Describe equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20).**
[C, CN, R, V]
PR4: Record equalities using the equal symbol.
[C, CN, PS, V]

Instructional Strategies

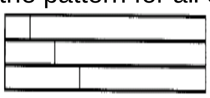
Consider the following strategies when planning lessons:

- Provide students with many opportunities to represent number sentences concretely.
- Students need many opportunities to come to understand that the equal sign represents a relation, not an operation. Use of the words "the same as" for the equal sign will help them see the relation.
- Balance activities form the basis for understanding equality. Using concrete materials, students can examine how a balance operates like the seesaw in the playground.
- Students need opportunities to create equations and to identify the equations visually.
- Expect students to explain, verbally, their answers about equalities and inequalities. Number sentences that demonstrate "is greater than" and "is less than" are known as "inequalities" and students should become familiar with that terminology. Number sentences using an equal sign are known as "equalities."
- Ensure that students learn to read number sentences from left to right and right to left.

Suggested Activities

- Ask the student to use Cuisenaire rods (or another suitable manipulative material such as linking cubes) to show the pattern for all of the facts for 8; for example:

  1 + 7 = 8
  2 + 6 = 8
  3 + 5 = 8



- Give students the following problems and ask, "Will the balance tilt?" If the balance will not tilt, then mark equal (=) in the middle of the balance.

  3 + 5 8 5 + 7 6 + 6

  4 + 8 2 + 9 3 + 9 7 + 5

- Create dots cards representing number sentences, like flash cards. Ask students to record the number sentence symbolically.
- Have students working in pairs and write as many different equations as possible for a number from 1 to 20. For example, one pair may have the number 13 and write the following:

  13 = 1 + 12 13 = 2 + 11 13 = 3 + 10 and so on.

SCO: **PR3: Describe equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20).**
[C, CN, R, V]
PR4: Record equalities using the equal symbol.
[C, CN, PS, V]

Assessment Strategies

- Use materials on an overhead to show a problem, such as:
  John saw five sparrows feeding at the birdfeeder. He went away and later he looked and counted 14 sparrows at the birdfeeder. How many more sparrows flew to the birdfeeder while he wasn't looking?
  Ask the students to write a number sentence to solve the problem.
- For each of the following, have students write an expression (either a whole number or a combination of numbers showing an operation) to complete the number sentence. Encourage student to explore these using materials.
  4 + 2 = ________   ________ = 7 + 11
  5 + 3 = ________   ________ = 8 + 10
- Ask the student to list all the pairs of numbers that can be added for a total of 4, then repeat the process for totals of 5 and 6. Have him/her arrange the lists so that the first number increases by one each time. Ask: What is the pattern?

SHAPE AND SPACE

SCO: **SS1 : Demonstrate an understanding of measurement as a process of comparing by:**
- **identifying attributes that can be compared**
- **ordering objects**
- **making statements of comparison**
- **filling, covering or matching.**

[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Scope and Sequence

Kindergarten	Grade One	Grade Two
SS1 Use direct comparison to compare two objects based on a single attribute, such as length (height), mass (weight) and volume (capacity).	**SS1** Demonstrate an understanding of measurement as a process of comparing by: • identifying attributes that can be compared • ordering objects • making statements of comparison • filling, covering or matching.	**SS3** Compare and order objects by length, height, distance around and mass (weight) using non-standard units, and make statements of comparison.

Elaboration

Measurement involves **identifying** and **comparing** similar **attributes**. Students should use a variety of words involving measurement including “longest,” “heaviest”, “most”, “least”. etc. It is important that students explore measurement in context throughout each day using **direct comparison**. For example, “Which bean plant grew the tallest?”.

Children should recognize that **length** tells about the extent of an object along one dimension. Direct measurement consists of comparing lengths by lining up items side by side, beginning at a common base. (Note: Students should be led to see why a common starting point is important.) Children should order objects from longest to shortest.

Children should recognize that **capacity** tells how much something will hold. They should investigate strategies to compare the capacities of two or more containers. Direct measurement involves filling one container and then pouring the contents into another to find which holds more.

In comparing **areas**, students are examining the amount of space taken up by an object. For example, one placemat might take up more of the table than another. Direct measurement involves placing one surface on top of another to see which “sticks out.”

Students should recognize that **mass** tells about the “heaviness” of an object. They should explore methods to compare and order masses. Direct measurement involves, for instance, placing two objects on a balance simultaneously and comparing the mass of one with that of the other.

SCO: **SS1 : Demonstrate an understanding of measurement as a process of comparing by:**
- **identifying attributes that can be compared**
- **ordering objects**
- **making statements of comparison**
- **filling, covering or matching.**

[C, CN, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Identify common attributes, such as length (height), mass (weight), volume (capacity) and area, which could be used to compare a given set of two objects.
- Compare two given objects and identify the attributes used to compare.
- Determine which of two or more given objects is longest/shortest by matching and explain the reasoning.
- Determine which of two or more given objects is heaviest/lightest by comparing and explain the reasoning.
- Determine which of two or more given objects holds the most/least by filling and explain the reasoning.
- Determine which of two or more given objects has the greatest/least area by covering and explain the reasoning.

SCO: **SS1 : Demonstrate an understanding of measurement as a process of comparing by:**
- **identifying attributes that can be compared**
- **ordering objects**
- **making statements of comparison**
- **filling, covering or matching.**

[C, CN, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- It would be valuable for students to participate in “dramas” in which someone measures incorrectly and the other students have to figure out what is wrong. For example, one student could play a part in which he/she lines up pencils of different lengths to measure an item, or uses uniform units, but counts, “1, 2, 4, 5...”
- Show a pan balance and two items. Ask the student how to use the balance to find out which item has the greater mass.
- Have students order objects from longest to shortest or by comparing other attributes. Include situations in which students are dealing with an extraneous variable, such as objects:
    - which are not straight
    - which are also wide or thick

Suggested Activities

- Provide the student with 3 containers (of various shapes) and filling material (e.g., beans, styrofoam packing material). Ask the student to order the containers based on how much they hold.
- Ask two children to perform standing long jumps. Encourage them to find a way to determine who jumped farther. Stress with the students the importance of a common starting point.
- Have the student make 3 play dough balls and determine which of the balls is the heaviest.
- Display a set of five objects of similar size and a sixth target object. Ask the student to sort them into groups with masses less than and greater than the target.
- Provide students with various sizes of storybooks. Have students compare the storybooks to determine which has the greatest area.

SCO: **SS1 : Demonstrate an understanding of measurement as a process of comparing by:**
- **identifying attributes that can be compared**
- **ordering objects**
- **making statements of comparison**
- **filling, covering or matching.**

[C, CN, PS, R, V]

Assessment Strategies

- Have students prepare a set of ribbons for first, second, and third places in a race, so that the faster runner gets a longer ribbon.
- Show the students a coffee mug and a drinking glass. Ask them how they would find out which holds more.
- Give the students sets of tangrams and have them compare the areas of the triangles in the sets.
- Provide students with "trains" of various lengths made from interlocking cubes. Ask them to order the trains from shortest to longest.
  Ask: What does "holds more" mean? Have the student explain his/her thinking.
- Ask the students to compare the mass of two sets of objects, for example, the mass of 10 pennies to 5 marbles. Have them identify which set is heavier or lighter and explain their thinking.
- Give a student a trapezoid (or other shape) and have them draw another shape with a larger area. Have them explain how they know it is larger.

SCO: **SS2 : Sort 3-D objects and 2-D shapes using one attribute, and explain the sorting rule.**
[C, CN, R, V]
SS3: Replicate composite 2-D shapes and 3-D objects.
[CN, PS, V]
SS4: Compare 2-D shapes to parts of 3-D objects in the environment.
[C, CN, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence

Kindergarten	Grade One	Grade Two
SS2 Sort 3-D objects using a single attribute. **SS3** Build and describe 3-D objects.	**SS2** Sort 3-D objects and 2-D shapes using one attribute, and explain the sorting rule. **SS3** Replicate composite 2-D shapes and 3-D objects. **SS4** Compare 2-D shapes to parts of 3-D objects in the environment.	**SS6** Sort 2-D shapes and 3-D objects using two attributes, and explain the sorting rule. **SS7** Describe, compare and construct 3-D objects, including: cubes, spheres, cones, cylinders, pyramids. **SS8** Describe, compare and construct 2-D shapes, including: triangles, squares, rectangles, circles. **SS9** Identify 2-D shapes as parts of 3-D objects in the environment.

Elaboration

The study of **two-dimensional shapes** and **three-dimensional objects** is essential as we strive to describe, analyze and understand the world we live in. Activities selected in geometry should provide students with the opportunity to explore. They need to see and feel, to build and take apart, to sort and identify their rule(s), and to share their observations with their classmates.

It is through such activities that students will become familiar with the names of 2-D shapes and 3-D objects and begin to recognize their characteristics. It is very important to encourage students to use accurate language when describing shapes. As pattern blocks are regularly used for geometric inquiry, it would seem reasonable that students become familiar with the terms that describe them as well. Students should be comfortable using such terms as **cylinder**, **sphere**, **cone**, **cube**, and may extend their exploration to rectangular prisms and square pyramids.

Explorations (sorting, building) with 2-D shapes involve the attributes of the number of sides and vertices and how shapes can be put together and taken apart to make other shapes. Students should be able to distinguish between shapes such as squares and rectangles and also to see that the squares are rectangles. Explorations with 3-D objects involve how these objects are alike and how they differ (for example: Will it roll?). They may observe other attributes, such as the number of faces and edges.

Children should recognize 2-D shapes and 3-D objects in their environment. These real-world associations are most important in the development of geometric concepts.

SCO: **SS2 : Sort 3-D objects and 2-D shapes using one attribute, and explain the sorting rule.**
[C, CN, R, V]
SS3: Replicate composite 2-D shapes and 3-D objects.
[CN, PS, V]
SS4: Compare 2-D shapes to parts of 3-D objects in the environment.
[C, CN, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Sort a given set of familiar 3-D objects or 2-D shapes using a given sorting rule.
- Sort a given set of familiar 3-D objects using a single attribute determined by the student and explain the sorting rule.
- Sort a given set of 2-D shapes using a single attribute determined by the student and explain the sorting rule.
- Determine the difference between two given pre-sorted sets of familiar 3-D objects or 2-D shapes and explain a possible sorting rule used to sort them.
- Select 2-D shapes from a given set of 2-D shapes to reproduce a given composite 2-D shape.
- Select 3-D objects from a given set of 3-D objects to reproduce a given composite 3-D object.
- Predict and select the 2-D shapes used to produce a composite 2-D shape, and verify by deconstructing the composite shape.
- Predict and select the 3-D objects used to produce a composite 3-D object, and verify by deconstructing the composite object.
- Identify 3-D objects in the environment that have parts similar to a given 2-D shape.

SCO: **SS2 : Sort 3-D objects and 2-D shapes using one attribute, and explain the sorting rule.**
[C, CN, R, V]
SS3: Replicate composite 2-D shapes and 3-D objects.
[CN, PS, V]
SS4: Compare 2-D shapes to parts of 3-D objects in the environment.
[C, CN, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students with many opportunities to represent 2-D shapes and 3-D objects concretely.
- Allow students to identify 2-D shapes and 3-D objects in their daily lives. This should include common everyday objects.

Suggested Activities

- Provide an assortment of 2-D shapes cut from tag board. Ask students to work in small groups to sort the shapes. Encourage discussion and have the groups share their sorting rules with their classmates.
- Provide several different 3-D objects. Ask the student to sort them and to explain the sorting criteria. Ask him/her to sort them again, using different criteria.
- Ask students to cut a square, rectangle, or triangle into three parts. Have them exchange their pieces and ask their partner to rearrange them to make the original shape. (Activities such as these, in which a student is required to assemble a figure from its parts, further develop figure-ground perception skills.)
- Have the students work in pairs with a geoboard to make a large square with a smaller square inside it.
- Display pictures of various 3-D objects, such as a rocket or sculpture. Ask students what 3-D objects were used to build the object. Students can then build their own composite 3-D objects from individual 3-D objects, such as small cardboard boxes or modeling clay. Once they are built, display the creations in class and ask students to identify the 3-D objects used to build the composite object.
- Have students examine a collection of objects found in their environment; e.g., cans, cereal boxes, ice cream cones, tissue boxes. Ask them to identify the shape of each face of each 3-D object. Ask, "What shape is the face? Do all the faces have the same shape?" Have students identify vertices and edges on the shapes.
- Invite children to hunt around the school to find various shapes (e.g., trapezoids, squares, triangles). Have them share their findings and speculate on why certain shapes are more common than others.

SCO: **SS2 : Sort 3-D objects and 2-D shapes using one attribute, and explain the sorting rule.**
[C, CN, R, V]
SS3: Replicate composite 2-D shapes and 3-D objects.
[CN, PS, V]
SS4: Compare 2-D shapes to parts of 3-D objects in the environment.
[C, CN, V]

Assessment Strategies

- Ask small groups of students to sort a collection 2-D shapes or 3-D objects and record their sorting rule.
- Give each student, or pair of students, a collection of pattern blocks. Say to them, "I am going to build a design with pattern blocks on the overhead projector. I want you to use your pattern blocks to build a design just like mine." Observe their construction.
- Give each student, or pair of students, a collection of pattern blocks. Say to them, "I want you to build the following shape with your pattern blocks. Place a red trapezoid on your desk. On top of the trapezoid place a green triangle. On the left place a blue rhombus and on the right place another blue rhombus." Observe their construction.
- Make an assortment of 2-D shapes from poster board. Ask the student to sort them and to give the sorting rule.
- Show a triangle. Ask the student to find three things in the classroom that make him/her think of that shape.
- Show the student a square that has been folded along the diagonal. Ask: What shape will this be when I unfold it?
- Ask the student to examine a variety of containers (such as yogurt container, cereal box, etc.). Ask: What shapes would be used to make this container? How do you know?

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