Prince Edward Island
CANADA

Education and Early Childhood Development
English Programs

Prince Edward Island Mathematics Curriculum

CURRICULUM

Mathematics

MAT521E

Prince Edward Island
CANADA

2012
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Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base. Developers have examined the curriculum proposed throughout Canada and secured the latest research in the teaching of mathematics, and the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for Grades 10-12 Mathematics* (2008) has been adopted as the basis for a revised high school mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories, and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP, and on the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➢ Essential Graduation Learnings

Essential Graduation Learnings (EGLs) are statements describing the knowledge, skills, and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills, and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work, and study today and in the future. Essential graduation learnings are cross curricular, and curriculum in all subject areas is focused to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will demonstrate knowledge, skills, and attitudes expressed as essential graduation learnings, and will be expected to

- respond with critical awareness to various forms of the arts, and be able to express themselves through the arts;
- assess social, cultural, economic, and environmental interdependence in a local and global context;
- use the listening, viewing, speaking, and writing modes of language(s), and mathematical and scientific concepts and symbols, to think, learn, and communicate effectively;
- continue to learn and to pursue an active, healthy lifestyle;
- use the strategies and processes needed to solve a wide variety of problems, including those requiring language, and mathematical and scientific concepts;
- use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate to all educational partners high expectations for students in mathematics education. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM, *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning;
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will

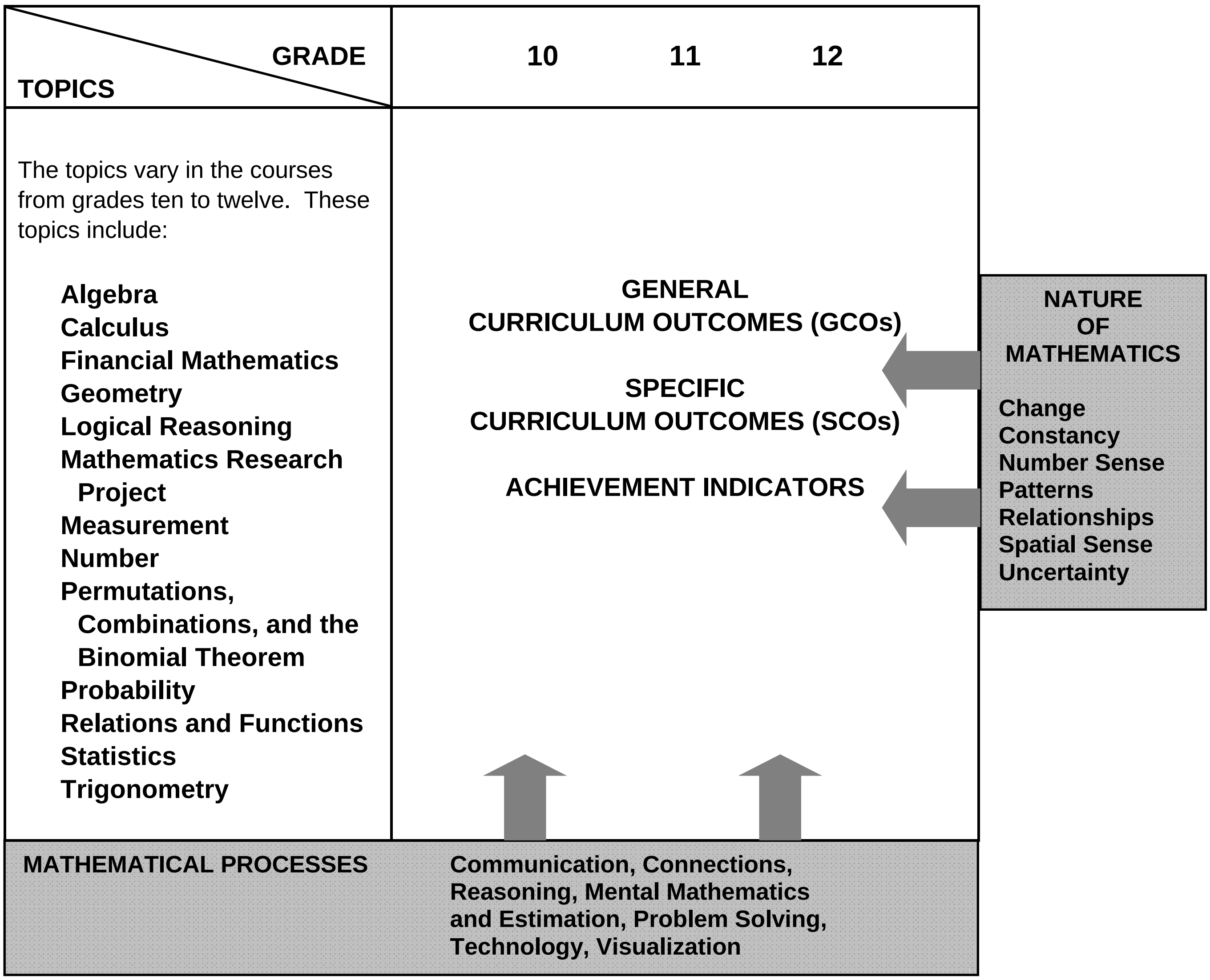
- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks;
- exhibit curiosity.

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the understanding of mathematical concepts by students and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, physical education, and other subject areas. Efforts should be made to make connections and use examples drawn from a variety of disciplines.

Conceptual Framework for 10-12 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



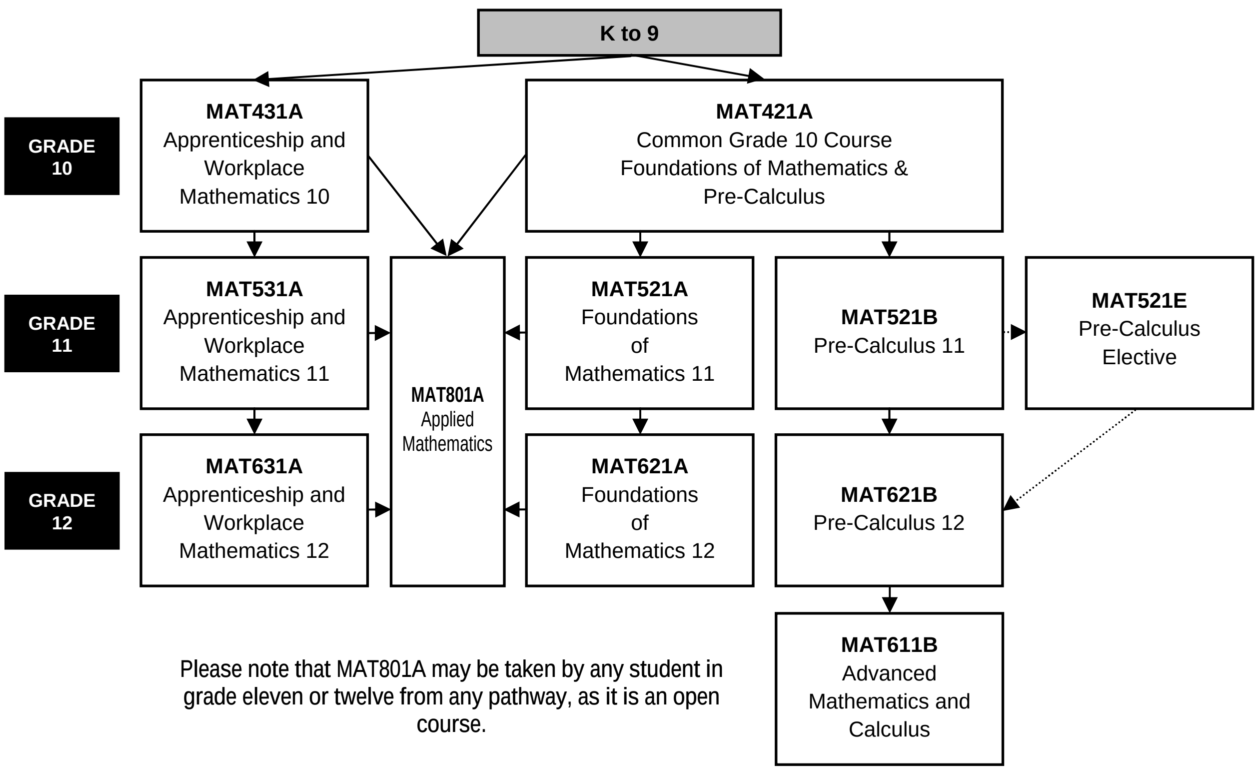
The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into a number of topics, as described above. These topics are not intended to be discrete units of instruction. The integration of outcomes across topics makes mathematical experiences meaningful. Students should make the connections among concepts both within and across topics. Consider the following when planning for instruction:

- Integration of the mathematical processes within each topic is expected.
- Decreasing emphasis on rote calculation, drill, and practice, and the size of numbers used in paper and pencil calculations makes more time available for concept development.

- Problem solving, reasoning, and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➢ Pathways and Topics

The Prince Edward Island 10-12 mathematics curriculum includes pathways with corresponding topics rather than strands, which are found in the Prince Edward Island K-9 mathematics curriculum. Three pathways are available: **Apprenticeship and Workplace Mathematics**, **Foundations of Mathematics**, and **Pre-Calculus**. A common grade ten course (Foundations of Mathematics and Pre-Calculus, Grade 10) is the starting point for the Foundations of Mathematics pathway and the Pre-Calculus pathway. Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings. These pathways are illustrated in the diagram below:



The goals of all three pathways are to provide the prerequisite knowledge, skills, understandings, and attitudes for specific post-secondary programs or direct entry into the work force. All three pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. Each pathway is designed to provide students with the mathematical understandings, rigour and critical-thinking skills that have been identified for specific post-secondary programs of study or for direct entry into the work force. When choosing a pathway, students should consider their interests, both current and future. Students, parents and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

Apprenticeship and Workplace Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into the majority of trades and for direct entry into the work force. Topics include algebra, geometry, measurement, number, statistics, and probability.

Foundations of Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of theoretical calculus. Topics include financial mathematics, geometry, measurement, algebra and number, logical reasoning, relations and functions, statistics, probability, and a mathematics research project.

Pre-Calculus

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Topics include algebra and number, measurement, relations and functions, trigonometry, combinatorics, and introductory calculus.

➢ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]**
- develop visualization skills to assist in processing information, making connections, and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

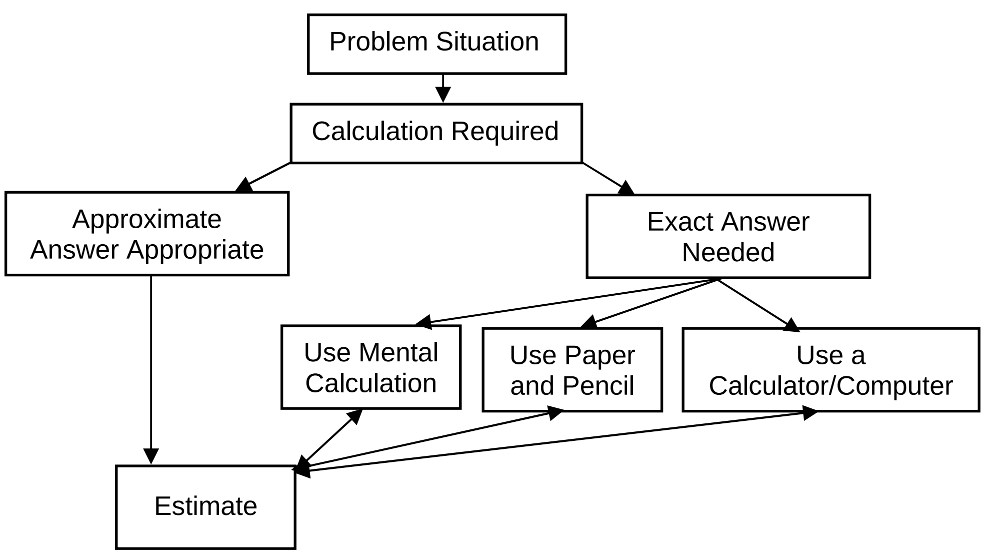
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves calculation without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need - more than ever before - with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001). Mental mathematics "provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers" (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know when to estimate, what strategy to use, and how to use it. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you. . . ?" or “How could you. . . ?” the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- using estimation
- guessing and checking
- looking for a pattern
- making an organized list or table
- using a model
- working backwards
- using a formula
- using a graph, diagram, or flow chart
- solving a simpler problem
- using algebra.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations;

- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K-3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform, and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and knowledge of several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

➢ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics which are woven throughout this document. These components include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180^0.

- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (*The Primary Program*, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, and flexible and intuitive with numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all topics in mathematics and it is important that connections are made among topics. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collecting and analysing of data, and describing relationships visually, symbolically, orally, or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to interpret representations of 2-D shapes and 3-D objects, and identify relationships to mathematical topics. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 2-D shapes and 3-D objects.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to use dimensions and make predictions about the results of changing dimensions.

- Knowing the dimensions of an object enables students to communicate about the object and create representations.
- The volume of a rectangular solid can be calculated from given dimensions.
- Doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Young children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Their mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Initial problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➢ Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding, and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should replace some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill-oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a teacher can give a parent clearer understanding of the mathematics curriculum and of the child's progress in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

➢ Diversity in Student Needs

Every class has students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters, but should be designed to help all students (whether strong, weak or average) to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson from which all students come away with a better understanding of what the solution to an equation really means.

➢ Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean not only that enrolments of students of both genders and various cultural backgrounds in public school mathematics courses should reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➢ Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English deficiencies and cultural differences must not be barriers to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and co-ordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication "as an essential part of mathematics and mathematics education" (p.60). The *Standards* elaborate that all students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading, and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers and will facilitate "communicating to learn mathematics and learning to communicate mathematically" (NCTM, p.60).

To this end,

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counsellors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not on other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated with appropriate language support to both students and parents;
- to verify that barriers have been removed, educators should monitor enrolment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➢ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development - such as poverty alleviation, human rights, health, environmental protection, and climate change - into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental, and economic perspective, and exploring how those factors are interrelated and interdependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database *Resources for Rethinking*, found at **http://r4r.ca/en**. It provides teachers with access to materials that integrate ecological, social, and economic spheres through active, relevant, interdisciplinary learning.

➢ Inquiry-Based Learning and Project Based Learning

Inquiry-based learning (IBL) allows students to explore, investigate, and construct new meaning from prior knowledge and from new information that is retrieved from other sources. It is not linear in nature, but promotes a continual looping back and forth throughout the process as students gather and process new information, redirect their inquiries, and continue through the process. Mathematical inquiry will require students to practise and refine their critical and creative-thinking skills. The terms *inquiry* and *research* are often used interchangeably within an educational context. While research often becomes the end-result of an inquiry process, it is the process itself that should be emphasized within an educational context. More information regarding the development of a mathematics research project is included in the appendix at the end of this document.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, teaching has been effective, or how best to address student learning needs. The quality of assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others.

➢ Assessment

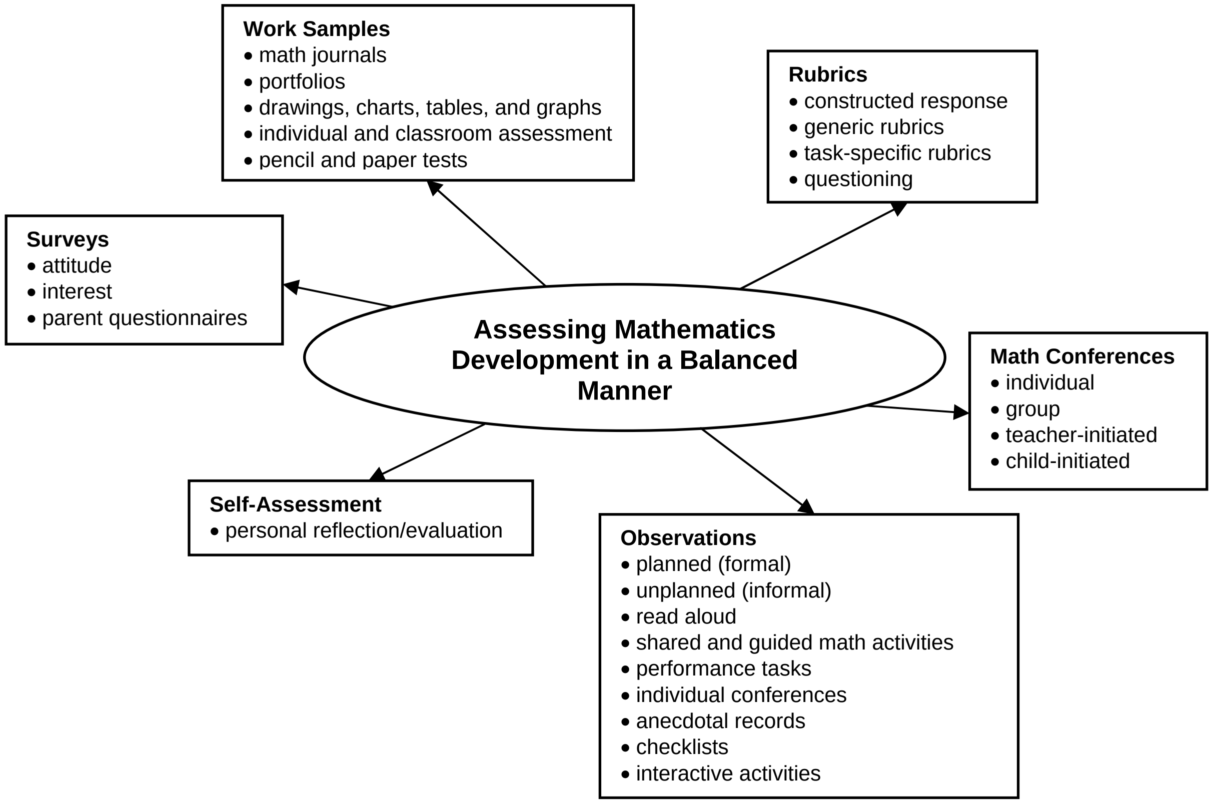
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children's learning;
- providing information to teachers on the effectiveness of their teaching, the program, and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources, including

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment.

This balanced approach for assessing mathematics development is illustrated in the diagram on the next page.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning, and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used

- to engage students in their own learning and self-assessment;
- to help students understand what is important in the mathematical concepts and particular tasks they encounter;
- to develop effective habits of metacognition and self-coaching;
- to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used

- to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
- to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
- to provide feedback to students about how they are doing and how they might improve;
- to provide feedback to other professionals and to parents about how to support students' learning.

Assessment *of* learning is used

- to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
- to facilitate reporting;

- to provide the basis for sound decision-making about next steps in a student's learning.

➢ Evaluation

Evaluation is the process of analysing, reflecting upon, and summarizing assessment information, and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires
- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information;
- using a high level of professional judgment in making decisions based upon that information.

➢ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes, phone calls, and electronic methods.

➢ Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration, and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles, as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes, or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.
- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.
- Assessment reports should be clear, accurate, and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that
- the best interests of the student are paramount;

- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes;
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he or she knows and can do.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island high school mathematics curriculum are organized into a number of topics across the grades from ten to twelve. Each topic has associated with it a general curriculum outcome (GCO). They are overarching statements about what students are expected to learn in each topic from grades ten to twelve.

Topic	General Curriculum Outcome (GCO)
Algebra (A)	Develop algebraic reasoning.
Algebra and Number (AN)	Develop algebraic reasoning and number sense.
Calculus (C)	Develop introductory calculus reasoning.
Financial Mathematics (FM)	Develop number sense in financial applications.
Geometry (G)	Develop spatial sense.
Logical Reasoning (LR)	Develop logical reasoning.
Mathematics Research Project (MRP)	Develop an appreciation of the role of mathematics in society.
Measurement (M)	Develop spatial sense and proportional reasoning. **(*Foundations of Mathematics and Pre-Calculus*)**
	Develop spatial sense through direct and indirect measurement. **(*Apprenticeship and Workplace Mathematics*)**
Number (N)	Develop number sense and critical thinking skills.
Permutations, Combinations and Binomial Theorem (PC)	Develop algebraic and numeric reasoning that involves combinatorics.
Probability (P)	Develop critical thinking skills related to uncertainty.
Relations and Functions (RF)	Develop algebraic and graphical reasoning through the study of relations.
Statistics (S)	Develop statistical reasoning.
Trigonometry (T)	Develop trigonometric reasoning.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific curriculum outcome.

In this curriculum guide, each specific curriculum outcome (SCO) is presented in a two-page format, and includes the following information:

- its corresponding topic and general curriculum outcome;
- the specific curriculum outcome, with a list of achievement indicators;
- a list of the sections in *Geometry*, *Mathematics of Data Management*, and *MathPower 11* which address the SCO, with specific achievement indicators highlighted in brackets;
- an elaboration for the SCO.

In the second half of this document, a curriculum guide supplement is presented which follows the primary resources, *Geometry*, *Mathematics of Data Management*, and *MathPower 11*. As well, an appendix is included which outlines the steps to follow in the development of an effective mathematics research project.

GEOMETRY

SPECIFIC CURRICULUM OUTCOMES

G1 – Demonstrate an understanding of basic geometric concepts, including
- **points;**
- **lines;**
- **line segments;**
- **planes;**
- **angles;**
- **two-dimensional figures.**

G2 – Demonstrate an understanding of parallel and perpendicular lines.

G3 – Demonstrate an understanding of triangles.

G4 – Demonstrate an understanding of congruent triangles.

MAT521E – Topic: Geometry (G)

GCO: Develop spatial sense.

SCO: **G1 – Demonstrate an understanding of basic geometric concepts, including**

- **points;**
- **lines;**
- **line segments;**
- **planes;**
- **angles;**
- **two-dimensional figures.**

[C, CN, ME, PS, R, V]

Students who have achieved this outcome should be able to:

A. Identify, model and interpret drawings involving points, lines, planes and angles.

B. Identify intersecting lines and planes.

C. Measure the length of a line segment.

D. Identify congruent line segments.

E. Find the distance between two points on a number line and on a coordinate plane.

F. Find the midpoint of a line segment on a number line and on a coordinate plane.

G. Determine the coordinates of an endpoint of a line segment.

H. Name, measure and classify angles.

I. Identify and use congruent angles and the bisector of an angle.

J. Identify and use special pairs of angles.

K. Find measures of line segments and angles by performing calculations, and by using algebraic reasoning.

L. Identify and name polygons.

M. Find the perimeter, circumference and/or area of two-dimensional figures.

Section(s) in Geometry *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.1 (A B)

1.2 (C D)

1.3 (E F G)

1.4 (H I)

1.5 (J K)

1.6 (L M)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: G1 – Demonstrate an understanding of basic geometric concepts, including

- **points;**
- **lines;**
- **line segments;**
- **planes;**
- **angles;**
- **two-dimensional figures.**

[C, CN, ME, PS, R, V]

Elaboration

Students will not have worked with many geometric concepts since grade nine, therefore it is important that they have a very good understanding of the basic geometric concepts highlighted in this specific curriculum outcome. The concepts of *point*, *line*, and *plane* are central in the teaching of geometry. While these concepts are intuitively easy to understand by most students, they should realize that these terms are actually undefined, and that all geometric definitions build from these three basic concepts. These concepts are connected by the following two statements.

- There is exactly one line through any two points.
- There is exactly one plane through any three noncollinear points.

A line segment is that part of a line consisting of two points on the line and all points between them. Since a line segment is measurable, its distance and midpoint can be found using the following formulas:

	NUMBER LINE	COORDINATE PLANE
Distance Formula	$PQ = \lvert x_1 - x_2 \rvert$	$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Midpoint Formula	$M = \frac{x_1 + x_2}{2}$	$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

An angle is formed by two noncollinear rays that have a common endpoint. Angles can be classified by their measures. There are a number of special pairs of angles that students should know.

- **Adjacent angles** – two coplanar angles that lie in the same plane, have a common vertex and a common side, but no common interior points
- **Vertical angles** – two nonadjacent angles formed by two intersecting lines
- **Linear pair** – a pair of adjacent angles with non-common sides that are opposite rays
- **Complementary angles** – two angles whose measures have a sum of 90^0
- **Supplementary angles** – two angles whose measures have a sum of 180^0

As well, students should be very familiar with the formulas for the perimeter, circumference, and area of common two-dimensional figures. These formulas are given in the following table.

	TRIANGLE	SQUARE	RECTANGLE	CIRCLE
Perimeter or Circumference	$P = a + b + c$	$P = 4s$	$P = 2l + 2w$	$C = 2\pi r$ or $C = \pi d$
Area	$A = \frac{1}{2}bh$	$A = s^2$	$A = lw$	$A = \pi r^2$

MAT521E – Topic: Geometry (G)

GCO: Develop spatial sense.

SCO: **G2 – Demonstrate an understanding of parallel and perpendicular lines.** [C, CN, PS, V]

Students who have achieved this outcome should be able to:

A. Identify the relationships between two lines or two planes.

B. Identify transversals, and classify angle pairs formed by parallel lines and transversals.

C. Use theorems about parallel lines to determine the relationships between specific pairs of angles.

D. Use algebra to find angle measurements.

E. Prove that two lines are parallel using angle relationships.

Section(s) in Geometry *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

3.1 (A B)

3.2 (C D)

3.5 (E)

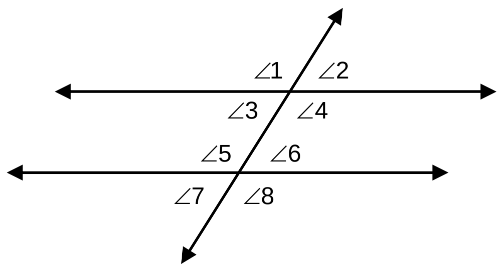
[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: G2 – Demonstrate an understanding of parallel and perpendicular lines. [C, CN, PS, V]

Elaboration

When a transversal line intersects a pair of parallel lines,

- the measures of the corresponding angles are equal: $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$, $\angle 4 = \angle 8$
- the measures of the alternate interior angles are equal: $\angle 3 = \angle 6$, $\angle 4 = \angle 5$
- the measures of the alternate exterior angles are equal: $\angle 1 = \angle 8$, $\angle 2 = \angle 7$
- the interior angles on the same side of the transversal are supplementary: $\angle 3 + \angle 5 = 180^0$, $\angle 4 + \angle 6 = 180^0$



Conversely, when a transversal intersects a pair of lines such that:

- the corresponding angles are congruent;
- the alternate interior angles are congruent;
- the alternate exterior angles are congruent; or
- the interior angles on the same side of the transversal are supplementary;

then the lines are parallel. Please note that when a transversal intersects a pair of non-parallel lines, none of the above relationships is true.

MAT521E – Topic: Geometry (G)

GCO: Develop spatial sense.

SCO: **G3 – Demonstrate an understanding of triangles.** [C, CN, ME, PS, V]

Students who have achieved this outcome should be able to:

A. Identify and classify triangles by angle and side measures.

B. Apply the Triangle Angle Sum Theorem to solve a problem.

C. Apply the Exterior Angle Theorem to solve a problem.

D. Find the missing side and angle values in a triangle.

Section(s) in Geometry *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.1 (A)

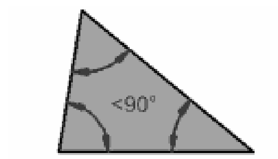
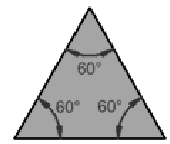
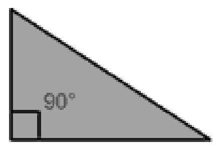
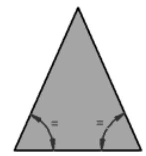
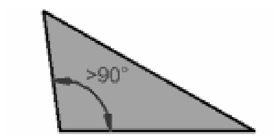
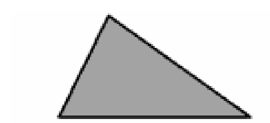
4.2 (B C D)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: **G3 – Demonstrate an understanding of triangles.** [C, CN, ME, PS, V]

Elaboration

Triangles can be classified by their angles as acute, obtuse, or right, and by their sides as scalene, isosceles, or equilateral.

CLASSIFICATION BY ANGLES		CLASSIFICATION BY SIDES	
Acute Triangle All angles are less than 90^0		**Equilateral Triangle** Three equal sides and three equal angles that are always 60^0	
Right Triangle Has a right angle (90^0)		**Isosceles Triangle** Two equal sides and two equal angles	
Obtuse Triangle Has an angle more than 90^0		**Scalene Triangle** No equal sides and no equal angles	

Triangle Angle-Sum Theorem

The sum of the measures of the three angles of a triangle is 180^0.

Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

MAT521E – Topic: Geometry (G)

GCO: Develop spatial sense.

SCO: **G4 – Demonstrate an understanding of congruent triangles.** [C, CN, PS, R, V]

Students who have achieved this outcome should be able to:

A. Identify and use the corresponding parts of two congruent polygons to solve problems.

B. Use the Third Angles Theorem to solve a problem.

C. Prove that two triangles are congruent using the definition of congruence.

D. Use the SSS, SAS, ASA and AAS postulates to prove that two triangles are congruent.

E. Apply triangle congruence in a real-world situation.

F. Use the properties of isosceles and equilateral triangles to find missing measures in a triangle.

G. Use the properties of isosceles and equilateral triangles within proofs.

Section(s) in Geometry *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.3 (A B C)

4.4 (D E)

4.5 (D E)

4.6 (F G)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: G4 – Demonstrate an understanding of congruent triangles. [C, CN, PS, R, V]

Elaboration

Third Angles Theorem

If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.

There are four basic ways in which two triangles can be proven to be congruent.

SSS	If all of the corresponding sides of two triangles are congruent, then the triangles are congruent.
SAS	If two pairs of corresponding sides of two triangles and the included angles are congruent, then the triangles are congruent.
ASA	If two pairs of corresponding angles of two triangles and the included sides are congruent, then the triangles are congruent.
AAS	If two pairs of corresponding angles of two triangles and a corresponding pair of non-included sides are congruent, then the triangles are congruent.

In an isosceles triangle, the base angles are congruent and the sides opposite the base angles are congruent. In an equilateral triangle, all three sides are congruent and all three angles are congruent, each with a measure of 60^0. Therefore, all equilateral triangles are also equiangular.

LOGICAL REASONING

SPECIFIC CURRICULUM OUTCOMES

LR1 – Analyse and prove conjectures, using inductive reasoning, to solve problems.

LR2 – Demonstrate an understanding of logical reasoning.

LR3 – Analyse and prove conjectures, using deductive reasoning, to solve problems.

LR4 – Use logical reasoning to write geometric and algebraic proofs.

LR5 – Analyse and prove conjectures, using indirect proof.

MAT521E – Topic: Logical Reasoning (LR)

GCO: Develop logical reasoning.

SCO: **LR1 – Analyse and prove conjectures, using inductive reasoning, to solve problems.** [C, CN, PS, R]

Students who have achieved this outcome should be able to:

A. Make a conjecture based on inductive reasoning, and use the conjecture to make predictions.

B. Find a counterexample to show that a conjecture is false.

C. Determine the truth values of negations, conjunctions and disjunctions, and represent them using Venn diagrams.

Section(s) in Geometry *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.1 (A B)

2.2 (C)

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

SCO: LR1 – Analyse and prove conjectures, using inductive reasoning, to solve problems. [C, CN, PS, R]

Elaboration

Inductive reasoning is being used when a conjecture is reached based on observations of a previous pattern. However, one cannot state that a conjecture is true based solely on a number of individual observations alone. Conjectures must be proven using deductive reasoning, but a conjecture can be disproved by simply finding a counterexample that is contrary to it.

A statement is a sentence that can either be true or false. Whether a statement is true or false is called the truth value of the statement. The negation of a statement has the opposite meaning. So, if a statement is true, then its negation is false, and vice versa.

Compound statements can be formed by combining statements with the words *and* or *or*. If two statements are combined with the word *and*, then the compound statement is called a conjunction. In order for a conjunction to be true, both statements in the conjunction must be true. Otherwise, the conjunction is false. If two statements are combined with the word *or*, then the compound statement is called a disjunction. In order for a disjunction to be true, only one of the statements in the disjunction has to be true. That is, the only way that a disjunction can be false is if both statements in the disjunction are false.

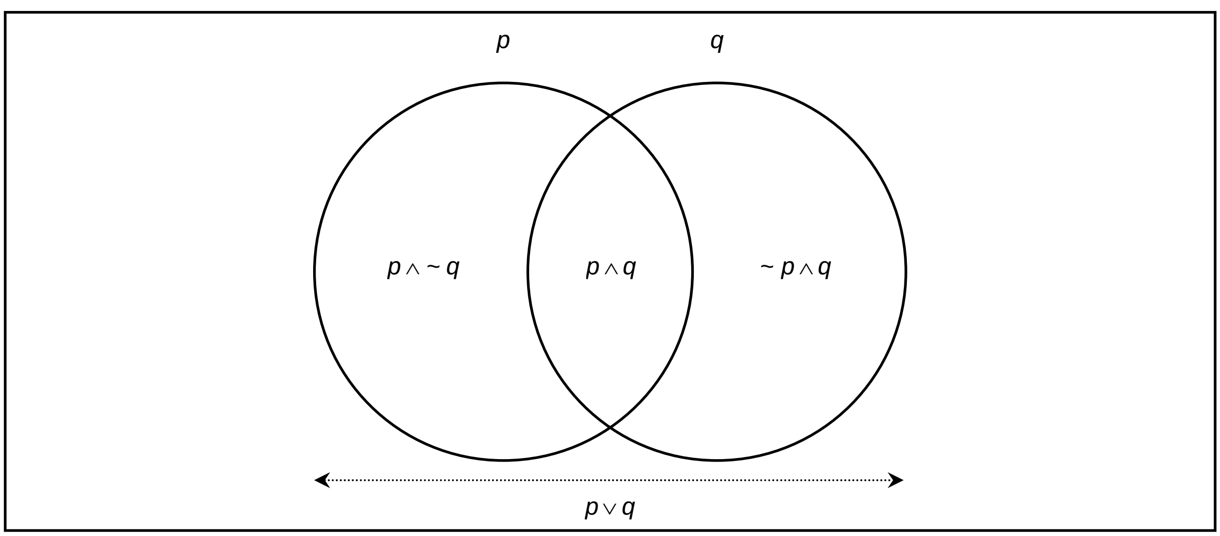
A convenient method for organizing the truth values of statements is to use a truth table. They are especially helpful when trying to determine the truth values of negations and compound statements.

NEGATION	
p	$\sim p$
T	F
F	T

CONJUNCTION		
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

DISJUNCTION		
p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Another effective way to visually organize compound statements is to use a Venn diagram. The relationships among various compound statements are shown below.



MAT521E – Topic: Logical Reasoning (LR)

GCO: Develop logical reasoning.

SCO: **LR2 – Demonstrate an understanding of logical reasoning.** [C, CN, PS, R]

Students who have achieved this outcome should be able to:

A. Use a Venn diagram to determine whether a conclusion is valid.

B. Write a compound statement for a given conjunction or disjunction.

C. Construct a truth table for a given situation.

D. Analyse a statement in "if-then" form by identifying the hypothesis and conclusion.

E. Write a conditional statement in "if-then" form.

F. Determine the truth value of a conditional statement.

G. Write the converse, inverse and contrapositive of an "if-then" statement.

Section(s) in Geometry *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.2 (A B C)

2.3 (D E F G)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: LR2 – Demonstrate an understanding of logical reasoning. [C, CN, PS, R]

Elaboration

Venn diagrams and truth tables are very helpful when trying to determine the validity of a logical argument. A Venn diagram is a diagram that is used to classify different sets that are discussed in an argument. A truth table provides a convenient method of organizing the truth values of statements in a given argument.

There are a number of important concepts that are used in the process of logical reasoning. Suppose that the statements are represented by the letters *p* and *q*. Then, we define the following concepts as follows:

STATEMENT	WORDS	SYMBOLS
Negation	A statement that has the opposite meaning and truth value of an original statement.	$\sim p$, read "not *p*"
Conjunction	A compound statement formed by joining two or more statements using the word *and*.	$p \wedge q$, read "*p* and *q*"
Disjunction	A compound statement formed by joining two or more statements using the word *or*.	$p \vee q$, read "*p* or *q*"

The statement, "if *p*, then *q*," is called a conditional statement. The statement *p* is called the hypothesis, and the statement *q* is called the conclusion. Symbolically, it can be written as $p \rightarrow q$. A true conditional statement can be converted to another conditional statement in one of four ways, each having its own truth value. It can be shown using truth tables that the conditional and contrapositive are logically equivalent, and that the converse and inverse are logically equivalent.

STATEMENT	WORDS	SYMBOLS
Conditional	A conditional statement is a statement that can be written in the form, "if *p*, then *q*."	$p \rightarrow q$
Converse	The converse is formed by exchanging the hypothesis and conclusion of the conditional statement.	$q \rightarrow p$
Inverse	The inverse is formed by negating both the hypothesis and conclusion of the conditional statement.	$\sim p \rightarrow \sim q$
Contrapositive	The contrapositive is formed by negating both the hypothesis and conclusion of the converse of the conditional statement.	$\sim q \rightarrow \sim p$

MAT521E – Topic: Logical Reasoning (LR)

GCO: Develop logical reasoning.

SCO: **LR3 – Analyse and prove conjectures, using deductive reasoning, to solve problems.** [C, CN, PS, R]

Students who have achieved this outcome should be able to:

A. Determine whether a conclusion is based on inductive or deductive reasoning.

B. Use the Law of Detachment to determine whether a conclusion is valid.

C. Use the Law of Syllogism to determine whether a conclusion is valid.

D. Apply the laws of deductive reasoning to draw a valid conclusion from given statements.

Section(s) in Geometry *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.4 (A B C D)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: LR3 – Analyse and prove conjectures, using deductive reasoning, to solve problems. [C, CN, PS, R]

Elaboration

Conjectures that are proved using deductive reasoning often use two very important properties called the Law of Detachment and the Law of Syllogism.

The Law of Detachment states that if $p \rightarrow q$ is a true statement and p is true, then q is true. Basically, as long as the facts that are given are true, then the conclusion reached using deductive reasoning will also be true.

The Law of Syllogism states that if $p \rightarrow q$ and $q \rightarrow r$ are true statements, then $p \rightarrow r$ is a true statement. When using the Law of Syllogism, it is very important to remember that if the conclusion of the first statement is not the hypothesis of the second statement, no valid conclusion can be drawn.

MAT521E – Topic: Logical Reasoning (LR)

GCO: Develop logical reasoning.

SCO: **LR4 – Use logical reasoning to write geometric and algebraic proofs.** [C, CN, ME, PS, R, V]

Students who have achieved this outcome should be able to:

A. Identify, analyse and use the basic postulates about points, lines and planes.

B. Justify each step when solving an equation.

C. Use algebra to write a two-column proof.

D. Use the properties of equality to write a two-column geometric proof.

E. Write proofs involving

- segment addition and angle addition;
- segment congruence and angle congruence;
- supplementary and complementary angles;
- congruent and right angles;
- vertical angles.

Section(s) in Geometry *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.5 (A)

2.6 (B C)

2.7 (D E)

2.8 (D E)

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

SCO: LR4 – Use logical reasoning to write geometric and algebraic proofs. [C, CN, ME, PS, R, V]

Elaboration

When writing a formal mathematical proof, there are a number of steps that must be followed.

- List the given information and, if possible, draw a diagram to illustrate this information.
- State the theorem or conjecture to be proven.
- Create a deductive argument by forming a logical chain of statements linking the given statement to what you are trying to prove.
- Justify each statement with a reason. Reasons include definitions, algebraic properties, postulates, and theorems.
- State what it is you have proven.

In geometry, there are a number of postulates that are accepted as true without proof. Following is a list of these postulates.

2.1 Through any two points, there is exactly one line.

2.2 Through any three noncollinear points, there is exactly one plane.

2.3 A line contains at least two points.

2.4 A plane contains at least three noncollinear points.

2.5 If two points lie in a plane, then the entire line containing those points lies in that plane.

2.6 If two lines intersect, then their intersection is exactly one point.

2.7 If two planes intersect, then their intersection is exactly one line.

2.8 The points on any line or line segment can be put into a one-to-one correspondence with real numbers.

2.9 If A, B, and C are collinear, then point B is between A and C if and only if $AB + BC = AC$.

2.10 Given any angle, the measure can be put into a one-to-one correspondence with real numbers between 0 and 180.

2.11 D is in the interior of $\angle ABC$ if and only if $m\angle ABD + m\angle DBC = m\angle ABC$.

In addition to the geometric postulates, there are a number of properties of real numbers that are also used quite frequently when constructing proofs.

PROPERTIES OF REAL NUMBERS	
Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $a \bullet c = b \bullet c$.
Division Property of Equality	If $a = b$, then $\frac{a}{c} = \frac{b}{c}$, for $c \neq 0$.
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$, then $b = a$.
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$.
Substitution Property of Equality	If $a = b$, then a may be replaced by b in any equation or expression.
Distributive Property	$a(b + c) = ab + ac$

MAT521E – Topic: Logical Reasoning (LR)

GCO: Develop logical reasoning.

SCO: **LR5 – Analyse and prove conjectures, using indirect proof.** [C, CN, PS, R]

Students who have achieved this outcome should be able to:

A. State the assumption for starting an indirect proof.

B. Write indirect algebraic and geometric proofs.

Section(s) in Geometry *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.4 (A B)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: LR5 – Analyse and prove conjectures, using indirect proof. [C, CN, PS, R]

Elaboration

In an indirect proof, or proof by contradiction, you begin by temporarily assuming that what you are trying to prove is false. By showing this assumption to be logically impossible, you can prove that your assumption is false, and as a result, the original conclusion must be true.

The following is the procedure to use when writing an indirect proof:

- Identify the conclusion you are asked to prove. Make the assumption that this conclusion is false by assuming that the opposite is true.
- Use logical reasoning to show that this assumption leads to a contradiction of the hypothesis, or of some other fact, such as a definition, postulate, theorem, or corollary.
- Point out that since the assumption leads to a contradiction, the original conclusion, what you were asked to prove, must be true.

PROBABILITY

SPECIFIC CURRICULUM OUTCOMES

P1 – Demonstrate an understanding of the probability of an event.

P2 – Interpret and assess the validity of odds and probability statements.

P3 – Demonstrate an understanding of events that are independent or dependent.

P4 – Demonstrate an understanding of events that are mutually exclusive and not mutually exclusive.

MAT521E – Topic: Probability (P)

GCO: Develop critical thinking skills related to uncertainty.

SCO: **P1 – Demonstrate an understanding of the probability of an event.** [C, ME, PS]

Students who have achieved this outcome should be able to:

A. Calculate the probability of an event.

B. Use a tree diagram to find the sample space and to compute the probability of an event.

C. Use a table to find the sample space and to compute the probability of an event.

Section(s) in Mathematics of Data Management *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.1 (A B C)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: P1 – Demonstrate an understanding of the probability of an event. [C, ME, PS]

Elaboration

A probability experiment is a well-defined process in which clearly identifiable outcomes are measured for each trial. An event is a collection of outcomes satisfying a particular condition. The probability of an event can range from 0 or 0% (impossible), to 1 or 100% (certain).

The empirical probability of an event is the number of times the event occurs divided by the total number of trials. The theoretical probability of an event *A* is given by $P(A)=\frac{n(A)}{n(S)}$, where $n(A)$ is the number of outcomes making up *A*, and $n(S)$ is the total number of outcomes in the sample space *S*, where all outcomes are equally likely to occur. A subjective probability is based on intuition and previous experience.

If the probability of event *A* is given by $P(A)$, then the probability of the complement of *A* is given by $P(A')=1-P(A)$.

MAT521E – Topic: Probability (P)

GCO: Develop critical thinking skills related to uncertainty.

SCO: **P2 – Interpret and assess the validity of odds and probability statements.** [C, CN, ME]

Students who have achieved this outcome should be able to:

A. Determine the probability of, or the odds for and against, an outcome in a situation.

B. Express odds as a probability and vice-versa.

C. Explain, using examples, the relationship between odds (part-part) and probability (part-whole).

D. Solve a contextual problem that involves odds or probability.

Section(s) in Mathematics of Data Management *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.2 (A B C D)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: P2 – Interpret and assess the validity of odds and probability statements. [C, CN, ME]

Elaboration

The odds in favour of *A* are given by the ratio $P(A):P(A')$. The odds against *A* are given by the ratio $P(A'):P(A)$. If the odds in favour of *A* are $h:k$, then $P(A)=\frac{h}{h+k}$.

For example, the odds in favour of rolling a number less than 3 on a die are $2:4$, and the odds against rolling a number less than 3 are $4:2$. Therefore, the probability of rolling a number less than 3 is $P(A)=\frac{2}{2+4}=\frac{1}{3}$.

MAT521E – Topic: Probability (P)

GCO: Develop critical thinking skills related to uncertainty.

SCO: **P3 – Demonstrate an understanding of events that are independent or dependent.** [C, CN, ME, PS, R]

Students who have achieved this outcome should be able to:

A. Determine whether two events are independent or dependent.

B. Compute the probability for events that are independent.

C. Explain what is meant by conditional probability.

D. Use conditional probability to find the probability of two dependent events.

Section(s) in Mathematics of Data Management *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.4 (A B C D)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: P3 – Demonstrate an understanding of events that are independent or dependent. [C, CN, ME, PS, R]

Elaboration

To determine whether two events are independent, determine whether one event will affect the probable outcome of the other event. If not, then the events are independent. If one event does not affect the other, then the events are dependent, and one must use conditional probability to calculate the probability of both events occurring.

If *A* and *B* are independent events, then the probability of both events occurring is given by

$$P(A \text{ and } B) = P(A) \bullet P(B)$$

If event *B* is dependent on event *A*, then the conditional probability of *B*, given *A*, is $P(B \mid A)$. In this case, the probability of both events occurring is

$$P(A \text{ and } B) = P(A) \bullet P(B \mid A)$$

In order to find $P(B \mid A)$, do not forget that the number of outcomes making up *A* and the total number of outcomes in the sample space *S* may be affected. For example, to determine the probability of selecting two hearts from a standard deck of 52 cards, without replacement, we would calculate the probability as follows:

$$\begin{aligned} P(A \text{ and } B) &= P(A) \bullet P(B \mid A) \\ &= \frac{13}{52} \bullet \frac{12}{51} \\ &= \frac{1}{17} \\ &\doteq 0.0588 \end{aligned}$$

Note that in the calculation of $P(B \mid A)$, there is one fewer heart (12) and one fewer card (51) in the deck, therefore $P(B \mid A) = \frac{12}{51}$ or $\frac{4}{17}$.

MAT521E – Topic: Probability (P)

GCO: Develop critical thinking skills related to uncertainty.

SCO: **P4 – Demonstrate an understanding of events that are mutually exclusive and not mutually exclusive.** [C, CN, ME, PS, R]

Students who have achieved this outcome should be able to:

A. Determine whether two events are mutually exclusive.

B. Compute the probability for events that are mutually exclusive.

C. Compute the probability for events that are not mutually exclusive.

Section(s) in Mathematics of Data Management *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.5 (A B C)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: P4 – Demonstrate an understanding of events that are mutually exclusive and not mutually exclusive. [C, CN, ME, PS, R]

Elaboration

To determine whether two events are mutually exclusive, it is necessary to determine whether there are elements that are in common to both *A* and *B*.

If *A* and *B* are mutually exclusive events, then the probability of either *A* or *B* occurring is given by

$$P(A \text{ or } B) = P(A) + P(B)$$

If *A* and *B* are non-mutually exclusive events, then the probability of either *A* or *B* occurring is given by

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

If *A* and *B* are non-mutually exclusive events, one must take into account the elements that are counted twice. That is why the two addition formulas above differ by the term $P(A \text{ and } B)$. This ensures that the elements in common are only counted once when calculating the probability.

For example, to determine the probability of rolling a double or rolling a sum of 8 when rolling two dice, we would calculate the probability as follows:

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= \frac{6}{36} + \frac{5}{36} - \frac{1}{36} \\ &= \frac{5}{18} \\ &\doteq 0.2778 \end{aligned}$$

Note that in the calculation of $P(A \text{ or } B)$, rolling two 4s is counted twice, therefore we subtract $\frac{1}{36}$ from the sum of $P(A) + P(B)$.

STATISTICS

SPECIFIC CURRICULUM OUTCOMES

S1 – Demonstrate an understanding of data and statistics.

S2 – Represent data visually in various ways.

S3 – Demonstrate an understanding of the measures of central tendency.

S4 – Demonstrate an understanding of the measures of dispersion.

S5 – Demonstrate an understanding of correlation and regression.

S6 – Demonstrate an understanding of the normal distribution.

MAT521E – Topic: Statistics (S)

GCO: Develop statistical reasoning.

SCO: **S1 – Demonstrate an understanding of data and statistics.** [C, CN, ME, R]

Students who have achieved this outcome should be able to:

A. Define data and statistics.

B. Explain the difference between a population and a sample.

C. Describe the four basic methods of sampling.

Section(s) in Mathematics of Data Management *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.1 (A)

2.3 (B C)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: S1 – Demonstrate an understanding of data and statistics. [C, CN, ME, R]

Elaboration

Statistical variables can be either continuous or discrete, depending on the context. Continuous variables can have any value within a given range, while discrete variables can only have certain values (usually integers).

A carefully selected sample can provide accurate information about a population. Selecting an appropriate sampling technique is important to ensure that the sample reflects the characteristics of the population. Randomly selected samples have a good chance of being representative of the population. The choice of sampling technique will depend on a number of factors, such as the nature of the population, cost, convenience, and reliability.

MAT521E – Topic: Statistics (S)

GCO: Develop statistical reasoning.

SCO: **S2 – Represent data visually in various ways.** [C, CN, ME, T, V]

Students who have achieved this outcome should be able to:

A. Construct a frequency distribution for a data set.

B. Draw a bar graph and a pie chart for a data set.

C. Draw a histogram and a frequency polygon for a data set.

D. Draw a time series graph for a data set.

Section(s) in Mathematics of Data Management *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.1 (A B C D)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: S2 – Represent data visually in various ways. [C, CN, ME, T, V]

Elaboration

Large amounts of data can be summarized by using frequency-distribution tables and diagrams. Sometimes the data is grouped into classes or intervals beforehand, particularly when the variables are continuous. A frequency diagram shows the frequencies of values in each individual interval, while a cumulative-frequency diagram shows the running total of frequencies from the lowest interval up. A relative-frequency diagram shows the frequency of each interval as a proportion or a percentage of the whole data set. Categorical data can be presented in various forms, including bar graphs, circle graphs and pictographs.

MAT521E – Topic: Statistics (S)

GCO: Develop statistical reasoning.

SCO: **S3 – Demonstrate an understanding of the measures of central tendency.** [C, PS, R, T]

Students who have achieved this outcome should be able to:

A. Determine the mean, median, and mode for a given set of data, and explain why these values may be the same or different from each other.

B. Provide a context in which the mean, median, or mode is the most appropriate measure of central tendency to use when reporting findings.

C. Solve a given problem involving measures of central tendency.

Section(s) in Mathematics of Data Management *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.5 (A B C)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: S3 – Demonstrate an understanding of the measures of central tendency. [C, PS, R, T]

Elaboration

The three principal measures of central tendency are the mean, median, and mode. These measures for a sample can differ from those of the whole population.

The mean is the sum of the values in a set of data divided by the number of data values in the set. The median is the middle value when the values are ranked in order. If there are two middle values, then the median is the mean of these two middle values. The mode is the most frequency occurring value. Outliers can have a dramatic effect on the mean if the sample size is small.

A weighted mean can be a useful measure when all of the data are not all of equal significance. For data grouped into intervals, the mean and median can be estimated using the midpoints and frequencies of the intervals.

MAT521E – Topic: Statistics (S)

GCO: Develop statistical reasoning.

SCO: **S4 – Demonstrate an understanding of the measures of dispersion.** [C, PS, R, T]

Students who have achieved this outcome should be able to:

A. Determine the variance and standard deviation for a given set of data, and explain why these values may be the same or different.

B. Provide a context in which the range, variance or standard deviation is the most appropriate measure of dispersion to use when reporting findings.

C. Solve a given problem involving quartiles or percentiles.

D. Calculate the *z*-score of a data item in a sample or a population.

Section(s) in Mathematics of Data Management *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.6 (A B C D)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: S4 – Demonstrate an understanding of the measures of dispersion. [C, PS, R, T]

Elaboration

The variance and the standard deviation are measures of how closely a set of data clusters around its mean. The variance and standard deviation of a sample may differ from those of the population from which the sample is drawn.

Quartiles are values that divide a set of ordered data into four intervals with equal numbers of data, while percentiles divide the data into 100 equal intervals. The interquartile range and semi-interquartile range are measures of how closely a set of data clusters around its median.

The *z*-score of a data item is a measure of how many standard deviations the data item is from the mean.

MAT521E – Topic: Statistics (S)

GCO: Develop statistical reasoning.

SCO: **S5 – Demonstrate an understanding of correlation and regression.** [CN, PS, T, V]

Students who have achieved this outcome should be able to:

A. Construct scatter plots for two data sets.

B. Calculate correlation coefficients.

C. Determine if correlation coefficients are significant.

D. Find a regression line for two data sets.

E. Use regression lines to make predictions.

Section(s) in Mathematics of Data Management *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

3.1 (A B C)

3.2 (D E)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: S5 – Demonstrate an understanding of correlation and regression. [CN, PS, T, V]

Elaboration

Statistical studies often find linear correlations between two variables. A scatter plot can often reveal the relationship between two variables. The independent variable is usually plotted on the horizontal axis and the dependent variable on the vertical axis. Two variables have a linear correlation if changes in one variable tend to be proportional to changes in the other variable. A linear correlation can be positive or negative, and its magnitude can vary in strength from zero (no correlation) to one (perfect correlation).

The correlation coefficient, r, is a quantitative measure of the correlation between two variables. Negative values indicate negative correlations while positive values indicate positive correlations. The greater the absolute value of r, the stronger the correlation. Manual calculations of correlation coefficients can be quite tedious, but a variety of powerful technology tools are available for such calculations.

Linear regression provides a means for analytically determining a line of best fit. In the least-squares method, the line of best fit is the line which minimizes the sum of the squares of the residuals while having the sum of the residuals equal zero. You can use the equation of the line of best fit to predict the value of one of the two variables given the value of the other variable.

The correlation coefficient is a measure of how well a regression line fits a set of data. Outliers and small sample sizes can lower the absolute value of the correlation coefficient, and reduce the accuracy of a linear model.

MAT521E – Topic: Statistics (S)

GCO: Develop statistical reasoning.

SCO: **S6 – Demonstrate an understanding of the normal distribution.** [CN, PS, T, V]

Students who have achieved this outcome should be able to:

A. Demonstrate an understanding of continuous probability distributions.

B. Explain, using examples, the properties of the normal curve, including the mean, median, mode, standard deviation, symmetry and area under the curve.

C. Determine if a data set approximates a normal distribution, and explain the reasoning.

D. Compare the properties of two or more normally distributed data sets.

E. Explain, using examples that represent multiple perspectives, the application of standard deviation for making decisions in applied situations, *e.g.*, warranties, insurance or opinion polls.

F. Solve a contextual problem that involves the interpretation of standard deviation.

G. Determine, with or without technology, and explain, the *z*-score for a given value in a normally distributed data set.

H. Solve a contextual problem that involves normal distribution.

I. Use the normal distribution to find percentages, probabilities and percentile ranks.

Section(s) in Mathematics of Data Management *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

8.1 (A)

8.2 (B C D E F G H I J)

8.3 (I J)

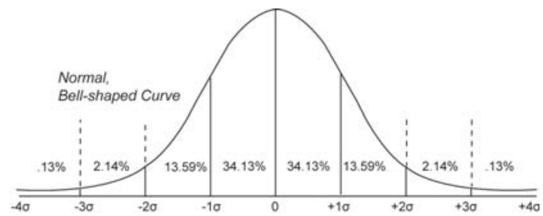
[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: S6 – Demonstrate an understanding of the normal distribution. [CN, PS, T, V]

Elaboration

Graphing a set of grouped data can help one determine whether the shape of the frequency polygon can be approximated by a normal distribution. The properties of a normal distribution are:

- The graph is symmetrical. The mean, median, and mode are equal and fall at the line of symmetry.
- The normal curve is shaped like a bell, peaking in the middle, sloping down toward the sides, and approaching zero at the extremes.
- About 68% of the data is within one standard deviation from the mean, about 95% of the data is within two standard deviations from the mean, and about 99.7% of the data is within three standard deviations from the mean.



- The area under the curve can be considered as one unit, since it represents 100% of the data.

The standard normal distribution is a normal distribution with a mean of 0 and a standard deviation of 1. The area under the curve of a standard normal distribution is 1. Data can be compared from different normally distributed sets by using *z*-scores. This process will convert any normal distribution to a standard normal distribution. A *z*-score indicates the number of standard deviations that a data value is from the mean. It is calculated using the formula

$$z = \frac{x - \bar{x}}{s}$$

where x is the data value, $\bar{x}$ is the mean, and s is the standard deviation. A positive *z*-score indicates that the data value lies above the mean. A negative *z*-score indicates that the data value lies below the mean.

FINANCIAL MATHEMATICS

SPECIFIC CURRICULUM OUTCOMES

FM1 – Demonstrate an understanding of income, including
- **wages;**
- **salary;**
- **contracts;**
- **commission;**
- **piecework**

to calculate gross and net pay.

FM2 – Solve problems that involve compound interest in financial decision making.

FM3 – Analyse costs and benefits of renting, leasing and buying.

FM4 – Develop a budget for a given set of circumstances.

MAT521E – Topic: Financial Mathematics (FM)

GCO: Develop number sense in financial applications.

SCO: **FM1 – Demonstrate an understanding of income, including**

- **wages;**
- **salary;**
- **contracts;**
- **commission;**
- **piecework**

to calculate gross and net pay. [C, CN, R, T]

Students who have achieved this outcome should be able to:

A. Describe, using examples, various methods of earning income.

B. Determine gross pay from given or calculated hours worked when given:
- the base hourly wage, with and without tips;
- the base hourly wage, plus overtime (time and a half, double time).

C. Determine gross pay for earnings acquired by:
- base wage, plus commission;
- single commission rate.

D. Determine the Canadian Pension Plan (CPP), Employment Insurance (EI) and income tax deductions for a given gross pay.

E. Determine the net pay when given deductions, *e.g.,* health plans, uniforms, union dues, charitable donations, payroll tax.

Section(s) in MathPower 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

9.1 (A B C)

9.2 (D E)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: **FM1 – Demonstrate an understanding of income, including**

- **wages;**
- **salary;**
- **contracts;**
- **commission;**
- **piecework**

to calculate gross and net pay. [C, CN, R, T]

Elaboration

The focus of this outcome is to give students a clear understanding of the difference between gross and net pay. Focus should be placed on the various methods of earning a wage when determining a person's gross pay, such as earning an annual salary, earning an hourly wage, tips, straight commission, base salary plus commission, graduated commission, and piecework. The discussion should include how gross income is calculated in each case.

In determining a person's net pay, particular attention should be paid to the various deductions that an employee is subject to, such as income tax, Canada Pension Plan, employment insurance, and other deductions.

MAT521E – Topic: Financial Mathematics (FM)

GCO: Develop number sense in financial applications.

SCO: **FM2 – Solve problems that involve compound interest in financial decision making.** [C, CN, PS, T, V]

Students who have achieved this outcome should be able to:

A. Explain the advantages and disadvantages of compound interest and simple interest.

B. Identify situations that involve compound interest.

C. Compare, in a given situation, the total interest paid or earned for different compounding periods.

D. Determine, given the principal, interest rate and number of compounding periods, the total interest of a loan.

E. Determine, using technology, the total cost of a loan under a variety of conditions, *e.g.*, different amortization periods, interest rates, compounding periods and terms.

F. Compare and explain, using technology, different credit options that involve compound interest, including bank and store credit cards, and special promotions.

G. Calculate the effective annual rate of interest for a given situation.

Section(s) in MathPower 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

9.3 (A B C D E F)

9.4 (G)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: FM2 – Solve problems that involve compound interest in financial decision making. [C, CN, PS, T, V]

Elaboration

Interest is a fee paid on borrowed assets. It is the price paid for the use of borrowed money, or money earned by deposited funds. Assets that are sometimes lent with interest include money, shares, consumer goods through hire purchase, major assets such as aircraft, and even entire factories in finance lease arrangements. The interest is calculated upon the value of the assets in the same manner as upon money.

Interest can be thought of as the “rent of money”. It is defined as the compensation paid by the borrower of money to the lender of money. When money is deposited in a bank, interest is typically paid to the depositor as a percentage of the amount deposited; when money is borrowed, interest is typically paid to the lender as a percentage of the amount owed. The percentage of the principal that is paid as a fee over a certain period of time (typically one month or year), is called the interest rate.

Simple interest is calculated only on the principal amount, or on that portion of the principal amount that remains unpaid. It is calculated using the formula $I = Prt$, where *I* is the interest, *P* is the principal, *r* is the annual rate of interest, and *t* is the time in years.

Compound interest arises when interest is added to the principal, so that from that moment on, the interest that has been added also itself earns interest. This addition of interest to the principal is called compounding. A bank account, for example, may have its interest compounded every year. As an example, an account with \$1000 initial principal and 5% interest per year would have a balance of \$1050 at the end of the first year, \$1102.50 at the end of the second year, and so on. It is calculated using the formula $A = P\left(1+i\right)^n$, where *A* is the amount, *P* is the principal, *i* is the rate of interest per compounding period, and *n* is the number of compounding periods.

The effective annual rate of interest is the simple interest rate that would produce the same interest in a year as the stated, or nominal interest rate. It can be calculated using the formula $r = \left(1+\frac{i}{n}\right)^n - 1$, where *i* is the nominal interest rate and *n* is the number of compounding periods.

MAT521E – Topic: Financial Mathematics (FM)

GCO: Develop number sense in financial applications.

SCO: **FM3 – Analyse costs and benefits of renting, leasing and buying.** [CN, PS, R, T]

Students who have achieved this outcome should be able to:

A. Identify and describe examples of assets that appreciate or depreciate.

B. Compare, using examples, renting, leasing and buying, and determine which would be the best choice for a specific set of circumstances.

C. Calculate the mortgage payments for a given set of circumstances.

Section(s) in MathPower 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

9.5 (A B)

9.6 (B C)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: FM3 – Analyse costs and benefits of renting, leasing and buying. [CN, PS, R, T]

Elaboration

To compare renting, leasing and buying, we can look at the example of acquiring a house.

When you buy a house, the house is yours. You hold the deed (which your loaning bank holds onto until you can pay the full price) stating it is completely yours. You are the owner and can do whatever you want to it and no one can take it away from you. That also means if anything goes wrong or needs fixing, you are responsible for all of its maintenance.

When you rent or lease, someone else owns the property but you are paying to use it and stay there. You are under whatever rules the actual owners have regarding what you can do to the accommodation, but usually that also means if anything needs fixing, you call upon the owners to fix the problems.

The major difference between renting and leasing is that renting is usually on a month-to-month basis, which is good if you want the flexibility to stay a short time and the freedom to leave whenever you want. But that also means the owners can raise the rent whenever they want.

Leasing is based on a signed contract for a specific length of time, usually in years. The good part is if you know you are staying long term, the lease price is usually fixed so the owners can't raise the price, but it also means you are bound to the length of that contract. It doesn't matter if you move before the end of the contract lease – you still have to pay for the full time of the lease.

MAT521E – Topic: Financial Mathematics (FM)

GCO: Develop number sense in financial applications.

SCO: **FM4 – Develop a budget for a given set of circumstances.** [C, CN, ME, PS, R, T, V]

Students who have achieved this outcome should be able to:

A. Identify income and expenses that should be included in a personal budget.

B. Explain considerations that must be made when developing a budget, *e.g.*, prioritizing, recurring and unexpected expenses.

C. Create a personal budget based on given income and expense data.

D. Collect income and expense data, and create a budget.

E. Modify a budget to achieve a set of personal goals.

F. Investigate and analyze, with or without technology, "what if …" questions related to personal budgets.

Section(s) in MathPower 11 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

9.7 (A B C D E F)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: FM4 – Develop a budget for a given set of circumstances. [C, CN, ME, PS, R, T, V]

Elaboration

A budget is a list of all planned expenses and revenues. It is a plan for saving and spending. A budget is an important concept in microeconomics, which uses a budget line to illustrate the trade-offs between two or more goods. In other terms, a budget is an organizational plan stated in monetary terms. It can be applied to a business, a household, or an individual.

Curriculum Guide Supplement

This supplement to the *Prince Edward Island MAT521E Mathematics Curriculum Guide* is designed to parallel the primary resources, *Geometry*, *Mathematics of Data Management*, and *MathPower 11*.

For each of the units, an approximate timeframe is suggested to aid teachers with planning. The timeframe is based on a total of 80 classes, each with an average length of 75 minutes:

UNIT	SUGGESTED TIME
Unit 1 – Tools of Geometry	11 classes
Unit 2 – Reasoning and Proof	12 classes
Unit 3 – Parallel and Perpendicular Lines	5 classes
Unit 4 – Congruent Triangles	12 classes
Unit 5 – Statistics of One Variable	11 classes
Unit 6 – Statistics of Two Variables	6 classes
Unit 7 – Introduction to Probability	8 classes
Unit 8 – The Normal Distribution	7 classes
Unit 9 – Personal Finance	8 classes

Each unit is divided into a number of sections. In this document, each section is supported by a one-page presentation, which includes the following information:

- the name and pages of the section in *Geometry*, *Mathematics of Data Management*, or *MathPower 11*;
- the specific curriculum outcome(s) and achievement indicator(s) addressed in the section (see the first half of the curriculum guide for an explanation of symbols);
- the student expectations for the section, which are associated with the SCO(s);
- the new concepts introduced in the section;
- other key ideas developed in the section;
- suggested problems in *Geometry*, *Mathematics of Data Management*, or *MathPower 11*;
- possible instructional and assessment strategies for the section.

UNIT 1
TOOLS OF GEOMETRY

SUGGESTED TIME
11 classes

Section 1.1 – Points, Lines and Planes (pp. 5-13)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

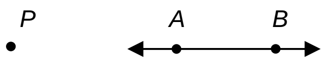
- **G1 (A B)**

After this lesson, students will be expected to:

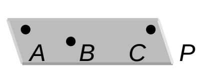
- identify and model points, lines and planes
- identify intersecting lines and planes

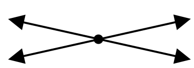
After this lesson, students should understand the following concepts:

- **undefined term –** words, usually readily understood, that are not formally explained by means of more basic words and concepts; the basic undefined terms of geometry are *point*, *line* and *plane*
- **point –** an undefined term; a location; in a figure, points are represented by a dot; points are named by capital letters



- **line –** an undefined term; made up of points and has no thickness or width; in a figure, a line is shown with an arrowhead at each end; lines are usually named by lowercase script letters or by writing capital letters for two points on the line, with a double arrow over the pair of letters
- **collinear –** points that line on the same line (see above right)
- **plane –** an undefined term; a flat surface made up of points that has no depth and extends indefinitely in all directions; in a figure, a plane is often represented by a shaded, slanted four-sided figure; planes are usually named by a capital script letter or by three noncollilnear points on the plane





- **coplanar –** points that lie in the same plane (see above left)
- **intersection –** a set of points common to two or more geometric figures (see above right)
- **defined term –** terms that are explained using undefined terms and/or other defined terms
- **space –** a boundless three-dimensional set of all points

Suggested Problems in *Geometry*:

- **pp. 8-12:** #1-56

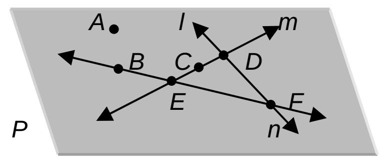
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Have a discussion with the class regarding why it is necessary to have undefined terms in geometry.

Possible Assessment Strategies:

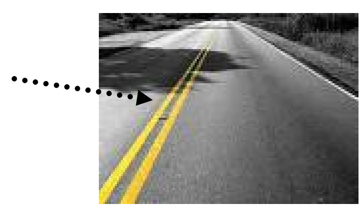
- Use the figure below to answer each of the following.



  a. Name the intersection of lines *l* and *m*.
  b. Give another name for line *n*.
  c. Name a point that is not contained in any of lines *l*, *m* or *n*.
  d. Give another name for plane *DEF*.

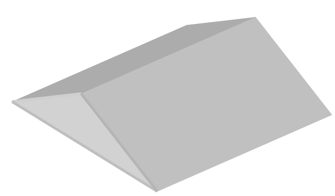
- Name the geometric term that is best modelled by each item.

  a.



  b.

- Draw and label a figure for each relationship.
  a. Line *l* lies in plane *P*.
  b. Two planes intersect at line *l*.
- How many planes are represented in the figure below?



Section 1.2 – Linear Measure (pp. 14-24)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **G1 (C D)**

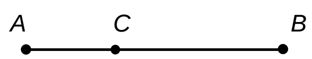
After this lesson, students will be expected to:

- measure segments
- calculate with measures

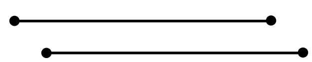
After this lesson, students should understand the following concepts:

- **line segment –** a measurable part of a line that consists of two points, called the endpoints, and all the points between them

- **between –** for any two points *A* and *B* on a line, there is another point *C* between *A* and *B* if and only if *A*, *B*, and *C* are collinear and $AC + CB = AB$



- **congruent segments –** segments that have the same measure



- **construction –** a method of creating geometric figures without the benefit of measuring tools; generally, only a pencil, straightedge, and compass are used

Suggested Problems in *Geometry*:

- **pp. 18-21:** #1-36

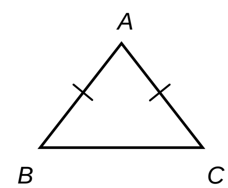
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

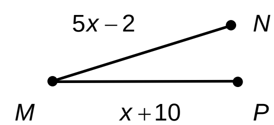
- Ensure that students understand that line segments can be measured, but not lines.

Possible Assessment Strategies:

- Find the value of the variable and *XP*, if *X* is between *P* and *Q*.
  a. $XQ = 10, \quad XP = 3x + 1, \quad PQ = 20$
  b. $XQ = 3x + 5, \quad XP = 2x + 1, \quad PQ = 4x + 10$
- Determine whether each pair of segments is congruent.
  a. $\overline{AB}$, $\overline{AC}$



  b. $\overline{MN}$, $\overline{MP}$



$x = 3$

- The distance from Tignish to Elmsdale is 2 km more than twice the distance from Elmsdale to Bloomfield. If Elmsdale is between Tignish and Bloomfield, and the distance from Tignish to Bloomfield is 23 km, how far is it from Tignish to Elmsdale?
- A 40-foot length of rope is cut into three pieces. The first piece is one-quarter as long as the second piece of rope. The third piece is 5 feet less than the second piece of rope. How long is each piece of rope?
- Five points *A*, *B*, *C*, *D*, and *E* are on a number line. If $AE = 10, \; AB = CD = 2, \; BC = DE,$ and point *A* is at coordinate 7, determine the coordinates of points *B*, *C*, *D*, and *E*.

Section 1.3 – Distance and Midpoints (pp. 25-35)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

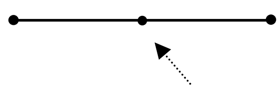
- **G1 (E F G)**

After this lesson, students will be expected to:

- find the distance between two points
- find the midpoint of a segment

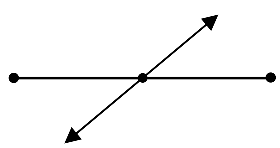
After this lesson, students should understand the following concepts:

- **distance between two points –** the length of the segment between two points
- **midpoint –** the point on a segment exactly halfway between the endpoints of the segment



midpoint

- **segment bisector –** a segment, line, or plane that intersects a segment at its midpoint



DISTANCE FORMULAS	
Number Line	$PQ = \lvert x_1 - x_2 \rvert$
Coordinate Plane	$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

MIDPOINT FORMULAS	
Number Line	$M = \dfrac{x_1 + x_2}{2}$
Coordinate Plane	$M\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Suggested Problems in *Geometry*:

- **pp. 30-35:** #1-67

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Derive the distance formula between two points on the coordinate plane from the distance formula between two points on a number line.
- Derive the midpoint formula between two points on the coordinate plane from the midpoint formula between two points on a number line.

Possible Assessment Strategies:

- Find the distance between each pair of points on a coordinate plane.
  a. $A(-2,0)$, $B(6,-1)$
  b. $C(1,1)$, $D(3,-5)$
- Find the coordinates of the midpoint of a line segment with the given endpoints on a coordinate plane.
  a. $A(7,2)$, $B(-1,0)$
  b. $C(-3,4)$, $D(-1,1)$
- Find the coordinates of the missing endpoint, Y, if M is the midpoint of $\overline{XY}$.
  a. $X(8,4)$, $M(10,-2)$
  b. $X(-9,3)$, $M(2,-4)$
- Jack and Jill are both in Calgary. The streets in Calgary are set up in a coordinate grid pattern. Jack's location is at $(2,3)$ and Jill's location is at $(8,9)$.
  a. Find the straight-line distance between them. Round off the answer to one decimal place. Assume that the units are in blocks.
  b. Find the coordinates of the point halfway between their locations.

Section 1.4 – Angle Measure (pp. 36-44)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

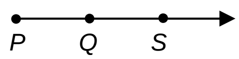
- **G1 (H I)**

After this lesson, students will be expected to:

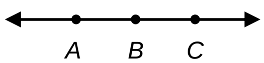
- measure and classify angles
- identify and use congruent angles and the bisector of an angle

After this lesson, students should understand the following concepts:

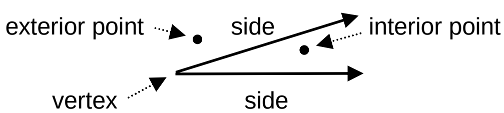
- **ray –** $\overrightarrow{PQ}$ is a ray if it is the set of points consisting of $\overline{PQ}$ and all points *S* for which *Q* is between *P* and *S*



- **opposite rays –** two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ such that *B* is between *A* and *C*



- **angle –** the intersection of two noncollinear rays at a common endpoint; the rays are called sides and the common endpoint is called the vertex



- **sides of an angle –** the rays of an angle
- **vertex of an angle –** the common endpoint of an angle
- **interior point –** a point is in the interior of an angle if it does not lie on the angle itself, and it lies on a segment with endpoints that are on the sides of the angle (see above)
- **exterior point –** a point is in the exterior of an angle if it is neither on the angle nor in the interior of the angle (see above)
- **degree –** a unit of measure used in measuring angles and arcs
- **right angle –** an angle with a measure of 90^0
- **acute angle –** an angle with a measure less than 90^0
- **obtuse angle –** an angle with a measure greater than 90^0 and less than 180^0
- **angle bisector –** a ray that divides an angle into two congruent angles

Suggested Problems in *Geometry*:

- **pp. 41-44:** #1-48

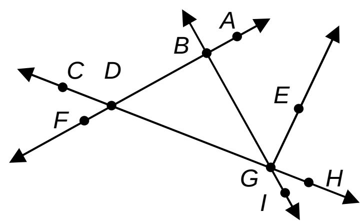
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Remind students that the length of the rays which form an angle do not affect the measure of the angle.
- Show students examples of acute, right, and obtuse angles to help them understand the differences among the different classifications of angles.

Possible Assessment Strategies:

- Refer to the figure below.



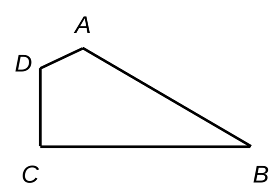
a. Name the vertex of $\angle CDF$.

b. Write another name for $\angle BDG$.

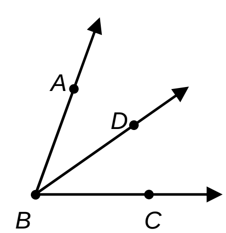
c. Name the sides of $\angle EGI$.

d. Name the ray opposite to $\overrightarrow{GB}$.

- Classify each angle of the following shape as **acute**, **right**, or **obtuse**.



- In the figure below, if $m\angle ABC = 70^0$, what must be the measure of $\angle ABD$ be in order for $\overrightarrow{BD}$ to be an angle bisector?



Section 1.5 – Angle Relationships (pp. 46-55)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

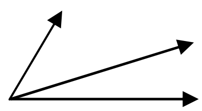
- **G1 (J K)**

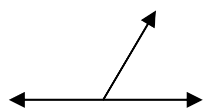
After this lesson, students will be expected to:

- identify and use special pairs of angles
- identify perpendicular lines

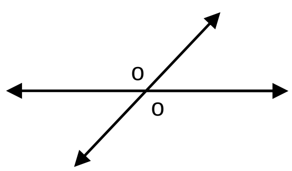
After this lesson, students should understand the following concepts:

- **adjacent angles –** two angles that lie in the same plane, have a common vertex and a common side, but no common interior points (see below left)

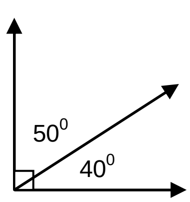


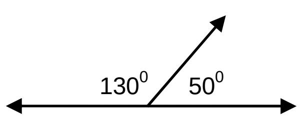


- **linear pair –** a pair of adjacent angles whose non-common sides are opposite rays (see above right)
- **vertical angles –** two nonadjacent angles formed by two intersecting lines

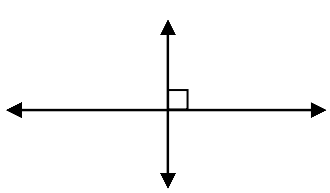


- **complementary angles –** two angles with measures that have a sum of 90^0 (see below left)





- **supplementary angles –** two angles with measures that have a sum of 180^0 (see above right)
- **perpendicular lines –** lines that form right angles



Suggested Problems in *Geometry*:

- **pp. 50-54:** #1-48

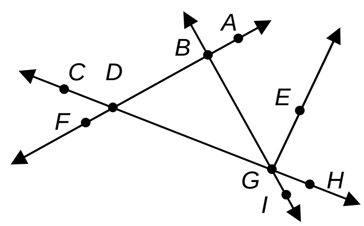
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Ensure that students are able to recognize pairs of angles, such as adjacent angles, linear pairs of angles, vertical angles, complementary angles, and supplementary angles.

Possible Assessment Strategies:

- Refer to the figure below.



a. Name an angle supplementary to $\angle CDF$.

b. Name a pair of vertical angles with vertex *G.*

c. If $m\angle EGH = 6x - 30$, find the value of x so that $\overline{EG} \perp \overline{GH}$.

d. If $m\angle FDG = 132^0$, what is the measure of $\angle BDG$?

- At 9:00 A.M., the small hand of a clock is in a horizontal position. Between 9:00 A.M. and 10:00 A.M., the small hand sweeps through an angle of 30^0. Through how many more degrees will the small hand have to turn so that it is vertical?

Section 1.6 – Two-Dimensional Figures (pp. 56-66)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

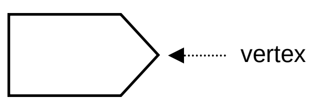
- **G1 (L M)**

After this lesson, students will be expected to:

- identify and name polygons
- find the perimeter, circumference, and/or area of two-dimensional figures

After this lesson, students should understand the following concepts:

- **polygon –** a two-dimensional closed figure formed by a finite number of segments called sides



- **vertex of a polygon –** the vertex of each angle of a polygon (see above)
- **concave polygon –** a polygon for which there is a line containing a side of the polygon that also contains a point in the interior of the polygon (see below right)

- **convex polygon –** a polygon for which there is no line that contains both a side of the polygon and a point in the interior of the polygon (see above left)
- ***n*-gon –** a polygon with *n* sides
- **equilateral polygon –** a polygon with all sides congruent
- **equiangular polygon –** a polygon with all angles congruent
- **regular polygon –** a convex polygon in which all of the sides are congruent and all of the angles are congruent
- **perimeter –** the sum of the lengths of the sides of a polygon
- **circumference –** the distance around a circle
- **area –** the number of square units needed to cover a surface

Suggested Problems in *Geometry*:

- **pp. 61-64:** #1-43

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

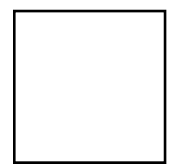
Possible Instructional Strategies:

- Ensure that students are familiar with the perimeter, circumference and area formulas for triangles, squares, rectangles, and circles.

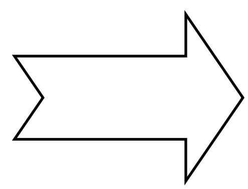
Possible Assessment Strategies:

- Name each polygon by its number of sides. Then classify it as **convex** or **concave**, and **regular** or **irregular**.

  a.



  b.



- Find the perimeter of quadrilateral *ABCD* with vertices at $A(-2,-1)$, $B(3,5)$, $C(5,1)$, and $D(4,-3)$. Round off the answer to one decimal place.
- A farmer has 100 m of fencing available to enclose an area for his cattle. He determines that his two best options to maximize the area are to either enclose a square or a circular field. Using all of the fencing that he has available to him, which option will enclose the greater area? Explain.

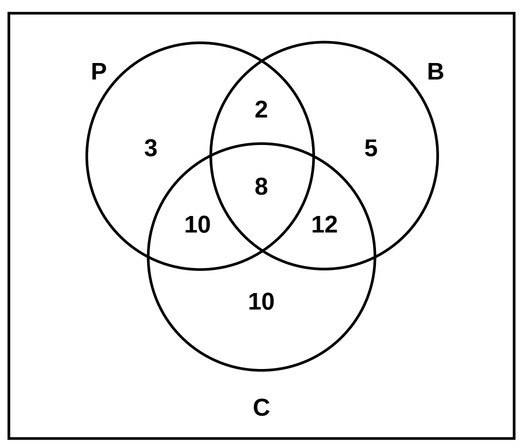
UNIT 2
REASONING AND PROOF

SUGGESTED TIME
12 classes

Section 2.1 – Inductive Reasoning and Conjecture (pp. 89-96)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **LR1 (A B)** **After this lesson, students will be expected to:** • make conjectures based on inductive reasoning • find counterexamples **After this lesson, students should understand the following concepts:** • **inductive reasoning –** reasoning that uses a number of specific examples to arrive at a plausible generalization or prediction; conclusions arrived at by inductive reasoning lack the logical certainty of those arrived at by deductive reasoning • **conjecture –** an educated guess based on known information • **counterexample –** an example used to show that a given statement is not always true **Suggested Problems in *Geometry*:** • **pp. 92-96:** #1-49	**Possible Instructional Strategies:** • Remind students that when using inductive reasoning, a conjecture can only be disproven, by means of a counterexample. Trying to prove a conjecture by showing that a number of examples are true does not suffice as a proof. **Possible Assessment Strategies:** • Determine whether each conjecture is **true** or **false**. If it is false, give a counterexample. a. All multiples of 8 are divisible by 4. b. The sum of two perfect squares is an even number. c. For any real number, $\sqrt{n^2} = n$. d. In a quadrilateral, if three angles are right angles, then the quadrilateral is a rectangle. e. If line segments $\overline{AB}$ and $\overline{BC}$ have the same length, then B is the midpoint of line segment $\overline{AB}$. • Give a counterexample to disprove each of the following conjectures. a. All provinces of Canada share a border with the United States. b. All birds can fly. c. Even numbers are divisible by 4. d. For all real numbers n, n^2 is positive.

Section 2.2 – Logic (pp. 97-104)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **LR1 (C)** • **LR2 (A B C)** **After this lesson, students will be expected to:** • determine truth values of negations, conjunctions, and disjunctions, and represent them using Venn diagrams • find counterexamples **After this lesson, students should understand the following concepts:** • **statement –** any sentence that is either true or false, but not both • **truth value –** the truth or falsity of a statement • **negation –** if a statement is represented by *p*, then "not *p*" is the negation of the statement • **compound statement –** a statement formed by joining two or more statements • **conjunction –** a compound statement formed by joining two or more statements with the word *and* • **disjunction –** a compound statement formed by joining two or more statements with the word *or* • **truth table –** a table used as a convenient method for organizing the truth values of statements **Suggested Problems in *Geometry*:** • **pp. 101-104:** #1-40	**Possible Instructional Strategies:** • Remind students that a statement can be true or false, but not both at the same time. • Remind students that in order for the conjunction $p \wedge q$ to be true, both *p* and *q* must be true. • Remind students that in order for the disjunction $p \vee q$ to be true, either *p* must be true, *q* must be true, or both *p* and *q* must be true. **Possible Assessment Strategies:** • Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value. Explain. *p: The capital of Prince Edward Island is Charlottetown.* *q: Prince Edward Island is the smallest province in Canada.* *r: There are two cities in Prince Edward Island.* a. $\sim p \vee r$ b. $p \wedge \sim q$ c. $\sim q \vee \sim r$ • The Venn diagram shows the results of a survey of 50 grade twelve science students to determine the courses in which they have enrolled. In the diagram, B represents biology, C represents chemistry, and P represents physics.  a. How many grade twelve science students are only taking biology? b. How many grade twelve science students are only taking physics and chemistry? c. How many grade twelve science students are not taking physics?

Section 2.3 – Conditional Statements (pp. 105-114)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **LR2 (D E F G)** **After this lesson, students will be expected to:** • analyse statements in *if-then* form • write the converse, inverse, and contrapositive of *if-then* statements **After this lesson, students should understand the following concepts:** • **conditional statement –** a statement that can be written in *if-then* form • ***if-then* statement –** a compound statement of the form "if *p*, then *q*," where *p* and *q* are statements • **hypothesis –** in a conditional statement, the statement that immediately follows the word *if* • **conclusion –** in a conditional statement, the statement that immediately follows the word *then* • **related conditionals –** statements that are based on a given conditional statement • **converse –** the statement formed by exchanging the hypothesis and conclusion of a conditional statement • **inverse –** the statement formed by negating both the hypothesis and conclusion of a conditional statement • **contrapositive –** the statement formed by negating both the hypothesis and conclusion of the converse of a conditional statement • **logically equivalent –** statements that have the same truth values • **biconditional –** the conjunction of a conditional statement and its converse **Suggested Problems in *Geometry*:** • **pp. 109-113:** #1-62	**Possible Instructional Strategies:** • Ensure that students are able to convert statements to "if-then" form. • Ensure that students understand the relationships between a statement and its converse, inverse, and contrapositive. **Possible Assessment Strategies:** • Determine the truth value of each conditional statement. If **true**, explain your reasoning. If **false**, give a counterexample. a. If two opposite sides of a quadrilateral are parallel, and one interior angle is 90^0, then the quadrilateral is a rectangle. b. If you determine the reciprocal of a real number, then the result is a real number. c. If a point lies in the third quadrant, then its *y*-coordinate is negative. d. If a triangle is a right triangle, then it does not contain an obtuse angle. • Write the converse, inverse, and contrapositive of each of the following true conditional statements. Then determine whether each related conditional is **true** or **false**. If a statement is false, find a counterexample. a. If it is raining, then it is cloudy. b. If a real number is positive, then its absolute value is positive. c. If $b > 1$, then $b^2 > 1$. d. If a right triangle is isosceles, then it has two angles measuring 45^0.

Section 2.4 – Deductive Reasoning (pp. 115-123)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **LR3 (A B C D)** **After this lesson, students will be expected to:** • use the Law of Detachment • use the Law of Syllogism **After this lesson, students should understand the following concepts:** • **deductive reasoning –** a system of reasoning that uses facts, rules, definitions, or properties to reach logical conclusions • **valid –** logically correct • **Law of Detachment –** if $p \rightarrow q$ is a true conditional, and p is true, then q is also true • **Law of Syllogism –** if $p \rightarrow q$ and $q \rightarrow r$ are true conditionals, then $p \rightarrow r$ is also true **Suggested Problems in *Geometry*:** • **pp. 119-123:** #1-41	**Possible Instructional Strategies:** • Ensure that students understand the role of the Law of Detachment and the Law of Syllogism when using deductive reasoning. **Possible Assessment Strategies:** • Draw a valid conclusion from the given statements, if possible. Then state whether your conclusion was drawn from the Law of Detachment or the Law of Syllogism. If no valid conclusion can be drawn, state **no valid conclusion**, and explain your reasoning. a. Given: If a polygon has five sides, then it is a pentagon. Polygon *ABCDE* has five sides. b. Given: If Johnny studies for 3 hours, then he will pass his test. Johnny passes his test. c. Given: If at least 15 cm of snow is forecast, then a heavy snowfall warning is issued. If a heavy snowfall warning is issued, then school will be cancelled. Twenty centimetres of snow are forecast. • Determine whether the stated conclusion is valid based on the given information. If not, write **invalid**. Explain. Given: In a particular high school, if a student's average is 80% or higher and no mark is below 75%, then that student will be on the honour roll. Janie had an average of 85%. Conclusion: Janie was on the honour roll. • Determine the statement that logically follows from the given statements. If you buy a special at the cafeteria, then you will get a free milk. John bought a special at the cafeteria.

Section 2.5 – Postulates and Paragraph Proof (pp. 124-132)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **LR4 (A)**

After this lesson, students will be expected to:

- identify and use basic postulates about points, lines and planes
- write paragraph proofs

After this lesson, students should understand the following concepts:

- **postulate –** a statement that describes a fundamental relationship between the basic terms of geometry; postulates are accepted as true without proof
- **axiom –** a statement that is accepted as true
- **proof –** a logical argument in which each statement you make is supported by a statement that is accepted to be true
- **theorem –** a statement or conjecture that can be proven true by undefined terms, definitions, and/or postulates
- **deductive argument –** a proof formed by a group of algebraic steps used to solve a problem
- **paragraph proof –** an informal proof written in the form of a paragraph that explains why a conjecture for a given situation is true

POSTULATES & THEOREMS	
P2.1	Through any two points, there is exactly one line.
P2.2	Through any three noncollinear points, there is exactly one plane.
P2.3	A line contains at least two points.
P2.4	A plane contains at least three noncollinear points.
P2.5	If two points lie in a plane, then the entire line containing that point lies in that plane.
P2.6	If two lines intersect, then their intersection is exactly one point.
P2.7	If two planes intersect, then their intersection is a line.
T2.1	**Midpoint Theorem** If M is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$.

Suggested Problems in *Geometry*:

- **pp. 128-132:** #1-44

Possible Instructional Strategies:

- Ensure students understand that, in geometry, postulates are accepted to be true without proof.
- Remind students that, when trying to prove a statement, the conclusion is never to be used as a statement in the proof.

Possible Assessment Strategies:

- Determine whether each statement is **always true**, **sometimes true**, or **never true**. Explain.
  a. Two lines intersect at a point.
  b. Two points are contained in more than one line.
  c. If two angles are supplementary, then they form a linear pair.
- If A, B and C are collinear and $\overline{AB} \cong \overline{BC}$, write a paragraph proof to show that B is the midpoint of $\overline{BC}$.

Section 2.6 – Algebraic Proof (pp. 134-141)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **LR4 (B C)**

After this lesson, students will be expected to:

- use algebra to write two-column proofs
- use properties of equality to write geometric proofs

After this lesson, students should understand the following concepts:

- **algebraic proof –** a proof that is made up of a series of algebraic statements; the properties of equality provide justification for many statements in algebraic proofs
- **two-column proof –** a formal proof that contains statements and reasons organized in two columns; each step is called a statement, and the properties that justify each step are called the reasons
- **formal proof –** a two-column proof containing statements and reasons

PROPERTIES OF REAL NUMBERS	
Addition Property of Equality	If $a=b$, then $a+c=b+c$.
Subtraction Property of Equality	If $a=b$, then $a-c=b-c$.
Multiplication Property of Equality	If $a=b$, then $a \bullet c=b \bullet c$.
Division Property of Equality	If $a=b$, then $\frac{a}{c}=\frac{b}{c}$, for $c \neq 0$.
Reflexive Property of Equality	$a=a$
Symmetric Property of Equality	If $a=b$, then $b=a$.
Transitive Property of Equality	If $a=b$ and $b=c$, then $a=c$.
Substitution Property of Equality	If $a=b$, then a may be replaced by b in any equation or expression.
Distributive Property	$a(b+c)=ab+ac$

Suggested Problems in *Geometry*:

- **pp. 137-141:** #1-35

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Ensure that students understand the properties of real numbers.

Possible Assessment Strategies:

- State the property that justifies each statement.
  a. If $5(x-2)=20$, then $20=5(x-2)$.
  b. If $6x+9=21$, then $6x=12$.
  c. $-2(x-5)=-2x+10$
  d. $9x+7=9x+7$
  e. If $-4=8(3-2x)$, and $8(3-2x)=2y$, then $-4=2y$.
- Complete the following proof.

  Given: $9(x-1)=27$

  Prove: $x=4$
- Write a two-column proof to show that if $AB=AC$, $AB=8x-7$, and $AC=7x-5$, then $x=2$.
- In the 2010-2011 NHL season, Sidney Crosby scored as many goals as Alex Ovechkin, and Alex Ovechkin scored as many goals as Jonathan Toewes. What property would show that Sidney Crosby and Jonathan Toewes scored the same number of goals?

Section 2.7 – Proving Segment Relationships (pp. 142-148)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **LR4 (D E)**

After this lesson, students will be expected to:

- write proofs involving segment addition
- write proofs involving segment congruence

POSTULATES & THEOREMS

P2.8 Ruler Postulate

The points on any line or line segment can be put into one-to-one correspondence with real numbers.

P2.9 Segment Addition Postulate

If A, B, and C are collinear, then point B is between A and C if and only if $AB + BC = AC$.

T2.2 Reflexive Property of Congruence

$\overline{AB} \cong \overline{AB}$

Symmetric Property of Congruence

If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive Property of Congruence

If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Suggested Problems in *Geometry*:

- **pp. 145-148:** #1-16

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Make the distinction between congruence and equality. Geometric figures are congruent, and their measures are equal.

Possible Assessment Strategies:

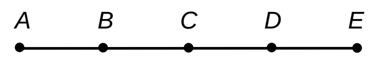
- Prove each of the following.

a. Given: B is the midpoint of $\overline{AC}$

C is the midpoint of $\overline{BD}$

D is the midpoint of $\overline{CE}$

Prove: $\overline{AB} \cong \overline{DE}$



b. Given: $\overline{AC} \cong \overline{BD}$

Prove: $\overline{AB} \cong \overline{CD}$

Section 2.8 – Proving Angle Relationships (pp. 149-157)

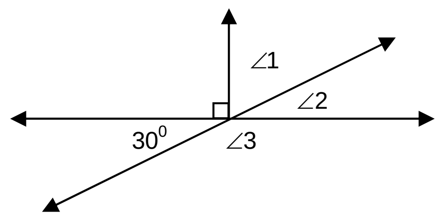
ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **LR4 (D E)** **After this lesson, students will be expected to:** • write proofs involving supplementary and complementary angles • write proofs involving congruent and right triangles	**Possible Instructional Strategies:** • Make the distinction between congruence and equality. Geometric figures are congruent, and their measures are equal. **Possible Assessment Strategies:** • Find the measure of each angle.

POSTULATES & THEOREMS

P2.10	**Protractor Postulate** Given an angle, the measure can be put into one-to-one correspondence with real numbers between 0 and 180.
P2.11	**Angle Addition Postulate** D is in the interior of $\angle ABC$ if and only if $m\angle ABD + m\angle DBC = m\angle ABC$.
T2.3	**Supplement Theorem** If two angles form a linear pair, then they are supplementary angles.
T2.4	**Complement Theorem** If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles.
T2.5	**Reflexive Property of Congruence** $\angle 1 \cong \angle 1$ **Symmetric Property of Congruence** If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$. **Transitive Property of Congruence** If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.
T2.6	**Congruent Supplements Theorem** Angles supplementary to the same angle or to congruent angles are congruent.
T2.7	**Congruent Complements Theorem** Angles complementary to the same angle or to congruent angles are congruent.
T2.8	**Vertical Angles Theorem** If two angles are vertical angles, then they are congruent.
T2.9	Perpendicular lines intersect to form four right angles.
T2.10	All right angles are congruent.
T2.11	Perpendicular lines form congruent adjacent angles.
T2.12	If two angles are congruent and supplementary, then each angle is a right angle.
T2.13	If two congruent angles form a linear pair, then they are right angles.

Suggested Problems in *Geometry*:

- **pp. 154-157:** #1-30



a. $\angle 1$

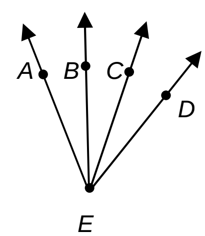
b. $\angle 2$

c. $\angle 3$

- Prove each of the following.

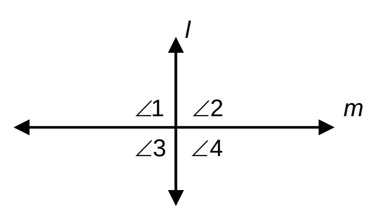
a. Given: $\angle AEC \cong \angle BED$

Prove: $\angle AEB \cong \angle CED$



b. Given: $\angle 2$ is a right angle

Prove: $\angle 3$ is a right angle



UNIT 3
PARALLEL AND PERPENDICULAR LINES

SUGGESTED TIME
5 classes

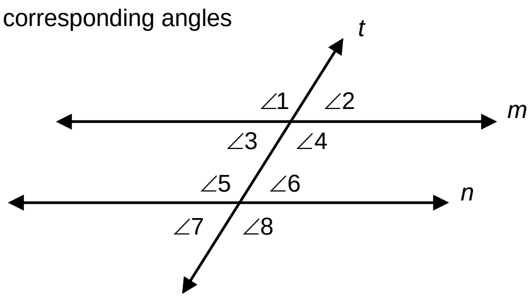
Section 3.1 – Parallel Lines and Transversals (pp. 171-176)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:
- **G2 (A B)**

After this lesson, students will be expected to:
- identify the relationships between two lines or two planes
- name angle pairs formed by parallel lines and transversals

After this lesson, students should understand the following concepts:
- **parallel lines –** coplanar lines that do not intersect; in the diagram, lines *m* and *n* are parallel lines
- **skew lines –** lines that do not intersect and are not coplanar
- **parallel planes –** planes that do not intersect
- **transversal –** a line that intersects two or more lines in a plane at different points; in the diagram, line *t* is a transversal
- **interior angles –** angles that lie between two parallel lines that intersect a transversal; in the diagram, $\angle 3$, $\angle 4$, $\angle 5$, and $\angle 6$ are interior angles
- **exterior angles –** angles that do not lie between two parallel lines that intersect a transversal; in the diagram, $\angle 1$, $\angle 2$, $\angle 7$, and $\angle 8$ are exterior angles
- **consecutive interior angles –** interior angles that lie on the same side of a transversal; in the diagram, $\angle 3$ and $\angle 5$, and $\angle 4$ and $\angle 6$ are consecutive interior angles
- **alternate interior angles –** nonadjacent interior angles that lie on opposite sides of a transversal; in the diagram, $\angle 4$ and $\angle 5$, and $\angle 3$ and $\angle 6$ are alternate interior angles
- **alternate exterior angles –** nonadjacent exterior angles that lie on opposite sides of a transversal; in the diagram, $\angle 1$ and $\angle 8$, and $\angle 2$ and $\angle 7$ are alternate exterior angles
- **corresponding angles –** angles that lie on the same side of a transversal and on the same side of the lines that cross it; in the diagram, $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, and $\angle 4$ and $\angle 8$ are corresponding angles

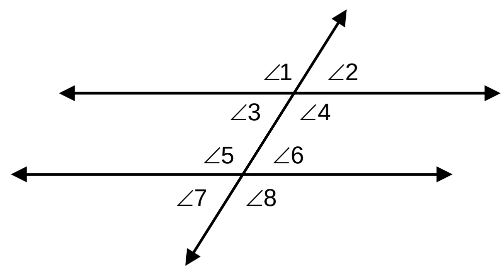


Suggested Problems in *Geometry*:
- **pp. 173-176:** #1-45

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:
- Review with students the various pairs of angles such as vertically opposite, corresponding, interior, and exterior angles.

Possible Assessment Strategies:
- Classify the relationship between each pair of angles as **alternate interior**, **alternate exterior**, **corresponding**, or **consecutive interior** angles.



a. $\angle 4$ and $\angle 8$

b. $\angle 2$ and $\angle 7$

c. $\angle 4$ and $\angle 5$

d. $\angle 3$ and $\angle 5$

Section 3.2 – Angles and Parallel Lines (pp. 177-184)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **G2 (C D)**

After this lesson, students will be expected to:

- use theorems to determine the relationships between specific pairs of angles
- use algebra to find angle measurements

POSTULATES & THEOREMS	
P3.1	**Corresponding Angles Postulate** If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.
T3.1	**Alternate Interior Angles Theorem** If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.
T3.2	**Consecutive Interior Angles Theorem** If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.
T3.3	**Alternate Exterior Angles Theorem** If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.
T3.4	**Perpendicular Transversal Theorem** In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.

Suggested Problems in *Geometry*:

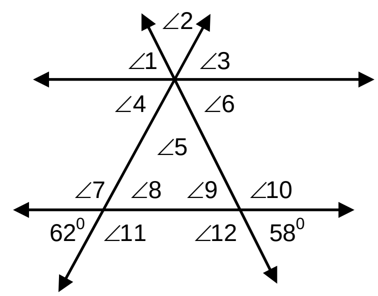
- **pp. 181-184:** #1-41

Possible Instructional Strategies:

- Ensure that students are familiar with the theorems regarding parallel lines and a transversal.

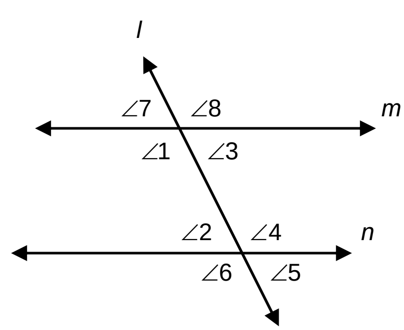
Possible Assessment Strategies:

- Find the measure of each of the indicated angles.



a. $\angle 1$
b. $\angle 2$
c. $\angle 3$
d. $\angle 4$
e. $\angle 5$
f. $\angle 6$
g. $\angle 7$
h. $\angle 8$
i. $\angle 9$
j. $\angle 10$
k. $\angle 11$
l. $\angle 12$

- Given $m \parallel n$, and l is a transversal, show that $\angle 1$ and $\angle 2$ are supplementary, and $\angle 3$ and $\angle 4$ are supplementary.



Section 3.5 – Proving Lines Parallel (pp. 205-212)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **G2 (E)**

After this lesson, students will be expected to:

- recognize angle pairs that occur with parallel lines
- prove that two lines are parallel using angle relationships

POSTULATES & THEOREMS	
P3.4	**Converse of Corresponding Angles Postulate** If two lines are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.
P3.5	**Parallel Postulate** If given a line and a point not on the line, then there exists exactly one line through the point that is parallel to the given line.
T3.5	**Alternate Exterior Angles Converse** If two lines in a plane are cut by a transversal so that a pair of alternate exterior angles is congruent, then the two lines are parallel.
T3.6	**Consecutive Interior Angles Converse** If two lines in a plane are cut by a transversal so that a pair of consecutive interior angles is supplementary, then the lines are parallel.
T3.7	**Alternate Interior Angles Converse** If two lines in a plane are cut by a transversal so that a pair of alternate interior angles is congruent, then the lines are parallel.
T3.8	**Perpendicular Transversal Converse** In a plane, if two lines are perpendicular to the same line, then they are parallel.

Suggested Problems in *Geometry*:

- **pp. 208-212:** #1-36

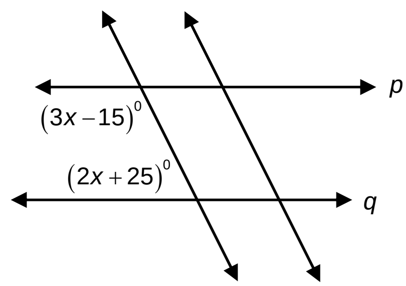
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Ensure that students understand that the converse of the statements involving parallel lines and a transversal are also true.

Possible Assessment Strategies:

- Find x so that $p \parallel q$.

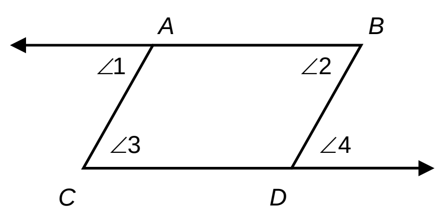


- Prove each of the following.

  a. Given: $\overline{AC} \parallel \overline{BD}$

  $\angle 2 \cong \angle 3$

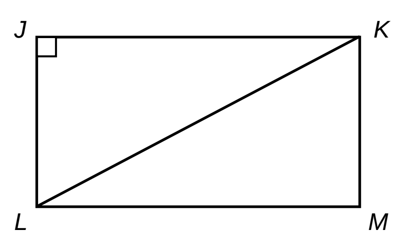
  Prove: $\overline{AB} \parallel \overline{CD}$



  b. Given: $\angle JLK \cong \angle LKM$

  $\overline{JK} \perp \overline{JL}$

  Prove: $\overline{JK} \perp \overline{KM}$



UNIT 4
CONGRUENT TRIANGLES

SUGGESTED TIME

12 classes

Section 4.1 – Classifying Triangles (pp. 235-242)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

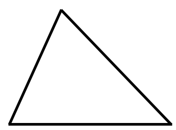
- **G3 (A)**

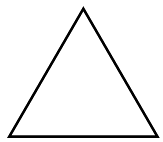
After this lesson, students will be expected to:

- identify and classify triangles by angle measures
- identify and classify triangles by side measures

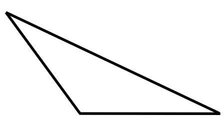
After this lesson, students should understand the following concepts:

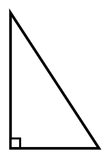
- **acute triangle –** a triangle in which all of the angles are acute angles (see below left)



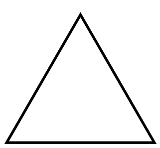


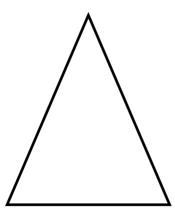
- **equiangular triangle –** a triangle with all angles congruent (see above right)
- **obtuse triangle –** a triangle with an obtuse angle (see below left)



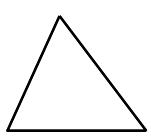


- **right triangle –** a triangle with a right angle; the side opposite the right angle is called the hypotenuse; the two other sides are called the legs (see above right)
- **equilateral triangle –** a triangle with all sides congruent (see below left)





- **isosceles triangle –** a triangle with at least two sides congruent; the congruent sides are called legs; the angles opposite the legs are base angles; the angle formed by the two legs is called the vertex angle; the side opposite the vertex is called the base (see above right)
- **scalene triangle –** a triangle with no two sides congruent



Suggested Problems in *Geometry*:

- **pp. 238-242:** #1-55

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

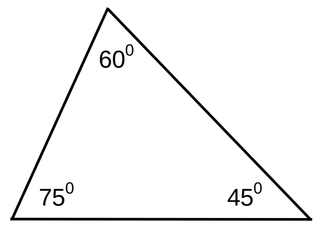
Possible Instructional Strategies:

- Ensure that students understand the two types of triangle classification, by angle measures and by side measures.

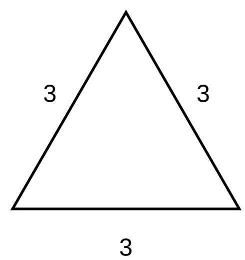
Possible Assessment Strategies:

- Classify each of the following triangles by angle measures and by side measures.

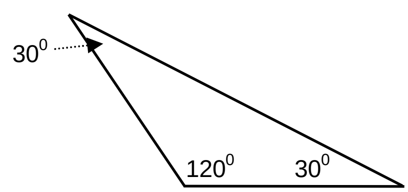
a.



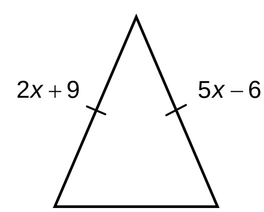
b.



c.



- Determine the value of *x*.



Section 4.2 – Angles of Triangles (pp. 243-252)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

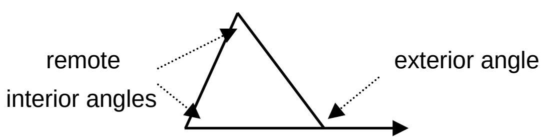
- **G3 (B C D)**

After this lesson, students will be expected to:

- apply the Triangle Angle-Sum Theorem
- apply the Exterior Angle Theorem

After this lesson, students should understand the following concepts:

- **auxiliary line –** an extra line or segment drawn in a figure to help complete a proof
- **exterior angle –** an angle formed by one side of a triangle and the extension of another side



- **remote interior angles –** the angles of a triangle that are not adjacent to the given exterior angle
- **corollary –** a statement that can be easily proved using a theorem is called a corollary of that theorem

THEOREMS & COROLLARIES	
T4.1	**Triangle Angle-Sum Theorem** The sum of the measures of the three angles of a triangle is 180^0.
C1	The acute angles of a right triangle are complementary.
C2	The can be at most one right or obtuse angle in a triangle.
T4.2	**Exterior Angle Theorem** The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

Suggested Problems in *Geometry*:

- **pp. 248-252:** #1-45

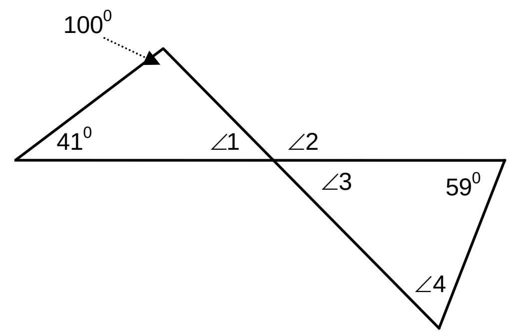
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Ensure that students understand the Triangle Angle-Sum Theorem and the Exterior Angle Theorem.

Possible Assessment Strategies:

- Find the measure of each indicated angle.



a. $\angle 1$

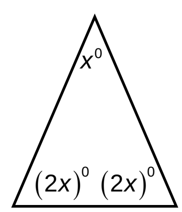
b. $\angle 2$

c. $\angle 3$

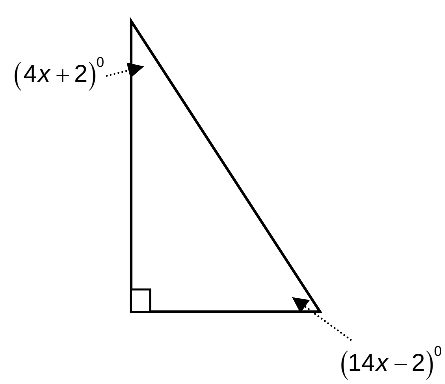
d. $\angle 4$

- Find the value of x. Then find the measure of each angle.

a.



b.



Section 4.3 – Congruent Triangles (pp. 253-261)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **G4 (A B C)**

After this lesson, students will be expected to:

- name and use corresponding parts of congruent polygons
- prove triangles congruent using the definition of congruence

After this lesson, students should understand the following concepts:

- **congruent –** having the same measure
- **congruent polygons –** polygons in which all matching parts are congruent
- **corresponding parts –** matching parts of congruent polygons

THEOREMS	
T4.3	**Third Angles Theorem** If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.
T4.4	**Reflexive Property of Triangle Congruence** $\triangle ABC \cong \triangle ABC$ **Symmetric Property of Triangle Congruence** If $\triangle ABC \cong \triangle EFG$, then $\triangle EFG \cong \triangle ABC$. **Transitive Property of Triangle Congruence** If $\triangle ABC \cong \triangle EFG$ and $\triangle EFG \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.

Suggested Problems in *Geometry*:

- **pp. 256-261:** #1-35

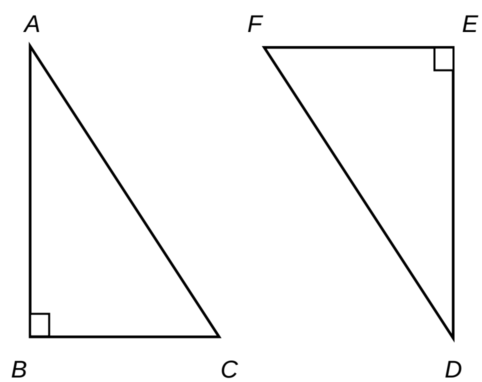
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Provide students examples of triangles with different orientations to help them recognize when two triangles are congruent.
- Ensure that students label congruent triangles with corresponding vertices in the same relative position.

Possible Assessment Strategies:

- Identify all corresponding parts of the following congruent triangles. Then write a congruence statement.

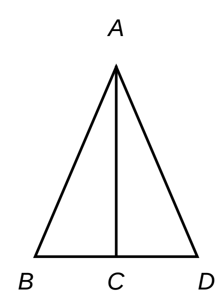


- Prove.

Given: $\angle B \cong \angle D$

$\overline{AC} \perp \overline{BD}$

Prove: $\angle BAC \cong \angle DAC$



Section 4.4 – Proving Triangles Congruent – SSS, SAS (pp. 262-271)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **G4 (D E)**

After this lesson, students will be expected to:

- use the SSS postulate to test for triangle congruence
- use the SAS postulate to test for triangle congruence

After this lesson, students should understand the following concept:

- **included angle –** in a triangle, the angle formed by two sides is the included angle for those two sides

POSTULATES
P4.1 Side-Side-Side (SSS) Congruence If three sides of one triangle are congruent to three sides of a second triangle, then the triangles are congruent.
P4.2 Side-Angle-Side (SAS) Congruence If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.

Suggested Problems in *Geometry*:

- **pp. 266-270:** #1-27

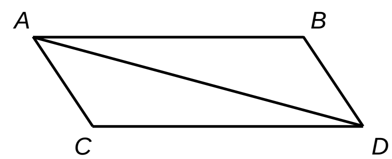
Possible Instructional Strategies:

- Ensure that students label congruent triangles with corresponding vertices in the same relative position.
- Ensure that students are able to recognize the SSS and the SAS cases for congruent triangles.

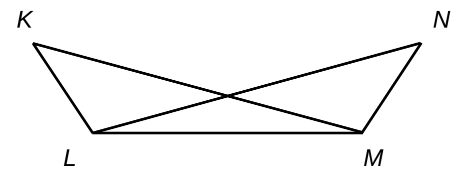
Possible Assessment Strategies:

- Prove that $\Delta ABC \cong \Delta ZXY$ if the coordinates of ΔABC are $A(0,0)$, $B(3,0)$, and $C(0,4)$, and the coordinates of ΔZXY are $Z(8,6)$, $X(5,6)$ and $Y(8,2)$.
- Prove each of the following.

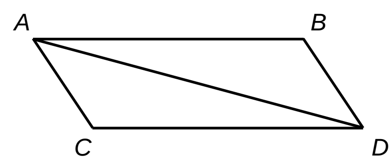
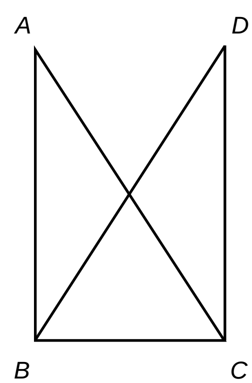
  a. Given: $\overline{AB} \cong \overline{DC}$
  $\overline{BD} \cong \overline{CA}$
  Prove: $\Delta ABD \cong \Delta DCA$



  b. Given: $\overline{KL} \cong \overline{NM}$
  $\angle KLM \cong \angle NML$
  Prove: $\Delta KLM \cong \Delta NML$



Section 4.5 – Proving Triangles Congruent – ASA, AAS (pp. 273-282)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **G4 (D E)** **After this lesson, students will be expected to:** • use the ASA postulate to test for triangle congruence • use the AAS postulate to test for triangle congruence **After this lesson, students should understand the following concept:** • **included side –** the side of a triangle that is a side of each of two angles	**Possible Instructional Strategies:** • Ensure that students label congruent triangles with corresponding vertices in the same relative position. • Ensure that students are able to recognize the ASA and the AAS congruent triangles cases. **Possible Assessment Strategies:** • Prove each of the following. a. Given: $\angle BAD \cong \angle CDA$ $\angle BDA \cong \angle CAD$ Prove: $\Delta ABD \cong \Delta DCA$  b. Given: $\angle A \cong \angle D$ $\overline{AB} \perp \overline{BC}$ $\overline{AB} \parallel \overline{DC}$ Prove: $\Delta ABC \cong \Delta DCB$ 

POSTULATES & THEOREMS

P4.3	**Angle-Side-Angle (ASA) Congruence** If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.
T4.5	**Angle-Angle-Side (AAS) Congruence** If two angles and the non-included side of one triangle are congruent to two angles and a side of a second triangle, then the two triangles are congruent.

Suggested Problems in *Geometry*:
- **pp. 276-280:** #1-21

Section 4.6 – Isosceles and Equilateral Triangles (pp. 283-291)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

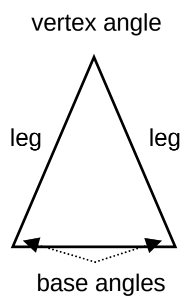
- **G4 (F G)**

After this lesson, students will be expected to:

- use properties of isosceles triangles
- use properties of equilateral triangles

After this lesson, students should understand the following concepts:

- **legs of an isosceles triangle –** the two congruent sides of an isosceles triangle
- **vertex angle of an isosceles triangle –** the angle formed by the two legs of an isosceles triangle
- **base angles of an isosceles triangle –** the angles opposite the legs of an isosceles triangle



THEOREMS & COROLLARIES	
T4.10	**Isosceles Triangle Theorem** If two sides of a triangle are congruent, then the angles opposite those sides are congruent.
C1	A triangle is equilateral if and only if it is equiangular.
C2	Each angle of an equilateral triangle measures 60^0.
T4.11	**Converse of Isosceles Triangle Theorem** If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Suggested Problems in *Geometry*:

- **pp. 287-291:** #1-44

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

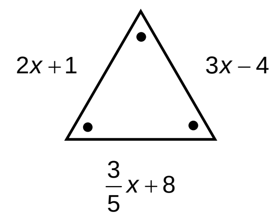
Possible Instructional Strategies:

- Students should be able to recognize the relationship between sides and angles in isosceles and equilateral triangles.

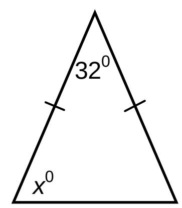
Possible Assessment Strategies:

- Find the value of each variable.

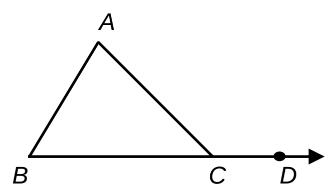
  a.



  b.



- In the figure, $\angle ACD = 110^0$, and $\angle B = 40^0$. Prove that ΔABC is isosceles.



Section 5.4 – Indirect Proof (pp. 351-358)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **LR5 (A B)** **After this lesson, students will be expected to:** • write indirect algebraic proofs • write indirect geometric proofs **After this lesson, students should understand the following concepts:** • **indirect reasoning –** reasoning that assumes that the conclusion is false and then shows that this assumption leads to a contradiction of the hypothesis by using a postulate, theorem or corollary; then, since the assumption has been proved false, the conclusion must be true • **indirect proof –** in an indirect proof, one assumes that the statement to be proved is false; one then uses logical reasoning to deduce that a statement contradicts a postulate, theorem, or one of the assumptions; once a contradiction is obtained, one concludes that the statement assumed false must in fact be true; also called *proof by contradiction* **Suggested Problems in *Geometry*:** • **pp. 354-358:** #1-39	**Possible Instructional Strategies:** • Review with the class what is meant by the converse, the inverse and the contrapositive of a statement. • Explain to the class that indirect proof is equivalent to proving the contrapositive of a statement. **Possible Assessment Strategies:** • State the assumption that you would make to start an indirect proof of each statement. a. In a group of 13 people, at least two were born in the same month. b. There is no greatest whole number. c. The sum of two odd integers is an even integer. d. There is only one perpendicular to a line that passes through a point not on the line. • Prove each of the following statements using an indirect proof. a. If x and y are positive integers such that their product is an odd number, then both x and y are odd numbers. b. If $\frac{1}{a} > 0$, then a must be positive. c. The sum of the measures of the acute angles in a right triangle is 90^0. d. A triangle cannot contain two obtuse angles.

UNIT 5
STATISTICS OF ONE VARIABLE

SUGGESTED TIME
11 classes

Section 2.1 – Data Analysis with Graphs (pp. 91-103)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **S1 (A)**
- **S2 (A B C D)**

After this lesson, students will be expected to:

- organize and summarize data from secondary sources, with or without technology

After this lesson, students should understand the following concepts:

- **statistics –** the gathering, organizing, analysis, and presentation of numerical information
- **raw data –** unprocessed information
- **variable –** a quantity that can have any of a set of values
- **continuous variable –** a variable that can have an infinite number of possible values in a given interval; a continuous function can be graphed as a smooth curve
- **discrete variable –** a variable that can only take on certain values within a given range
- **frequency table –** a table which states the frequency of each value of the variable in question
- **histogram –** a bar graph in which the area of the bars are proportional to the frequencies for various values of the variable
- **bar graph –** a chart or diagram that represents quantities with horizontal or vertical bars whose lengths are proportional to the quantities
- **frequency polygon –** a plot of frequencies versus variable values with the resulting points joined by line segments
- **cumulative-frequency graph –** a graph that shows the running total of the frequencies for values of the variable, starting from the lowest value; also called an *ogive*
- **class –** a set of values whose values lie within a given range or interval; also called an *interval*
- **range –** the difference between the highest and lowest values in a set of data
- **relative-frequency table –** a table which shows the frequencies of values or groups of values expressed as a fraction or percent of the whole data set
- **circle graph –** a graph that represents quantities with segments of a circle that are proportional to the quantities; also called a *pie chart*
- **pictograph –** a chart or diagram that represents quantities with symbols

Suggested Problems in *Mathematics of Data Management*:

- **pp. 101-103:** #1-13

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- It may be necessary to review with the class the various types of statistical graphs, as these were last discussed in grade nine. At the intermediate level, students would have learned about histograms, bar graphs, line graphs, circle graphs, pictographs, double bar graphs and double line graphs.

Possible Assessment Strategies:

- The examination scores for a math class are shown below.

69	79	94	70	57	63	86	100	86
64	76	59	45	58	72	80	83	58
92	49	68	98	50	86	66	87	62

  a. Determine the range for these data.

  b. Determine a reasonable interval size and the number of intervals for the data.

  c. Produce a frequency table for the grouped data.

  d. Produce a histogram and a frequency polygon for the grouped data.

- The speeds of 24 motorists ticketed for exceeding a 60 km/h limit are listed below.

74	70	66	76	70	66	67	74
69	71	66	73	84	72	74	79
75	95	79	74	72	79	68	72

  a. Construct a frequency table for the grouped data.

  b. Construct a histogram and a frequency polygon for the grouped data.

  c. How many of the motorists exceeded the speed limit by 15 km/h or less?

  d. how many of the motorists exceeded the speed limit by over 20 km/h?

Section 2.3 – Sampling Techniques (pp. 113-118)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **S1 (B C)** **After this lesson, students will be expected to:** • demonstrate an understanding of the purpose and the use of a variety of sampling techniques **After this lesson, students should understand the following concepts:** • **population –** all individuals that belong to a group being studied • **sample –** a group of items or people being selected from a population • **sampling frame –** the members of a population that actually have a chance of being selected for a sample • **simple random sample** – a sample in which every member of a population has an equal and independent chance of being selected • **systematic sample** – a sample selected by listing a population sequentially and choosing members at regular intervals • **strata** – groups whose members share common characteristics, which may differ from the rest of the population • **stratified sample** – a sample in which each stratum or group is represented in the same proportion as it appears in the population • **cluster sample** – a survey of selected groups within a population; this sampling technique can save time and expense, but may not give reliable results unless the clusters are representative of the population • **multi-stage sample –** a sample that uses several levels of random sampling • **voluntary-response sample –** a sample in which the participation is at the discretion or initiative of the respondent • **convenience sample –** a sample selected simply because it is easily accessible; such samples may not be random, so their results are not always reliable **Suggested Problems in *Mathematics of Data Management*:** • **pp. 117-118:** #1-9	**Possible Instructional Strategies:** • Some students may not understand the difference between a population and a sample. Reinforce the meaning of each term using a familiar example. Stress the importance of the context to help determine whether they are working with a population or a sample. **Possible Assessment Strategies:** • Classify the sampling method used in each of the following scenarios. a. An Internet website invites readers to leave their comments with respect to their political views. b. The Western School Board selects a sample of students such that the proportions from each school are the same as for all students last year. c. A student stops people in the school hallway to ask what they think of the school's cafeteria food. d. A pollster calls every 25th person in the phone book to ask them about their political preferences.

Section 2.5 – Measures of Central Tendency (pp. 125-135)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **S3 (A B C)** **After this lesson, students will be expected to:** • compute measures of central tendency, and demonstrate an understanding of the appropriate use of each measure • interpret measures of central tendency to describe characteristics of a data set • assess the validity of conclusions made using measures of central tendency **After this lesson, students should understand the following concepts:** • **measures of central tendency –** the values around which a set of data tends to cluster • **mean –** the sum of the values of a set of data divided by the number of values; $\bar{x} = \frac{\sum x_i}{n}$ • **median –** the middle value of a set of data ranked from highest to lowest; if there is an even number of data, the median in the midpoint between the two middle values • **mode –** the value in a distribution or a set of data that occurs most frequently • **outliers –** points in a set of data that are significantly far from the majority of the other data • **weighted mean –** a measure of central tendency that reflects the greater significance of certain data; $\bar{x} = \frac{\sum w_i x_i}{\sum w_i}$ **Suggested Problems in *Mathematics of Data Management*:** • **pp. 133-135:** #1-14	**Possible Instructional Strategies:** • It may be necessary to review with the class the various types of measures of central tendency, as these were last discussed in grade nine. **Possible Assessment Strategies:** • For the following set of quiz marks out of 10, calculate the mean, median, and mode. Round off to one decimal place, where necessary. 8 7 9 10 8 6 5 6 8 9 8 10 7 7 7 8 6 10 10 4 7 9 7 9 • Suppose the mean of a set of test scores is 89. One of the scores is erased from the report card, and the other four scores are 90, 95, 85 and 100. Determine the value of the missing test score. • When Mr. Brown gave a science test, he found the following: o The mean for the test was 72%. o The mode for the test was 65%. o The median for the test was 65%. When he gave back the test, it was determined that his answer key was wrong, and all of the students had a certain question correct which was valued at 5%. He was then compelled to increase all the marks by 5%. How did this affect the mean, median and mode? • In a particular math course, the final examination is worth 40%, the class tests are worth 50%, and the class assignments are worth 10%. Jack received the following marks: Final exam: 82% Class tests: 89%, 92%, 78%, 84%, 97% Class assignments: 97%, 99%, 92%, 91%, 100% Use a weighted mean to determine Jack's final mark for the course. Round off the answer to the nearest whole number.

Section 2.6 – Measures of Spread (pp. 136-150)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **S4 (A B C D)**

After this lesson, students will be expected to:

- compute measures of spread, and demonstrate an understanding of the appropriate use of each measure
- interpret measures of spread to describe characteristics of a data set

After this lesson, students should understand the following concepts:

- **measures of spread –** quantities that indicate how closely a set of data clusters around its central value; also called *measures of dispersion*
- **deviation –** the difference in value between a data item and the mean
- **standard deviation –** the square root of the mean of the squares of the deviations of a set of data; the standard deviation is given by the formulas

  $\sigma = \sqrt{\dfrac{\sum(x-\mu)^2}{N}}$ for a population, and

  $s = \sqrt{\dfrac{\sum\left(x-\bar{x}\right)^2}{n-1}}$ for a sample
- **variance –** the mean of the squares of the deviations for a set of data; variance is denoted by σ^2 for a population and s^2 for a sample
- **quartile –** one of a set of values that divide a set of data into four groups with equal numbers of data
- **interquartile range –** the range of the central half of a set of data when the data are arranged in numerical order
- **semi-interquartile range –** one half of the interquartile range
- **box-and-whisker plot –** a graph that summarizes a set of data by representing the first quartile, the median, and the third quartile with a box, and the lowest and highest data with the ends of lines extending from the box
- **modified box-and-whisker plot –** a box-and-whisker plot that shows outliers as separate points instead of including them in the whiskers
- ***z*-score –** the number of standard deviations from a data item to the mean; the *z*-score of a data item is given by the formula $z = \dfrac{x-\bar{x}}{s}$

Suggested Problems in *Mathematics of Data Management*:

- **pp. 148-150:** #1-15

Possible Instructional Strategies:

- When calculating the standard deviation without technology, it is important that students are able to understand each of the steps involved in arriving at the final answer.
- Students should understand what the concept of standard deviation represents for a given data set.

Possible Assessment Strategies:

- Determine the mean, variance and standard deviation for each of the following sets of data. Round off the answers to one decimal place.
  a. The scores on a math quiz of a sample of 16 randomly chosen students (marked out of 10):

6	9	10	8	6	8	9	4
9	10	8	9	7	7	6	10

  b. The ages of a sample of 27 randomly chosen university graduates on graduation day:

25	23	24	27	27	23	24	24	25
39	32	22	25	26	31	24	25	25
23	23	25	29	57	21	24	23	25

- The students in a class were asked how many hours they studied for their math final exam. The results are shown below.

4	2	8	5	3	4	4	5	6
4	10	6	7	2	3	3	4	9

  a. Determine the median, the first and third quartiles, and the interquartile and semi-interquartile range.
  b. Construct a box plot for the data.
- A manufacturer builds containers that have a mean capacity of 250 mL with a standard deviation of 5 mL. A container is selected at random and is found to have a capacity of 252 mL. Determine its *z*-score.

UNIT 6
STATISTICS OF TWO VARIABLES

SUGGESTED TIME

6 classes

Section 3.1 – Scatter Plots and Linear Correlation (pp. 159-170)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **S5 (A B C)**

After this lesson, students will be expected to:

- define the correlation coefficient as a measure of the fit of a scatter plot to a model
- calculate the correlation coefficient for a set of data, using technology
- demonstrate an understanding of the distinction between cause-effect relationships and the mathematical correlation between variables

After this lesson, students should understand the following concepts:

- **dependent variable –** a variable whose value is affected by another variable
- **independent variable –** a variable that affects the value of another variable
- **linear correlation –** a relationship in which changes in one variable tend to be proportional to the changes in another
- **perfect positive linear correlation –** a relationship in which one variable increases at a constant rate as the other variable increases; a graph of the first variable versus the second variable is a straight line with a positive slope
- **perfect negative linear correlation –** a relationship in which one variable increases at a constant rate as the other variable decreases; a graph of the first variable versus the second variable is a straight line with a negative slope
- **scatter plot –** a graph in which data are plotted with one variable on the *x*-axis and the other on the *y*-axis; the pattern of the resulting points can show the relationship between the two variables
- **line of best fit –** the straight line that passes closest to the data points on a scatter plot and best represents the relationship between the two variables
- **correlation coefficient –** a summary statistic that gives a quantified measure of the linear relationship between two variables; sometimes referred to as the Pearson product-moment coefficient of correlation, this coefficient is denoted by *r* and can be calculated using the formula

$$r = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{\sqrt{\left[n\sum x^2 - \left(\sum x\right)^2\right]\left[n\sum y^2 - \left(\sum y\right)^2\right]}}$$

Suggested Problems in *Mathematics of Data Management*:

- **pp. 168-170:** #1-11

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

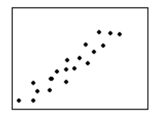
Possible Instructional Strategies:

- Some students think that a negative correlation indicates that there is very little correlation between two variables. Remind students that the strength of the correlation depends on the absolute value of the correlation only, but not its sign.

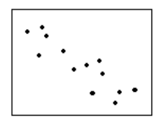
Possible Assessment Strategies:

- Describe the correlation in each of the following scatter plots.

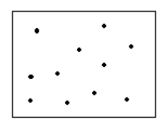
  a.



  b.



  c.



- A researcher wants to see if there is a correlation between the number of hours studied for a math test and the mark on the test. Here are the results for a sample of ten students.

HOURS OF STUDY	MARK
5	85
3	82
8	93
6	78
2	86
9	95
5	91
3	76
2	70
3	72

  Compute the correlation coefficient, correct to two decimal places. What can the researcher conclude about the relationship between the number of hours studied and the test mark?

Section 3.2 – Linear Regression (pp. 171-183)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **S5 (D E)**

After this lesson, students will be expected to:

- perform a linear regression on a set of data
- describe the possible misuses of regression

After this lesson, students should understand the following concepts:

- **regression analysis –** an analytic technique for determining the relationship between two variables
- **interpolation –** an estimation of a value between two known values
- **extrapolation –** an estimation of a value beyond the range of the data
- **least-squares fit –** an analytic technique for determining the line of best fit by minimizing the sum of the squares of the deviations of the data from the line
- **residual –** the difference between the observed value of a variable and the corresponding value predicted by the regression equation

Suggested Problems in *Mathematics of Data Management*:

- **pp. 180-183:** #1-13

Possible Instructional Strategies:

- Review with the class the properties of linear functions including slope and y-intercept.

Possible Assessment Strategies:

- The table shows the MAT521B and the MAT621B marks for ten randomly chosen students.

MAT521B	MAT621B
78	82
89	87
97	98
66	63
80	81
98	99
75	75
86	84
78	86
52	54

a. Find the equation of the line of best fit. Round off the values to two decimal places.

b. Use the line of best fit to predict the MAT621B mark for a student who received a mark of 90 in MAT521B. Round off the answer to the nearest whole number.

UNIT 7
INTRODUCTION TO PROBABILITY

SUGGESTED TIME
8 classes

Section 6.1 – Basic Probability Concepts (pp. 304-313)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **P1 (A B C)**

After this lesson, students will be expected to:

- interpret probability statements from a variety of sources
- represent complex tasks or issues using diagrams

After this lesson, students should understand the following concepts:

- **probability of an event –** a quantified measure of the likelihood that an event will occur; the probability of an event is always a value between 0 and 1, inclusive
- **probability experiment –** a well-defined process consisting of a number of trials in which clearly distinguishable outcomes are observed
- **trial –** a step in a probability experiment in which an outcome is produced and tallied
- **outcome –** a possible result, a component of an event
- **sample space –** the set of all possible outcomes in a probability experiment
- **event –** a group of outcomes
- **empirical probability –** the number of times that an event occurs in an experiment divided by the number of trials; the empirical probability is also known as the experimental or relative-frequency probability
- **theoretical probability –** the probability of an event deduced from analysis of the possible outcomes; theoretical probability is also called classical or *a priori* probability
- **statistical fluctuation –** a case in which events occur more or less frequently than the predicted theoretical probability
- **complement of an event –** the set of all outcomes that are not included in an event; the complement of an event A is the event that event A does not happen, and is denoted as A' or $\sim A$
- **subjective probability –** an estimate of the likelihood of an event based on intuition and experience; an educated guess

Suggested Problems in *Mathematics of Data Management*:

- **pp. 312-313:** #1-8

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Review benchmarks and how they relate to probability with students:

← less often →← more often →

0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
0.0	0.25	0.50	0.75	1.0
0%	25%	50%	75%	100%
impossible	unlikely	half of the time	likely	certain

Possible Assessment Strategies:

- Letter tiles for the word CANADIAN are placed in a bag.
  a. What is the probability of drawing a letter A from the bag?
  b. What is the probability of drawing a consonant from the bag?
- Find the theoretical probability for each of the following situations which involve a six-faced die:
  a. the probability of tossing a 4
  b. the probability of tossing an even number
  c. the probability of tossing a number greater than 2
- What is the theoretical probability:
  a. of randomly pointing to a prime number on a hundreds chart with the numbers from 1 to 100?
  b. that a 2-digit number that ends in 3 is also divisible by 3?
- A coin is flipped and a four-sided die is rolled.
  a. Use a tree diagram to show all possible outcomes. Write the sample space.
  b. What is the probability of getting a heads together with an even number on the die?
- What is the probability that a randomly drawn whole number between 1 and 100, inclusive, is not a multiple of 9?

Section 6.2 – Odds (pp. 314-319)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **P2 (A B C D)** **After this lesson, students will be expected to:** • interpret statements about odds from a variety of sources **After this lesson, students should understand the following concepts:** • **odds –** a measure of the probability of an event versus its complement • **odds in favour –** the ratio of the probability that an event will occur to the probability that it will not occur • **odds against –** the ratio of the probability that an event will not occur to the probability that it will occur **Suggested Problems in *Mathematics of Data Management*:** • **pp. 318-319:** #1-12	**Possible Instructional Strategies:** • Ensure that students clearly understand the difference between odds and probability. **Possible Assessment Strategies:** • Calculate the odds in favour and the odds against each event. a. Christmas falling on a Monday b. tossing exactly three heads with three coins c. randomly drawing a face card from a standard deck of 52 cards d. a random digit being odd e. rolling two dice and getting a sum of 7 f. winning Lotto 6/49 if there are 13,983,816 different six-number combinations • The odds that the Toronto Blue Jays will beat the Boston Red Sox are $3:4$. What is the probability that Toronto will beat Boston in their next game? • Boomer Gallant gives a 30% probability of precipitation for tomorrow. a. What are the odds in favour of precipitation for tomorrow? b. What are the odds against precipitation for tomorrow?

Section 6.4 – Dependent and Independent Events (pp. 327-335)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **P3 (A B C D)**

After this lesson, students will be expected to:

- solve probability problems involving combinations of simple events, both dependent and independent

After this lesson, students should understand the following concepts:

- **compound event –** an event consisting of two or more events
- **independent event –** an event whose probability is not affected by the outcome of another event
- **product rule for independent events –** the principle that the probability of events *A* and *B* both occurring is

$$P(A \text{ and } B) = P(A) \bullet P(B)$$

- **dependent event –** an event whose outcome depends directly on the outcome of another event
- **conditional probability –** the probability that event *B* occurs, given that event *A* has already occurred
- **product rule for dependent events –** the principle that the probability of event *B* occurring after event *A* has occurred is

$$P(A \text{ and } B) = P(A) \bullet P(B \mid A)$$

where $P(B \mid A)$ is the conditional probability of event *B*

Suggested Problems in *Mathematics of Data Management*:

- **pp. 334-335:** #1-14

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Ensure that students understand the difference between dependent and independent events.

Possible Assessment Strategies:

- Determine each of the following probabilities.
  a. A die is rolled and a 6 comes up. What is the probability that the next roll will be a 6?
  b. A die is rolled four times and a 6 comes up each time. What is the probability that the fifth roll will be a 6?
  c. Find the probability of rolling five sixes in a row.
  d. Why are the answers to part (b) and (c) different?
- A coin is flipped and a die is rolled. What is the probability of flipping tails and rolling an even number in a single trial?
- At work, John determines that there is a 0.8 probability that he will talk to his friend and a 0.4 chance that he will have a meeting. What is the probability that John will talk to his friend and not have a meeting?
- Two cards are drawn from a standard deck, without replacement. What is the probability that two face cards will be drawn?
- A recent survey asked 100 people if they thought women in the armed forces should be permitted to participate in combat. The results of the survey are shown.

GENDER	YES	NO	TOTAL
MALE	32	18	50
FEMALE	8	42	50
TOTAL	40	60	100

  a. What is the probability that the respondent answered yes, given that the respondent was a female?
  b. What is the probability that the respondent was a male, given that the respondent answered no?

Section 6.5 – Mutually Exclusive Events (pp. 336-343)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **P4 (A B C)** **After this lesson, students will be expected to:** • solve probability problems involving mutually exclusive events **After this lesson, students should understand the following concepts:** • **mutually exclusive events –** events that cannot occur at the same time • **addition rule for mutually exclusive events –** the principle relating the probabilities of events that cannot occur at the same time; if events *A* and *B* are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$ • **non-mutually exclusive events –** events that can occur simultaneously • **addition rule for non-mutually exclusive events –** the principle relating the probabilities of events that can occur at the same time; if events *A* and *B* are not mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ **Suggested Problems in *Mathematics of Data Management*:** • **pp. 340-343:** #1-12	**Possible Instructional Strategies:** • Ensure that students understand the difference between mutually exclusive and non-mutually exclusive events. **Possible Assessment Strategies:** • Determine which events are mutually exclusive and which are not, when a single die is rolled. a. Rolling an odd number or rolling an even number. b. Rolling a 3 or rolling an odd number. c. Rolling an odd number or rolling an number less than 4. d. Rolling a number greater than 4 or rolling a number less than 4. • A box contains 3 glazed doughnuts, 4 jelly doughnuts, and 5 chocolate doughnuts. If a person selects a doughnut at random, find the probability that it is either a glazed doughnut or a chocolate doughnut. • At a political rally, there are 20 Liberals, 13 Conservatives, and 6 New Democrats. If a person is selected at random, find the probability that he or she is either a Liberal or a New democrat. • A single card is drawn from an ordinary deck of cards. Find the probability that it is either an ace or a black card. • In a hospital unit, there are 8 nurses and 5 physicians. Of the staff in the unit, 7 nurses and 3 physicians are female. If a staff person is selected at random, find the probability that the person is a nurse or a male. • On New Year's Eve, the probability of a person driving while intoxicated is 0.15, the probability of a person having a driving accident is 0.03, and the probability of a person having a driving accident while intoxicated is 0.02. What is the probability of a person driving while intoxicated or having a driving accident?

UNIT 8
THE NORMAL DISTRIBUTION

SUGGESTED TIME

7 classes

Section 8.1 – Continuous Probability Distributions (pp. 414-421)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **S6 (A)** **After this lesson, students will be expected to:** • identify situations that give rise to common distributions **After this lesson, students should understand the following concepts:** • **positive skew –** the pulling to the right of the tail in an asymmetric probability distribution • **negative skew –** the pulling to the left of the tail in an asymmetric probability distribution • **unimodal –** having only one mode, or “hump” • **bimodal –** having two modes, or “humps” • **probability density –** the probability per unit of a continuous model **Suggested Problems in *Mathematics of Data Management*:** • **pp. 419-421:** #1-6	**Possible Instructional Strategies:** • Ensure that the students understand the difference between the terms *discrete* and *continuous*. • Omit the problems that deal with an exponential distribution, as the number *e* is not introduced to students until grade twelve. **Possible Assessment Strategies:** • Represent graphically the probability distribution for rolling a single die. • Represent graphically the probability distribution for the sum when rolling two dice. • Represent graphically the probability distribution for the number of heads when tossing three coins.

Section 8.2 – Properties of the Normal Distribution (pp. 422-431)

<table>
<tr><th>ELABORATIONS &
SUGGESTED PROBLEMS</th><th>POSSIBLE INSTRUCTIONAL &
ASSESSMENT STRATEGIES</th></tr>
<tr><td>

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **S6 (B C D E F G H I J)**

After this lesson, students will be expected to:

- demonstrate an understanding of the properties of the normal distribution
- make probability statements about normal distributions
- assess the validity of some simulation results by comparing them with the theoretical probabilities

After this lesson, students should understand the following concepts:

- **normal distribution –** a common continuous probability distribution in which the data are distributed symmetrically and unimodally about the mean
- **probability-density function –** an equation that describes or defines the curve for a probability distribution
- **standard normal distribution –** a normal distribution in which the mean is equal to 0 and the standard deviation is equal to 1
- **cumulative probability –** the probability that a variable is less than a certain value

Suggested Problems in *Mathematics of Data Management*:

- **pp. 430-431:** #1-10

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Possible Instructional Strategies:

- Discuss with the class examples of natural phenomena that follow normal distributions, such as the physical characteristics of plants and animals, like height and weight.
- Discuss with the class what areas between standard deviations represent under the normal distribution curve.

Possible Assessment Strategies:

- Using the *z*-score table, determine the percent of data that lies
  a. to the left of $z = 0.57$
  b. to the right of $z = 1.24$
  c. between $z = -2.06$ and $z = 1.10$
- What *z*-score is required for each situation?
  a. 25% of the data is to the left of the *z*-score
  b. 30% of the data is to the right of the *z*-score
  c. 92% of the data is to the left of the *z*-score
- The scores on a history test follow a normal distribution with an average score of 72% and a standard deviation of 9%. What percentage of the scores were
  a. between 63% and 81%
  b. between 54% and 72%
  c. between 72% and 94.5%
  d. less than 63%
- The daily sales at Pizza Delight have a mean of $2500 and a standard deviation of $150. What percent of the time will the daily sales be less than $2200?
- The average heights of teenage boys are normally distributed, with a mean of 175 cm and a standard deviation of 8 cm.
  a. What is the probability that a teenage boy's height is greater than 171 cm?
  b. What range of heights would occur about 95% of the time? Round off the values to one decimal place.
- The RCMP is measuring the speed of cars on a highway. The speeds are normally distributed with a mean of 80 km/h and a standard deviation of 10 km/h. What is the probability that a car picked at random is travelling more than 95 km/h?

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Section 8.3 – Normal Sampling and Modelling (pp. 432-441)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **S6 (I J)**

After this lesson, students will be expected to:

- determine probabilities, using the normal distribution

After this lesson, students should understand the following concept:

- **continuity correction –** treating the values of a discrete variable as continuous intervals in order to use a normal approximation for a binomial distribution

Suggested Problems in *Mathematics of Data Management*:

- **pp. 439-441:** #1-12

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Discuss with the class examples of phenomena that follow normal distributions.

Possible Assessment Strategies:

- The data shown represent the number of goals scored by a random sample of NHL players during the 2010-2011 season. Assume that the number of goals is normally distributed.

14	18	5	11	11
0	22	6	0	7
1	15	9	0	18
5	1	3	11	0
12	10	0	4	0

a. Calculate the mean and the standard deviation of these data. Round off the answers to one decimal place.

b. What is the probability that a player scored at least 20 goals in the 2010-2011 season?

c. What is the probability that a player scored 5 goals or fewer in the 2010-2011 season?

UNIT 9
PERSONAL FINANCE

SUGGESTED TIME

8 classes

Section 9.1 – Earning Income (pp. 527-531)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **FM1 (A B C)** **After this lesson, students will be expected to:** • calculate gross income **After this lesson, students should understand the following concepts:** • **gross income –** an amount of money earned through employment • **straight commission –** income that is a percent of the value of sales • **sales quota –** an amount of sales that a salesperson is expected to make before the salesperson starts to earn commission • **graduated commission –** a pay scheme in which the rate of commission increases as sales increase • **piecework –** a method of earning income in which a worker is paid for each item produced or each service provided **Suggested Problems in *MathPower 11*:** • **pp. 530-531:** #1-25	**Possible Instructional Strategies:** • Ask students to research the minimum wage of each Canadian province. Which province has the highest and lowest minimum wage? • Remind students that being paid bi-weekly (26 times/year) and semi-monthly (24 times/year) are not the same. **Possible Assessment Strategies:** • Paul’s regular rate of pay is $9.40 per hour and he works 25 hours per week. What is his gross pay per week? • Jennie worked for 38 hours and earned $446.50. What is her hourly wage? • Jim earns $39,000 per year. If he is paid bi-weekly, what is his gross pay for each pay period? • Mary earns $8.70 per hour. She is paid time and a half for overtime hours over 40 hours. How much will she earn for the week if she works 44 hours? • Jason is paid $1200 per month plus a commission of 4% of the first $2,500 of his sales, and 6% of his sales over $2,500. Last month his sales totaled $22,100. Find Jason's wages for the month. • Jake works at a call centre promoting low-rate credit cards. He earns $10.50 per hour plus $10 for every sale he makes. This past week, he worked 37.5 hours and was able to sign up 36 customers. What was his gross pay for the week? • Elena works part-time at an electronics store. Her weekly salary is $350 plus 5% commission on her total sales. Find her total sales when her gross pay for the week is $450.

Section 9.2 – Net Income (pp. 532-537)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **FM1 (D E)** **After this lesson, students will be expected to:** • calculate net income **After this lesson, students should understand the following concepts:** • **net income –** the amount an employee receives after all deductions are subtracted from the employee's gross revenue • **taxable income –** the amount obtained by subtracting tax-exempt deductions from gross income • **tax-exempt deductions –** union or professional dues, registered Pension Plan contributions, and child care expenses that are deducted from gross income to obtain taxable income **Suggested Problems in *MathPower 11*:** • **pp. 536-537:** #1-33	**Possible Instructional Strategies:** • Remind students that an employee's CPP, EI and income tax deduction depends on his or her salary. • Actual CPP and EI rates vary from year to year. The values given in the textbook are for illustrative purposes only. • Current income tax tables for each province can be found at http://www.cra-arc.gc.ca/. **Possible Assessment Strategies:** • Bob Sloan's weekly salary is $570.00. Use 52 pay periods in a year to answer these questions. a. What is his weekly CPP deduction if the contribution rate is 4.95% of any annual gross earnings above $3500.00. Round off the answer to the nearest cent. b. What is his weekly EI deduction if the EI premium rate is 1.73% of gross earnings. Round off the answer to the nearest cent. • Paula DuMont works a 40-hour work week and earns $11.25 per hour with time and a half for overtime. Over the last two weeks, she has worked 45 hours each week. Calculate her net pay for the two weeks if all of her deductions add up to 35% of her gross pay. Round off the answer to the nearest cent. • Stuart works as a waiter and earns $800.00 bi-weekly. Each pay period, his employer deducts $59.50 for federal tax, $16.91 for provincial tax, and 1.73% for EI premiums. The CPP contribution rate is 4.95% of any annual gross earnings above $3500.00. What is Stuart's bi-weekly net pay? Use 26 pay periods for the year. Round off the answer to the nearest cent.

Section 9.3 – Interest and Annuities (pp. 538-543)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **FM2 (A B C D E F)** **After this lesson, students will be expected to:** • calculate interest and annuities **After this lesson, students should understand the following concepts:** • **simple interest –** interest calculated using the formula $I = Prt$ • **compound interest –** interest that is calculated at regular intervals and is added to the principal for the next interest period; it is calculated using the formula $A = P(1+i)^n$ • **annuity –** a sequence of equal payments made at equal intervals of time **Suggested Problems in *MathPower 11*:** • **pp. 542-543:** #1-33	**Possible Instructional Strategies:** • Demonstrate to students that the compound interest formula is an example of a geometric sequence, when *n* takes on whole number values. **Possible Assessment Strategies:** • Calculate the final value of each of the following. Round off the answers to the nearest cent. a. \$2500 at 4%/a, compounded annually for 5 years b. \$1350 at 2.5%/a, compounded quarterly for 3 years c. \$250 at 3.6%/a, compounded monthly for 18 months • John invests \$1000 into a bank account at 4.5%/a for 5 years. Calculate the final value of his investment under each of the following conditions. Round off the answers to the nearest cent. a. compounded annually b. compounded quarterly c. compounded monthly d. compounded daily • Steven opened a savings account on January 1, 2010 with a deposit of \$500. At the beginning of each month in 2010, he deposited another \$500. If the account paid interest at 4.8%/a, compounded monthly, how much money did he have in his account on January 1, 2011? Round off the answer to the nearest cent.

Section 9.4 – Effective Annual Rate of Interest (pp. 547-550)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **FM2 (G)** **After this lesson, students will be expected to:** • calculate the effective annual rate of interest **After this lesson, students should understand the following concepts:** • **nominal interest rate –** the stated, or named, rate of interest of an investment or loan • **effective annual interest rate –** the simple interest rate that would produce the same interest in one year as the nominal interest rate **Suggested Problems in *MathPower 11*:** • **pp. 549-550:** #1-15	**Possible Instructional Strategies:** • Ensure that students understand the difference between the nominal interest rate and the effective annual interest rate. **Possible Assessment Strategies:** • What is the effective annual rate for an investment earning a nominal interest rate of 4.8%/a, compounded monthly? Round off the answer to two decimal places. • Kathy has $10,000 to invest. Which option is better? Why? Option A: 6%/a, compounded quarterly Option B: 6.2%/a, compounded semi-annually

Section 9.5 – Consumer Credit (pp. 551-556)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **FM3 (A B)** **After this lesson, students will be expected to:** • develop an understanding of consumer credit **After this lesson, students should understand the following concept:** • **credit –** a contractual agreement in which a borrower receives something of value now and agrees to repay the lender at some later date **Suggested Problems in *MathPower 11*:** • **pp. 555-556:** #1-29	**Possible Instructional Strategies:** • Discuss with the class the role that credit plays in the economy. **Possible Assessment Strategies:** • Josh bought an entertainment system with a retail price of $1500, including taxes. He paid 20% down and financed the balance. He agreed to pay the store $125/month for 12 months. a. What amount was financed? b. What was the finance charge? c. How much did the entertainment system cost Josh? • One credit card charges 18%/a on the unpaid balance. Jessica has an unpaid balance of $450 on her card. How much of her $50 monthly payment is due to interest for that month?

Section 9.6 – Housing Costs (pp. 557-561)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **FM3 (B C)** **After this lesson, students will be expected to:** • develop an understanding of housing costs **After this lesson, students should understand the following concepts:** • **mortgage –** an agreement between a money lender and a borrower to purchase property • **amortization period –** the period of time in which a mortgage would be fully repaid with equal periodic payments • **mill rate –** the amount of tax levied annually for each $1000 of assessed value of a property • **mill –** one tenth of a cent, or $0.001 **Suggested Problems in *MathPower 11*:** • **pp. 560-561:** #1-33	**Possible Instructional Strategies:** • Discuss with the class the process involved in purchasing and financing a house. **Possible Assessment Strategies:** • Graham bought a house for $200,000. He paid 20% of the value of the house as a down payment and arranged a 5-year mortgage at 6%/a, amortized over 25 years, on the balance of the house. a. How much was Graham's down payment? b. How much is his monthly mortgage payment? • The assessed value of a house is $85,000. If the residential mill rate is 18.755, what is the annual property tax on the house? Round off the answer to the nearest cent. • Lindsay has just renewed the mortgage on her house. She arranged a 5-year $120,000 mortgage at 6.5%/a, amortized over 20 years. Her house has an assessed value of $175,000, and the residential mill rate in her town is 14.525. Find her monthly housing costs. Round off the answer to the nearest cent.

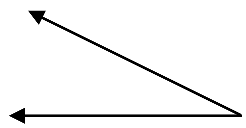
Section 9.7 – Balancing a Budget (pp. 562-566)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **FM4 (A B C D E F)** **After this lesson, students will be expected to:** • develop an understanding of balancing a budget **After this lesson, students should understand the following concepts:** • **budget –** a spending plan, or record of money earned and of money spent • **balanced budget –** a spending plan in which the total expenditure equals the total income • **fixed expenses –** expenses that occur regularly and are difficult to control • **variable expenses –** expenses that occur regularly, but can be controlled • **occasional expenses –** expenses that occur occasionally or unexpectedly **Suggested Problems in *MathPower 11*:** • **pp. 565-566:** #1-25	**Possible Instructional Strategies:** • Discuss with the class the process required in order to balance a budget. **Possible Assessment Strategies:** • Joanna has a net monthly income of \$3000. Her expenses are: rent \$840, food \$400, transportation \$300, phone and internet \$120, clothing \$200, and entertainment \$200. How much is she able to save each month?

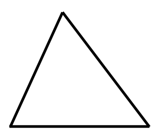
GLOSSARY OF MATHEMATICAL TERMS

A

- **acute angle –** an angle with a measure less than 90^0



- **acute triangle –** a triangle in which all of the angles are acute angles



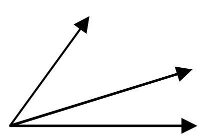
- **addition rule for mutually exclusive events –** the principle relating the probabilities of events that cannot occur at the same time; if events *A* and *B* are mutually exclusive, then

$$P(A \text{ or } B) = P(A) + P(B)$$

- **addition rule for non-mutually exclusive events –** the principle relating the probabilities of events that can occur at the same time; if events *A* and *B* are not mutually exclusive, then

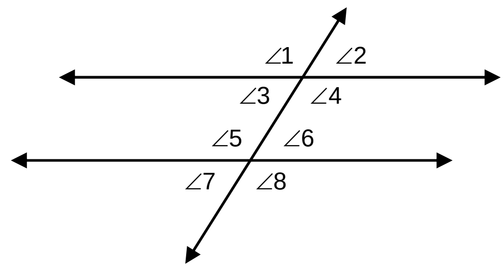
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

- **adjacent angles –** two angles that lie in the same plane, have a common vertex and a common side, but no common interior points



- **algebraic proof –** a proof that is made up of a series of algebraic statements; the properties of equality provide justification for many statements in algebraic proofs

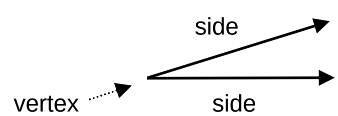
- **alternate exterior angles –** nonadjacent exterior angles that lie on opposite sides of a transversal; in the diagram, $\angle 1$ and $\angle 8$, and $\angle 2$ and $\angle 7$ are alternate exterior angles



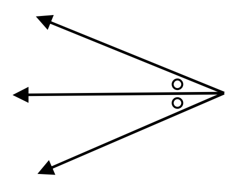
- **alternate interior angles –** nonadjacent interior angles that lie on opposite sides of a transversal; in the diagram above, $\angle 4$ and $\angle 5$, and $\angle 3$ and $\angle 6$ are alternate interior angles

- **amortization period –** the period of time in which a mortgage would be fully repaid with equal periodic payments

- **angle –** the intersection of two noncollinear rays at a common endpoint; the rays are called sides and the common endpoint is called the vertex



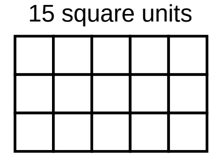
- **angle bisector –** a ray that divides an angle into two congruent angles



- **annuity –** a sequence of equal payments made at equal intervals of time

- **area –** the number of square units needed to cover a surface

15 square units

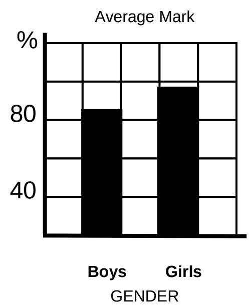


- **auxiliary line –** an extra line or segment drawn in a figure to help complete a proof

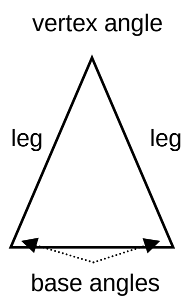
- **axiom –** a statement that is accepted as true

B

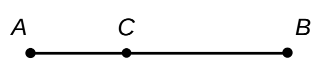
- **balanced budget –** a spending plan in which the total expenditure equals the total income
- **bar graph –** a chart or diagram that represents quantities with horizontal or vertical bars whose lengths are proportional to the quantities



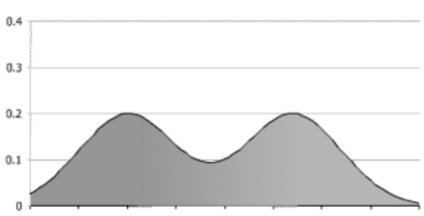
- **base angles of an isosceles triangle –** the angles opposite the legs of an isosceles triangle



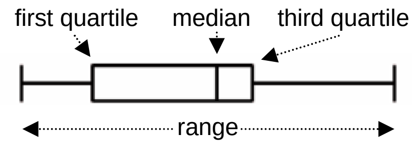
- **between –** for any two points *A* and *B* on a line, there is another point *C* between *A* and *B* if and only if *A*, *B*, and *C* are collinear and $AC + CB = AB$



- **biconditional –** the conjunction of a conditional statement and its converse
- **bimodal –** having two modes, or two "humps"



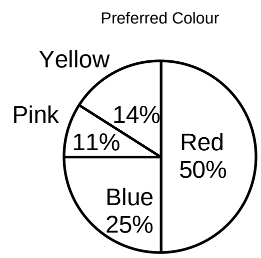
- **box-and-whisker plot –** a graph that summarizes a set of data by representing the first quartile, the median, and the third quartile with a box, and the lowest and highest data with the ends of lines extending from the box



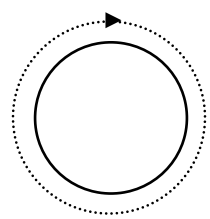
- **budget –** a spending plan, or record of money earned and of money spent

C

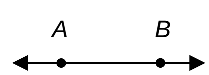
- **circle graph –** a graph that represents quantities with segments of a circle that are proportional to the quantities; also called a *pie chart*



- **circumference –** the distance around a circle

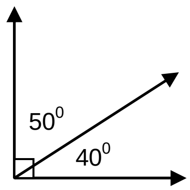


- **class –** a set of values whose values lie within a given range or interval; also called an *interval*
- **cluster sample** – a survey of selected groups within a population; this sampling technique can save time and expense, but may not give reliable results unless the clusters are representative of the population
- **collinear –** points that line on the same line



- **complement of an event –** the set of all outcomes that are not included in an event; the complement of an event *A* is the event that event *A* does not happen, and is denoted as A' or $\sim A$

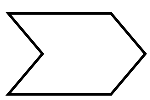
- **complementary angles –** two angles with measures that have a sum of 90^0



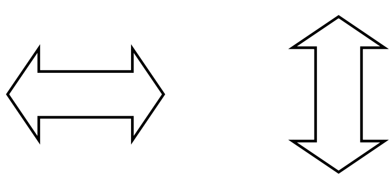
- **compound event –** an event consisting of two or more events
- **compound interest –** interest that is calculated at regular intervals and is added to the principal for the next interest period; it is calculated using the formula

$$A = P\left(1+i\right)^n$$

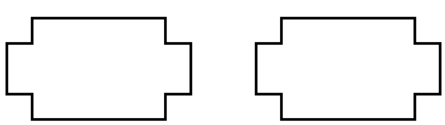
- **compound statement –** a statement formed by joining two or more statements
- **concave polygon –** a polygon for which there is a line containing a side of the polygon that also contains a point in the interior of the polygon



- **conclusion –** in a conditional statement, the statement that immediately follows the word *then*
- **conditional probability –** the probability that event *B* occurs, given that event *A* has already occurred
- **conditional statement –** a statement that can be written in *if-then* form
- **congruent –** having the same measure

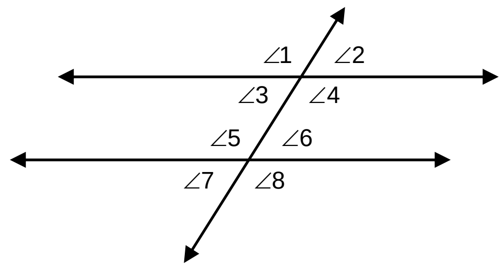


- **congruent polygons –** polygons in which all matching parts are congruent

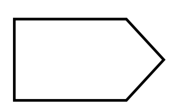


- **congruent segments –** segments that have the same measure

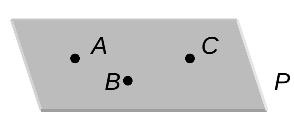
- **conjecture –** an educated guess based on known information
- **conjunction –** a compound statement formed by joining two or more statements with the word *and*
- **consecutive interior angles –** interior angles that lie on the same side of a transversal; in the diagram, $\angle 3$ and $\angle 5$, and $\angle 4$ and $\angle 6$ are consecutive interior angles



- **construction –** a method of creating geometric figures without the benefit of measuring tools; generally, only a pencil, straightedge, and compass are used
- **continuity correction –** treating the values of a discrete variable as continuous intervals in order to use a normal approximation for a binomial distribution
- **continuous variable –** a variable that can have an infinite number of possible values in a given interval; a continuous function can be graphed as a smooth curve
- **contrapositive –** the statement formed by negating both the hypothesis and conclusion of the converse of a conditional statement
- **convenience sample –** a sample selected simply because it is easily accessible; such samples may not be random, so their results are not always reliable
- **converse –** the statement formed by exchanging the hypothesis and conclusion of a conditional statement
- **convex polygon –** a polygon for which there is no line that contains both a side of the polygon and a point in the interior of the polygon



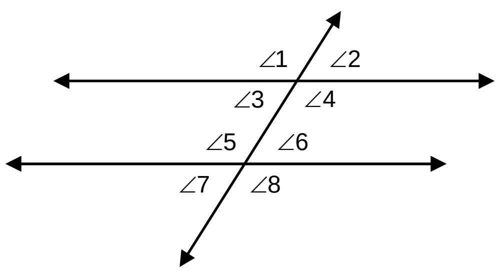
- **coplanar –** points that lie in the same plane



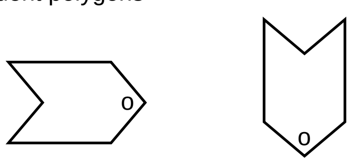
- **corollary –** a statement that can be easily proved using a theorem is called a corollary of that theorem
- **correlation coefficient –** a summary statistic that gives a quantified measure of the linear relationship between two variables; sometimes referred to as the Pearson product-moment coefficient of correlation, this coefficient is denoted by *r* and can be calculated using the formula

$$r = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{\sqrt{\left[n\sum x^2 - \left(\sum x\right)^2\right]\left[n\sum y^2 - \left(\sum y\right)^2\right]}}$$

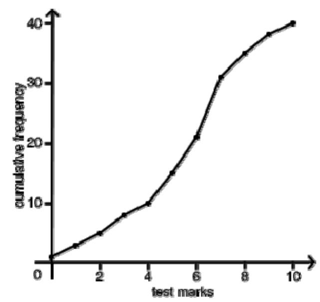
- **corresponding angles –** angles that lie on the same side of a transversal and on the same side of the lines that cross it; in the diagram, $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, and $\angle 4$ and $\angle 8$ are corresponding angles



- **corresponding parts –** matching parts of congruent polygons



- **counterexample –** an example used to show that a given statement is not always true
- **credit –** a contractual agreement in which a borrower receives something of value now and agrees to repay the lender at some later date
- **cumulative-frequency graph –** a graph that shows the running total of the frequencies for values of the variable, starting from the lowest value; also called an *ogive*



- **cumulative probability –** the probability that a variable is less than a certain value

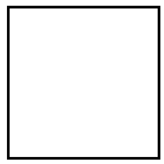
D

- **deductive argument –** a proof formed by a group of algebraic steps used to solve a problem
- **deductive reasoning –** a system of reasoning that uses facts, rules, definitions, or properties to reach logical conclusions
- **defined term –** terms that are explained using undefined terms and/or other defined terms
- **degree –** a unit of measure used in measuring angles and arcs
- **dependent event –** an event whose outcome depends directly on the outcome of another event
- **dependent variable –** a variable whose value is affected by another variable
- **deviation –** the difference in value between a data item and the mean
- **discrete variable –** a variable that can only take on certain values within a given range
- **disjunction –** a compound statement formed by joining two or more statements with the word *or*
- **distance between two points –** the length of the segment between two points

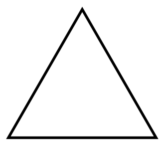
E

- **effective annual interest rate –** the simple interest rate that would produce the same interest in one year as the nominal interest rate
- **empirical probability –** the number of times that an event occurs in an experiment divided by the number of trials; the empirical probability is also known as the experimental or relative-frequency probability

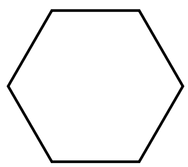
- **equiangular polygon –** a polygon with all angles congruent



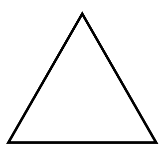
- **equiangular triangle –** a triangle with all angles congruent



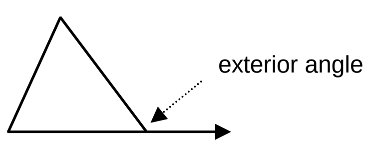
- **equilateral polygon –** a polygon with all sides congruent



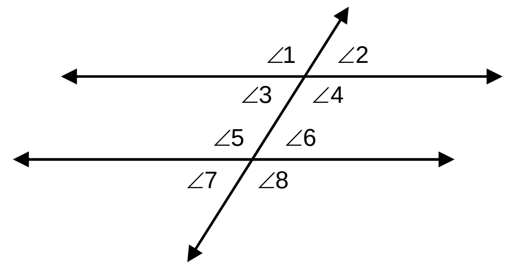
- **equilateral triangle –** a triangle with all sides congruent



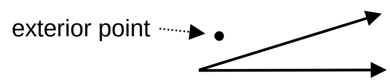
- **event –** a group of outcomes
- **exterior angle to a triangle –** an angle formed by one side of a triangle and the extension of another side



- **exterior angles –** angles that do not lie between two parallel lines that intersect a transversal; in the diagram, $\angle 1$, $\angle 2$, $\angle 7$, and $\angle 8$ are exterior angles



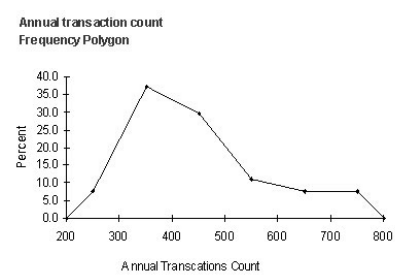
- **exterior point –** a point is in the exterior of an angle if it is neither on the angle nor in the interior of the angle



- **extrapolation –** an estimation of a value beyond the range of the data

F

- **fixed expenses –** expenses that occur regularly and are difficult to control
- **formal proof –** a two-column proof containing statements and reasons
- **frequency polygon –** a plot of frequencies versus variable values with the resulting points joined by line segments



- **frequency table –** a table which states the frequency of each value of the variable in question

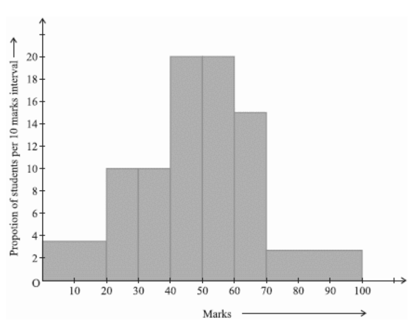
SCORE	TALLY	FREQUENCY
1	|	1
2	|	1
3	|||	3
4	|	1
5	||||	4
6	~~||||~~	5
7	~~||||~~ |	6
8	~~||||~~	5
9	|||	3
10	|	1

G

- **graduated commission –** a pay scheme in which the rate of commission increases as sales increase
- **gross income –** an amount of money earned through employment

H

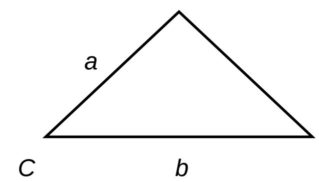
- **histogram –** a bar graph in which the area of the bars are proportional to the frequencies for various values of the variable



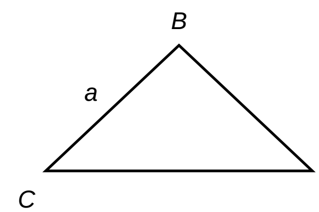
- **hypothesis –** in a conditional statement, the statement that immediately follows the word *if*

I

- ***if-then* statement –** a compound statement of the form “if *p*, then *q*,” where *p* and *q* are statements
- **included angle –** in a triangle, the angle formed by two sides is the included angle for those two sides; in the diagram, *C* is the included angle for sides *a* and *b*

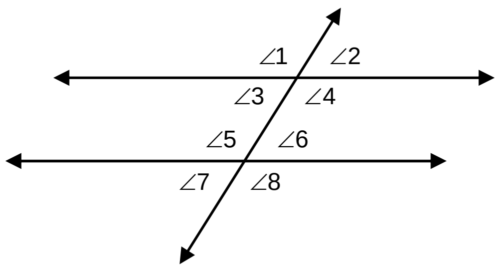


- **included side –** the side of a triangle that is a side of each of two angles; in the diagram, *a* is the included side for angles *B* and *C*

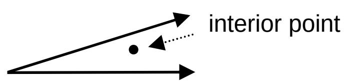


- **independent event –** an event whose probability is not affected by the outcome of another event
- **independent variable –** a variable that affects the value of another variable
- **indirect proof –** in an indirect proof, one assumes that the statement to be proved is false; one then uses logical reasoning to deduce that a statement contradicts a postulate, theorem, or one of the assumptions; once a contradiction is obtained, one concludes that the statement assumed false must in fact be true; also called *proof by contradiction*
- **indirect reasoning –** reasoning that assumes that the conclusion is false and then shows that this assumption leads to a contradiction of the hypothesis by using a postulate, theorem or corollary; then, since the assumption has been proved false, the conclusion must be true
- **inductive reasoning –** reasoning that uses a number of specific examples to arrive at a plausible generalization or prediction; conclusions arrived at by inductive reasoning lack the logical certainty of those arrived at by deductive reasoning

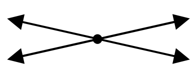
- **interior angles –** angles that lie between two parallel lines that intersect a transversal; in the diagram, $\angle 3$, $\angle 4$, $\angle 5$, and $\angle 6$ are interior angles



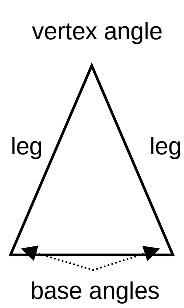
- **interior point –** a point is in the interior of an angle if it does not lie on the angle itself, and it lies on a segment with endpoints that are on the sides of the angle



- **interpolation –** an estimation of a value between two known values
- **interquartile range –** the range of the central half of a set of data when the data are arranged in numerical order
- **intersection –** a set of points common to two or more geometric figures

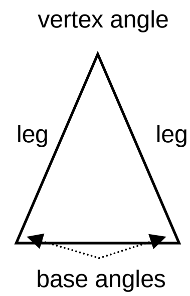


- **inverse –** the statement formed by negating both the hypothesis and conclusion of a conditional statement
- **isosceles triangle –** a triangle with at least two sides congruent; the congruent sides are called legs; the angles opposite the legs are base angles; the angle formed by the two legs is called the vertex angle; the side opposite the vertex is called the base

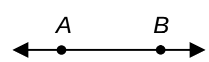


L

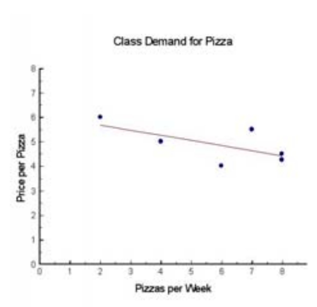
- **Law of Detachment –** if $p \rightarrow q$ is a true conditional, and *p* is true, then *q* is also true
- **Law of Syllogism –** if $p \rightarrow q$ and $q \rightarrow r$ are true conditionals, then $p \rightarrow r$ is also true
- **least-squares fit –** an analytic technique for determining the line of best fit by minimizing the sum of the squares of the deviations of the data from the line
- **legs of an isosceles triangle –** the two congruent sides of an isosceles triangle



- **line –** an undefined term; made up of points and has no thickness or width; in a figure, a line is shown with an arrowhead at each end; lines are usually named by lowercase script letters or by writing capital letters for two points on the line, with a double arrow over the pair of letters

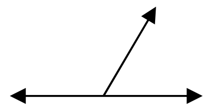


- **line of best fit –** the straight line that passes closest to the data points on a scatter plot and best represents the relationship between the two variables



- **line segment –** a measurable part of a line that consists of two points, called the endpoints, and all the points between them

- **linear correlation –** a relationship in which changes in one variable tend to be proportional to the changes in another
- **linear pair –** a pair of adjacent angles whose non-common sides are opposite rays



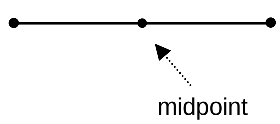
- **logically equivalent –** statements that have the same truth values

M

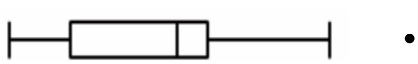
- **mean –** the sum of the values of a set of data divided by the number of values;

$$\bar{x} = \frac{\sum x_i}{n}$$

- **measures of central tendency –** the values around which a set of data tends to cluster
- **measures of spread –** quantities that indicate how closely a set of data clusters around its central value; also called *measures of dispersion*
- **median –** the middle value of a set of data ranked from highest to lowest; if there is an even number of data, the median in the midpoint between the two middle values
- **midpoint –** the point on a segment exactly halfway between the endpoints of the segment



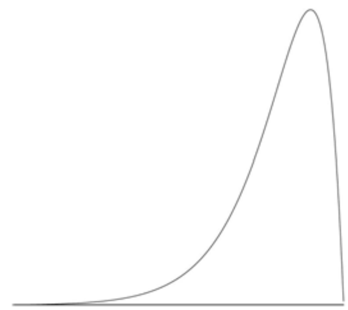
- **mill –** one tenth of a cent, or \$0.001
- **mill rate –** the amount of tax levied annually for each \$1000 of assessed value of a property
- **mode –** the value in a distribution or a set of data that occurs most frequently
- **modified box-and-whisker plot –** a box-and-whisker plot that shows outliers as separate points instead of including them in the whiskers



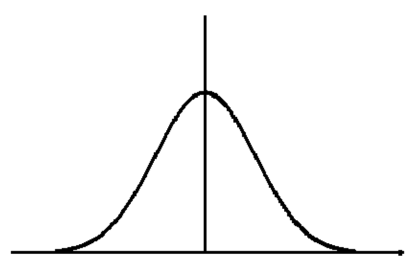
- **mortgage –** an agreement between a money lender and a borrower to purchase property
- **multi-stage sample –** a sample that uses several levels of random sampling
- **mutually exclusive events –** events that cannot occur at the same time

N

- ***n*-gon –** a polygon with *n* sides
- **negation –** if a statement is represented by *p*, then "not *p*" is the negation of the statement
- **negative skew –** the pulling to the left of the tail in an asymmetric probability distribution

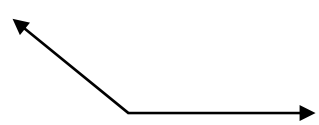


- **net income –** the amount an employee receives after all deductions are subtracted from the employee's gross revenue
- **nominal interest rate –** the stated, or named, rate of interest of an investment or loan
- **non-mutually exclusive events –** events that can occur simultaneously
- **normal distribution –** a common continuous probability distribution in which the data are distributed symmetrically and unimodally about the mean

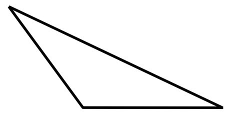


O

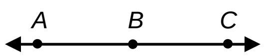
- **obtuse angle –** an angle with a measure greater than 90^0 and less than 180^0



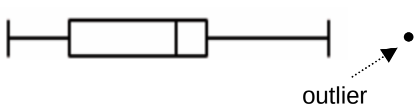
- **obtuse triangle –** a triangle with an obtuse angle



- **occasional expenses –** expenses that occur occasionally or unexpectedly
- **odds –** a measure of the probability of an event versus its complement
- **odds against –** the ratio of the probability that an event will not occur to the probability that it will occur
- **odds in favour –** the ratio of the probability that an event will occur to the probability that it will not occur
- **opposite rays –** two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ such that B is between A and C

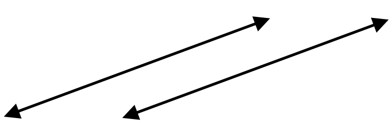


- **outcome –** a possible result, a component of an event
- **outliers –** points in a set of data that are significantly far from the majority of the other data

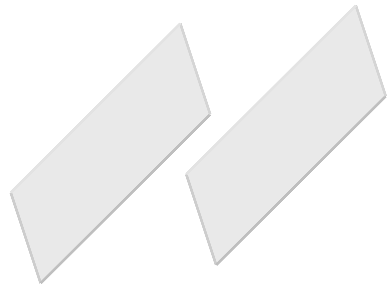


P

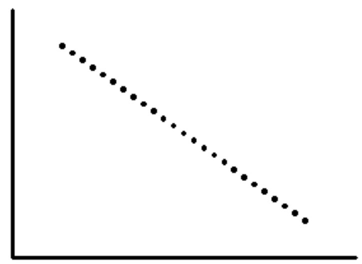
- **paragraph proof –** an informal proof written in the form of a paragraph that explains why a conjecture for a given situation is true
- **parallel lines –** coplanar lines that do not intersect; in the diagram, lines *m* and *n* are parallel lines



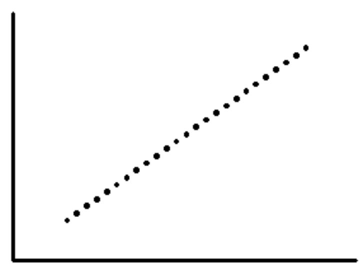
- **parallel planes –** planes that do not intersect



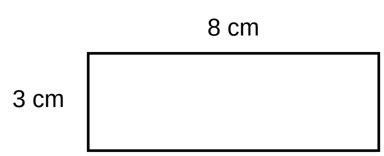
- **perfect negative linear correlation –** a relationship in which one variable increases at a constant rate as the other variable decreases; a graph of the first variable versus the second variable is a straight line with a negative slope



- **perfect positive linear correlation –** a relationship in which one variable increases at a constant rate as the other variable increases; a graph of the first variable versus the second variable is a straight line with a positive slope

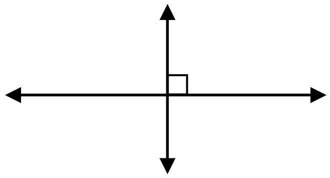


- **perimeter –** the sum of the lengths of the sides of a polygon



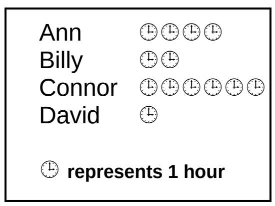
Perimeter = 22 cm

- **perpendicular lines –** lines that form right angles

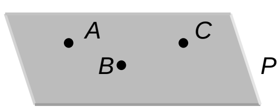


- **pictograph –** a chart or diagram that represents quantities with symbols

Hours Spend Studying
Per Week

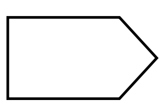


- **piecework –** a method of earning income in which a worker is paid for each item produced or each service provided
- **plane –** an undefined term; a flat surface made up of points that has no depth and extends indefinitely in all directions; in a figure, a plane is often represented by a shaded, slanted four-sided figure; planes are usually named by a capital script letter or by three noncollilnear points on the plane

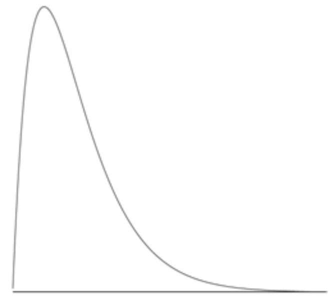


- **point –** an undefined term; a location; in a figure, points are represented by a dot; points are named by capital letters

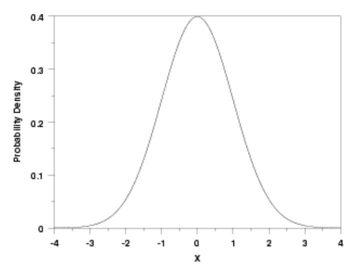
- **polygon –** a two-dimensional closed figure formed by a finite number of segments called sides



- **population –** all individuals that belong to a group being studied
- **positive skew –** the pulling to the right of the tail in an asymmetric probability distribution



- **postulate –** a statement that describes a fundamental relationship between the basic terms of geometry; postulates are accepted as true without proof
- **probability density –** the probability per unit of a continuous model
- **probability-density function –** an equation that describes or defines the curve for a probability distribution



- **probability experiment –** a well-defined process consisting of a number of trials in which clearly distinguishable outcomes are observed
- **probability of an event –** a quantified measure of the likelihood that an event will occur; the probability of an event is always a value between 0 and 1, inclusive
- **product rule for dependent events –** the principle that the probability of event *B* occurring after event *A* has occurred is

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

where $P(B \mid A)$ is the conditional probability of event *B*

- **product rule for independent events –** the principle that the probability of events *A* and *B* both occurring is

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

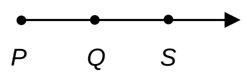
- **proof –** a logical argument in which each statement you make is supported by a statement that is accepted to be true

Q

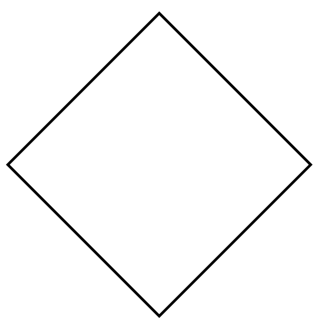
- **quartile –** one of a set of values that divide a set of data into four groups with equal numbers of data

R

- **range –** the difference between the highest and lowest values in a set of data
- **raw data –** unprocessed information
- **ray –** $\overrightarrow{PQ}$ is a ray if it is the set of points consisting of $\overline{PQ}$ and all points *S* for which *Q* is between *P* and *S*



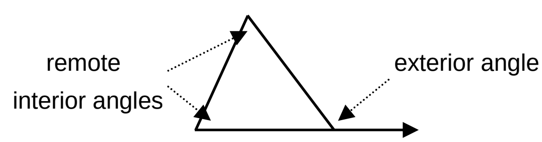
- **regression analysis –** an analytic technique for determining the relationship between two variables
- **regular polygon –** a convex polygon in which all of the sides are congruent and all of the angles are congruent



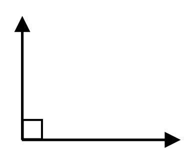
- **related conditionals –** statements that are based on a given conditional statement
- **relative-frequency table –** a table which shows the frequencies of values or groups of values expressed as a fraction or percent of the whole data set

SCORE	TALLY	FREQUENCY	RELATIVE FREQUENCY
1	𝍷	1	4%
2	𝍷𝍷𝍷	3	12%
3	𝍷	4	16%
4	𝍷𝍷𝍷𝍷	5	20%
5	𝍸	6	24%
6	𝍸𝍷	5	20%
7	𝍸	1	4%

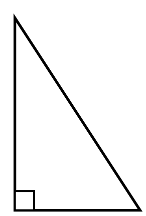
- **remote interior angles –** the angles of a triangle that are not adjacent to the given exterior angle



- **residual –** the difference between the observed value of a variable and the corresponding value predicted by the regression equation
- **right angle –** an angle with a measure of 90^0



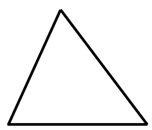
- **right triangle –** a triangle with a right angle; the side opposite the right angle is called the hypotenuse; the two other sides are called the legs



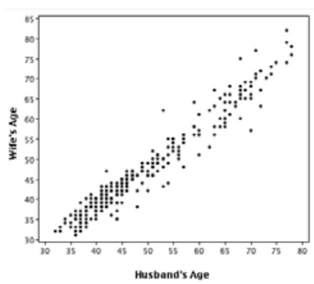
S

- **sales quota –** an amount of sales that a salesperson is expected to make before the salesperson starts to earn commission

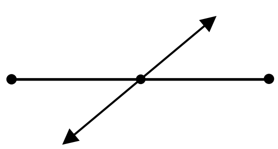
- **sample –** a group of items or people being selected from a population
- **sample space –** the set of all possible outcomes in a probability experiment
- **sampling frame –** the members of a population that actually have a chance of being selected for a sample
- **scalene triangle –** a triangle with no two sides congruent



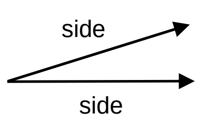
- **scatter plot –** a graph in which data are plotted with one variable on the *x*-axis and the other on the *y*-axis; the pattern of the resulting points can show the relationship between the two variables



- **segment bisector –** a segment, line, or plane that intersects a segment at its midpoint



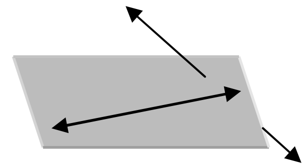
- **semi-interquartile range –** one half of the interquartile range
- **sides of an angle –** the rays of an angle



- **simple interest –** interest calculated using the formula

$$I = Prt$$

- **simple random sample** – a sample in which every member of a population has an equal and independent chance of being selected
- **skew lines –** lines that do not intersect and are not coplanar

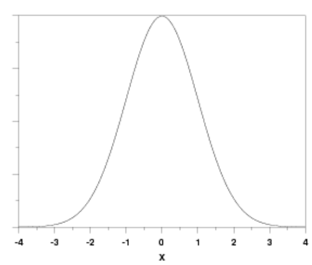


- **space –** a boundless three-dimensional set of all points
- **standard deviation –** the square root of the mean of the squares of the deviations of a set of data; the standard deviation is given by the formulas

$$\sigma = \sqrt{\frac{\sum (x-\mu)^2}{N}} \text{ for a population, and}$$

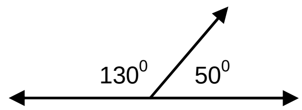
$$s = \sqrt{\frac{\sum \left(x-\bar{x}\right)^2}{n-1}} \text{ for a sample}$$

- **standard normal distribution –** a normal distribution in which the mean is equal to 0 and the standard deviation is equal to 1



- **statement –** any sentence that is either true or false, but not both
- **statistical fluctuation –** a case in which events occur more or less frequently than the predicted theoretical probability
- **statistics –** the gathering, organizing, analysis, and presentation of numerical information
- **straight commission –** income that is a percent of the value of sales
- **strata** – groups whose members share common characteristics, which may differ from the rest of the population
- **stratified sample** – a sample in which each stratum or group is represented in the same proportion as it appears in the population
- **subjective probability –** an estimate of the likelihood of an event based on intuition and experience; an educated guess

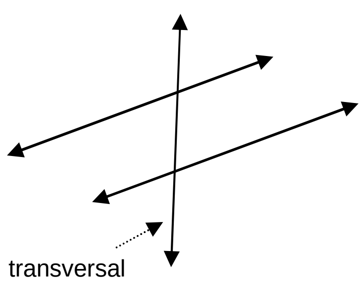
- **supplementary angles –** two angles with measures that have a sum of 180^0



- **systematic sample** – a sample selected by listing a population sequentially and choosing members at regular intervals

T

- **tax-exempt deductions –** union or professional dues, registered Pension Plan contributions, and child care expenses that are deducted from gross income to obtain taxable income
- **taxable income –** the amount obtained by subtracting tax-exempt deductions from gross income
- **theorem –** a statement or conjecture that can be proven true by undefined terms, definitions, and/or postulates
- **theoretical probability –** the probability of an event deduced from analysis of the possible outcomes; theoretical probability is also called classical or *a priori* probability
- **transversal –** a line that intersects two or more lines in a plane at different points



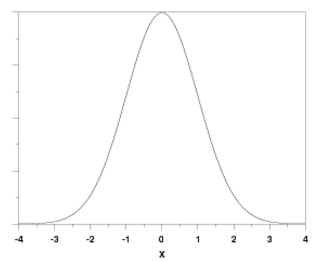
- **trial –** a step in a probability experiment in which an outcome is produced and tallied
- **truth table –** a table used as a convenient method for organizing the truth values of statements

p	***q***	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- **truth value –** the truth or falsity of a statement
- **two-column proof –** a formal proof that contains statements and reasons organized in two columns; each step is called a statement, and the properties that justify each step are called the reasons

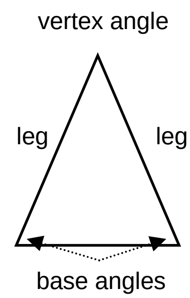
U

- **undefined term –** words, usually readily understood, that are not formally explained by means of more basic words and concepts; the basic undefined terms of geometry are *point*, *line* and *plane*
- **unimodal –** having only one mode, or "hump"

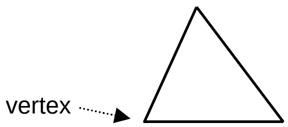


V

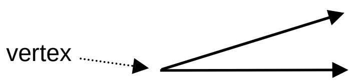
- **valid –** logically correct
- **variable –** a quantity that can have any of a set of values
- **variable expenses –** expenses that occur regularly, but can be controlled
- **variance –** the mean of the squares of the deviations for a set of data; variance is denoted by σ^2 for a population and s^2 for a sample
- **vertex angle of an isosceles triangle –** the angle formed by the two legs of an isosceles triangle



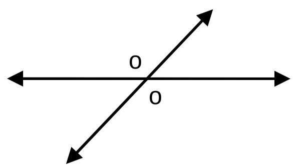
- **vertex of a polygon –** the vertex of each angle of a polygon



- **vertex of an angle –** the common endpoint of an angle



- **vertical angles –** two nonadjacent angles formed by two intersecting lines



- **voluntary-response sample –** a sample in which the participation is at the discretion or initiative of the respondent

W

- **weighted mean –** a measure of central tendency that reflects the greater significance of certain data

$$\bar{x} = \frac{\sum w_i x_i}{\sum w_i}$$

Z

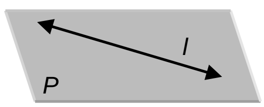
- ***z*-score –** the number of standard deviations from a data item to the mean; the *z*-score of a data item is given by the formula

$$z = \frac{x - \bar{x}}{s}$$

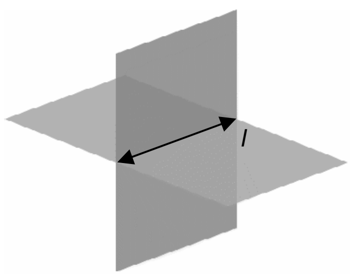
SOLUTIONS TO POSSIBLE ASSESSMENT STRATEGIES

SECTION 1.1

- a. *D*
  b. $\overleftrightarrow{BE}$, $\overleftrightarrow{BF}$, or $\overleftrightarrow{EF}$
  c. *A*
  d. plane P; any three non-collinear points on the plane is also correct
- a. line
  b. plane
- a.



  b.



- 5

SECTION 1.2

- a. $x = 3$; $XP = 10$
  b. $x = 4$; $XP = 9$
- a. congruent
  b. congruent
- 16 km
- 5 ft, 15 ft, 20 ft
- $B = 9,\ C = 12,\ D = 14,\ E = 17$

SECTION 1.3

- a. $\sqrt{65}$
  b. $2\sqrt{10}$
- a. $(3,1)$
  b. $\left(-2,\dfrac{5}{2}\right)$
- a. $(12,-8)$
  b. $(13,-11)$
- a. 8.5 blocks
  b. $(5,6)$

SECTION 1.4

- a. *D*
  b. $\angle ADG$, $\angle ADH$, or $\angle BDH$
  c. $\overrightarrow{GE}$ and $\overrightarrow{GI}$
  d. $\overrightarrow{GI}$
- $\angle A$ is obtuse, $\angle B$ is acute, $\angle C$ is right, and $\angle D$ is obtuse
- 35^0

SECTION 1.5

- a. $\angle CDB$ or $\angle FDG$
  b. $\angle BGD$ and $\angle HGI$, or $\angle BGH$ and $\angle DGI$
  c. $x = 20$
  d. 48^0
- 60^0

SECTION 1.6

- a. quadrilateral, convex, regular
  b. octagon, concave, irregular
- 22.7
- The fencing will enclose a square with side length of 25 m, giving an area of 625 m^2, and a circle with a radius of $\dfrac{50}{\pi}$ m (or 15.92 m) giving an area of 795.8 m^2. Therefore, the circular field will enclose a greater area.

SECTION 2.1

- a. true
  b. false; Any pair of perfect squares such that one square is even and one square is odd will produce an odd sum.
  c. false; If $n < 0$, the statement is false.
  d. true
  e. false; If *A*, *B*, and *C* are not collinear, then *B* is not the midpoint of $\overline{AC}$.

- a. Nova Scotia, Prince Edward Island, and Newfoundland and Labrador do not share a border with the United States.

  b. Birds such as ostriches and emus cannot fly.

  c. The even numbers 2, 6, 10, 14, … are not divisible by 4.

  d. If $n = 0$, the conjecture is not true.

SECTION 2.2

- a. The capital of Prince Edward Island is not Charlottetown, or there are two cities in Prince Edward Island. This is a true statement because the second part is true and it is a disjunction.

  b. The capital of Prince Edward Island is Charlottetown, and Prince Edward island is not the smallest province in Canada. This is a false statement because the second part is false and it is a conjunction.

  c. Prince Edward island is not the smallest province in Canada, or there are not two cities in Prince Edward Island. This is a false statement because both parts are false.

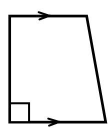
- a. 5 students

  b. 10 students

  c. 27 students

SECTION 2.3

- a. false; The quadrilateral could be a trapezoid, such as the one pictured below.



  b. false; The number zero has no reciprocal.

  c. true; Any point in the third quadrant is of the form $(\text{negative, negative})$.

  d. true; If a triangle is right, then one of its angles measures 90^0. Therefore, the sum of the measures of the other two angles is 90^0, which means that each of the other two angles must have a measure less than 90^0, which means they are acute. Therefore, there cannot be any obtuse angle in a right triangle.

- a. **Converse:** If it is cloudy, then it is raining. False; If it is cloudy, other forms of precipitation could be falling, or there may not be any precipitation falling.

  Inverse: If it is not raining, then it is not cloudy. False; If it is not raining, it can still be cloudy.

  Contrapositive: If it is not cloudy, then it is not raining. True; If it is not cloudy, no rain can fall.

  b. **Converse:** If the absolute value of a real number is positive, then the real number is positive. False; This statement is false for any negative number or zero.

  Inverse: If a real number is not positive, then its absolute value is not positive. False; This statement is false for negative numbers.

  Contrapositive: If the absolute value of a real number is not positive, then the real number is not positive. True; If the absolute value of a real number is not positive, then the number must be zero. The absolute value of zero is zero, which is not positive.

  c. **Converse:** If $b^2 > 1$, then $b > 1$. False; If $b < -1$, then the hypothesis is true, but the conclusion is false.

  Inverse: If $b \le 1$, then $b^2 \le 1$. False; If $b < -1$, then the hypothesis is true, but the conclusion is false.

  Contrapositive: If $b^2 \le 1$, then $b \le 1$. True; If $b^2 \le 1$, then $-1 \le b \le 1$, which is within the solution set of $b \le 1$.

  d. **Converse:** If a right triangle has two angles measuring 45^0, then it is isosceles. True; Since this triangle has two congruent angles, it would have two equal sides, making it isosceles.

  Inverse: If a right triangle is not isosceles, then it does not have two angles measuring 45^0. True; If a right triangle is not isosceles, then it must be scalene, since it cannot be equilateral. Scalene triangles have three angles which are all different, therefore the conclusion is true.

  Contrapositive: If a right triangle does not have two angles measuring 45^0, then it is not isosceles. True; The sum of the measures of the two non-right angles in a right triangle is 90^0. Since these angles cannot measure 45^0, these two acute angles cannot be congruent. Therefore, the measures of the three angles of the triangle must all be different, which means that the triangle cannot be isosceles.

SECTION 2.4

- a. Polygon *ABCDE* is a pentagon. Law of Detachment
  b. no valid conclusion; Even though Johnny passed the test, he may have studied for a longer or a shorter period of time.
  c. School will be cancelled. Law of Syllogism
- not valid; No mention is made of whether she had any marks below 75%.
- John will get a free milk.

SECTION 2.5

- a. sometimes true; Parallel lines and skew lines do not intersect.
  b. never true; Postulate 2.1
  c. always true; Two angles form a linear pair if their measures have a sum of 180^0, and two angles are supplementary if their measures have a sum of 180^0.
- If $\overline{AB} \cong \overline{BC}$, then line segments $\overline{AB}$ and $\overline{BC}$ have the same length. We can represent that length by *x*. Since *A*, *B* and *C* are collinear, we can model the points on a number line. If point *A* is at coordinate 0, then point *B* must be at coordinate *x* and point *C* at 2*x*. Using the midpoint formula, the midpoint of $\overline{AC}$ can be found by calculating $\frac{0+2x}{2} = x$, which is the coordinate of point *B*.

SECTION 2.6

- a. Symmetric Property of Equality
  b. Subtraction Property of Equality
  c. Distributive Property
  d. Reflexive Property of Equality
  e. Transitive Property of Equality
-

Statement	Justification
$9(x-1) = 27$	Given
$9x - 9 = 27$	Distributive Property
$9x = 36$	Addition Property of Equality
$x = 4$	Division Property of Equality

-

Statement	Justification
$AB = AC$	Given
$AB = 8x - 7$	
$AC = 7x - 5$	
$8x - 7 = 7x - 5$	Substitution Property of Equality
$x - 7 = -5$	Subtraction Property of Equality
$x = 2$	Addition Property of Equality

- Transitive Property of Equality

SECTION 2.7

-

Statement	Justification
B is the midpoint of $\overline{AC}$	Given
C is the midpoint of $\overline{BD}$	
D is the midpoint of $\overline{CE}$	
$\overline{AB} \cong \overline{BC}$	Midpoint Theorem
$\overline{BC} \cong \overline{CD}$	
$\overline{CD} \cong \overline{DE}$	
$\overline{AB} \cong \overline{CD}$	Transitive Property of Congruence
$\overline{AB} \cong \overline{DE}$	Transitive Property of Congruence

•

Statement	Justification
$\overline{AC} \cong \overline{BD}$	Given
$AC = BD$	Definition of Congruent Segments
$AC = AB + BC$	Segment Addition Postulate
$BD = BC + BD$	
$AB + BC = BC + CD$	Substitution Property of Equality
$AB = CD$	Subtraction Property of Equality
$\overline{AB} \cong \overline{CD}$	Definition of Congruent Segments

SECTION 2.8

- a. 60^0
  b. 30^0
  c. 150^0
- a.

Statement	Justification
$\angle AEC \cong \angle BED$	Given
$m\angle AEC = m\angle BED$	Definition of Congruent Angles
$m\angle AEC = m\angle AEB + m\angle BEC$	Angle Addition Postulate
$m\angle BED = m\angle BEC + m\angle CED$	
$m\angle AEB + m\angle BEC = m\angle BEC + m\angle CED$	Substitution Property of Equality
$m\angle AEB = m\angle CED$	Subtraction Property of Equality
$\angle AEB \cong \angle CED$	Definition of Congruent Angles

b.

Statement	Justification
$\angle 2$ is a right angle	Given
$m\angle 2 = 90^0$	Definition of Right Angle
$\angle 2 \cong \angle 3$	Vertical Angles Theorem
$m\angle 2 = m\angle 3$	Definition of Congruent Angles
$m\angle 3 = 90^0$	Transitive Property of Equality
$\angle 3$ is a right angle	Definition of Right Angle

SECTION 3.1

- a. corresponding angles
  b. alternate exterior angles
  c. alternate interior angles
  d. consecutive angles

SECTION 3.2

- a. 58^0
  b. 60^0
  c. 62^0
  d. 62^0
  e. 60^0
  f. 58^0
  g. 118^0
  h. 62^0
  i. 58^0
  j. 122^0
  k. 118^0
  l. 122^0

•

Statement	Justification
$m \parallel n$	Given
l is a transversal	
$m\angle 1 + m\angle 7 = 180^0$	Definition of Linear Pair
$m\angle 3 + m\angle 8 = 180^0$	
$\angle 2 \cong \angle 7$	Corresponding Angles Postulate
$\angle 4 \cong \angle 8$	
$m\angle 2 \cong m\angle 7$	Definition of Congruent Angles
$m\angle 4 \cong m\angle 8$	
$m\angle 1 + m\angle 2 = 180^0$	Substitution Property of Equality
$m\angle 3 + m\angle 4 = 180^0$	
$\angle 1$ and $\angle 2$ are supplementary	Definition of Supplementary Angles
$\angle 3$ and $\angle 4$ are supplementary	

SECTION 3.5

- $x = 34$
- a.

Statement	Justification
$\overline{AC} \parallel \overline{BD}$	Given
$\angle 2 \cong \angle 3$	
$\angle 1 \cong \angle 2$	Corresponding Angles Postulate
$\angle 1 \cong \angle 3$	Transitive Property of Equality
$AB \parallel CD$	Alternate Interior Angles Converse

b.

Statement	Justification
$\angle JLK \cong \angle LKM$	Given
$\overline{JK} \perp \overline{JL}$	
$m\angle LJK = 90^0$	Definition of Perpendicular Lines
$\overline{JL} \parallel \overline{KM}$	Alternate Interior Angles Theorem
$\angle LJK$ and $\angle JKM$ are supplementary angles	Consecutive Interior Angles Theorem
$m\angle LJK + m\angle JKM = 180^0$	Definition of Supplementary Angles
$90^0 + m\angle JKM = 180^0$	Substitution Property of Equality
$m\angle JKM = 90^0$	Subtraction Property of Equality
$\overline{JK} \perp \overline{KM}$	Definition of Perpendicular Lines

SECTION 4.1

- a. acute, scalene
  b. acute or equiangular, equilateral
  c. obtuse, isosceles
- a. $x = 5$

SECTION 4.2

- a. 39^0
  b. 141^0
  c. 39^0
  d. 82^0
- a. $x = 36$; 36^0, 72^0, 72^0
  b. $x = 5$; 22^0, 68^0, 90^0

SECTION 4.3

- $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$
  $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$
  $\Delta ABC \cong \Delta DEF$

- a.

Statement	Justification
$\angle B \cong \angle D$	Given
$\overline{AC} \perp \overline{BD}$	
$\angle ACB$ and $\angle ACD$ are right angles	Definition of Perpendicular Lines
$m\angle ACB = 90^0$	Definition of Right Angles
$m\angle ACD = 90^0$	
$m\angle ACB = m\angle ACD$	Transitive Property of Equality
$\angle ACB \cong \angle ACD$	Definition of Congruent Angles
$\angle BAC \cong \angle DAC$	Third Angles Theorem

SECTION 4.4

- $AB = \sqrt{3^2 + 0^2} = \sqrt{9+0} = \sqrt{9} = 3$

  $AC = \sqrt{0^2 + 4^2} = \sqrt{0+16} = \sqrt{16} = 4$

  $BC = \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$

  $ZX = \sqrt{(-3)^2 + 0^2} = \sqrt{9+0} = \sqrt{9} = 3$

  $ZY = \sqrt{0^2 + (-4)^2} = \sqrt{0+16} = \sqrt{16} = 4$

  $XY = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

  Since $AB = ZX$, $AC = ZY$, and $BC = XY$, we can see that corresponding sides are congruent. As a result, $\Delta ABC \cong \Delta ZXY$ by the SSS Congruence Postulate.
- a.

Statement	Justification
$\overline{AB} \cong \overline{DC}$	Given
$\overline{BD} \cong \overline{CA}$	
$\overline{AD} \cong \overline{DA}$	Reflexive Property of Congruence
$\Delta ABD \cong \Delta DCA$	SSS Congruence Postulate

b.

Statement	Justification
$\overline{KL} \cong \overline{NM}$	Given
$\angle KLM \cong \angle NML$	
$\overline{LM} \cong \overline{ML}$	Reflexive Property of Congruence
$\Delta KLM \cong \Delta NML$	SAS Congruence Postulate

SECTION 4.5

- a.

Statement	Justification
$\angle BAD \cong \angle CDA$	Given
$\angle BDA \cong \angle CAD$	
$\overline{AD} \cong \overline{DA}$	Reflexive Property of Congruence
$\Delta ABD \cong \Delta DCA$	ASA Congruence Postulate

b.

Statement	Justification
$\angle A \cong \angle D$	Given
$\overline{AB} \perp \overline{BC}$	
$\overline{AB} \parallel \overline{DC}$	
$\angle ABC$ is a right angle	Definition of Perpendicular Lines
$m\angle ABC = 90^0$	Definition of Right Angle
$\angle ABC$ and $\angle DCB$ are supplementary angles	Consecutive Interior Angles Theorem
$m\angle ABC + m\angle DCB = 180^0$	Definition of Supplementary Angles
$90^0 + m\angle DCB = 180^0$	Substitution Property of Equality
$m\angle DCB = 90^0$	Subtraction Property of Equality
$m\angle ABC = m\angle DCB$	Transitive Property of Equality
$\angle ABC \cong \angle DCB$	Definition of Congruent Angles
$\overline{BC} \cong \overline{CB}$	Reflexive Property of Congruence
$\Delta ABC \cong \Delta DCB$	AAS Congruence Theorem

SECTION 4.6

- a. $x = 5$

  b. $x = 74$

-

Statement	Justification
$m\angle ACD = 110^0$	Given
$m\angle B = 40^0$	
$\angle ACB$ and $\angle ACD$ form a linear pair	Definition of Linear Pair
$m\angle ACB + m\angle ACD = 180^0$	Definition of Linear Pair
$m\angle ACB + 110^0 = 180^0$	Substitution Property of Equality
$m\angle ACB = 70^0$	Subtraction Property of Equality
$m\angle A + m\angle B + m\angle ACB = 180^0$	Triangle Angle-Sum Theorem
$m\angle A + 40^0 + 70^0 = 180^0$	Substitution Property of Equality
$m\angle A = 70^0$	Subtraction Property of Equality
$\overline{AB} \cong \overline{BC}$	Converse of Isosceles Triangle Theorem
ΔABC is isosceles	Definition of Isosceles Triangle

SECTION 5.4

- a. Assume that in a group of 13 people, no two people were born in the same month.

  b. Assume that there is a greatest whole number.

  c. Assume that the sum of two odd integers is an odd number.

  d. Assume that there are two perpendiculars to a line that pass through a point not on the line.

- a. Assume that at least one of two positive integers, x and y, is even. If we let x be the even integer, then it can be represented by $2k$, where k is an integer. The product of x and y will be $2ky$, which is a multiple of 2, which means that the product is even. Since the assumption that at least one of two positive integers, x and y, is even leads to a contradiction of the given statement, the original conclusion that both x and y are odd numbers must be true.

  b. Assume that a is a negative number. Then $\frac{1}{a}$, which will be a positive number divided by a negative number, will be negative, that is $\frac{1}{a} < 0$. Since the assumption that a is a negative number leads to a contradiction of the given statement, the original conclusion that a must be positive must be true.

  c. Assume that the sum of the acute angles in a right triangle is not 90^0. The sum of the measures of three angles in a triangle is 180^0, and the non-acute angle in a right triangle is a right angle, with a measure of 90^0. By subtracting the measure of the right angle from the sum of the measures of the three angles of a triangle, we get 90^0 for the sum of the other two acute angles. Since the assumption that the sum of the acute angles in a right triangle is not 90^0 leads to a contradiction of the given statement, the original conclusion that the sum of the acute angles in a right triangle is 90^0 must be true.

  d. Assume that a triangle contains two obtuse angles. An obtuse angle has a measure between 90^0 and 180^0. Two obtuse angles would have measures with a total between 180^0 and 360^0. This contradicts the fact that the sum of the measures of the three angles in a triangle is 180^0, therefore the original conclusion that a triangle cannot contain two obtuse angles must be true.

SECTION 2.1

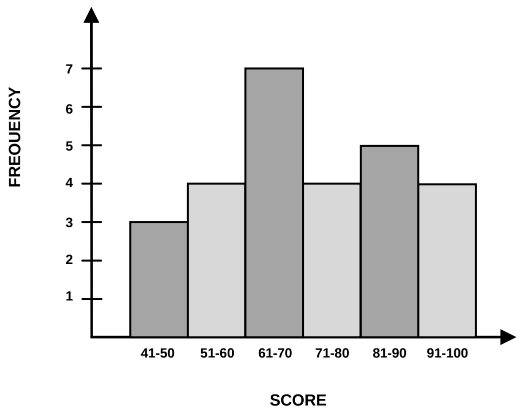
- a. 55

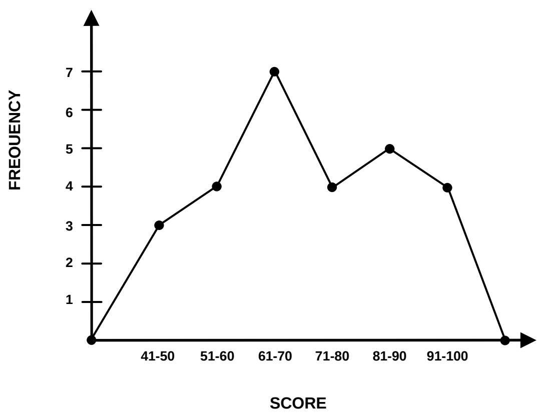
  b. Answers may vary. A reasonable interval size would be 10 with 6 intervals.

c.

SCORE	FREQUENCY
41 – 50	3
51 – 60	4
61 – 70	7
71 – 80	4
81 – 90	5
91 – 100	4

d.

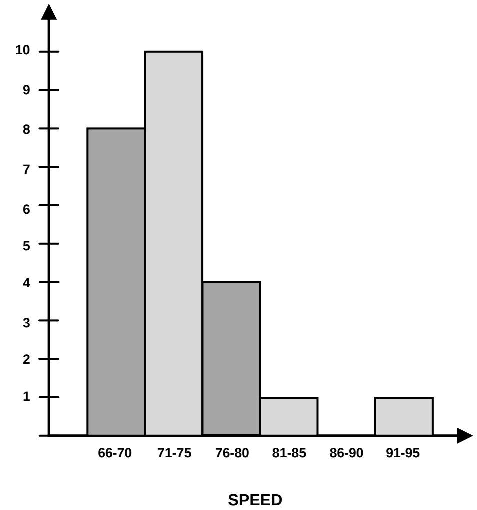


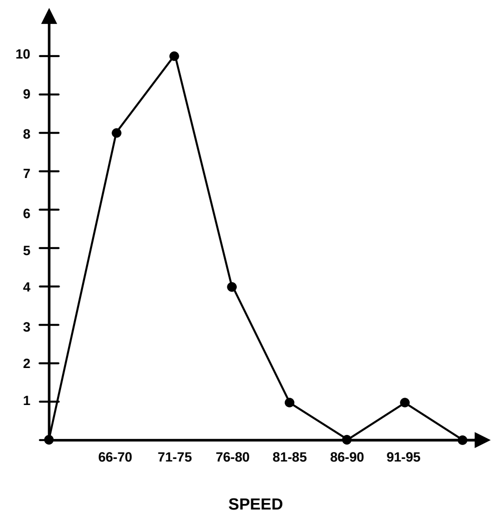


- a.

SCORE	FREQUENCY
66 – 70	8
71 – 75	10
76 – 80	4
81 – 85	1
86 – 90	0
91 – 95	1

b.





c. 18 motorists

d. 2 motorists

SECTION 2.3

- a. voluntary-response sample
  b. stratified sample
  c. convenience sample
  d. systematic sample

SECTION 2.5

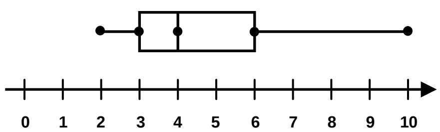
- mean: 7.7; median: 8; mode: 7
- 75
- All three quantities increased by 5%.
- 86%

SECTION 2.6

- a. mean: 7.9; variance: 3.1; standard deviation: 1.7

  b. mean: 26.7; variance: 50.1; standard deviation: 7.1
- a. median: 4; first quartile: 3; third quartile: 6; interquartile range: 3; semi-quartile range: 1.5

  b.



- 0.40

SECTION 3.1

- a. positive correlation

  b. negative correlation

  c. no correlation
- $r = 0.73$; There is a strong positive correlation between the number of hours of study and the test mark.

SECTION 3.2

- a. $y = 0.98x + 2.47$, where x represents the MAT521B mark, and y represents the MAT621B mark.

  b. 91

SECTION 6.1

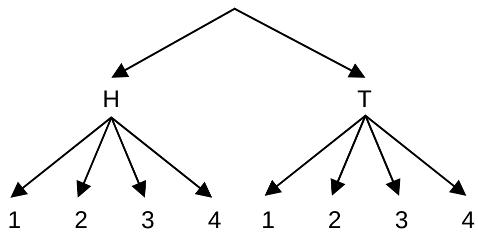
- a. $\frac{3}{8}$

  b. $\frac{1}{2}$
- a. $\frac{1}{6}$

  b. $\frac{1}{2}$

  c. $\frac{2}{3}$
- a. $\frac{1}{4}$

  b. $\frac{1}{3}$
- a.



$$\left\{ \begin{matrix} (H,1),\ (H,2),\ (H,3),\ (H,4) \\ (T,1),\ (T,2),\ (T,3),\ (T,4) \end{matrix} \right\}$$

  b. $\frac{1}{4}$
- $\frac{89}{100}$

SECTION 6.2

- a. odds in favour: $1:6$; odds against: $6:1$

  b. odds in favour: $1:7$; odds against: $7:1$

  c. odds in favour: $3:10$; odds against: $10:3$

  d. odds in favour: $1:1$; odds against: $1:1$

  e. odds in favour: $1:5$; odds against: $5:1$

  f. odds in favour: $1:13{,}983{,}815$; odds against: $13{,}983{,}815:1$
- $\frac{3}{7}$
- a. $3:7$

  b. $7:3$

SECTION 6.4

- a. $\frac{1}{6}$

  b. $\frac{1}{6}$

  c. $\frac{1}{7776}$

  d. In part (b), since the first five rolls do not affect the sixth roll, the sample space is $\{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$. In part (c), each element of the sample space consists of 5 parts, one for each roll, with each part having 6 possibilities, giving 7776 elements in the sample space. Therefore, the probabilities are different.

- $\frac{1}{4}$
- 0.48
- $\frac{11}{221}$
- a. $\frac{4}{25}$

  b. $\frac{3}{10}$

SECTION 6.5

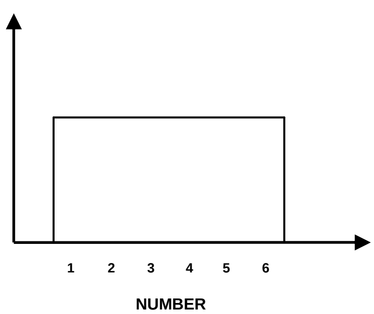
- a. mutually exclusive

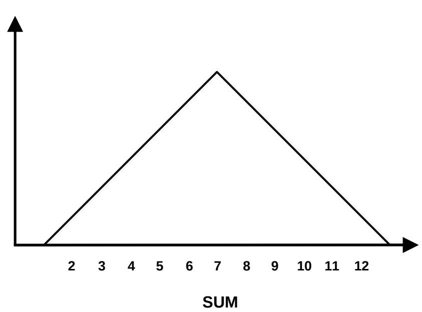
  b. not mutually exclusive

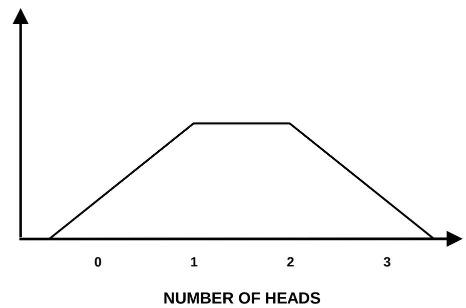
  c. not mutually exclusive

  d. mutually exclusive
- $\frac{2}{3}$
- $\frac{2}{3}$
- $\frac{7}{13}$
- $\frac{10}{13}$
- 0.16

SECTION 8.1

- 

- 

- 

SECTION 8.2

- a. 71.57%

  b. 10.75%

  c. 84.46%
- a. –0.67

  b. 0.52

  c. 1.41
- a. 68.26%

  b. 47.72%

  c. 49.38%

  d. 15.87%
- 2.28%
- a. 0.6915

  b. 159.3 cm to 190.7 cm
- 0.0668

SECTION 8.3

- a. mean: 7.3; standard deviation: 6.7

  b. 0.0287

  c. 0.3669

SECTION 9.1

- $235
- $11.75
- $1500
- $400.20
- $2476
- $753.75
- $2000

SECTION 9.2

- a. $24.88

  b. $9.86
- $694.69

- $676.81

SECTION 9.3

- a. $3041.63
  b. $1454.80
  c. $263.85
- a. $1246.18
  b. $1250.75
  c. $1251.80
  d. $1252.31
- $6158.31

SECTION 9.4

- 4.91%
- Option A pays $10,613.64 in its first year and Option B pays $10,629.61 in its first year. Option B is better.

SECTION 9.5

- a. $1200
  b. $300
  c. $1800
- $6.75

SECTION 9.6

- a. $40,000
  b. $1024
- $1594.18
- $1101.02

SECTION 9.7

- $940

APPENDIX

MATHEMATICS RESEARCH PROJECT

Introduction

A research project can be a very important part of a mathematics education. Besides the greatly increased learning intensity that comes from personal involvement with a project, and the chance to show universities, colleges, and potential employers the ability to initiate and carry out a complex task, it gives the student an introduction to mathematics as it is – a living and developing intellectual discipline where progress is achieved by the interplay of individual creativity and collective knowledge. A major research project must successfully pass through several stages. Over the next few pages, these stages are highlighted, together with some ideas that students may find useful when working through such a project.

Creating an Action Plan

As previously mentioned, a major research project must successfully pass though several stages. The following is an outline for an action plan, with a list of these stages, a suggested time, and space for students to include a probable time to complete each stage.

STAGE	SUGGESTED TIME	PROBABLE TIME
Select the topic to explore.	1 – 3 days	
Create the research question to be answered.	1 – 3 days	
Collect the data.	5 – 10 days	
Analyse the data.	5 – 10 days	
Create an outline for the presentation.	2 – 4 days	
Prepare a first draft.	3 – 10 days	
Revise, edit and proofread.	3 – 5 days	
Prepare and practise the presentation.	3 – 5 days	

Completing this action plan will help students organize their time and give them goals and deadlines that they can manage. The times that are suggested for each stage are only a guide. Students can adjust the time that they will spend on each stage to match the scope of their projects. For example, a project based on primary data (data that they collect) will usually require more time than a project based on secondary data (data that other people have collected and published). A student will also need to consider his or her personal situation – the issues that each student deals with that may interfere with the completion of his or her project. Examples of these issues may include:

- a part-time job;
- after-school sports and activities;
- regular homework;
- assignments for other courses;
- tests in other courses;
- time they spend with friends;
- family commitments;
- access to research sources and technology.

➢ Selecting the Research Topic

To decide what to research, a student can start by thinking about a subject and then consider specific topics. Some examples of subjects and topics may be:

SUBJECT	TOPIC
Entertainment	• effects of new electronic devices • file sharing
Health care	• doctor and/or nurse shortages • funding
Post-secondary education	• entry requirements • graduate success
History of Western and Northern Canada	• relations among First Nations • immigration

It is important to take the time to consider several topics carefully before selecting a topic for a research project. The following questions will help a student determine if a topic that is being considered is suitable.

- **Does the topic interest the student?**

  Students will be more successful they choose a topic that interests them. They will be more motivated to do research, and they will be more attentive while doing the research. As well, they will care more about the conclusions they make.

- **Is the topic practical to research?**

  If a student decides to use first-hand data, can the data be generated in the time available, with the resources available? If a student decides to use second-hand data, are there multiple sources of data? Are the sources reliable, and can they be accessed in a timely manner?

- **Is there an important issue related to the topic?**

  A student should think about the issues related to the topic that he or she has chosen. If there are many viewpoints on an issue, they may be able to gather data that support some viewpoints but not others. The data they collect from all viewpoints should enable them to come to a reasoned conclusion.

- **Will the audience appreciate the presentation?**

  The topic should be interesting to the intended audience. Students should avoid topics that may offend some members of their audience.

➢ Creating the Research Question or Statement

A well-written research question or statement clarifies exactly what the project is designed to do. It should have the following characteristics:

- The research topic is easily identifiable.

- The purpose of the research is clear.
- The question or statement is focused. The people who are listening to or reading the question or statement will know what the student is going to be researching.

A good question or statement requires thought and planning. Below are three examples of initial questions or statements and how they were improved.

UNACCEPTABLE QUESTION OR STATEMENT	WHY?	ACCEPTABLE QUESTION OR STATEMENT
Is mathematics used in computer technology?	Too general	What role has mathematics played in the development of computer animation?
Water is a shared resource.	Too general	Homes, farms, ranches, and businesses east of the Rockies all use runoff water. When there is a shortage, that water must be shared.
Do driver's education programs help teenagers parallel park?	Too specific, unless the student is generating his or her own data	Do driver's education programs reduce the incidence of parking accidents?

The following checklist can be used to determine if the research question or statement is effective.

- Does the question or statement clearly identify the main objective of the research? After the question or statement is read to someone, can they tell what the student will be researching?
- Is the student confident that the question or statement will lead him or her to sufficient data to reach a conclusion?
- Is the question or statement interesting? Does it make the student want to learn more?
- Is the topic that the student chose purely factual, or is that student likely to encounter an issue, with different points of view?

➢ Carrying Out the Research

As students continue with their projects, they will need to conduct research and collect data. The strategies that follow will help them in their data collection.

There are two types of data that students will need to consider – primary and secondary. Primary data is data that the student collects himself or herself using surveys, interviews and direct observations. Secondary data is data that the student obtains through other sources, such as online publications, journals, magazines, and newspapers.

Both primary and secondary data have their advantages and disadvantages. Primary data provide specific information about the research question or statement, but may take time to collect and process. Secondary data is usually easier to obtain and can be analysed in less time. However, because the data was originally gathered for other purposes, a student may need to sift through it to find exactly what he or she is looking for.

The type of data chosen can depend on many factors, including the research question, the skills of the student, and available time and resources. Based on these and other factors, the student may chose to use primary data, secondary data, or both.

When collecting primary data, the student must ensure the following:

- For surveys, the sample size must be reasonably large and the random sampling technique must be well designed.
- For surveys and interviews, the questionnaires must be designed to avoid bias.
- For experiments and studies, the data must be free from measurement bias.
- The data must be compiled accurately.

When collecting secondary data, the student should explore a variety of resources, such as:

- textbooks, and other reference books;
- scientific and historical journals, and other expert publications;
- the Internet;
- library databases.

After collecting the secondary data, the student must ensure that the source of the data is reliable:

- If the data is from a report, determine what the author’s credentials are, how recent the data is, and whether other researchers have cited the same data.
- Be aware that data collection is often funded by an organization with an interest in the outcome or with an agenda that it is trying to further. Knowing which organization has funded the data collection may help the student decide how reliable the data is, or what type of bias may have influenced the collection or presentation of the data.
- If the data is from the Internet, check it against the following criteria:
    - authority – the credentials of the author should be provided;
    - accuracy – the domain of the web address may help the student determine the accuracy;
    - currency – the information is probably being accurately managed if pages on a site are updated regularly and links are valid.

➢ Analysing the Data

Statistical tools can help a student analyse and interpret the data that is collected. Students need to think carefully about which statistical tools to use and when to use them, because other people will be scrutinizing the data. A summary of relevant tools follows.

Measures of central tendency will give information about which values are representative of the entire set of data. Selecting which measure of central tendency (mean, median, or mode) to use depends on the distribution of the data. As the researcher, the student must decide which measure most accurately describes the tendencies of the population. The following criteria should be considered when deciding upon which measure of central tendency best describes a set of data.

- Outliers affect the mean the most. If the data includes outliers, the student should use the median to avoid misrepresenting the data. If the student chooses to use the mean, the outliers should be removed before calculating the mean.

- If the distribution of the data is not symmetrical, but instead strongly skewed, the median may best represent the set of data.
- If the distribution of the data is roughly symmetrical, the mean and median will be close, so either may be appropriate to use.
- If the data is not numeric (for example, colour), or if the frequency of the data is more important than the values, use the mode.

Both the range and the standard deviation will give the student information about the distribution of data in a set. The range of a set of data changes considerably because of outliers. The disadvantage of using the range is that it does not show where most of the data in a set lies – it only shows the spread between the highest and the lowest values. The range is an informative tool that can be used to supplement other measures, such as standard deviation, but it is rarely used as the only measure of dispersion.

Standard deviation is the measure of dispersion that is most commonly used in statistical analysis when the mean is used to calculate central tendency. It measures the spread relative to the mean for most of the data in the set. Outliers can affect standard deviation significantly. Standard deviation is a very useful measure of dispersion for symmetrical distributions with no outliers. Standard deviation helps with comparing the spread of two sets of data that have approximately the same mean. For example, the set of data with the smaller standard deviation has a narrower spread of measurement around the mean, and therefore has comparatively fewer high or low scores, than a set of data with a higher standard deviation.

When working with several sets of data that approximate normal distributions, you can use *z*-scores to compare the data values. A *z*-score table enables a student to find the area under a normal distribution curve with a mean of zero and a standard deviation of one. To determine the *z*-score for any data value in a set that is normally distributed, the formula $z = \frac{x - \bar{x}}{s}$ can be used where x is any observed data value in the set, $\bar{x}$ is the mean of the set, and is s is the standard deviation of the set.

When analysing the results of a survey, a student may need to interpret and explain the significance of some additional statistics. Most surveys and polls draw their conclusions from a sample of a larger group. The margin of error and the confidence level indicate how well a sample represents a larger group. For example, a survey may have a margin of error of plus or minus 3% at a 95% level of confidence. This means that if the survey were conducted 100 times, the data would be within 3 percent points above or below the reported results in 95 of the 100 surveys.

The size of the sample that is used for a poll affects the margin of error. If a student is collecting data, he or she must consider the size of the sample that is needed for a desired margin of error.

➢ Identifying Controversial Issues

While working on a research project, a student may uncover some issues on which people disagree. To decide on how to present an issue fairly, he or she should consider some questions to ask as the research proceeds.

- **What is the issue about?**

  The student should identify which type of controversy has been uncovered. Almost all controversy revolves around one or more of the following:
  - Values – What should be? What is best?
  - Information – What is the truth? What is a reasonable interpretation?

  - Concepts – What does this mean? What are the implications?

- **What positions are being taken on the issue?**

  The student should determine what is being said and whether there is reasonable support for the claims being made. Some questions to ask are:
  - Would you like that done to you?
  - Is the claim based on a value that is generally shared?
  - Is there adequate information?
  - Are the claims in the information accurate?
  - Are those taking various positions on the issue all using the same term definitions?

- **What is being assumed?**

  Faulty assumptions reduce legitimacy. The student can ask:
  - What are the assumptions behind an argument?
  - Is the position based on prejudice or an attitude contrary to universally held human values, such as those set out in the United Nations Declaration of Human Rights?
  - Is the person who is presenting a position or an opinion an insider or an outsider?

- **What are the interests of those taking positions?**

  The student should try to determine the motivations of those taking positions on the issue. What are their reasons for taking their positions? The degree to which the parties involved are acting in self-interest could affect the legitimacy of their opinions.

➢ The Final Product and Presentation

The final presentation should be more than just a factual written report of the information gathered from the research. To make the most of the student's hard work, he or she should select a format for the final presentation that suits his or her strengths, as well as the topic.

To make the presentation interesting, a format should be chosen that suits the student's style. Some examples are:

- a report on an experiment or an investigation;
- a summary of a newspaper article or a case study;
- a short story, musical performance, or play;
- a web page;
- a slide show, multimedia presentation, or video;
- a debate;
- an advertising campaign or pamphlet;
- a demonstration or the teaching of a lesson.

Sometimes, it is also effective to give the audience an executive summary of the presentation. This is a one-page summary of the presentation that includes the research question and the conclusions that were made.

Before giving the presentation, the student can use these questions to decide if the presentation will be effective.

- Did I define my topic well? What is the best way to define my topic?
- Is my presentation focused? Will my classmates find it focused?
- Did I organize my information effectively? Is it obvious that I am following a plan in my presentation?
- Am I satisfied with my presentation? What might make it more effective?
- What unanswered questions might my audience have?

➢ Peer Critiquing of Research Projects

After the student has completed his or her research for the question or statement being studied, and the report and presentation have been delivered, it is time to see and hear the research projects developed by other students. However, rather than being a passive observer, the student should have an important role – to provide feedback to his or her peers about their projects and presentations.

Critiquing a project does not involve commenting on what might have been or should have been. It involves reacting to what is seen and heard. In particular, the student should pay attention to:

- strengths and weaknesses of the presentation;
- problems or concerns with the presentation.

While observing each presentation, students should consider the content, the organization, and the delivery. They should take notes during the presentation, using the following rating scales as a guide. These rating scales can also be used to assess the presentation.

Content

Shows a clear sense of audience and purpose.	1	2	3	4	5
Demonstrates a thorough understanding of the topic.	1	2	3	4	5
Clearly and concisely explains ideas.	1	2	3	4	5
Applies knowledge and skills developed in this course.	1	2	3	4	5
Justifies conclusions with sound reasoning.	1	2	3	4	5
Uses vocabulary, symbols and diagrams correctly.	1	2	3	4	5

Organization

Presentation is clearly focused.	1	2	3	4	5
Engaging introduction includes the research question, clearly stated.	1	2	3	4	5
Key ideas and information are logically presented.	1	2	3	4	5
There are effective transitions between ideas and information.	1	2	3	4	5
Conclusion follows logically from the analysis and relates to the question.	1	2	3	4	5

Delivery

Speaking voice is clear, relaxed, and audible.	1	2	3	4	5
Pacing is appropriate and effective for the allotted time.	1	2	3	4	5
Technology is used effectively.	1	2	3	4	5
Visuals and handouts are easily understood.	1	2	3	4	5
Responses to audience's questions show a thorough understanding of the topic.	1	2	3	4	5

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