



Prince Edward Island Mathematics Curriculum

Education and Early
Childhood Development
English Programs

Mathematics

Grade 8

CURRICULUM



2009
Prince Edward Island
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Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base. Developers have examined the curriculum proposed throughout Canada and secured the latest research in the teaching of mathematics, and the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for K-9 Mathematics* (2006) has been adopted as the basis for a revised mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories, and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP, and on the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➤ Essential Graduation Learnings

Essential graduation learnings (EGLs) are statements describing the knowledge, skills, and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills, and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work, and study today and in the future. Essential graduation learnings are cross curricular, and curriculum in all subject areas is focussed to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will demonstrate knowledge, skills, and attitudes expressed as essential graduation learnings, and will be expected to

- respond with critical awareness to various forms of the arts, and be able to express themselves through the arts;
- assess social, cultural, economic, and environmental interdependence in a local and global context;
- use the listening, viewing, speaking, and writing modes of language(s), and mathematical and scientific concepts and symbols, to think, learn, and communicate effectively;
- continue to learn and to pursue an active, healthy lifestyle;
- use the strategies and processes needed to solve a wide variety of problems, including those requiring language and mathematical and scientific concepts;
- use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

➤ **Curriculum Focus**

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate to all educational partners high expectations for students in mathematics education. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM, *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning;
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will

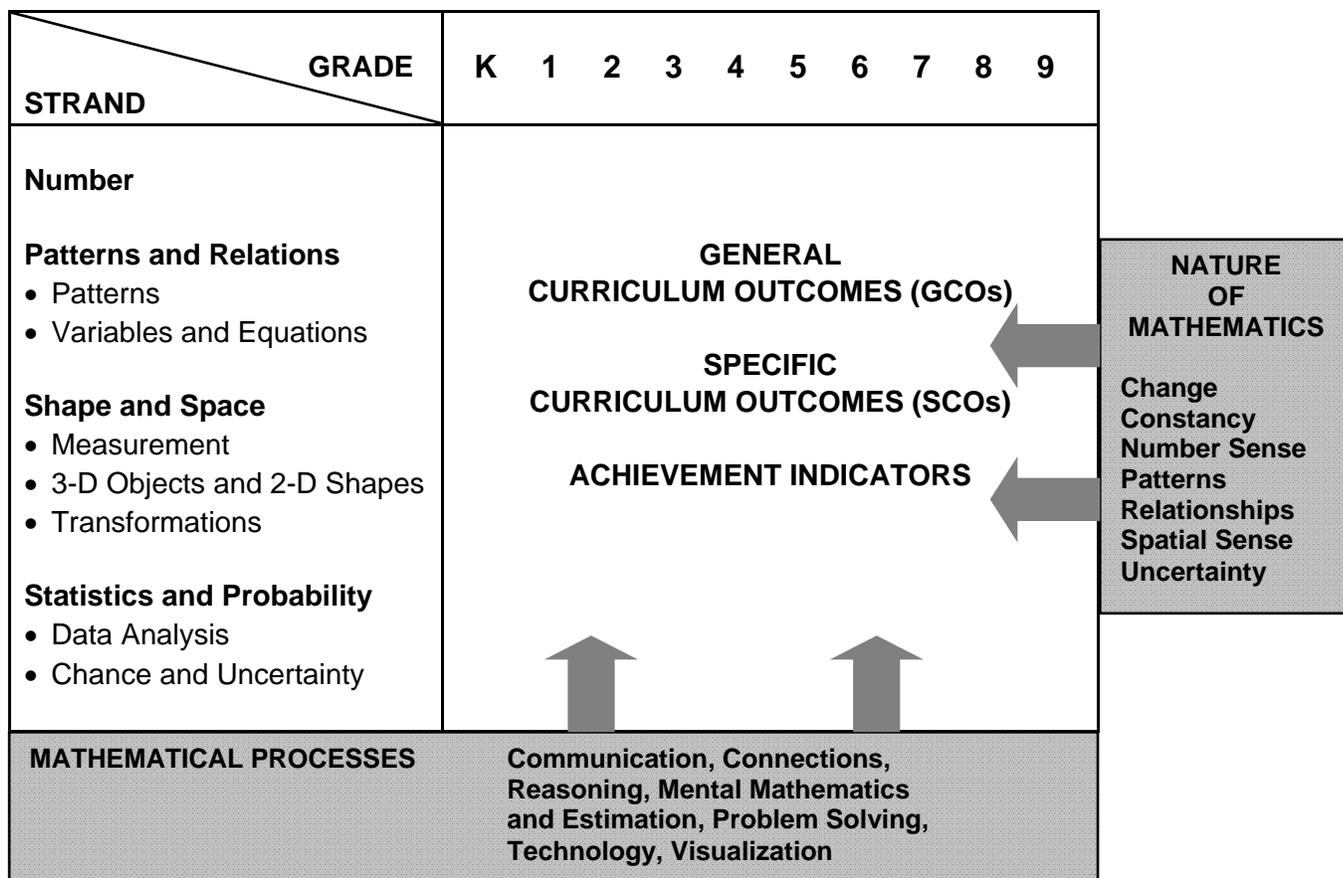
- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks;
- exhibit curiosity.

➤ **Connections across the Curriculum**

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the understanding of mathematical concepts by students and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, physical education, and other subject areas. Efforts should be made to make connections and use examples drawn from a variety of disciplines.

Conceptual Framework for K-9 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into four strands, namely Number, Patterns and Relations, Shape and Space, and Statistics and Probability. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connections among concepts both within and across strands. Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- Decreasing emphasis on rote calculation, drill, and practice, and the size of numbers used in paper and pencil calculations makes more time available for concept development.
- Problem solving, reasoning, and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➤ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]**
- develop visualization skills to assist in processing information, making connections, and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

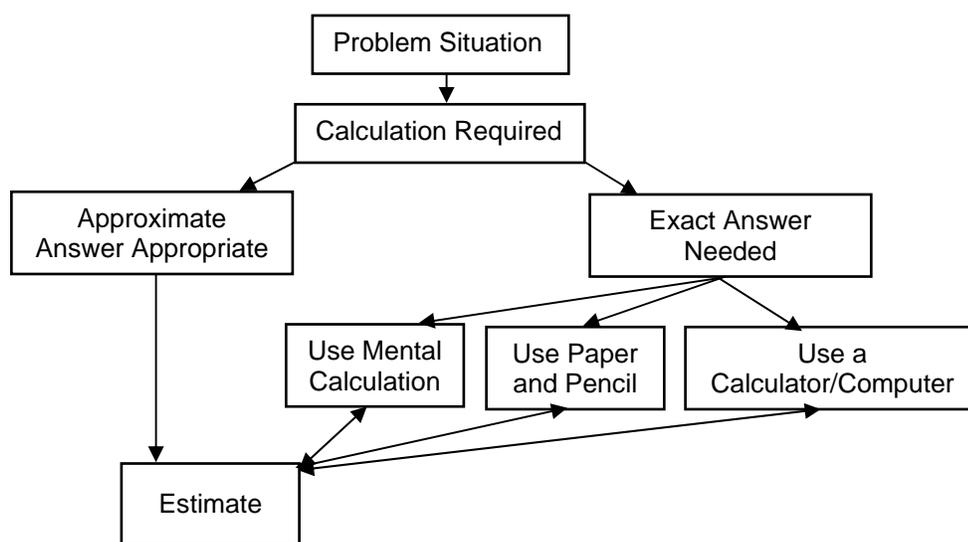
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves calculation without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need - more than ever before - with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein, 2001). Mental mathematics “provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers” (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know when to estimate, what strategy to use, and how to use it. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you. . . ?” or “How could you. . . ?” the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not

a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- using estimation
- guessing and checking
- looking for a pattern
- making an organized list or table
- using a model
- working backwards
- using a formula
- using a graph, diagram, or flow chart
- solving a simpler problem
- using algebra.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations;
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K-3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform, and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and knowledge of several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

➤ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics which are woven throughout this document. These components include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, ... can be described as

- skip counting by 2s, starting from 4;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180° .
- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (*The Primary Program*, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, and flexible and intuitive with numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collecting and analysing of data, and describing relationships visually, symbolically, orally, or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to interpret representations of 2-D shapes and 3-D objects, and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 2-D shapes and 3-D objects.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to use dimensions and make predictions about the results of changing dimensions.

- Knowing the dimensions of an object enables students to communicate about the object and create representations.
- The volume of a rectangular solid can be calculated from given dimensions.
- Doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of

probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Young children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Their mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Initial problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➤ Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding, and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should replace some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill-oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a teacher can give a parent clearer understanding of the mathematics curriculum and of the child's progress in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

➤ **Diversity in Student Needs**

Every class has students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters, but should be designed to help all students (whether strong, weak or average) to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson from which all students come away with a better understanding of what the solution to an equation really means.

➤ **Gender and Cultural Equity**

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean not only that enrolments of students of both genders and various cultural backgrounds in public school mathematics courses should reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➤ **Mathematics for EAL Learners**

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English deficiencies and cultural differences must not be barriers to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and co-ordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication “as an essential part of mathematics and mathematics education (p.60).” The *Standards* elaborate that all

students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading, and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers and will facilitate “communicating to learn mathematics and learning to communicate mathematically” (NCTM, p.60).

To this end,

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counsellors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not on other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated with appropriate language support to both students and parents;
- to verify that barriers have been removed, educators should monitor enrolment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➤ **Education for Sustainable Development**

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development - such as poverty alleviation, human rights, health, environmental protection, and climate change - into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental, and economic perspective, and exploring how those factors are interrelated and interdependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database *Resources for Rethinking*, found at <http://r4r.ca/en>. It provides teachers with access to materials that integrate ecological, social, and economic spheres through active, relevant, interdisciplinary learning.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, or teaching has been effective, or how best to address student learning needs. The quality of the assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others.

➤ **Assessment**

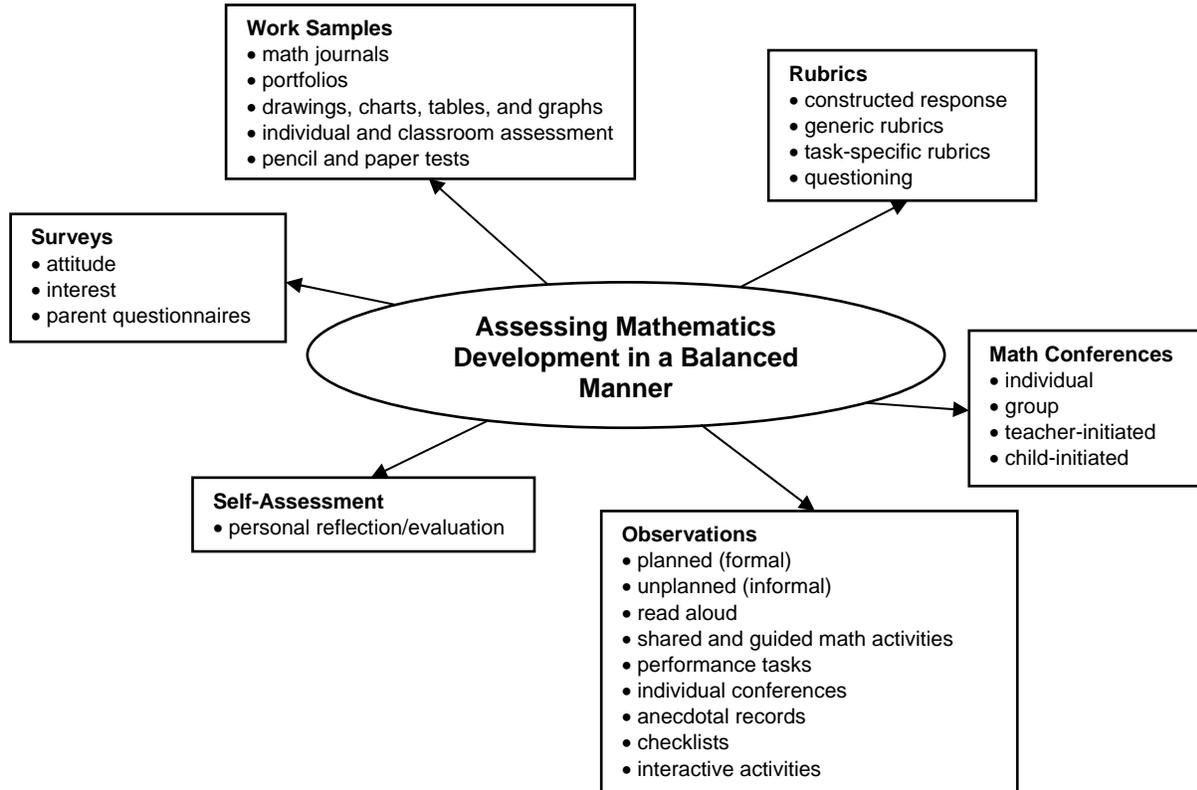
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children's learning;
- providing information to teachers on the effectiveness of their teaching, the program, and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources, including

- | | |
|------------------------------------|------------------------------|
| • formal and informal observations | • portfolios |
| • work samples | • learning journals |
| • anecdotal records | • questioning |
| • conferences | • performance assessment |
| • teacher-made and other tests | • peer- and self-assessment. |

This balanced approach for assessing mathematics development is illustrated in the diagram on the next page.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning, and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used

- to engage students in their own learning and self-assessment;
- to help students understand what is important in the mathematical concepts and particular tasks they encounter;
- to develop effective habits of metacognition and self-coaching;
- to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used

- to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
- to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
- to provide feedback to students about how they are doing and how they might improve;
- to provide feedback to other professionals and to parents about how to support students' learning.

Assessment of learning is used

- to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
- to facilitate reporting;
- to provide the basis for sound decision-making about next steps in a student's learning.

➤ Evaluation

Evaluation is the process of analysing, reflecting upon, and summarizing assessment information, and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires

- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information;
- using a high level of professional judgment in making decisions based upon that information.

➤ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes, and phone calls.

➤ Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration, and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles, as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes, or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.
- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.
- Assessment reports should be clear, accurate, and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that

- the best interests of the student are paramount;
- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes;
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he/she knows and can do.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island mathematics curriculum are organized into four strands across the grades K-9. They are Number, Patterns and Relations, Shape and Space, and Statistics and Probability. These strands are further subdivided into sub-strands, which are the general curriculum outcomes (GCOs). They are overarching statements about what students are expected to learn in each strand or sub-strand from grades K-9.

Strand	General Curriculum Outcome (GCO)
Number (N)	Number: Develop number sense.
Patterns and Relations (PR)	Patterns: Use patterns to describe the world and solve problems.
	Variables and Equations: Represent algebraic expressions in multiple ways.
Shape and Space (SS)	Measurement: Use direct and indirect measure to solve problems.
	3-D Objects and 2-D Shapes: Describe the characteristics of 3-D objects and 2-D shapes, and analyse the relationships among them.
	Transformations: Describe and analyse position and motion of objects and shapes.
Statistics and Probability (SP)	Data Analysis: Collect, display, and analyse data to solve problems.
	Chance and Uncertainty: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific outcome.

In this curriculum guide, each specific curriculum outcome (SCO) is presented in a two-page format, and includes the following information:

- its corresponding strand and general curriculum outcome;
- the scope and sequence of the specific curriculum outcome(s) from grades seven to nine which correspond to this SCO;
- the specific curriculum outcome, with a list of achievement indicators;
- a list of the sections in *MathLinks 8* which address the SCO, with specific achievement indicators highlighted in brackets;
- an elaboration for the SCO.

In the second half of this document, a curriculum guide supplement is presented which follows the primary resource, *MathLinks 8*.

NUMBER

SPECIFIC CURRICULUM OUTCOMES

N1 – Demonstrate an understanding of perfect square and square root, concretely, pictorially and symbolically (limited to whole numbers).

N2 – Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers).

N3 – Demonstrate an understanding of percents greater than or equal to 0%.

N4 – Demonstrate an understanding of ratio and rate.

N5 – Solve problems that involve rates, ratios and proportional reasoning.

N6 – Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.

N7 – Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically.

Grade 8 – Strand: Number (N)

GCO: Develop number sense.

GRADE 7	GRADE 8	GRADE 9
<p>N1 Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and why a number cannot be divided by 0.</p>	<p>N1 Demonstrate an understanding of perfect square and square root, concretely, pictorially and symbolically (limited to whole numbers).</p>	<p>N5 Determine the square root of positive rational numbers that are perfect squares.</p> <p>N6 Determine an approximate square root of positive rational numbers that are non-perfect squares.</p>

SCO: **N1 – Demonstrate an understanding of perfect square and square root, concretely, pictorially and symbolically (limited to whole numbers).** [C, CN, R, V]

Students who have achieved this outcome should be able to:

- A. Represent a given perfect square as a square region using materials, such as grid paper or square shapes.
- B. Determine the factors of a given perfect square, and explain why one of the factors is the square root and the others are not.
- C. Determine whether or not a given number is a perfect square using materials and strategies, such as square shapes, grid paper or prime factorization, and explain the reasoning.
- D. Determine the square root of a given perfect square and record it symbolically.
- E. Determine the square of a given number.

Section(s) in MathLinks 8 text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:

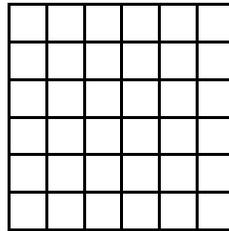
3.1 (A B C D E)

[C] Communication	[ME] Mental Mathematics and Estimation	[PS] Problem Solving	[T] Technology
[CN] Connections		[R] Reasoning	[V] Visualization

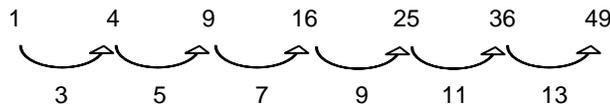
SCO: N1 – Demonstrate an understanding of perfect square and square root, concretely, pictorially and symbolically (limited to whole numbers). [C, CN, R, V]

Elaboration

Students should be able to model perfect squares (any whole number squared) and square roots through the use of color tiles or grid paper. They should make a link between these concrete or pictorial representations of square roots and their numerical representations. In the figure below, students should be encouraged to view the area as a perfect square, and either dimension of the square as the square root.



Students should be able to recognize automatically each of the perfect squares from 1 to 144. It is also valuable to bring out the patterns that emerge from a list of perfect squares; that is, students should recognize that the differences between the perfect squares increase in a consistent way as shown in the pattern below:



In working with patterns, they should also be exposed to, and predict, other perfect squares. Prime factorization is a method used to find the square root of perfect squares. This will build on what students learned in grade six on prime factors and factor trees. For example, consider $\sqrt{144}$:

$$\begin{aligned}
 \text{Since } 144 &= 2 \times 72 \\
 &= 2 \times 2 \times 36 \\
 &= 2 \times 2 \times 6 \times 6 && \text{[This process could be stopped at this point if} \\
 & && \text{students recognize this as } 12 \times 12: (2 \times 6) \times (2 \times 6).] \\
 &= 2 \times 2 \times 2 \times 3 \times 2 \times 3 \\
 &= (2 \times 2 \times 3) \times (2 \times 2 \times 3) && \text{[Group factors into two equal groups.]} \\
 &= 12 \times 12, \text{ therefore, } \sqrt{144} = 12.
 \end{aligned}$$

Grade 8 – Strand: Number (N)

GCO: Develop number sense.

GRADE 7	GRADE 8	GRADE 9
<p>N1 Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and why a number cannot be divided by 0.</p>	<p>N2 Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers).</p>	<p>N5 Determine the square root of positive rational numbers that are perfect squares.</p> <p>N6 Determine an approximate square root of positive rational numbers that are non-perfect squares.</p>

SCO: N2 – Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers). [C, CN, ME, R, T]

Students who have achieved this outcome should be able to:

- A. Estimate the square root of a given number that is not a perfect square using the roots of perfect squares as benchmarks.
- B. Approximate the square root of a given number that is not a perfect square using technology, e.g., calculator, computer.
- C. Explain why the square root of a number shown on a calculator may be an approximation.
- D. Identify a number with a square root that is between two given numbers.

Section(s) in MathLinks 8 text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:

3.3 (A B C D)

[C] Communication	[ME] Mental Mathematics and Estimation	[PS] Problem Solving	[T] Technology
[CN] Connections		[R] Reasoning	[V] Visualization

SCO: N2 – Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers). [C, CN, ME, R, T]

Elaboration

It is very important to emphasize the difference between an exact square root and a decimal approximation. The square root of any non-perfect square will be an irrational number (any number that cannot be converted to the form $\frac{a}{b}$, or a non-terminating, non-repeating decimal). Regardless of the number of decimal places retained in an irrational number, it is still an approximation (e.g., $\pi \approx 3.1416$).

Students will develop a greater intuitive understanding of square root through practicing estimation skills. For numbers between 1 and 144, students should use benchmarks (roots of perfect square numbers) to identify between which two whole numbers the square root will fall and to which whole number it is closer. For example, students should know that the square root of 22 is between 4 and 5, and that it is closer to 5. Given a choice, students should also realize that $\sqrt{22}$ will be closer to 4.7 than to 4.2.

Grade 8 – Strand: Number (N)

GCO: Develop number sense.

GRADE 7	GRADE 8	GRADE 9
<p>N3 Solve problems involving percents from 1% to 100%.</p> <p>SP3 Construct, label and interpret circle graphs to solve problems.</p>	<p>N3 Demonstrate an understanding of percents greater than or equal to 0%.</p>	

SCO: **N3 – Demonstrate an understanding of percents greater than or equal to 0%.** [CN, PS, R, V]

Students who have achieved this outcome should be able to:

- A. Provide a context where a percent may be more than 100% or between 0% and 1%.
- B. Represent a given fractional percent using grid paper.
- C. Represent a given percent greater than 100 using grid paper.
- D. Determine the percent represented by a given shaded region on a grid, and record it in decimal, fractional and percent form.
- E. Express a given percent in decimal or fractional form.
- F. Express a given decimal in percent or fractional form.
- G. Express a given fraction in decimal or percent form.
- H. Solve a given problem involving percents.
- I. Solve a given problem involving combined percents.
- J. Solve a given problem that involves finding the percent of a percent, e.g., A population increased by 10% one year and then increased by 15% the next year. Explain why there was not a 25% increase in population over the two years.

Section(s) in MathLinks 8 text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:

4.1 (A B C D)

4.2 (A D E F G)

4.3 (H)

4.4 (H I J)

[C] Communication	[ME] Mental Mathematics and Estimation	[PS] Problem Solving	[T] Technology
[CN] Connections		[R] Reasoning	[V] Visualization

SCO: N3 – Demonstrate an understanding of percents greater than or equal to 0%. [CN, PS, R, V]

Elaboration

Percents are ratios or fractions where the second term or denominator is 100. The term percent is simply another name for *hundredths*. Percents can be written as low as 0, but can go higher than 100. In grade seven, students worked with percents from 1% to 100%. In grade eight, students examine contexts where percents can be greater than 100% or less than 1% (fractional percents).

Students should be able to move flexibly between percent, fraction and decimal equivalents in problem solving situations. For example, when finding 25% of a number, it is often much easier to use $\frac{1}{4}$ and then divide by 4 as a means of finding or estimating the percent. If students can express fractions and decimals as hundredths, the term percent can be substituted for the term hundredths. The fraction $\frac{3}{2}$ can be expressed in hundredths, $\frac{150}{100}$, which has a decimal equivalent of 1.5, and is equivalent to 150%.

Fractional and decimal percents can be related to benchmark percents. For example, 0.25% means a quarter of 1%. If you know that 1% of 400 is 4, then 0.25% of 400 would be a quarter of 4 or 1. It is also important to recognize that 1% can be a little or a lot depending on the size of the whole. For example, 1% of all of the population of a city is a lot of people compared to 1% of the students in a class.

Students will continue to create and solve problems that they explored in grade seven, which involve finding a , b or c in the relationship of $a\%$ of $b = c$ using estimation and calculation. They will also be required to apply percentage increase and decrease in problem situations in which percents greater than 100 or fractional percents are meaningful. In these situations, is important for students to recognize that 100% is still the whole.

Grade 8 – Strand: Number (N)

GCO: Develop number sense.

GRADE 7	GRADE 8	GRADE 9
<p>N3 Solve problems involving percents from 1% to 100%.</p> <p>SP4 Express probabilities as ratios, fractions and percents.</p>	<p>N4 Demonstrate an understanding of ratio and rate.</p>	<p>N3 Demonstrate an understanding of rational numbers by:</p> <ul style="list-style-type: none"> • comparing and ordering rational numbers; • solving problems that involve arithmetic operations on rational numbers.

SCO: **N4 – Demonstrate an understanding of ratio and rate.** [C, CN, V]

Students who have achieved this outcome should be able to:

- A.** Express a two-term ratio from a given context in the forms 3 : 5 or 3 to 5.
- B.** Express a three-term ratio from a given context in the forms 4 : 7 : 3 or 4 to 7 to 3.
- C.** Express a part to part ratio as a part to whole fraction, e.g., frozen juice to water; 1 can of concentrate to 4 cans of water can be represented as $\frac{1}{5}$, which is the ratio of concentrate to solution, or $\frac{4}{5}$, which is the ratio of water to solution.
- D.** Identify and describe ratios and rates from real-life examples, and record them symbolically.
- E.** Express a given rate using words or symbols, e.g., 20 L per 100 km or 20 L/100 km.
- F.** Express a given ratio as a percent and explain why a rate cannot be represented as a percent.

Section(s) in MathLinks 8 text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:

2.1 (A B C D F)

2.2 (D E F)

[C] Communication	[ME] Mental Mathematics and Estimation	[PS] Problem Solving	[T] Technology
[CN] Connections		[R] Reasoning	[V] Visualization

SCO: N4 – Demonstrate an understanding of ratio and rate. [C, CN, V]

Elaboration

A ratio is a comparison of at least two quantities. Ratios can express comparisons of a part to a whole (every fraction, percent, and probability is a ratio) or compare part of a whole to other parts of the same whole. Part-to-whole and part-to-part ratios compare two or more measures of the same type. A map scale is a common application of ratios.

A ratio that compares measures of two different types is called a rate (e.g., a comparison of distance to time). A unit rate is an equivalent rate where the second term is one. This rate can be used to determine the better buy when comparing prices. Percent cannot be considered a rate because a percent is a ratio that compares quantities expressed in the same units.

Grade 8 – Strand: Number (N)

GCO: Develop number sense.

GRADE 7	GRADE 8	GRADE 9
<p>N3 Solve problems involving percents from 1% to 100%.</p> <p>SP4 Express probabilities as ratios, fractions and percents.</p>	<p>N5 Solve problems that involve rates, ratios and proportional reasoning.</p>	<p>N3 Demonstrate an understanding of rational numbers by:</p> <ul style="list-style-type: none"> • comparing and ordering rational numbers; • solving problems that involve arithmetic operations on rational numbers.

SCO: **N5 – Solve problems that involve rates, ratios and proportional reasoning.** [C, CN, PS, R]

Students who have achieved this outcome should be able to:

- A.** Explain the meaning of $\frac{a}{b}$ within a given context.
- B.** Provide a context in which $\frac{a}{b}$ represents a:
- fraction;
 - rate;
 - ratio;
 - quotient;
 - probability.
- C.** Solve a given problem involving rate, ratio or percent.

Section(s) in MathLinks 8 text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:

- 2.1 (A B C)**
2.2 (A B C)
2.3 (A B C)

[C] Communication	[ME] Mental Mathematics and Estimation	[PS] Problem Solving	[T] Technology
[CN] Connections		[R] Reasoning	[V] Visualization

SCO: N5 – Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]

Elaboration

Proportional reasoning is the ability to think about and compare multiplicative relationships between quantities. These relationships are represented symbolically as ratios. A proportion is a statement of equality between two ratios. Different notations for proportions can be used:

$$2:5 = 4:10 \text{ or } 2 \text{ to } 5 = 4 \text{ to } 10 \text{ or } \frac{2}{5} = \frac{4}{10}$$

These can be read as “two to five” and mean that for every 2 items there will be 5 items.

Finding one number in a proportion when the other three numbers are known is called solving a proportion. For example, how many girls are in a class when the ratio of boys to girls in a class is 3 : 5 and there are 12 boys. To solve, set up the proportion: $\frac{3}{5} = \frac{12}{?}$. The students must think multiplicatively to solve the proportion in the same way

they would to determine equivalent fractions. Also, a student who knows that a runner who runs at a rate of $\frac{1 \text{ km}}{7 \text{ min}}$

will win a 10 km race over a runner who runs at a rate of $\frac{1 \text{ km}}{8 \text{ min}}$ is thinking proportionally.

Students may need as much as three years worth of opportunities to reason in multiplicative situations to order to adequately develop proportional reasoning skills. Premature use of rules encourages students to apply rules without thinking and, thus, the ability to reason proportionally does not develop (Van de Walle & Lovin, vol. 3, 2006; p. 157).

Grade 8 – Strand: Number (N)

GCO: Develop number sense.

GRADE 7	GRADE 8	GRADE 9
N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).	N6 Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.	N3 Demonstrate an understanding of rational numbers by: <ul style="list-style-type: none"> • comparing and ordering rational numbers; • solving problems that involve arithmetic operations on rational numbers.

SCO: **N6 – Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.** [C, CN, ME, PS]

Students who have achieved this outcome should be able to:

- A. Identify the operation required to solve a given problem involving positive fractions.
- B. Provide a context that requires the multiplying of two given positive fractions.
- C. Provide a context that requires the dividing of two given positive fractions.
- D. Estimate the product of two given positive proper fractions to determine if the product will be closer to 0, $\frac{1}{2}$ or 1.
- E. Estimate the quotient of two given positive fractions and compare the estimate to whole number benchmarks.
- F. Express a given positive mixed number as an improper fraction and a given positive improper fraction as a mixed number.
- G. Model multiplication of a positive fraction by a whole number concretely or pictorially and record the process.
- H. Model multiplication of a positive fraction by a positive fraction concretely or pictorially using an area model and record the process.
- I. Model division of a positive proper fraction by a whole number concretely or pictorially and record the process.
- J. Model division of a positive proper fraction by a positive proper fraction pictorially and record the process.
- K. Generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers.
- L. Solve a given problem involving positive fractions, taking into consideration order of operations (limited to problems with positive solutions).

Section(s) in MathLinks 8 text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:

- 6.1 (A G L)**
- 6.2 (A I L)**
- 6.3 (B D H K L)**
- 6.4 (B F H K)**
- 6.5 (C E J K)**
- 6.6 (K L)**

[C] Communication	[ME] Mental Mathematics and Estimation	[PS] Problem Solving	[T] Technology
[CN] Connections		[R] Reasoning	[V] Visualization

SCO: N6 – Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

Elaboration

Students have added and subtracted positive fractions and mixed numbers in grade seven. Multiplying and dividing fractions should be learned in relation to each other.

The following guidelines should be kept in mind when developing computational strategies for fractions. It is important to not rush to computational rules.

- Begin with simple contextual tasks (include sets, area models, distance).
- Connect the meaning of fraction computation with whole number computation.
- Let estimation and informal methods play a big role in the development of strategies.
- Explore each of the operations using models.

(Van de Walle & Lovin, vol. 3, 2006; p. 88)

It is important for students to understand what multiplication means: 3×5 means 3 groups of 5. The same is true for fractions. When multiplying a fraction by a whole number, it can be thought of as groups of the fraction or fractions of a group. For example, $3 \times \frac{1}{3}$ equals three groups of $\frac{1}{3}$ or $\frac{1}{3}$ of a group of three.

When multiplying a proper fraction by another number, some students struggle with the product being less than one or both of the factors. Students need to keep in mind they are multiplying by a number that is less than one. Language is very important. Models should be explored consistently throughout the teaching of this outcome. The area model should be used as a key method for exploring fraction multiplication.

For division of a fraction by a whole number, ask students what each part represents (e.g., $\frac{1}{2} \div 3$ is telling us to break the half into 3 equal parts, so the answer is $\frac{1}{6}$). For division of a whole number by a fraction, ask students how many parts there are in the whole number (e.g., to find $4 \div \frac{1}{2}$, ask how many halves there are in 4, giving an answer of 8.)

When the denominators are the same, the numerators can be divided to find the answer. If the simple fractions do not have common denominators, one strategy is to make them common and then divide the numerators: $\frac{4}{3} \div \frac{1}{2} = \frac{8}{6} \div \frac{3}{6} = \frac{8}{3}$. This approach is easier for students to conceptualize rather than following the traditional method of inverting the second fraction and multiplying. The number line can provide a useful model for division to help students visualize division.

Estimation is important for students to determine whether their products and quotients are reasonable. There are many real world examples where students can apply these skills and they should be encouraged to estimate either before or after any computation.

Grade 8 – Strand: Number (N)

GCO: Develop number sense.

GRADE 7	GRADE 8	GRADE 9
<p>N6 Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.</p>	<p>N7 Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically.</p>	<p>N3 Demonstrate an understanding of rational numbers by:</p> <ul style="list-style-type: none"> • comparing and ordering rational numbers; • solving problems that involve arithmetic operations on rational numbers.

SCO: **N7 – Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically.** [C, CN, PS, R, V]

Students who have achieved this outcome should be able to:

- A. Identify the operation required to solve a given problem involving integers.
- B. Provide a context that requires multiplying two integers.
- C. Provide a context that requires dividing two integers.
- D. Model the process of multiplying two integers using concrete materials or pictorial representations and record the process.
- E. Model the process of dividing an integer by an integer using concrete materials or pictorial representations and record the process.
- F. Solve a given problem involving the division of integers (2-digit by 1-digit) without the use of technology.
- G. Solve a given problem involving the division of integers (2-digit by 2-digit) with the use of technology.
- H. Generalize and apply a rule for determining the sign of the product and quotient of integers.
- I. Solve a given problem involving integers taking into consideration order of operations.

Section(s) in MathLinks 8 text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:

8.1 (A B D)

8.2 (D H)

8.3 (A C E)

8.4 (E F G H)

8.5 (I)

[C] Communication	[ME] Mental Mathematics and Estimation	[PS] Problem Solving	[T] Technology
[CN] Connections		[R] Reasoning	[V] Visualization

SCO: N7 – Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically. [C, CN, PS, R, V]

Elaboration

Addition of integers, which is a grade seven outcome, helps to establish some of the initial groundwork for the multiplication of integers. Multiplication of integers should start with examining $4 \times (-3)$ as 4 groups of -3 , which equals $(-3) + (-3) + (-3) + (-3)$, or -12 .

It is important to remember that integers can be multiplied in any order (commutative property) without affecting the product. Using this property helps students to deal with the multiplication $(-4) \times 5$ because they can think of it as 5 groups of -4 .

Patterning can be used to justify the result for a negative multiplied by a negative:

$$3 \times (-2) = -6$$

$$2 \times (-2) = -4$$

$$1 \times (-2) = -2$$

$$0 \times (-2) = 0$$

$$(-1) \times (-2) = ?$$

$$(-2) \times (-2) = ?$$

$$(-3) \times (-2) = ?$$

Comparison of multiplication and division situations can also be very useful in helping students understand division of integers. After multiplication has been fully developed, the fact that multiplication and division are inverse operations can be utilized. For example, since $(-4) \times 3 = -12$, it must be true that the product divided by either factor should equal the other factor; therefore, $(-12) \div (-4) = 3$ and $(-12) \div 3 = -4$. Likewise, if $(-4) \times (-3) = 12$, then $12 \div (-4) = -3$ and $12 \div (-3) = -4$.

Using a missing factor can also be useful. For example, in the case of $(-16) \div (-4)$, ask, what multiplied by -4 gives -16 .

Once multiplication and division of integers have been addressed, students should be exposed to questions involving all four operations and the application of the order of operations.

PATTERNS AND RELATIONS

SPECIFIC CURRICULUM OUTCOMES

PR1 – Graph and analyse two-variable linear relations.

PR2 – Model and solve problems using linear equations of the form:

- $ax = b$;
- $\frac{x}{a} = b, a \neq 0$;
- $ax + b = c$;
- $\frac{x}{a} + b = c, a \neq 0$;
- $a(x + b) = c$

concretely, pictorially and symbolically, where a , b and c are integers.

Grade 8 – Strand: Patterns and Relations (PR)

GCO: Use patterns to describe the world and solve problems.

GRADE 7	GRADE 8	GRADE 9
<p>PR1 Demonstrate an understanding of oral and written patterns and their equivalent linear relations.</p> <p>PR2 Create a table of values from a linear relation, graph the table of values, and analyse the graph to draw conclusions and solve problems.</p> <p>SS4 Identify and plot points in the four quadrants of a Cartesian plane using integral ordered pairs.</p>	<p>PR1 Graph and analyse two-variable linear relations.</p>	<p>PR1 Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.</p> <p>PR2 Graph linear relations, analyse the graph and interpolate or extrapolate to solve problems.</p>

SCO: PR1 – Graph and analyse two-variable linear relations. [C, ME, PS, R, T, V]

Students who have achieved this outcome should be able to:

- A.** Determine the missing value in an ordered pair for a given equation.
- B.** Create a table of values by substituting values for a variable in the equation of a given linear relation.
- C.** Construct a graph from the equation of a linear relation (limited to discrete data).
- D.** Describe the relationship between the variables of a given graph.

Section(s) in MathLinks 8 text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:

- 9.1 (A B D)**
- 9.2 (B C D)**
- 9.3 (B C D)**

[C] Communication	[ME] Mental Mathematics and Estimation	[PS] Problem Solving	[T] Technology
[CN] Connections		[R] Reasoning	[V] Visualization

SCO: PR1 – Graph and analyse two-variable linear relations. [C, ME, PS, R, T, V]

Elaboration

The graphs constructed from the equation of a given linear equation will be limited to discrete data. Discrete data can only have a finite or limited number of possible values. Generally discrete data are countable: for example, the number of students in class, the number of tickets sold or the number of Christmas trees that were purchased. Continuous data can have an infinite number of possible values within a selected range, such as the quantities of temperature or time. A graph of discrete data has plotted points, but they are not joined together.

Students should observe when looking at tabular data that, when an equal spacing between the values of one variable produces an equal spacing between values of the other variable, the relationship will be linear. Students should recognize that for linear relationships, the ratio of vertical change to horizontal change is consistent anywhere along the line. However, it is not necessary to discuss slope of a line at this point.

Many resources will show continuous data graphs (all points connected) displayed as though they are discrete (no points connected). For example, any graph with time on the horizontal axis is actually displaying continuous data. The analysis of graphs should include creating stories that describe the relationship depicted and constructing graphs based on a story which involves changes in related quantities. For example, as the temperature rises, the number of people at the beach increases.

When students are describing a relationship in a graph they should use language like “as this increases that decreases” or “as one quantity drops, the other also drops.” When students are attempting to find a missing value in an ordered pair, they should use either patterning or substitution into the equation if the equation has been provided.

Students need to be able to transition between given information whether it is presented as a table of values, a graph, a linear relation or a set of ordered pairs.

Grade 8 – Strand: Patterns and Relations (PR)

GCO: Represent algebraic expressions in multiple ways.

GRADE 7	GRADE 8	GRADE 9
<p>PR3 Demonstrate an understanding of preservation of equality by:</p> <ul style="list-style-type: none"> modeling preservation of equality, concretely, pictorially and symbolically; applying preservation of equality to solve equations. <p>PR6 Model and solve problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially and symbolically, where a and b are integers.</p> <p>PR7 Model and solve problems that can be represented by linear equations of the form:</p> <ul style="list-style-type: none"> $ax + b = c$; $ax = b$; $\frac{x}{a} = b, a \neq 0$ <p>concretely, pictorially and symbolically, where a, b and c are whole numbers.</p>	<p>PR2 Model and solve problems using linear equations of the form:</p> <ul style="list-style-type: none"> $ax = b$; $\frac{x}{a} = b, a \neq 0$; $ax + b = c$; $\frac{x}{a} + b = c, a \neq 0$; $a(x + b) = c$ <p>concretely, pictorially and symbolically, where a, b and c are integers.</p>	<p>PR3 Model and solve problems using linear equations of the form:</p> <ul style="list-style-type: none"> $ax = b$; $\frac{x}{a} = b, a \neq 0$; $ax + b = c$; $\frac{x}{a} + b = c, a \neq 0$; $ax = b + cx$; $a(x + b) = c$; $ax + b = cx + d$; $a(bx + c) = d(ex + f)$; $\frac{a}{x} = b, x \neq 0$ <p>where a, b, c, d, e and f are rational numbers.</p>

SCO: **PR2 – Model and solve problems using linear equations of the form:**

- $ax = b$;
- $\frac{x}{a} = b, a \neq 0$;
- $ax + b = c$;
- $\frac{x}{a} + b = c, a \neq 0$;
- $a(x + b) = c$

concretely, pictorially and symbolically, where a, b and c are integers. [C, CN, PS, V]

Students who have achieved this outcome should be able to:

- A. Model a given problem with a linear equation and solve the equation using concrete models, e.g., counters, integer tiles.
- B. Verify the solution to a given linear equation using a variety of methods, including concrete materials, diagrams and substitution.
- C. Draw a visual representation of the steps used to solve a given linear equation and record each step symbolically.
- D. Solve a given linear equation symbolically.
- E. Identify and correct an error in a given incorrect solution of a linear equation.
- F. Apply the distributive property to solve a given linear equation, e.g., $2(x + 3) = 8$ becomes $2x + 6 = 8$.
- G. Solve a given problem using a linear equation and record the process.

Section(s) in MathLinks 8 text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:

10.1 (A B C D E G)

10.3 (A B C D E G)

10.2 (A B C D E G)

10.4 (A B C D E F G)

[C] Communication	[ME] Mental Mathematics and Estimation	[PS] Problem Solving	[T] Technology
[CN] Connections		[R] Reasoning	[V] Visualization

SCO: PR2 – Model and solve problems using linear equations of the form:

- $ax = b$;
- $\frac{x}{a} = b, a \neq 0$;
- $ax + b = c$;
- $\frac{x}{a} + b = c, a \neq 0$;
- $a(x + b) = c$

concretely, pictorially and symbolically, where a , b and c are integers. [C, CN, PS, V]

Elaboration

Students have experience solving one-step equations in the form of $x + a = b$ and two-step equations in the forms of $ax + b = c$, $ax = b$, and $\frac{x}{a} = b$ ($a \neq 0$), where a , b and c are whole numbers. In grade eight, students will continue to solve equations that will now include integers as well as fractions for the values of a , b and c .

In problem solving situations, students should consider in advance what might be a reasonable solution, and be aware that once they acquire a solution, it can be checked for accuracy by substitution into the original equation.

In order for students to solve linear equations of the forms $ax = b$, $\frac{x}{a} = b$ ($a \neq 0$), $ax + b = c$, $\frac{x}{a} + b = c$ ($a \neq 0$), and $a(x + b) = c$, they must recognize that the idea of “balancing” or “moving from one side to another” by using opposite operation(s) is required. The zero principle is an important aspect of finding equality between the two sides. This, in fact, allows for the preservation of balance and equality in the equation (where left side equals right side). In the form $ax + b = c$, for example, students need to perform a two-step elimination process to solve for the variable whereas in some other equations, only a single-step process is needed.

There are many methods for solving linear equation such as inspection, systematic trial (guess and test), rewriting the equation, creating models using algebra tiles and using illustrations of balances to show equality. Students should be encouraged to choose the most appropriate method for solving a given problem. Emphasis at this level should be on solving problems concretely, pictorially, and symbolically.

SHAPE AND SPACE

SPECIFIC CURRICULUM OUTCOMES

SS1 – Develop and apply the Pythagorean theorem to solve problems.

SS2 – Draw and construct nets for 3-D objects.

SS3 – Determine the surface area of:

- right rectangular prisms;
- right triangular prisms;
- right cylinders

to solve problems.

SS4 – Develop and apply formulas for determining the volume of right prisms and right cylinders.

SS5 – Draw and interpret top, front and side views of 3-D objects composed of right rectangular prisms.

SS6 – Demonstrate an understanding of tessellation by:

- explaining the properties of shapes that make tessellating possible;
- creating tessellations;
- identifying tessellations in the environment.

Grade 8 – Strand: Shape and Space (SS)

GCO: Use direct or indirect measurement to solve problems.

GRADE 7	GRADE 8	GRADE 9
<p>SS2 Develop and apply a formula for determining the area of:</p> <ul style="list-style-type: none"> • triangles; • parallelograms; • circles. 	<p>SS1 Develop and apply the Pythagorean theorem to solve problems.</p>	

SCO: SS1 – Develop and apply the Pythagorean theorem to solve problems. [CN, PS, R, T, V]

Students who have achieved this outcome should be able to:

- A.** Model and explain the Pythagorean theorem concretely, pictorially or using technology.
- B.** Explain, using examples, that the Pythagorean theorem applies only to right triangles.
- C.** Determine whether or not a given triangle is a right triangle by applying the Pythagorean theorem.
- D.** Determine the measure of the third side of a right triangle, given the measures of the other two sides, to solve a given problem.
- E.** Solve a given problem that involves Pythagorean triples, e.g., 3, 4, 5 or 5, 12, 13.

Section(s) in MathLinks 8 text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:

3.2 (A B C E)

3.4 (D)

3.5 (D)

[C] Communication	[ME] Mental Mathematics and Estimation	[PS] Problem Solving	[T] Technology
[CN] Connections		[R] Reasoning	[V] Visualization

SCO: SS1 – Develop and apply the Pythagorean theorem to solve problems. [CN, PS, R, T, V]

Elaboration

Pythagoras of Samos, c. 560 BC – c. 480 BC, was a Greek philosopher who is credited with providing the first proof of the Pythagorean relationship. It states that the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides. The conventional formula for the Pythagorean relationship, $c^2 = a^2 + b^2$, should be developed through investigations. It is also important for students to recognize that the Pythagorean relationship can be labelled differently from the conventional a - b - c notation. The hypotenuse, or the longest side, is c and two shorter sides, or legs, are a and b .

A Pythagorean triple is any set of three whole numbers a , b and c , for which $a^2 + b^2 = c^2$. It is believed that the Egyptians and other ancient cultures used a 3-4-5 rule ($a = 3$, $b = 4$, $c = 5$) in construction to ensure buildings were square. The 3-4-5 rule allowed them a quick method of establishing a right angle. This method is still used today in construction.

In presenting diagrams of right triangles, it is important to give diagrams of the triangles in various orientations. Students should recognize the hypotenuse as being the side opposite the right angle, regardless of the orientation of the figure. Whenever a triangle has a right angle and two known side lengths, the Pythagorean relationship should be recognized by students. Students should be given experiences with side lengths of triangles that do not make right angle triangles. Students should also be provided with experiences that involve finding the length of the hypotenuse, as well as situations where the hypotenuse and one side is known and the other side is to be found. Also, it is important for students to realize that they can use the Pythagorean relationship when only one side is known, as long as the right triangle is isosceles. Finally, students should be able to use the Pythagorean relationship to determine if three given side lengths are, or are not, the sides of a right triangle. There are many opportunities to use the Pythagorean relationship to solve other problems, such as determining the height of a building or finding the shortest distance across a rectangular field.

Students need to be provided with opportunities to model and explain the Pythagorean theorem concretely, pictorially, and symbolically:

- **Concretely** – by cutting up areas represented by a^2 , b^2 and fitting the two areas onto c^2
- **Pictorially** – using grid paper or technology
- **Symbolically** – by confirming that $a^2 + b^2 = c^2$ forms a right triangle

Grade 8 – Strand: Shape and Space (SS)

GCO: Use direct or indirect measurement to solve problems.

GRADE 7	GRADE 8	GRADE 9
	SS2 Draw and construct nets for 3-D objects.	

SCO: **SS2 – Draw and construct nets for 3-D objects.** [C, CN, PS, V]

Students who have achieved this outcome should be able to:

- A.** Match a given net to the 3-D object it represents.
- B.** Construct a 3-D object from a given net.
- C.** Draw nets for a given right circular cylinder, right rectangular prism and right triangular prism, and verify by constructing the 3-D objects from the nets.
- D.** Predict 3-D objects that can be created from a given net and verify the prediction.

Section(s) in MathLinks 8 text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:

5.2 (A B C D)

5.3 (C)

5.4 (C)

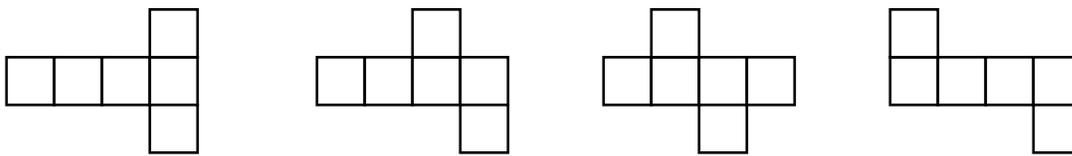
[C] Communication	[ME] Mental Mathematics and Estimation	[PS] Problem Solving	[T] Technology
[CN] Connections		[R] Reasoning	[V] Visualization

SCO: SS2 – Draw and construct nets for 3-D objects. [C, CN, PS, V]

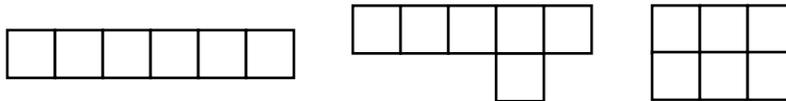
Elaboration

A net is a 2-D representation of a 3-D shape that can be folded to recreate the shape. A net shows all of the faces of an object. A net can be used to make an object called a polyhedron. Two faces meet at an edge. Three or more faces meet at a vertex. When students are making nets, they should focus on the faces, and how the faces fit together to form the shape.

It is important for students to realize that there can be many different nets for a single shape. Even though the faces do not change, they can be connected in different ways. Please note that it is not a different net if it is a reflection or rotation of one you already have. For example, these four nets will all produce a cube.



Students cannot assume that because a cube has six square faces, any grouping of six squares will create a net. For example, the following are not nets for a cube:



A regular pyramid has a regular polygon as its base. The other faces are triangles. Many students are surprised to find that pyramids with different heights can be created on the same base.

Please note that this outcome is closely related to SS3.

Grade 8 – Strand: Shape and Space (SS)

GCO: Use direct or indirect measurement to solve problems.

GRADE 7	GRADE 8	GRADE 9
<p>SS1 Demonstrate an understanding of circles by:</p> <ul style="list-style-type: none"> • describing the relationships among radius, diameter and circumference of circles; • relating circumference to π; • determining the sum of the central angles; • constructing circles with a given radius or diameter; • solving problems involving the radii, diameters and circumferences of circles. <p>SS2 Develop and apply a formula for determining the area of:</p> <ul style="list-style-type: none"> • triangles; • parallelograms; • circles. 	<p>SS3 Determine the surface area of:</p> <ul style="list-style-type: none"> • right rectangular prisms; • right triangular prisms; • right cylinders to solve problems. 	<p>SS2 Determine the surface area of composite 3-D objects to solve problems.</p>

SCO: SS3 – Determine the surface area of:

- **right rectangular prisms;**
 - **right triangular prisms;**
 - **right cylinders**
- to solve problems.** [C, CN, PS, R, V]

Students who have achieved this outcome should be able to:

- A. Explain, using examples, the relationship between the area of 2-D shapes and the surface area of a given 3-D object.
- B. Identify all the faces of a given prism, including right rectangular and right triangular prisms.
- C. Describe and apply strategies for determining the surface area of a given right rectangular or right triangular prism.
- D. Describe and apply strategies for determining the surface area of a given right cylinder.
- E. Solve a given problem involving surface area.

Section(s) in MathLinks 8 text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:

5.3 (A B C E)

5.4 (D E)

[C] Communication	[ME] Mental Mathematics and Estimation	[PS] Problem Solving	[T] Technology
[CN] Connections		[R] Reasoning	[V] Visualization

SCO: SS3 – Determine the surface area of:

- **right rectangular prisms;**
 - **right triangular prisms;**
 - **right cylinders**
- to solve problems.** [C, CN, PS, R, V]

Elaboration

It is important for students to be able to visualize the net of a 3-D object to calculate the surface area of that object efficiently. It is important to use concrete materials to help students visualize the relationship between the 2-D net and the 3-D object. Surface area is the sum of the area of all the faces of a 3-D object. Show students a centimetre cube. Make sure to explain that square units are used to measure area and surface area.

In calculating surface area, students should start with objects such as cereal or cracker boxes for rectangular prisms, boxes from some types of chocolate bars for triangular prisms, and cylinders such as those found in bathroom tissue, paper towels or gift paper. These objects can be cut open, and the shape of each net determined. Students can then estimate the area of each face and total the areas to find the surface area. Students can compare and discuss similarities and differences in their approaches. Teachers should validate the different methods but encourage students to realize that some methods can do the job more efficiently.

One possible net for a cylinder is shown below. One dimension of the rectangle is the circumference of the circle, and the other is the height of the cylinder.



Please note that this outcome is closely related to SS2 and SS4.

Grade 8 – Strand: Shape and Space (SS)

GCO: Use direct or indirect measurement to solve problems.

GRADE 7	GRADE 8	GRADE 9
<p>SS1 Demonstrate an understanding of circles by:</p> <ul style="list-style-type: none"> • describing the relationships among radius, diameter and circumference of circles; • relating circumference to π; • determining the sum of the central angles; • constructing circles with a given radius or diameter; • solving problems involving the radii, diameters and circumferences of circles. <p>SS2 Develop and apply a formula for determining the area of:</p> <ul style="list-style-type: none"> • triangles; • parallelograms; • circles. 	<p>SS4 Develop and apply formulas for determining the volume of right prisms and right cylinders.</p>	

SCO: SS4 – Develop and apply formulas for determining the volume of right prisms and right cylinders.
[C, CN, PS, R, V]

Students who have achieved this outcome should be able to:

- A.** Determine the volume of a given right prism, given the area of the base.
- B.** Generalize and apply a rule for determining the volume of right cylinders.
- C.** Explain the connection between the area of the base of a given right 3-D object and the formula for the volume of that object.
- D.** Demonstrate that the orientation of a given 3-D object does not affect its volume.
- E.** Apply a formula to solve a given problem involving the volume of a right cylinder or a right prism.

Section(s) in MathLinks 8 text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:

- 7.1 (A B C D)**
- 7.2 (A C E)**
- 7.3 (A B C E)**
- 7.4 (E)**

[C] Communication	[ME] Mental Mathematics and Estimation	[PS] Problem Solving	[T] Technology
[CN] Connections		[R] Reasoning	[V] Visualization

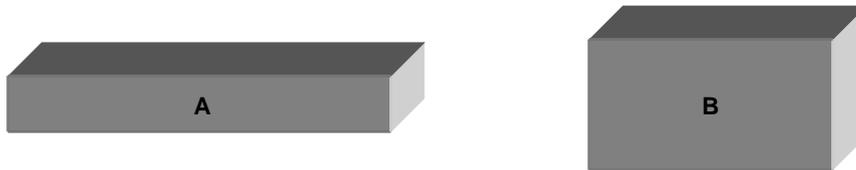
SCO: SS4 – Develop and apply formulas for determining the volume of right prisms and right cylinders.
[C, CN, PS, R, V]

Elaboration

The volume of a shape is a measure that describes the amount of space that an object occupies. Connections should be made between calculating the area of an object's base and calculating its volume. The volume of a shape should be thought of as the area of the base multiplied by its height rather than having students memorize different formulas for different shapes. Objects should be placed in various orientations so students can see that the volume is not affected by orientation of the shape.

Estimation of volume can be done in a variety of situations. For example, it may be desirable to find out how many cans or smaller packages will fit into a larger box or to estimate the volume of a package when the dimensions are not accurately known. Often, for rough estimations, cylinders can be treated as if they were rectangular prisms so that a rough estimation of volume would be $\text{length} \times \text{width} \times \text{height}$. In this case, the diameter of the circular base is treated as both length and width, and all dimensions are rounded for convenience of calculating mentally.

Some students will only use one dimension to calculate volume. For example, many students will say that prism A has more volume than prism B because prism A is longer.



Please note that this outcome is closely related to SS3.

Grade 8 – Strand: Shape and Space (SS)

GCO: Describe the characteristics of 3-D objects and 2-D shapes, and analyse the relationships among them.

GRADE 7	GRADE 8	GRADE 9
<p>SS3 Perform geometric constructions, including:</p> <ul style="list-style-type: none"> • perpendicular line segments; • parallel line segments; • perpendicular bisectors; • angle bisectors. 	<p>SS5 Draw and interpret top, front and side views of 3-D objects composed of right rectangular prisms.</p>	<p>SS2 Determine the surface area of composite 3-D objects to solve problems.</p>

SCO: **SS5 – Draw and interpret top, front and side views of 3-D objects composed of right rectangular prisms.** [C, CN, R, T, V]

Students who have achieved this outcome should be able to:

- A.** Draw and label the top, front and side views for a given 3-D object on isometric dot paper.
- B.** Compare different views of a given 3-D object to the object.
- C.** Predict the top, front and side views that will result from a described rotation (limited to multiples of 90 degrees) and verify predictions.
- D.** Draw and label the top, front and side views that result from a given rotation (limited to multiples of 90 degrees).
- E.** Build a 3-D block object, given the top, front and side views, with or without the use of technology.
- F.** Sketch and label the top, front and side views of a 3-D object in the environment with or without the use of technology.

Section(s) in MathLinks 8 text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:

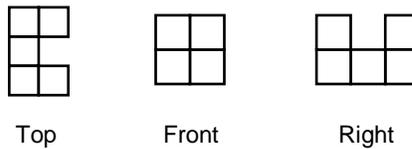
5.1 (A B C D E F)

[C] Communication	[ME] Mental Mathematics and Estimation	[PS] Problem Solving	[T] Technology
[CN] Connections		[R] Reasoning	[V] Visualization

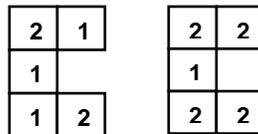
SCO: SS5 – Draw and interpret top, front and side views of 3-D objects composed of right rectangular prisms. [C, CN, R, T, V]

Elaboration

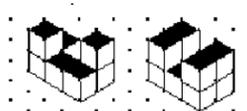
Observing and learning to represent 2-D and 3-D figures in various positions by drawing and construction helps students to develop spatial sense. Students’ mathematical experience with 3-D is often derived from 2-D pictures. It is important that students be able to interpret information from 2-D pictures of the world, as well as to represent real-world information in 2-D. Students can be given a series of 2-D views of a 3-D object, such as the following, and be asked to construct, using cubes, a building that adheres to the plans. Such plans are often referred to as orthographic plans or drawings.



To make a mat plan, you show height of each part of the structure on the top view. When interpreting orthographic drawings, it is sometimes the case that not all students will produce exactly the same building structure. For example, with the set of plans above, each of the following mat plans would apply.



This outcome can also be explored using isometric drawings. For example, the orthographic plans at the top are satisfied by each of the following isometric drawings.



Students can discover that, when they are given only one view of an isometric drawing, they often cannot see all the cubes because some are hidden. Students can be given an isometric drawing, such as the one shown below, and be asked to create a building from it. Generally, not all students make the same structure, and they realize that one drawing can lead to more than one 3-D object. They can again explore the maximum, minimum, and variety of structures which can support a given drawing.



Visualization of the movement of 3-D objects is an important life skill, as anyone who has attempted to move furniture around in a living room or move a sofa through a door will quickly realize. The purpose of this topic is to provide students with some experiences in visualizing and recording the movement of 3-D objects.

Students should be given a 3-D object, asked to sketch it from a particular point of view, predict what views will result from the rotation, perform the indicated rotation, and re-sketch the object, all on isometric paper. Students are also expected to re-sketch the orthographic drawings after such rotations.

Grade 8 – Strand: Shape and Space (SS)

GCO: Describe and analyse position and motion of objects and shapes.

GRADE 7	GRADE 8	GRADE 9
<p>SS5 Perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices).</p>	<p>SS6 Demonstrate an understanding of tessellation by:</p> <ul style="list-style-type: none"> • explaining the properties of shapes that make tessellating possible; • creating tessellations; • identifying tessellations in the environment. 	<p>SS4 Draw and interpret scale diagrams of 2-D shapes.</p> <p>SS5 Demonstrate an understanding of line and rotation symmetry.</p>

SCO: **SS6 – Demonstrate an understanding of tessellation by:**

- **explaining the properties of shapes that make tessellating possible;**
- **creating tessellations;**
- **identifying tessellations in the environment.**

[C, CN, PS, T, V]

Students who have achieved this outcome should be able to:

- A.** Identify, in a given set of regular polygons, those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices, e.g., squares, regular n -gons.
- B.** Identify, in a given set of irregular polygons, those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices.
- C.** Identify a translation, reflection or rotation in a given tessellation.
- D.** Identify a combination of transformations in a given tessellation.
- E.** Create a tessellation using one or more 2-D shapes, and describe the tessellation in terms of transformations and conservation of area.
- F.** Create a new tessellating shape (polygon or non-polygon) by transforming a portion of a given tessellating polygon, e.g., one by M.C. Escher, and describe the resulting tessellation in terms of transformations and conservation of area.
- G.** Identify and describe tessellations in the environment.

Section(s) in MathLinks 8 text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:

12.1 (A B E G)

12.2 (C E F)

12.3 (C E F G)

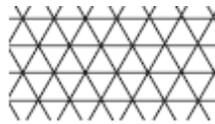
12.4 (C D E F)

[C] Communication	[ME] Mental Mathematics and Estimation	[PS] Problem Solving	[T] Technology
[CN] Connections		[R] Reasoning	[V] Visualization

- SCO: SS6 – Demonstrate an understanding of tessellation by:**
- explaining the properties of shapes that make tessellating possible;
 - creating tessellations;
 - identifying tessellations in the environment.
- [C, CN, PS, T, V]

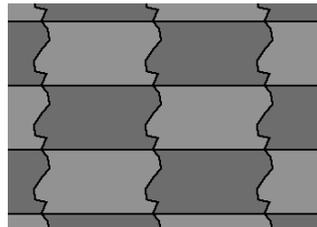
Elaboration

A 2-D figure is said to tessellate if an arrangement of replications of it can cover a surface without gaps or overlapping. For example, if a number of triangular pattern blocks were used, they could be used to cover a surface; therefore, this triangle is said to tessellate (see below left). Investigations should include some shapes such as regular pentagons and regular octagons that will not tessellate. When octagons are used in flooring and tiles, squares fill the gaps because octagonal tiles will not tessellate (see below right).



For a regular figure to tessellate, the angle measurement must be a factor of 360° . Only three regular tessellations exist – those made up of equilateral triangles, squares, or hexagons. There are combinations of polygons that will tessellate. Shapes will tessellate if the angles that meet at a central point fill a total of 360° . For this reason, any triangle or quadrilateral will tessellate because the angles can be combined to fill 360° and congruent sides will match.

This topic can provide an avenue for students to demonstrate their creativity. The designs produced can make interesting wall hangings for the classroom. The works of M.C. Escher would make an interesting research project using the Internet. A simple application of Escher-like tessellations might look like the following:



STATISTICS AND PROBABILITY

SPECIFIC CURRICULUM OUTCOMES

SP1 – Critique ways in which data is presented.

SP2 – Solve problems involving the probability of independent events.

Grade 8 – Strand: Statistics and Probability (SP)

GCO: Collect, display and analyse data to solve problems.

GRADE 7	GRADE 8	GRADE 9
<p>SP1 Demonstrate an understanding of central tendency and range by:</p> <ul style="list-style-type: none"> • determining the measures of central tendency (mean, median, mode) and range; • determining the most appropriate measures of central tendency to report findings. <p>SP2 Determine the effect on the mean, median and mode when an outlier is included in a data set.</p> <p>SP3 Construct, label and interpret circle graphs to solve problems.</p>	<p>SP1 Critique ways in which data is presented.</p>	<p>SP1 Describe the effect of:</p> <ul style="list-style-type: none"> • bias; • use of language; • ethics; • cost; • time and timing; • privacy; • cultural sensitivity on the collection of data. <p>SP2 Select and defend the choice of using either a population or a sample of a population to answer a question.</p>

SCO: **SP1 – Critique ways in which data is presented.** [C, R, T, V]

Students who have achieved this outcome should be able to:

- A. Compare the information that is provided for the same data set by a given set of graphs, including circle graphs, line graphs, bar graphs, double bar graphs and pictographs, to determine the strengths and limitations of each graph.
- B. Identify the advantages and disadvantages of different graphs, including circle graphs, line graphs, bar graphs, double bar graphs and pictographs, in representing a specific given set of data.
- C. Justify the choice of a graphical representation for a given situation and its corresponding data set.
- D. Explain how the format of a given graph, such as the size of the intervals, the width of bars and the visual representation, may lead to misinterpretation of the data.
- E. Explain how a given formatting choice could misinterpret the data.
- F. Identify conclusions that are inconsistent with a given data set or graph and explain the misinterpretation.

Section(s) in MathLinks 8 text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:

1.1 (A B C)

1.2 (D E F)

1.3 (A B C D E F)

[C] Communication	[ME] Mental Mathematics and Estimation	[PS] Problem Solving	[T] Technology
[CN] Connections		[R] Reasoning	[V] Visualization

SCO: SP1 – Critique ways in which data is presented. [C, R, T, V]

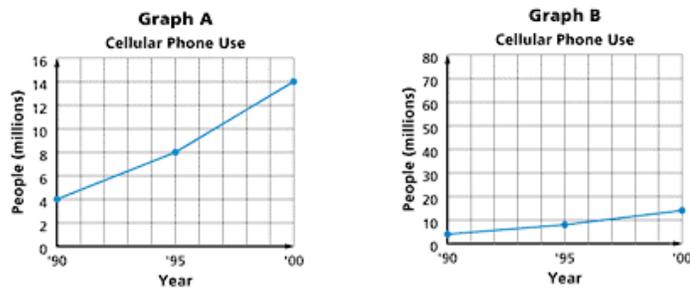
Elaboration

Students can compare various methods of displaying data and evaluating their effectiveness. Comparisons of scale adjustments to indicate such things as degree of growth or loss should be explored. Discussion should take place regarding how the choice of certain graphs can lead to inaccurate judgments. Students' understanding of statistics is enhanced by evaluating the arguments of others. This is particularly important since advertising, forecasting, and public policy are frequently based on data analysis. The media is full of representations of data to support statistical claims. These can be used to stimulate discussion.

It is important for students to be asked to evaluate various situations to determine and debate why a particular display is best suited to a specific type of data, or to a given context. Students should be able to discuss this in terms of continuous versus discrete data sets. For example, given a bar graph and a line graph, students should determine which is most appropriate to display the amount of water flowing into a container and justify their choice.

Students should also be aware of the characteristics of a good graph: it accurately shows the facts, complements or demonstrates arguments presented in the text, has a title and labels, shows data without altering the message of the data and clearly shows any trends or differences in the data.

A common cause of misleading information on graphs stems from the choice of intervals on the vertical axis. Another cause is to begin the vertical axis numbering with something other than zero. Both situations may either over- or under-exaggerate increases or decreases. For example, the graphs below depict a situation where the choice of scale on the vertical axis impacts the effect of the graph.



Grade 8 – Strand: Statistics and Probability (SP)

GCO: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

GRADE 7	GRADE 8	GRADE 9
<p>SP4 Express probabilities as ratios, fractions and percents.</p> <p>SP5 Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.</p> <p>SP6 Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or another graphic organizer) and experimental probability of two independent events.</p>	<p>SP2 Solve problems involving the probability of independent events.</p>	<p>SP4 Demonstrate an understanding of the role of probability in society.</p>

SCO: **SP2 – Solve problems involving the probability of independent events.** [C, CN, PS, T]

Students who have achieved this outcome should be able to:

- A.** Determine the probability of two given independent events and verify the probability using a different strategy.
- B.** Generalize and apply a rule for determining the probability of independent events.
- C.** Solve a given problem that involves determining the probability of independent events.

Section(s) in MathLinks 8 text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:

- 11.1 (A)**
- 11.2 (A)**
- 11.3 (A B C)**

[C] Communication	[ME] Mental Mathematics and Estimation	[PS] Problem Solving	[T] Technology
[CN] Connections		[R] Reasoning	[V] Visualization

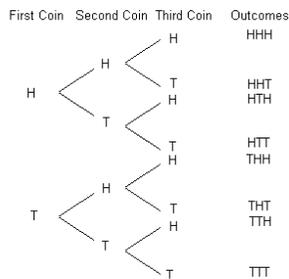
SCO: SP2 – Solve problems involving the probability of independent events. [C, CN, PS, T]

Elaboration

Probability questions presented will be limited to those involving independent events. Tossing heads on a coin and rolling a 5 on a number cube are independent events, where the outcome of one event has no effect on the outcome of another. Although the focus is on independent events, it is still important for students to understand the difference between independent and dependent events. Selecting a heart from a deck of cards, not replacing the card, and then selecting another heart would be an example of dependent events, where the outcome of the second event is affected by the first.

Students should already be familiar with constructing tree diagrams (for two or more events) and tables (limited to two events) for determining the sample space of all possible outcomes for an event, such as the examples given below.

All possible outcomes when 3 coins are tossed



All possible sums generated when two number cubes are rolled

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

The probability that an event will occur is denoted as $P(E)$, and is found by:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Students should be adept at expressing probability outcomes as fractions, decimals or percents.



Curriculum Guide Supplement

This supplement to the *Prince Edward Island Grade 8 Mathematics Curriculum Guide* is designed to parallel the primary resource, *MathLinks 8*.

For each of the chapters in *MathLinks 8*, an approximate timeframe is suggested to aid teachers with planning. The timeframe is based on a total of 176 classes, each with an average length of 40 minutes:

CHAPTER	SUGGESTED TIME
Chapter 1 – Representing Data	12 classes
Chapter 2 – Ratios, Rates and Proportional Reasoning	12 classes
Chapter 3 – Pythagorean Relationship	16 classes
Chapter 4 – Understanding Percent	19 classes
Chapter 5 – Surface Area	15 classes
Chapter 6 – Fraction Operations	13 classes
Chapter 7 – Volume	15 classes
Chapter 8 – Integers	16 classes
Chapter 9 – Linear Relations	15 classes
Chapter 10 – Solving Linear Equations	15 classes
Chapter 11 – Probability	12 classes
Chapter 12 – Tessellations	16 classes

Each chapter of *MathLinks 8* is divided into a number of sections. In this document, each section is supported by a one-page presentation, which includes the following information:

- the name and pages of the section in *MathLinks 8*;
- the specific curriculum outcome(s) and achievement indicator(s) addressed in the section (see the first half of the curriculum guide for an explanation of symbols);
- the student expectations for the section, which are associated with the SCO(s);
- the new concepts introduced in the section;
- literacy links, which reinforce previously learned concepts and highlight the language of mathematics;
- suggested problems in *MathLinks 8*;
- possible instructional and assessment strategies for the section.



CHAPTER 1
REPRESENTING DATA

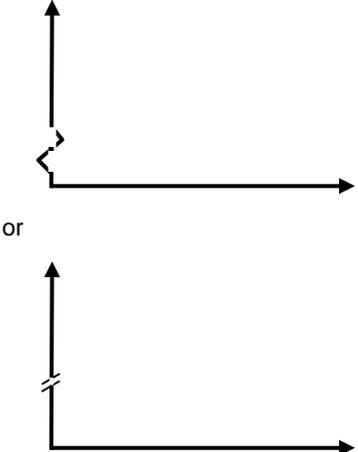
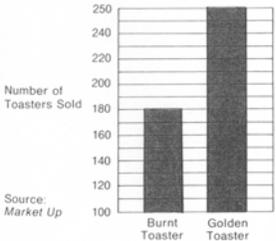
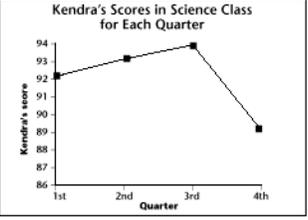
SUGGESTED TIME

12 classes

Section 1.1 – Advantages and Disadvantages of Different Graphs (pp. 6-17)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES								
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> SP1 (A B C) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> compare information from given graphs identify the advantages and disadvantages of different types of graphs <p>After this lesson, students should understand the following concept:</p> <ul style="list-style-type: none"> interval – the spread between the smallest and the largest numbers in a range of numbers <p>Literacy Links:</p> <ul style="list-style-type: none"> Types of Graphs <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>Bar Graph Average Mark</p> <p>Boys Girls CATEGORY</p> </div> <div style="text-align: center;"> <p>Line Graph Average Temperature</p> <p>Jun Jul Aug Sep Oct MONTH</p> </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> <p>Circle Graph Preferred Colour</p> <p>Yellow Pink 14% 11% Red 50% Blue 25%</p> </div> <div style="text-align: center;"> <p>Pictograph Hours Spend Studying Per Week</p> <table border="1" style="margin: auto;"> <tr> <td>Ann</td> <td>⊕⊕⊕⊕</td> </tr> <tr> <td>Billy</td> <td>⊕⊕</td> </tr> <tr> <td>Connor</td> <td>⊕⊕⊕⊕⊕⊕</td> </tr> <tr> <td>David</td> <td>⊕</td> </tr> </table> <p>⊕ represents 1 hour</p> </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> <p>Double Bar Graph Average Mark</p> <p>Boys Girls CLASS</p> </div> <div style="text-align: center;"> <p>Double Line Graph Average Temperature</p> <p>Jun Jul Aug Sep Oct MONTH</p> </div> </div> <ul style="list-style-type: none"> Trend - A trend is the general direction that a line graph is going. Decibel – A decibel is a measure of the intensity of a sound. The abbreviation is dB. <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> pp. 13-17: #1-8, 10, 12-15, Math Link 	Ann	⊕⊕⊕⊕	Billy	⊕⊕	Connor	⊕⊕⊕⊕⊕⊕	David	⊕	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> As a class, have students list advantages and disadvantages of the different graphs, and contribute to a master list. Provide some examples of large quantities of data for students. Examples include population trends, attendance at sporting events, CD sales, Olympic medal standings and box office records for movies. Have students summarize what kinds of data each type of graph best displays. They could generate a context for using each type of graph. Encourage students to refer to the textbook for ideas. Given a graph from a text, magazine or newspaper, convert the graph to some other display form. Discuss which is the better way to display this data and why. Ask students which type of graph they would use to display a student council budget. Ask why they chose that graph. Provide an untitled and unlabelled graph such as the one below and ask students to come up with different sets of data that might realistically be represented by the graph. <div style="text-align: center; margin: 20px 0;"> </div> <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Choose the type of graph that you would recommend to represent each situation. Justify your choice. <ol style="list-style-type: none"> Ray wants to compare the percent of students who go home for lunch to the percent of students who have lunch at school. Ben's mother has put marks on a wall to track his growth since he was three years old.
Ann	⊕⊕⊕⊕								
Billy	⊕⊕								
Connor	⊕⊕⊕⊕⊕⊕								
David	⊕								

Section 1.2 – Misrepresenting Data (pp. 18-27)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES														
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • SP1 (D E F) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • explain how the size of the intervals on a graph could be misleading • explain how the visual representation of a graph could represent data • explain how the size of the bars on a graph could be misleading • identify conclusions that do not agree with a given data set or graph and explain the misinterpretation <p>Literacy Links:</p> <ul style="list-style-type: none"> • Notation – A break in the y-axis of a graph means the length of the axis has been shortened. The break can be shown as: <div style="text-align: center; margin: 10px 0;">  <p style="margin: 0;">or</p> </div> <ul style="list-style-type: none"> • Majority – Majority means more than 50%. <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> • pp. 23-27: #1-18, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Encourage students to think of ways and times that the media have published misleading graphs. • Ask students why the following statement is incorrect: “Sales of Golden Toaster were about double the sales of Burnt Toaster.” Discuss what could be changed on the graph, or added to make it less misleading. <div style="text-align: center; margin: 10px 0;">  <p style="font-size: small; margin: 0;">Source: Market Up</p> </div> <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • At an annual meeting for a certain company, the following information on profits was presented: <table border="1" style="margin: 10px auto; border-collapse: collapse; text-align: center;"> <thead> <tr style="background-color: #cccccc;"> <th style="padding: 5px;">YEAR</th> <th style="padding: 5px;">PROFIT</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">1993</td> <td style="padding: 5px;">\$340,000</td> </tr> <tr> <td style="padding: 5px;">1994</td> <td style="padding: 5px;">\$350,000</td> </tr> <tr> <td style="padding: 5px;">1995</td> <td style="padding: 5px;">\$370,000</td> </tr> <tr> <td style="padding: 5px;">1996</td> <td style="padding: 5px;">\$410,000</td> </tr> <tr> <td style="padding: 5px;">1997</td> <td style="padding: 5px;">\$450,000</td> </tr> <tr> <td style="padding: 5px;">1998</td> <td style="padding: 5px;">\$465,000</td> </tr> </tbody> </table> <p style="margin: 10px 0;">Make a graph to help support each of the following:</p> <ol style="list-style-type: none"> a. The company for the past six years has experienced a very small profit increase. b. The company for the past six years shows a large profit increase. <ul style="list-style-type: none"> • This graph shows that Kendra received a much lower grade in science class during the fourth quarter of the year. Do you think Kendra should be worried by what appears to be such a large drop in her grades? Explain your reasoning. <div style="text-align: center; margin: 10px 0;">  </div>	YEAR	PROFIT	1993	\$340,000	1994	\$350,000	1995	\$370,000	1996	\$410,000	1997	\$450,000	1998	\$465,000
YEAR	PROFIT														
1993	\$340,000														
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1996	\$410,000														
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1998	\$465,000														

Section 1.3 – Critiquing Data Presentation (pp. 28-35)

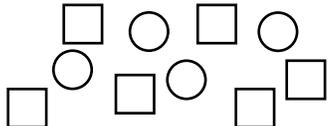
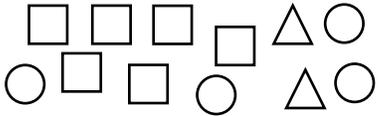
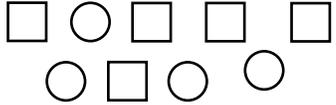
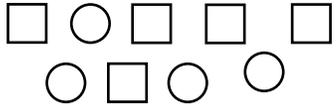
ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES															
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • SP1 (A B C D E F) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • explain how a graph is used to represent the data from a given situation <p>Literacy Links:</p> <ul style="list-style-type: none"> • Stacked Bar Graph – A stacked bar graph has bars stacked instead of side-by-side. <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> • pp. 31-35: #1-10, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Have students brainstorm the features of effective graphs and record their ideas on the board. As a group, develop a master list for critiquing a graph. Include the following headings: graph type, graph format and graph usefulness. • Check that students have considered factors for critiquing a graph such as whether the type of graph and the format of the graph (e.g., intervals) are appropriate to represent the data, and what information can be obtained from the graph. • Make a list of all the graphs students have studied. Assign each group a different type of graph. Ask students to start with raw data and construct a graph to represent the data. Ask each group to prepare a presentation to explain to the class how they went about constructing their graph. Each group can also make a wall display. Ask students to critique the display method chosen by classmates in terms of its suitability for the type of data with which the group was working. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • In a certain country the defence budget was 30 million dollars for 1980. The total budget for that country was 500 million dollars. The following year the defence budget was 35 million dollars, whereas the total budget was 605 million dollars. <ul style="list-style-type: none"> a. You are invited to give a presentation for a pacifist society. You want to explain that the defence budget has decreased this year. How would you do this? b. You are invited to give a presentation at a military academy. You want to explain that the defence budget has increased this year. Explain how you would do this. • Use the data displayed on the pictograph to create a different type of graph that you believe might be a more suitable representation. <div style="text-align: center;"> <p>Favorite Pizza</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Pizza Topping</th> <th>Number of Icons</th> <th>Number of Students</th> </tr> </thead> <tbody> <tr> <td>Mushroom</td> <td>1</td> <td>4</td> </tr> <tr> <td>Pepperoni</td> <td>4</td> <td>16</td> </tr> <tr> <td>Sausage</td> <td>2</td> <td>8</td> </tr> <tr> <td>Cheese</td> <td>3</td> <td>12</td> </tr> </tbody> </table> <p>Each icon means 4 students.</p> </div>	Pizza Topping	Number of Icons	Number of Students	Mushroom	1	4	Pepperoni	4	16	Sausage	2	8	Cheese	3	12
Pizza Topping	Number of Icons	Number of Students														
Mushroom	1	4														
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CHAPTER 2
RATIOS, RATES AND PROPORTIONAL REASONING

SUGGESTED TIME

12 classes

Section 2.1 – Two-Term and Three-Term Ratios (pp. 46-54)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • N4 (A B C D F) • N5 (A B C) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • represent two-term and three-term ratios • identify, describe and record ratios from real-life examples • represent a ratio as a fraction or as a percent • solve problems using ratios <p>After this lesson, students should understand the following concepts:</p> <ul style="list-style-type: none"> • two-term ratio – compares two quantities measured in the same units; written as $a : b$ or a to b <div style="text-align: center;">  <p>squares : circles is 6 : 4</p> </div> <ul style="list-style-type: none"> • three-term ratio – compares three quantities measured in the same units; written as $a : b : c$ or a to b to c <div style="text-align: center;">  <p>squares : circles : triangles is 6 : 4 : 2</p> </div> <ul style="list-style-type: none"> • part-to-part ratio – compares different parts of a group to each other; 5 : 4 is the part-to-part ratio of squares to circles <div style="text-align: center;">  </div> <ul style="list-style-type: none"> • part-to-whole ratio – compares one part of a group to the whole group; 5 : 9 is the part-to-whole ratio of squares to the total number of shapes <div style="text-align: center;">  </div> <p>Literacy Link:</p> <ul style="list-style-type: none"> • Reading Prime – A' is read as “A prime.” A' labels the point in the reduction that corresponds to point A. <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> • pp. 51-54: #1-19, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Some students may benefit from being shown examples in which the ratio is greater than one, equal to one and less than one. Ask them to describe what each type of ratio means. • As a mini-project, teachers might consider asking students to make a scale drawing, or even a model, of the school. The class can be divided into groups where each group takes responsibility for measurement of a section of the school. The groups will bring their information together to produce the final product. • Present a situation in which students will mix fruit juice using different ratios of liquids. One is made using one part fruit juice to three parts water and the other is made using one part fruit juice and four parts water. Ask the students which one will taste sweeter. Continue with activities of mixing juice in various proportions. • Provide students with a hundreds grid. Have them shade in part of the grid. Ask them to exchange their grids with a partner and then represent the portion shaded on their partner’s grid in more than one way. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • Walter and Pat have the same ratio of cats to dogs in their kennels, Walter has 3 cats for every 5 dogs. <ol style="list-style-type: none"> In September, Pat had 25 dogs. How many cats did she have? In January, Pat had 48 cats and dogs altogether. How many of pat’s animals are dogs? • What is the scale of a map if 7.2 cm on the map represents a distance of 1800 km? • The ratio of girls to boys in Mr. Gosse’s homeroom is 18 to 12. What proportion of the students in Mr. Gosse’s homeroom are boys? • In the ratio $3 : 8 = 17 : \square$, can the value of \square be a whole number? • Explain why 1 : 20,000,000 is another way to describe the ratio of 1 cm representing 200 km on a map. • Select three different colours so that the colours show the following ratios: <ol style="list-style-type: none"> 4 to 3 2 : 1 $\frac{1}{3}$ 2 : 3 : 5

Section 2.2 – Rates (pp. 55-62)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • N4 (D E F) • N5 (A B C) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • express rates using words and symbols • identify, describe and record rates from real-life examples • solve problems using rates <p>After this lesson, students should understand the following concepts:</p> <ul style="list-style-type: none"> • rate – compares two quantities measured in different units; \$1.69 per 100 g or \$1.69/100 g is a rate for purchasing bulk food; 72 beats per minute or 72 beats/min is a heart rate • unit rate – a rate in which the second term is one; for example, 20 km/h and 64 beats/min • unit price – a unit rate used when shopping; often shown per 100 g or per 100 mL; makes it easier for shoppers to compare costs of similar items <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> • pp. 59-62: #1-16, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Remind students that the numbers must be in the same units in order to compare rates. • Have students bring in samples of unit pricing information and use the information to provide additional problems to the class. • Suggest to students the use of sports statistics as a basis for a project. Rates, such as points per game, runs batted in and goals against are often applied in sports. Such statistics are usually available in the sports section of major newspapers. • Ensure that students understand why rate cannot be represented as a percent. For example, if 2 out of 5 students are going to a dance, we could also say that 40% will be attending. In this case, the ratio is comparing the part (students attending) against the whole (the entire school population). This is different from a rate, which compares two different things, like speed in kilometres per hour. Since rates compare different things, they cannot be represented as a percent, which compares part to whole of only one thing. • Explain how you would use speed of travel and a watch to determine distance travelled. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • Joan's family drove at 80 kilometres per hour for 8 hours and 12 minutes. How far did they drive? What assumptions did you make? • A certain brand of grass seed makes the claim that a 1-kg bag will seed an area of 50 m². How many bags of grass seed would be required for seeding a rectangular field which is rectangular and measures 120 m by 70 m. • Use the unit-rate method to solve the following: If a package of 6 bottles of sports drink costs \$4.50 and an individual bottle costs \$1.50, how much would you save per bottle by purchasing a package? • Determine who would get the bigger portion of pizza if 9 girls share 4 pepperoni pizzas while 7 boys share 3 vegetarian pizzas. Explain your reasoning. What assumptions are you making? • If a tap is dripping at a rate of 50 mL per hour, can you describe that as a percentage? Explain why or why not?

Section 2.3 – Proportional Reasoning (pp. 63-69)

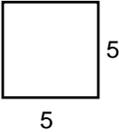
ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> N5 (A B C) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> solve problems using proportional reasoning use more than one method to solve proportional reasoning problems <p>After this lesson, students should understand the following concept:</p> <ul style="list-style-type: none"> proportion – a relationship that says that two ratios or two rates are equal; can be written in fraction form $\frac{2}{3} = \frac{6}{9}$ $\frac{2 \text{ km}}{3 \text{ h}} = \frac{6 \text{ km}}{9 \text{ h}}$ <p>Literacy Link:</p> <ul style="list-style-type: none"> Slough – In Western Canada, a slough is a small lake or pond formed by rain or melted snow. <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> pp. 67-69: #1-18, 21-23, 25, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Encourage students to use their knowledge of equivalent fractions or unit rates to solve proportion problems. Students should select the method that is most efficient to them and appropriate to solve problems. Have each student bring in a picture of himself or herself standing beside a person or an object. Measure the height of the images in the picture (in cm) and use a ratio to find the actual height of the other person or object in the photo. One of the actual heights must be known. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> A recipe uses 500 mL of flour for every 125 mL of sugar. How much flour would be needed when 500 mL of sugar is used? Suzelle found a good deal on pop. She could buy 12 cans for \$2.99. She needs 72 cans for her party. Explain how she can calculate the cost. When making lemonade, Sue used 5 scoops of powder for 6 cups of water, and Sarah uses 4 scoops of powder for 5 cups of water. <ol style="list-style-type: none"> Are the two situations proportional to each other? Explain why or why not. In which situation is it likely that the lemonade will be more flavourful? What assumptions did you make? Can the following problem be solved as a proportion: David is 6 years old and Ellen is 2 years old. How old will Ellen be when David is 12 years old? Explain your reasoning. Study each of the proportions and estimate which of the three problems represents the largest value. Solve to verify your estimate. <ol style="list-style-type: none"> $\frac{3}{7} = \frac{a}{28}$ $\frac{b}{9} = \frac{3}{4}$ $\frac{5}{c} = \frac{15}{33}$ Use proportions to find which is the better buy: 1.2 L of orange juice for \$2.00, or 0.75 L of orange juice for \$1.40. Explain why it is the better buy. During a very heavy rain storm, 40 mm of rain fell in 30 min. How much rain would you expect to fall in 1 hour? in 3 hours? What assumption(s) are you making? Determine the amount of each type of fruit in a salad if the ratio of grapes to melon to pineapple is 3 : 2 : 4 and the salad bowl holds 4.5 litres.

CHAPTER 3
PYTHAGOREAN RELATIONSHIP

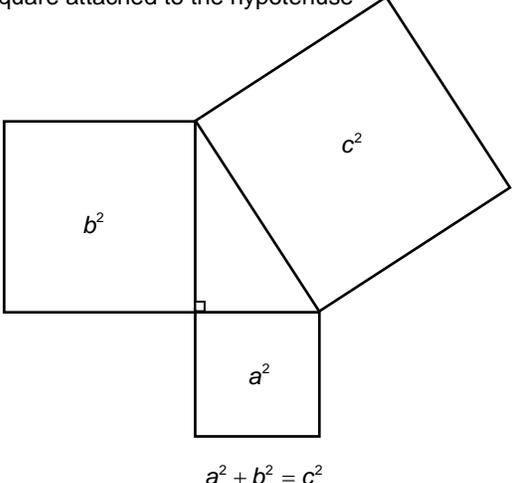
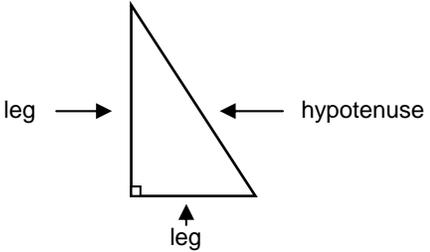
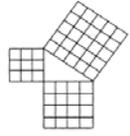
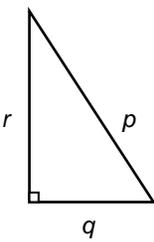
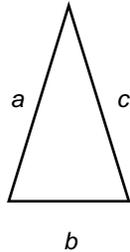
SUGGESTED TIME

16 classes

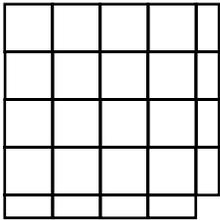
Section 3.1 – Squares and Square Roots (pp. 80-87)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> N1 (A B C D E) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> determine the square of a whole number determine the square root of a perfect square <p>After this lesson, students should understand the following concepts:</p> <ul style="list-style-type: none"> prime factorization – a number written as a product of its prime factors; the prime factorization of 6 is 2×3 perfect square – a number that is the product of the same two factors; has only an even number of prime factors in its prime factorization; $5 \times 5 = 25$, so 25 is a perfect square <div style="text-align: center;">  </div> <ul style="list-style-type: none"> square root – a number that when multiplied by itself equals a given value; 6 is the square root of 36 because $6 \times 6 = 36$ <p>Literacy Links:</p> <ul style="list-style-type: none"> Square Numbers – A square number is the product of the same two numbers. Since $3 \times 3 = 9$, 9 is a square number. A square number is also known as a <i>perfect square</i>. A number that is not a perfect square is called a <i>non-perfect square</i>. Prime Numbers and Prime Factors – A prime number is natural number that has exactly two factors. Prime factors are factors that are prime numbers. For example, the prime factors of 10 are 2 and 5. Notation – You can write repeated multiplication like 13×13 as a square: $13 \times 13 = 13^2$. The expression 13^2 is read as “thirteen squared.” Reading Square Roots – The symbol for square root is $\sqrt{\quad}$. Read $\sqrt{9}$ as “the square root of 9,” “square root 9” or “root 9.” <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> pp. 85-87: #1-18, 23, 24, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Some students may benefit from examples of non-perfect squares. These counterexamples may assist students to understand what a perfect square is. Sometimes, it helps to see what something is not, rather than what it is. It may be helpful to point out to students that the side of a square is the square root and the area is the square. For natural numbers, the area is always larger than the square root. Reinforce the fact that the square root can be obtained by rearranging the prime factors into two equal groups. Investigate the inverse relationship between squares (3^2) and square roots ($\sqrt{9}$). Use patterns to determine that the square root of 1600 is 40 since the square root of 16 is 4. Verify this by $\sqrt{1600} = \sqrt{16} \times \sqrt{100} = 4 \times 10 = 40$. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Use grid paper or square tiles to show the square root of 36, 49 and 81. Use square roots to solve each of the following: <ul style="list-style-type: none"> A square has an area of 81 m^2. What are the dimensions? A cube has a surface area of 294 m^2. What are the dimensions? Find the square root of each of the following, using patterning and/or prime factorization. Justify your answer. <ul style="list-style-type: none"> 6400 12,100 900 676 Explain why $\sqrt{\square\square\square 8}$ cannot be a whole number. Simplify: <ul style="list-style-type: none"> $\sqrt{15 \times 15}$ $\sqrt{10^2}$ Lydia listed all of the factors of 7569 and wrote: 1, 3, 9, 87, 841, 2523, 7569 How can you determine the square root of 7569 using Lydia’s list of factors? Find the square root of 324 using prime factorization. The prime factorization of a number is $2 \times 2 \times 3 \times 3 \times 7 \times 7$. What is the number, and what is its square root?

Section 3.2 – Exploring the Pythagorean Relationship (pp. 88-94)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • SS1 (A B C E) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • model the Pythagorean relationship • describe how the Pythagorean relationship applies to right triangles <p>After this lesson, students should understand the following concepts:</p> <ul style="list-style-type: none"> • hypotenuse – the longest side of a right triangle; the side opposite the right angle • Pythagorean relationship – the relationship among the lengths of the sides of a right triangle; the sum of the areas of the squares attached to the legs of a right triangle equals the area of the square attached to the hypotenuse  <p style="text-align: center;">$a^2 + b^2 = c^2$</p> <p>Literacy Links:</p> <ul style="list-style-type: none"> • Right Triangle – a right triangle has a right angle (90°). The right angle may be marked with a small square. The two shorter sides that form the right angle are called the legs. The longest side is called the hypotenuse.  <ul style="list-style-type: none"> • Notation – The symbol \neq means “is not equal to.” • Area – The area of a circle is $A = \pi r^2$. <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> • pp. 91-94: #1-17, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Give or have students draw a variety of right triangles which have whole number sides, such as the 3-4-5, 6-8-10 or the 5-12-13 triangle. Have students cut out squares from centimetre grid paper so the sides of each square are the same as the side lengths for each square are the same as the side lengths for each triangle. Place the squares on the sides of the triangle as shown. Find the area of each square. Ask students what they notice.  <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • Determine if 3-4-5 is a Pythagorean triple. Multiply each number by 2. Show whether the resulting three numbers form a Pythagorean triple. Explore by multiplying by other whole numbers. Is there any whole number that does not make a Pythagorean triple when 3-4-5 is multiplied by it? • Explain how you can determine whether or not a triangle is a right triangle if you know that it has side lengths of 7 cm, 11 cm and 15 cm. • Determine whether each of the following student's work is correct and explain your thinking. <ol style="list-style-type: none"> Corey wrote the Pythagorean relationship as $r^2 = p^2 + q^2$.  <ol style="list-style-type: none"> Mia wrote the Pythagorean relationship as $a^2 = b^2 + c^2$. 

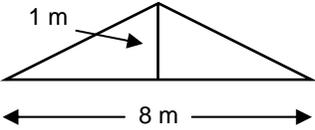
Section 3.3 – Estimating Square Roots (pp. 95-100)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> N2 (A B C D) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> estimate the square root of a number that is not a perfect square identify a number with a square root that is between two given numbers <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> pp. 98-100: #1-11, 13-16, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Ensure that students are comfortable with the perfect square benchmarks from 1 to 144 as these are used to establish an initial estimate when finding a square root. Use a calculator to estimate the square root of a non-perfect square without the $\sqrt{\quad}$ key. If students are asked to estimate the square root of 20, they should know that it is about halfway between 4 and 5, since 20 is about halfway between 16 and 25. They might try 4.4×4.4, then 4.5×4.5 on the calculator to determine which is closer to 20. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Jan used grid paper to show that the square root of 20 is not a whole number. She formed a square by using 16 blocks. The four additional blocks she cut in half and placed 4 on each of the dimensions of the 4×4 square. This produced a figure as shown: <div style="text-align: center;">  </div> <ol style="list-style-type: none"> Use the diagram to estimate the square root of 20. Find the square root of 20 on the calculator. Use the diagram to justify why there is a difference between the two answers. Use a similar diagram to estimate the square root of 30. <ul style="list-style-type: none"> Identify a whole number with a square root between 6 and 7. Estimate each square root to the nearest tenth. <ol style="list-style-type: none"> $\sqrt{14}$ $\sqrt{35}$ Estimate to determine whether each answer is reasonable. Check your prediction using your calculator. <ol style="list-style-type: none"> $\sqrt{11} \approx 3.3$ $\sqrt{27} \approx 5.9$ While Rebecca was shopping online, she found a square rug with an area of 17 m^2. The dimensions of her bedroom are $4 \text{ m} \times 5 \text{ m}$. Will the rug fit in her room? Explain.

Section 3.4 – Using the Pythagorean Relationship (pp. 101-105)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • SS1 (D) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • use the Pythagorean relationship to determine the missing side length of a right triangle <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> • pp. 103-105: #1-7, 9-14, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Students may benefit from seeing the square root of both sides of an equation so that it is clear how the solution is obtained. For example, $c^2 = 144$ $\sqrt{c^2} = \sqrt{144}$ $c = 12$ • Discuss with students the concept of assigning variables to the sides of a triangle. Make sure they understand that it does not matter what variables, a or b, they use for the legs, since it will not affect the answer as long as they do not confuse the lengths of the legs with the length of the hypotenuse. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • Use the Pythagorean relationship to find lengths of $\sqrt{2}$ and $\sqrt{8}$. • Find whole-number legs for the triangle so that the square on the hypotenuse has areas of each of the following. <ol style="list-style-type: none"> 34 37 68

Section 3.5 – Applying the Pythagorean Relationship (pp. 106-111)

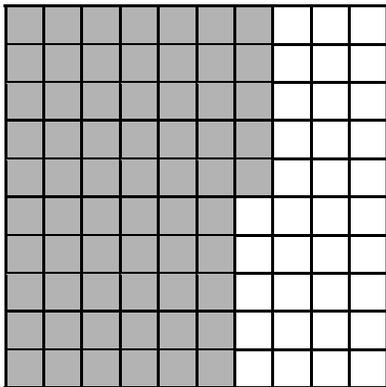
ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • SS1 (D) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • apply the Pythagorean relationship to solve problems • determine distances between objects <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> • pp. 109-111: #1-10, 13, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Use dot paper or geoboards and make squares of 1 square unit, 4 square units, 5 square units, 9 square units and 10 square units. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • For safety reasons, a construction company established the following rule: When placing a ladder against the side of a building, the distance of the base of the ladder from the wall should be at least $\frac{1}{3}$ of the length of the ladder. <ul style="list-style-type: none"> a. Can an 8 m ladder reach a 7 m window when this rule is followed? Explain. b. What is the shortest ladder that can be used to reach a 12 m window? The rule must apply. • A flower garden is created where two walkways intersect. The two walkways intersect at right angles, and the flower garden extends 2 m along one walkway and 1.5 m along the other. <ul style="list-style-type: none"> a. If Nick wants to put a border around the whole garden, what length of border will be required? b. If Nick wishes to spray the area for pests, he needs to know the area of the garden to determine the size of the pesticide container to purchase. What is the area of the garden? • For a particular rectangular house of dimensions 8 m by 18 m, the roof truss is as shown: <div style="text-align: center;">  </div> <ul style="list-style-type: none"> a. Find the area of the roof and use this area to determine the number of shingles necessary to cover the roof, if each shingle has an exposed area of 500 cm^2. b. What assumptions did you make?

CHAPTER 4
UNDERSTANDING PERCENT

SUGGESTED TIME

19 classes

Section 4.1 – Representing Percents (pp. 122-129)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> N3 (A B C D) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> show percents that are between 0% and 1% show percents that are greater than 100% show percents involving fractions <p>After this lesson, students should understand the following concepts:</p> <ul style="list-style-type: none"> percent – means <i>out of 100</i>; another name for hundredths; 65% means 65 out of 100 or $\frac{65}{100}$ or 0.65 <div style="text-align: center;">  </div> <ul style="list-style-type: none"> fractional percent – a percent that includes a portion of a percent, such as $\frac{1}{2}\%$, 0.42%, $7\frac{3}{8}\%$, $125\frac{3}{4}\%$, 4.5% <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> pp. 128-129: #1-15, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Some students may benefit from using base ten blocks or hundred grids to represent percents greater than 100%. Doing so will help them visualize the number of hundred grids needed. Show students how the hundreds place determines the number of hundred grids needed to represent percents greater than 100%. Show students who are struggling with representing less than 1% how to use a hundred grid to represent an enlarged square of one unit or 1%. Have students shade the appropriate number of units to show their understanding. Use the chalkboard or an overhead to write examples of percents greater than 100% and have students predict the number of grids needed to represent each one. Allow students to cut and paste hundred grids (or portions thereof) onto drawing paper. Encourage creative students to make mosaic representations of percents. They could then use any combination of oral and written work to note the similarities and differences. Consider posting samples of different percents in the classroom. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Assuming that a flat from set of base-ten blocks represents 100% of something, represent each of the following using base-ten blocks. <ul style="list-style-type: none"> 110% 125% 200% Sarah has a savings account that earns $\frac{1}{2}\%$ simple interest monthly. Jane has a savings account that earns $5\frac{3}{4}\%$ annually. Who do you think would have more money in the bank at the end of one year if they both start with the same amount? Why? When Jane saw the interest rate of 0.9% per year, she thought it was a reasonable return on her money. Jack advised her that it was a very low rate of return. <ul style="list-style-type: none"> Why do you think Jane thought it was a good rate of return? Why do you think Jack thought it was a bad rate of return?

Section 4.2 – Fractions, Decimals and Percents (pp. 130-137)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • N3 (A D E F G) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • convert among decimals, fractions and percents <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> • pp. 135-137: #1-18, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Have students share their procedures for converting among fractions, decimals and percents. They may benefit from viewing the methods of other students, as long as they make mathematical sense. • It may be helpful to provide a rule when converting decimals to percents, such as <i>multiplying by 100 moves the decimal to the right two places.</i> • It may be helpful to provide a rule when converting percent to decimals, such as <i>dividing by 100 moves the decimal to the left two places.</i> • Have students who need help converting between decimals and fractions verbalize the place values of each decimal. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • Estimate the percent for each fraction. Explain your reasoning. <ul style="list-style-type: none"> a. $\frac{125}{85}$ b. $\frac{99}{95}$ c. $\frac{2}{230}$

Section 4.3 – Percent of a Number (pp. 138-143)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> N3 (H) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> solve problems that involve percents less than 1% solve problems involving percents greater than 100% solve problems involving fractional percents <p>Literacy Links:</p> <ul style="list-style-type: none"> Profit – Profit is the amount of money left over after all expenses are paid. Halve and Double – Halve means to divide by two. Double means to multiply by two. Terminology – In math, the word <i>of</i> often means to multiply. <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> pp. 141-143: #1-13, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Encourage students to identify and use the technique that they are most comfortable with when finding the percent of a number mentally, such as halving, dividing repeatedly by tens, or calculating 1% and then multiplying by what is needed. If students use a calculator to find the percent of a number, encourage them to use mental math beforehand to check the reasonableness of the answer. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> A jacket is now selling for \$64. The sign above it indicates it was reduced by 20%. What was the original selling price. McDunphy's Burger Heaven has a sale on hamburgers. A hamburger is $\frac{1}{2}$ price when you buy a medium drink and a medium fries. The normal prices are as follows: hamburger – \$2.30; medium drink – \$1.29; and medium fries – \$1.39. What is the actual percentage off the regular price when you take into account what must be purchased to take advantage of the sale? A politician was elected with 2145 votes at a convention. If she received 60% of the votes cast, how many votes were cast? If 30 is close to 80% of a number, what do you know about the number? A certain number is between 10 and 100. Explain what can be concluded about 150% of the number. Explain why 60% is not a good estimate for what percentage of 30 is 70. Estimate the percent increase represented in the following problem. Derek's father said, "In my day, I could buy a chocolate bar and a soft drink for 25¢." If these items together cost \$2.25 today, what is the percent increase? Solve each of the following: <ul style="list-style-type: none"> Two percent of a certain number is 4. What is the number? What is 11.5% of 40? Approximately 0.6% of new Brunswick's population lives in Sackville. The population of New Brunswick is approximately 750,000. If the population of Sackville increases by 1000 when students attend Mount Allison University, what percent increase would this be?

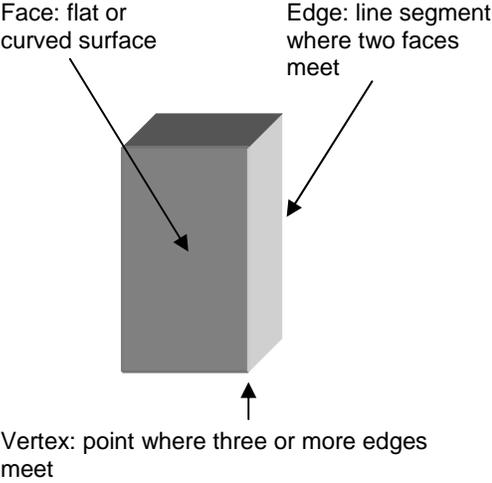
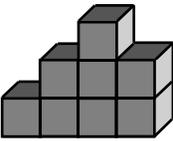
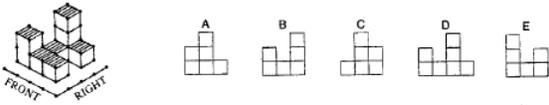
Section 4.4 – Combining Percents (pp. 144-149)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> N3 (H I J) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> solve problems involving combined percents <p>Literacy Links:</p> <ul style="list-style-type: none"> PST and GST – PST means provincial sales tax (currently 10% in P.E.I.); GST means goods and services tax (currently 5% in Canada) Combining Percents – You can combine percents by adding individual percent values together. <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> pp. 148-149: #1-12, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Explain to students that when calculating sales in Prince Edward Island, first calculate the GST (5%) on the price and then calculate the PST (10%) on the total of the sales price and the GST. This number is then added to the cost price. Please note that this is equivalent to finding 115.5% of the cost price. Also please note that these percentages are subject to change. Help students understand the concept that 25% off means that 75% remains. This concept can be shown on a hundred grid. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Elite basketball sneakers, which regularly sell for \$185, were marked down by 25%. To further improve sales, the discount price was reduced by another 15%. <ul style="list-style-type: none"> What was the final selling price? What was the total percent of discount on the original price? What was the total cost after 15% tax was added? Jane decided that, since the second discount was 15% and the tax was 15%, she would save herself a lot of work and let one balance out the other. This way, she could calculate the first discount and determine the total cost. Does her reasoning produce an accurate result? Explain why or why not? Sarah found out that the new car she bought would depreciate in value by 20% per year. Sarah paid \$20,000 for the car and planned to keep it for three years. She wanted to find the car's value at the end of three years and asked a friend to help. They decided to do their calculations independently and then compare answers. Sarah's answer was \$10,240, but her friend's answer was \$8000. <ul style="list-style-type: none"> How was each of the answers obtained? Who do you think is correct? Why? Solve each of the following problems. <ul style="list-style-type: none"> DVD players, which regularly sell for \$150, were marked down by 25%. To further improve sales, the discount price was reduced by another 15%. What was the final selling price? Explain why this is not the same as a 40% discount. What would the difference be? The price of a \$250 video game console is increased by 25%. After two weeks, the new price is reduced by 25%. Explain why the final price is not \$250.

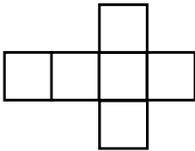
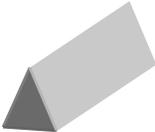
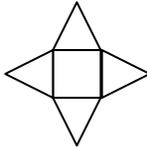
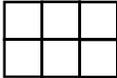
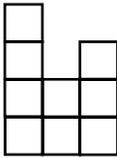
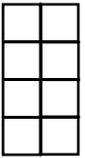
CHAPTER 5
SURFACE AREA

SUGGESTED TIME
15 classes

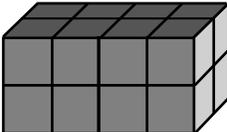
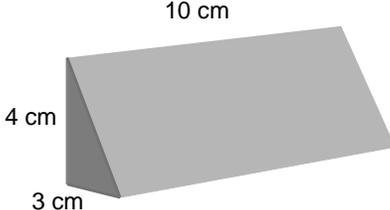
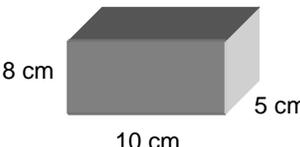
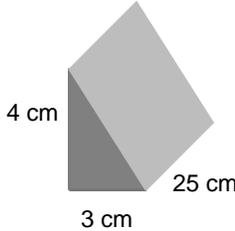
Section 5.1 – Views of Three-Dimensional Objects (pp. 164-169)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> SS5 (A B C D E F) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> draw and label top, front and side views of 3-D objects build 3-D objects when given top, front and side views <p>Literacy Link:</p> <ul style="list-style-type: none"> Describing a 3-D Object – To describe a 3-dimensional (3-D) object, count its <i>faces</i>, <i>edges</i> and <i>vertices</i>. <div style="text-align: center;">  </div> <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> pp. 168-169: #1-8, 9(a, b or c), Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Use mats to help students with 2-D drawings of 3-D objects. A square of plain paper appropriately marked with directions would be a simple mat for this purpose. This is particularly useful for drawing the orthographic views and rotating the object. Some students might find it helpful to close one eye and sit so that they are at eye level with the shape. They should then see only one face of the object. Have students compare structures so they can come to realize that there can be more than one structure that fulfils the information in a set of plans. Have students explore such questions as the following: What is the minimum number of cubes that can be used to fulfill the plans provided? What is the maximum? How many different objects can be built to fulfill the plans? Use linking cubes as the basic building blocks for 3-D objects, as they are very versatile. With the front of the object facing the students, have them turn it 90° clockwise and sketch the object. Now have them turn it another 90° clockwise and sketch it again. Have them turn it one more time 90° clockwise and produce the third sketch. Have them continue rotating at 90° intervals until the sketch looks identical to the one already drawn. Use interactive websites to aid with isometric drawings such as: http://illuminations.nctm.org/ActivityDetail.aspx?ID=125 <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Using blank paper, draw the top, front and side views of this object. <div style="text-align: center;">  </div> <ul style="list-style-type: none"> Examine this picture of a building drawn from its right-front corner. Which one of A – E is the right orthographic view? <div style="text-align: center;">  </div>

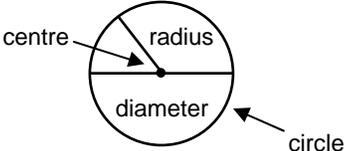
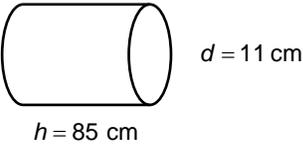
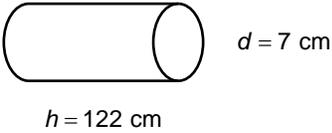
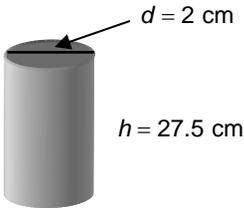
Section 5.2 – Nets of Three-Dimensional Objects (pp. 170-175)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • SS2 (A B C D) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • determine the correct nets for 3-D objects • build 3-D objects from nets • draw nets for 3-D objects <p>After this lesson, students should understand the following concepts:</p> <ul style="list-style-type: none"> • rectangular prism – a prism whose bases are congruent rectangles <div style="text-align: center; margin: 10px 0;">  </div> <ul style="list-style-type: none"> • net – a two-dimensional shape that, when folded, encloses a 3-D object <div style="display: flex; justify-content: space-around; align-items: center; margin: 10px 0;"> <div style="text-align: center;">  <p>net</p> </div> <div style="text-align: center;">  <p>cube</p> </div> </div> <ul style="list-style-type: none"> • triangular prism – a prism with two triangular bases, each the same size and shape <div style="text-align: center; margin: 10px 0;">  </div> <p>Literacy Link:</p> <ul style="list-style-type: none"> • Right Prism – A right prism has sides that are perpendicular to the base of the prism <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> • pp. 173-175: #1-8, 10, 13, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Provide a can for students to examine. Have them trace the bottom and carefully remove the label to trace the side. Have students explain why they drew the net the way they did. Clarify any misunderstandings. Clarify that the end (bottom and top) of the can cannot be the same size as the width of the label. Have them compare their tracing to the net drawn. • Have students cut along the edge of various shaped boxes (cereal boxes, tennis ball canister, potato chip cans) and unfold them to form a net. Students should predict what the net will look like before they cut it and explore for themselves. • Provide copies of nets for students to cut and fold up. They should be encouraged to unfold them and examine the 2-D shapes that are connected to make each net. • Have students find all the nets for the square based pyramid. Many students find the net of a square based pyramid easier to visualize as a 3-D object than the net of a cube. <div style="text-align: center; margin: 10px 0;">  </div> <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • Use the information provided to draw a 3-D shape using blocks. Is there only one shape which can be built to meet these specifications? <div style="display: flex; justify-content: space-around; align-items: center; margin: 10px 0;"> <div style="text-align: center;">  <p>Top</p> </div> <div style="text-align: center;">  <p>Front</p> </div> <div style="text-align: center;">  <p>Right</p> </div> </div> <ul style="list-style-type: none"> • Have students draw all the possible nets for a triangular pyramid with all faces equilateral triangles. Repeat for one with an equilateral base and three isosceles triangular faces. Did you get more nets for one of them? Why do you think this happened?

Section 5.3 – Surface Area of a Prism (pp. 176-181)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • SS2 (C) • SS3 (A B C E) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • link area to surface area • find the surface area of a right prism <p>After this lesson, students should understand the following concept:</p> <ul style="list-style-type: none"> • surface area – the number of square units needed to cover a 3-D object; the sum of the areas of all the faces of an object <div style="text-align: center;">  </div> <p>Literacy Link:</p> <ul style="list-style-type: none"> • Equilateral Triangle – An equilateral triangle has three equal sides and three equal angles. Equal sides are shown on diagrams by placing tick marks on them. <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> • pp. 179-181: #1-6, 8, 10, 12, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Students may benefit from working through an example using a 3-D object that you cut apart and manipulate. Alternatively, consider pairing students so they can explain their thinking to each other. • Have students measure any rectangular prism in the classroom and calculate its surface area. • Encourage students to estimate the surface area before calculating the exact answer to check the reasonableness of their calculations. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • What is the surface area of the wedge of cheese shown? <div style="text-align: center;">  </div> <ul style="list-style-type: none"> • The owners of a cracker factory are trying to choose a box to hold their new flavour of cracker. They want a box that uses the least amount of cardboard. Which box should they choose? <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <ul style="list-style-type: none"> • Determine which pentominoes can fold to make an open box. • How does drawing the net of a prism help you calculate its surface area? • Calculate the surface area of a DVD case to the nearest tenth of a square centimetre whose plastic covering measures 19 cm long, 12.5 cm wide and 1.7 cm thick. • Marilyn has 1 m^2 of paper to wrap a box 28 cm long, 24 cm wide and 12 cm high for a present. Does she have enough paper?

Section 5.4 – Surface Area of a Cylinder (pp. 182-187)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> SS2 (C) SS3 (D E) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> find the surface area of a cylinder <p>After this lesson, students should understand the following concept:</p> <ul style="list-style-type: none"> cylinder – a three-dimensional object with two parallel and congruent circular bases <div style="text-align: center;">  </div> <p>Literacy Links:</p> <ul style="list-style-type: none"> Circle <div style="text-align: center;">  </div> <ul style="list-style-type: none"> Notation – The abbreviation <i>S.A.</i> is often used as a short form for surface area. <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> Essential: #1-10, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Some students may benefit from having the surface area formula for the cylinder written using only radius so they do not have two different values (<i>d</i> and <i>r</i>) in the formula. $S.A. = 2 \cdot (\pi \cdot r^2) + 2 \cdot (\pi \cdot r \cdot h)$ Ask students how calculating the surface area of a cylinder and calculating the surface area of a prism are alike. Ask how they are different. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Jennifer and Jamie each bought a tube of candy. Both containers cost the same amount. Which container required more plastic to make? <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> Calculate the outside surface area of the paper towel tube with the following dimensions. Do not count the top or bottom circles <div style="text-align: center;">  </div> Calculate the surface area of the pencil sharpeners on Kay's desk. It is a cylinder with a diameter of 3.1 cm and a height of 5 cm. Brad is purchasing burlap to protect his three apple trees against the cold winter weather. He will wrap the burlap around the bottom 140 cm of each tree trunk. The trees are 22.1 cm, 24.7 cm and 33.2 cm in circumference. How much burlap will he need?

CHAPTER 6
FRACTION OPERATIONS

SUGGESTED TIME
13 classes

Section 6.1 – Multiplying a Fraction and a Whole Number (pp. 198-203)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> N6 (A G L) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> multiply a fraction and a whole number solve problems involving the multiplication of a fraction and a whole number <p>Literacy Links:</p> <ul style="list-style-type: none"> Understanding Multiplication – The product of 4 and 2 is 8, because $4 \times 2 = 8$. The equation $4 \times 2 = 8$ means that 4 groups of 2 make 8. You can also think of 4×2 as the repeated addition $2 + 2 + 2 + 2$. Classifying Fractions – In a <i>proper fraction</i>, such as $\frac{1}{2}$ or $\frac{5}{6}$, the denominator is greater than the numerator. In an <i>improper fraction</i>, such as $\frac{5}{2}$ or $\frac{4}{3}$, the numerator is greater than the denominator. A <i>mixed number</i>, such as $1\frac{1}{4}$ or $4\frac{3}{5}$, includes a whole number and a proper fraction. Terminology – In mathematics, the word <i>of</i> often indicates multiplication. Commutative Property – The commutative property states that $a \cdot b = b \cdot a$. <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> pp. 202-203: #1-12, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Check for understanding of fractions or parts of a whole using fraction blocks or rectangles to ensure the concept is clear before moving to repeated addition. Compare the following two situations: Two-thirds of John’s fifteen cars are red. Fifteen glasses are two-thirds full. Discuss the difference between how these are represented. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Sketch diagrams to illustrate how to find each of the following: <ul style="list-style-type: none"> a. $\frac{3}{4}$ of a set of 20 items b. $\frac{2}{5}$ of a set of 15 items At the school concert there were 600 people in attendance. Of those people, $\frac{1}{4}$ of those attending were men, $\frac{1}{3}$ of those attending were women and the rest were children. <ul style="list-style-type: none"> a. Determine how many children attended. b. Determine how much money was taken in at the door, if adults paid \$4.00 per ticket and children paid \$2.00 per ticket. Explain how you could estimate $\frac{3}{4}$ of a set of 21 items. Explain how to rewrite $\frac{1}{6}$ of 48 as a division problem. Mentally find the product of $\frac{4}{5} \times 20$ and explain your thinking. Use mental computation to answer each of the following: <ul style="list-style-type: none"> a. George had $\frac{2}{3}$ of the questions on the test correct. Since there were 30 questions on the test, how many did he get correct? b. Two-fifths of the students in Sarah’s class have to take the bus to school. There are 30 students in Sarah’s class. How many do not take the bus?

Section 6.2 – Dividing a Fraction By a Whole Number (pp. 204-209)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • N6 (A I L) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • divide a fraction by a whole number • solve problems involving the division of fractions by whole numbers <p>Literacy Link:</p> <ul style="list-style-type: none"> • Understanding Division – In the equation $6 \div 2 = 3$, the dividend is 6, the divisor is 2 and the quotient is 3. The equation $6 \div 2 = 3$ means that in 6 there are 3 groups of 2. This division statement also means that if 6 is separated into 2 equal groups, there are 3 in each group. <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> • pp. 208-209: #1-6, 8-13, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Demonstrate division to students by using a variety of models, but encourage them to use the method they feel most comfortable with and explain why. • Remind students that the denominator tells how many pieces the whole will be divided into and the divisor tells how many pieces an individual unit is divided into. • Present division of a fraction by a whole number as a sharing situation. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • Answer each of the following questions. <ul style="list-style-type: none"> a. If $\frac{1}{3}$ is divided into two equal parts, how large is each part? b. If $\frac{1}{6}$ is divided into two equal parts, how large is each part? c. If $\frac{1}{3}$ is divided into three equal parts, how large is each part? d. If $\frac{3}{4}$ is divided into three equal parts, how large is each part? • Five-sixths of the grade eight students in a school are in the band. These band students are divided into four equal groups. What fraction of the grade eight students is in each of these groups?

Section 6.3 – Multiplying Proper Fractions (pp. 210-215)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> N6 (B D H K L) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> multiply two proper fractions solve problems involving the multiplication of two proper fractions <p>Literacy Link:</p> <ul style="list-style-type: none"> Understanding Common Denominators – For $\frac{1}{2}$ and $\frac{2}{3}$, a common denominator is 6, which is a common multiple of 2 and 3. <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> pp. 214-215: #1-11, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Students should be encouraged to model multiplication of fractions using paper folding or other manipulatives. Ask students to explain what it means to multiply by a fraction such as $\frac{1}{3}$. Ask students to compare the solutions to 2×4 and $\frac{1}{2} \times \frac{1}{4}$, and discuss their observations. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Place the numbers 1, 2, 3 and 4 in the boxes to get the least possible answer: $\frac{\square}{\square} \times \frac{\square}{\square}$ Simplify $\frac{3}{4} \times \frac{4}{5}$, explaining your strategy. Draw a diagram to show that $\frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$. If you cut $\frac{1}{4}$ of your lawn before lunch and then $\frac{2}{3}$ of the remaining lawn after lunch, how much (if any) of the lawn remains to be cut?

Section 6.4 – Multiplying Improper Fractions and Mixed Numbers (pp. 216-221)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES									
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> N6 (B F H K) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> multiply two improper fractions or mixed numbers solve problems involving the multiplication of two improper fractions or mixed numbers <p>Literacy Links:</p> <ul style="list-style-type: none"> Mixed Numbers in Lowest Terms – A mixed number is in lowest terms when the fraction is in lowest terms. For example, $3\frac{4}{8}$ expressed in lowest terms is $3\frac{1}{2}$. Converting Improper Fractions and Mixed Numbers – Convert by using the denominator to decide the number of parts in one whole. In $\frac{11}{4}$, one whole is $\frac{4}{4}$, therefore: $\frac{11}{4} = \frac{4}{4} + \frac{4}{4} + \frac{3}{4}$$\frac{11}{4} = 2\frac{3}{4}$ In $3\frac{2}{5}$, one whole is $\frac{5}{5}$, therefore: $3\frac{2}{5} = \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{2}{5}$$3\frac{2}{5} = \frac{17}{5}$ Whole Numbers – A whole number can be written as a fraction with a denominator of 1. For example, $2 = \frac{2}{1}$. <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> pp. 219-221: #1-15, 17, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Use an area model or multiply the equivalent improper fractions for modelling multiplying mixed fractions, as demonstrated below. This model is also referred to as a Punnett square. <table style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td style="text-align: center;">3</td> <td style="text-align: center;">$\frac{1}{3}$</td> </tr> <tr> <td style="text-align: center;">2</td> <td style="border: 1px solid black; padding: 5px;">2×3</td> <td style="border: 1px solid black; padding: 5px;">$2 \times \frac{1}{3}$</td> </tr> <tr> <td style="text-align: center;">$\frac{1}{2}$</td> <td style="border: 1px solid black; padding: 5px;">$\frac{1}{2} \times 3$</td> <td style="border: 1px solid black; padding: 5px;">$\frac{1}{2} \times \frac{1}{3}$</td> </tr> </table> <p>The distributive property applies here:</p> $2\frac{1}{2} \times 3\frac{1}{3} = \left(2 + \frac{1}{2}\right) \times \left(3 + \frac{1}{3}\right)$ $= (2 \times 3) + \left(2 \times \frac{1}{3}\right) + \left(\frac{1}{2} \times 3\right) + \left(\frac{1}{2} \times \frac{1}{3}\right)$ $= 6 + \frac{2}{3} + 1\frac{1}{2} + \frac{1}{6}$ $= 8\frac{1}{3}$ <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Estimate each of the following and explain your thinking. <ul style="list-style-type: none"> a. $6\frac{1}{4} \times 8$ b. $4 \times 8\frac{3}{16}$ Simplify each of the following. <ul style="list-style-type: none"> a. $4 \times 8\frac{3}{8}$ b. $5\frac{1}{4} \times 8$ c. $4 \times 2\frac{1}{4}$ For each of the following, write two fractional numbers which have a product between the given numbers. <ul style="list-style-type: none"> a. 14 and 15 b. $\frac{1}{2}$ and $\frac{1}{3}$ 		3	$\frac{1}{3}$	2	2×3	$2 \times \frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2} \times 3$	$\frac{1}{2} \times \frac{1}{3}$
	3	$\frac{1}{3}$								
2	2×3	$2 \times \frac{1}{3}$								
$\frac{1}{2}$	$\frac{1}{2} \times 3$	$\frac{1}{2} \times \frac{1}{3}$								

Section 6.5 – Dividing Fractions and Mixed Numbers (pp. 222-229)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES												
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> N6 (C E J K) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> divide two fractions or mixed numbers solve problems involving the division of fractions or mixed numbers <p>After this lesson, students should understand the following concept:</p> <ul style="list-style-type: none"> reciprocal – the multiplier of a number to give a product of 1; $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$, because $\frac{3}{2} \times \frac{2}{3} = 1$; the result of switching the numerator and denominator in a fraction <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> pp. 227-229: #1-19, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Present examples that can be modeled concretely and pictorially and then move to the symbolic representation once students understand the process. The common denominator method for division of fractions relates well to whole number division; once the denominators are the same, the numerators can be divided. Have students model division questions by determining a common denominator. For example, $\frac{5}{3} \div \frac{1}{2} = \frac{10}{6} \div \frac{3}{6} = 10 \div 3 = \frac{10}{3}$. Ensure that students can compare the solutions of problems such as $8 \div \frac{1}{2}$ and $8 \times \frac{1}{2}$ as it is important for students to understand the concepts of multiplication and division of fractions. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Complete the following patterns, and extend them for two extra lines. What patterns do you observe? <table style="width: 100%; border: none;"> <tr> <td style="padding: 5px;">a. $9 \div 3 = \square$</td> <td style="padding: 5px;">b. $2 \div \frac{1}{2} = \square$</td> </tr> <tr> <td style="padding: 5px;">$9 \div 1 = \square$</td> <td style="padding: 5px;">$1 \div \frac{1}{2} = \square$</td> </tr> <tr> <td style="padding: 5px;">$9 \div \frac{1}{3} = \square$</td> <td style="padding: 5px;">$\frac{1}{2} \div \frac{1}{2} = \square$</td> </tr> <tr> <td style="padding: 5px;">$9 \div \frac{1}{9} = \square$</td> <td style="padding: 5px;">$\frac{1}{2} \div \frac{1}{4} = \square$</td> </tr> </table> Simplify each of the following. <table style="width: 100%; border: none;"> <tr> <td style="padding: 5px;">a. $30 \div 2\frac{7}{8}$</td> </tr> <tr> <td style="padding: 5px;">b. $24 \div 4\frac{1}{4}$</td> </tr> </table> Solve each of the following problems. <table style="width: 100%; border: none;"> <tr> <td style="padding: 5px;">a. Six containers of ice cream have been purchased for a birthday party. If each guest gets a serving of $\frac{3}{8}$ of a container of ice cream, how many guests can be served?</td> </tr> <tr> <td style="padding: 5px;">b. Casey had $5\frac{1}{4}$ metres of material to make headbands for 7 friends. How much material should she use for each headband if she wants to use the same length of material for each?</td> </tr> </table> 	a. $9 \div 3 = \square$	b. $2 \div \frac{1}{2} = \square$	$9 \div 1 = \square$	$1 \div \frac{1}{2} = \square$	$9 \div \frac{1}{3} = \square$	$\frac{1}{2} \div \frac{1}{2} = \square$	$9 \div \frac{1}{9} = \square$	$\frac{1}{2} \div \frac{1}{4} = \square$	a. $30 \div 2\frac{7}{8}$	b. $24 \div 4\frac{1}{4}$	a. Six containers of ice cream have been purchased for a birthday party. If each guest gets a serving of $\frac{3}{8}$ of a container of ice cream, how many guests can be served?	b. Casey had $5\frac{1}{4}$ metres of material to make headbands for 7 friends. How much material should she use for each headband if she wants to use the same length of material for each?
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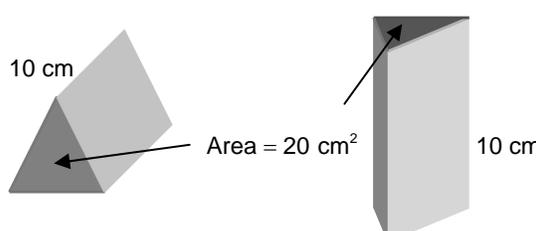
Section 6.6 – Applying Fraction Operations (pp. 230-235)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • N6 (K L) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • decide when to multiply fractions and when to divide fractions in solving problems • apply the order of operations to solve problems involving fractions <p>Literacy Links:</p> <ul style="list-style-type: none"> • Order of Operations – The order of operations for fractions is the same as for whole numbers and decimals: <ul style="list-style-type: none"> ➤ brackets first ➤ multiply and divide in order from left to right ➤ add and subtract in order from left to right • Time-and-a-Half – To earn time-and-a-half means to be paid for $1\frac{1}{2}$ h for each hour of work done. • Mean – The mean of a set of fractions is their sum divided by the number of fractions. The mean of $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{1}{8}$ is $\left(\frac{1}{4} + \frac{1}{2} + \frac{1}{8}\right) \div 3$, which equals $\frac{7}{24}$. <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> • pp. 233-235: #1-6, 11-13, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • It may be beneficial to help students recall the roll of the word <i>of</i> in this section. • For some students, an acronym, such as BDMAS, may help in remembering the order of operations. • Some students may need to review the basics of adding fractions and the rules for multiplying and changing mixed to improper fractions. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • Caitlin decided to make muffins for the school picnic. Her recipe requires $2\frac{1}{4}$ cups of flour to make 12 muffins. Caitlin found there were exactly 18 cups of flour in the canister, so she decided to use all of it. <ol style="list-style-type: none"> How many muffins can Caitlin expect to get? The principal of the school liked Caitlin's muffins and asked her to cater the school picnic next year, producing enough muffins for all 400 students. How many cups of flour will Caitlin require? • Insert one set of brackets to make the following statements true. <ol style="list-style-type: none"> $\frac{1}{2} + \frac{1}{4} \times \frac{2}{3} = \frac{1}{2}$ $\frac{3}{4} \times \frac{1}{5} + \frac{2}{3} \times \frac{5}{3} = 1\frac{1}{12}$

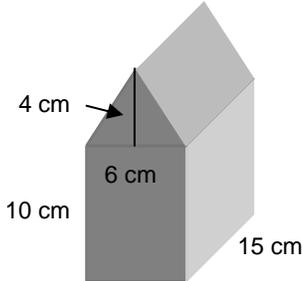
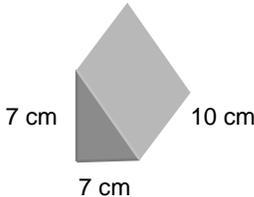
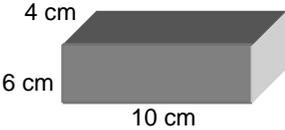
CHAPTER 7
VOLUME

SUGGESTED TIME
15 classes

Section 7.1 – Understanding Volume (pp. 246-253)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • SS4 (A B C D) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • explain the meaning of volume • determine the volume of a right rectangular prism, right triangular prism and right cylinder • show that orientation does not affect volume <p>After this lesson, students should understand the following concepts:</p> <ul style="list-style-type: none"> • base (of a prism or cylinder) – any face of a prism that shows the named shape of the prism; the base of a rectangular prism is any face; the base of a triangular prism is a triangular face; the base of a cylinder is a circular face • height (of a prism or a cylinder) – the perpendicular distance between the two bases of a prism or cylinder • volume – the amount of space an object occupies; measured in cubic units • orientation – the different position of an object formed by translating, rotating or reflecting the object <div style="text-align: center;">  <p>The diagram illustrates two prisms. On the left is a right triangular prism with a base of 10 cm and a height of 4 cm. An arrow points to the triangular base with the text 'Area = 20 cm²'. On the right is a right rectangular prism with a height of 10 cm, indicated by an arrow.</p> </div> <p>Literacy Links:</p> <ul style="list-style-type: none"> • Notation – Read 1 cm^3 as “one cubic centimetre.” • Right shapes – Prisms and cylinders in this chapter are <i>right</i> prisms and <i>right</i> cylinders. <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> • pp. 250-253: #1-13, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Some students may not understand the numeric relationship between the values for area of the base, height of a prism and volume of a prism. Help them to understand the relationships in order to generalize these relationship between the area of the base, height, and volume of prisms and cylinders. • Encourage students to observe that the volume of 3-D objects changes with changes in height or changes in area of the base, • Some students may benefit from holding a cylinder and moving it to change the orientation of the base, and then explaining how the volume does not change, no matter how they hold the cylinder. • Encourage students to turn their books to help them understand the orientation of a 3-D object. • Some students may benefit from being coached through a problem that addresses all three orientations. This will help reinforce that orientation does not affect the volume of a 3-D object. • Encourage visual learners to draw a sketch and label the dimensions of an object before determining the volume. • Have models of prisms available for students to use. Prompt students to realize that the base of a rectangular prism is the face from which the height is measured. Usually, a rectangular prism rests on its base. • Reinforce the idea that the shape of the base determines the type of prism. • Bring in small boxes of various shapes and sizes and have students use centimetre cubes to fill them to determine the volume of each box. • Have discussions with students about the different measures of volume they have encountered in their containers found at home. Show and discuss the centimetre cube. Explain that just as square units are used to measure area and surface area, cubic units are used to measure volume. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • Determine the height of each rectangular prism. <ol style="list-style-type: none"> volume = 108 cm^3, area of base = 12 cm^2 volume = 80 cm^3, area of base = 16 cm^2 • The Canola Oil Company is designing cans for its oil. Their cans hold 1 L, which is 1000 cm^3. The area of the base of their can is 80 cm^2. How tall is the can? Show your answer to one decimal place.

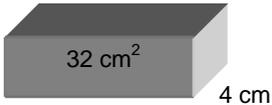
Section 7.2 – Volume of a Prism (pp. 254-261)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • SS4 (A C E) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • use a formula to determine the volume of a right rectangular prism ($V = lwh$) • use a formula to determine the volume of a right triangular prism <p>Literacy Link:</p> <ul style="list-style-type: none"> • Capacity – Capacity refers to the greatest volume that a container such as a tank, a truck or a measuring cup can contain. <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> • pp. 258-261: #1-15, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Some students may benefit from using diagrams labelled with the area of the base. Have students verbalize how the given base area was determined. Have them identify and label the dimensions of the prism, and then recalculate the volume. • Some students may indicate that the difference between the formulas for rectangular prisms and triangular prisms is the $\frac{1}{2}$ in the formula for the triangular prism. Prompt students to understand that this difference is related to the shape of the base. Visual learners may benefit from drawing and labelling a diagram of the base of a rectangular prism and a triangular prism with the dimensions and area formulas to help make this connection. • Some students may not make the connection that the volume of a cube is one of its sides cubed. Encourage these students to continue using the formula $V = l \cdot w \cdot h$. • Provide the students with linking cubes. Have students construct rectangular prisms with the following dimensions: $3 \times 5 \times 2$ and $6 \times 5 \times 2$. Have students find the volume of each. Ask them how they could have anticipated that the second volume would be twice the first. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • A certain cube has a surface area of 96 cm^2. What is the volume of the cube? • Find the volume of the following object. <div style="text-align: center;">  </div> <ul style="list-style-type: none"> • Each of the following pieces of cheese cost \$5.00. Which is the better deal? Why? <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div>

Section 7.3 – Volume of a Cylinder (pp. 262-267)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • SS4 (A B C E) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • determine the volume of a cylinder ($V = \pi r^2 h$) <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> • pp. 265-267: #1-8, 10, 12-14, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Check that students understand how to find the area of a circle and use 3.14 as a value for π. • Some students may benefit from drawing the base of a cylinder and calculating the area of the base first, before multiplying by the height. Labelling the radius and the diameter may help them avoid using the diameter to calculate the area of the circular base. • As the first step to solve a problem involving cylinders, have students develop a habit of writing, "The radius is _____." Doing this will reduce the chance of using the value of the diameter instead of the radius in applying the formula. • Encourage students to write out the formula for each calculation. • Make a variety of right prisms, cubes and cylinders available for students to help make the connection that cylinders and triangular prisms each have two faces that can be considered a base, while rectangular prisms and cubes have six faces that can be considered a base. • Having students verbally describe a formula for finding the volume of a 3-D object and then linking their explanation to the different shapes of cylinders and prisms may help them calculate the volume of each object. • Ensure that students understand that all 3-D objects (including prisms and cylinders) have volume. • Ask students to describe what makes each shape different and how that is reflected in the calculation of its volume. • Have students predict and explore whether the volume of a cylinder created by rolling a sheet of paper lengthwise or widthwise will be the same volume or a different volume. If the volumes are different, Which will have the greater volume? <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • A school is having a fundraiser by selling popcorn and the students are making their own containers to save on expenses. If they have sheets of cardboard for the sides with dimensions 27 cm by 43 cm, would the volume be greater if the sheets were folded to make cylindrical containers with a height of 27 cm or with a height of 43 cm? Justify your answer mathematically. • Which of the following cylinders will hold more water? Explain your answer. <ul style="list-style-type: none"> Cylinder A: height 7.0 cm; diameter 5.0 cm Cylinder B: height 5.0 cm; diameter 7.0 cm

Section 7.4 – Solving Problems Involving Prisms and Cylinders (pp. 268-275)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • SS4 (E) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • solve problems involving right rectangular prisms, right triangular prisms and right cylinders <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> • pp. 272-275: #1-7, 9-11, 13, 14, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Encourage students who need support in solving contextual problems to use the following strategies: <ul style="list-style-type: none"> ➤ Draw and label a diagram. ➤ Draw a second diagram in cases when a 3-D object sits inside another 3-D object. ➤ Write out all the steps to help track errors, since many problems require more than one calculation. • It may benefit students if you have them verbalize what they are trying to determine, what 3-D objects are involved and what they know about each 3-D object. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • A tube of chocolate chip cookie dough has a volume of 785 cm^3 and a diameter of 10 cm. Each cookie use 1 cm of dough. How many cookies can Nicole make? • Is there enough information presented to determine the volume of the box? <div style="text-align: center;">  </div>

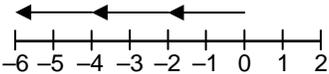
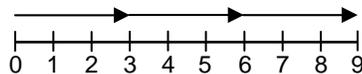
CHAPTER 8
INTEGERS

SUGGESTED TIME
16 classes

Section 8.1 – Exploring Integer Multiplication (pp. 286-292)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> N7 (A B D) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> multiply integers using integer chips <p>Literacy Links:</p> <ul style="list-style-type: none"> Representing Integers – Integer chips are coloured disks that represent integers. One black chip represents +1 and one white chip represents -1. <div style="text-align: center;">  <p style="margin-left: 40px;">+1 -1</p> </div> <ul style="list-style-type: none"> Understanding Multiplication – The product of 4 and 2 is 8, because $4 \times 2 = 8$. The equation $4 \times 2 = 8$ means that 4 groups of 2 make 8. You can also think of 4×2 as the repeated addition $2 + 2 + 2 + 2$. Modelling with Zero Pairs – A zero pair is a pair of integer chips with one chip representing +1 and one chip representing -1. <div style="text-align: center;">  <p style="margin-left: 40px;">zero pair</p> </div> <p>A zero pair represents zero because $(+1) + (-1) = 0$. Any whole number of zero pairs represents zero.</p> <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> pp. 290-292: #1-18 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Some students may benefit from guiding rules for understanding integer multiplication. For example: <ul style="list-style-type: none"> If the leading factor is positive, the question can be completed as repeated addition. If the leading factor has a negative sign, you need zero pairs. The negative sign tells you that you will be removing that quantity of the second factor, e.g., $(-2) \times (-3)$ means remove two groups of (-3) or 6 white integer chips. Use net worth as a context for multiplication. Consider, for example, the impact on net worth if a person owes \$6 to each of 3 friends, or if a debt of \$6 to each of three friends is forgiven. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Start with a neutrally charged container (equal number of positives and negatives). A glass container with two colours of marbles could serve as a model. <ol style="list-style-type: none"> Take out 4 groups of -2 from the container. What is the charge left in the container. Write a number sentence to model the situation described. Add 3 groups of -3 to the container. What is the charge in the container. Write a number sentence to model the situation described. Write a number sentence for each of the following problems. <ol style="list-style-type: none"> Fran lost 3 points in each round (hand) of cards that was played. If she played 4 rounds (hands), what was her score at the end of the game? Bill owed \$5 to each of three friends. What was his net worth, based on this situation? Name as many pairs of integers as possible that have a product of -16 and then a product of +16. What do you notice about the number of possible pairs for the positive product compared to the negative product. Sarah borrowed \$8 from each of her two friends, Chris and Jo. Because it was Sarah's birthday, her two friends each forgave Sarah's debt. Explain how this affected Sarah's net worth.

Section 8.2 – Multiplying Integers (pp. 293-299)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> N7 (D H) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> determine integer products using a number line apply a sign rule when multiplying integers <p>After this lesson, students should understand the following concept:</p> <ul style="list-style-type: none"> sign rule (for multiplication) – the product of two integers with the same sign is positive; the product of two integers with different signs is negative <p>Literacy Link:</p> <ul style="list-style-type: none"> Opposite Integers – Two integers with the same numeral but different signs are called opposite integers. Examples are +5 and -5. <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> pp. 296-299: #1-24, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Some students may need assistance generalizing the number-line method. Have them model their understanding with integer chips and compare their work to a number line. Have them verbalize the similarities and differences between the two methods. While teaching the sign rules for multiplication, reinforce the idea that the result is the same if integers are multiplied in either order. Students could write the sign rules in chart form and use the chart as a quick reference. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Multiply, using the number line: <ul style="list-style-type: none"> a. $(-3) \times (+2)$ b. $(+4) \times (-1)$ Find the answer to the problem $25 \times (-102) \times 4$ mentally and explain your strategy. The problem $-34 \times 17 \times 624 \times 0$ was written on the board, and before the teacher could turn around, Jill had the solution. Explain how Jill was able to solve the problem so quickly. Ask students how to explain how to mentally compute each of the following. <ul style="list-style-type: none"> a. $58 \times (-7)$ b. 8×73 Write number sentences for each of the following. <ul style="list-style-type: none"> a.  b.  Write a problem that can be solved, using each of the following number sentences. <ul style="list-style-type: none"> a. $4 \times (-5) = -20$ b. $(-5) \times (-6) = 30$

Section 8.3 – Exploring Integer Division (pp. 300-305)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES										
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> N7 (A C E) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> divide integers using integer chips <p>Literacy Link:</p> <ul style="list-style-type: none"> Understanding Division – In the equation $6 \div 2 = 3$, the dividend is 6, the divisor is 2 and the quotient is 3. The equation $6 \div 2 = 3$ means that in 6 there are 3 groups of 2. This division statement also means that if 6 is separated into 2 equal groups, there are 3 in each group. <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> pp. 303-305: #1-8, 10-14 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Students who may not understand how the same integer-chip model can represent two different division statements should be coached through both representations. Some students may benefit from guiding rules for understanding integer division, especially those having difficulty when knowing when the integer chip methods can be applied <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Complete the following patterns, and extend them for two extra lines. What patterns do you observe? <table style="width: 100%; border: none;"> <tr> <td style="padding: 5px;">a. $9 \div 3 = \square$</td> <td style="padding: 5px;">b. $(-9) \div (-3) = \square$</td> </tr> <tr> <td style="padding: 5px;">$6 \div 3 = \square$</td> <td style="padding: 5px;">$(-6) \div (-3) = \square$</td> </tr> <tr> <td style="padding: 5px;">$3 \div 3 = \square$</td> <td style="padding: 5px;">$(-3) \div (-3) = \square$</td> </tr> <tr> <td style="padding: 5px;">$0 \div 3 = \square$</td> <td style="padding: 5px;">$0 \div (-3) = \square$</td> </tr> <tr> <td style="padding: 5px;">$-3 \div 3 = \square$</td> <td style="padding: 5px;">$3 \div (-3) = \square$</td> </tr> </table> Write a number sentence to model the following situation. Chris and his three friends together owe \$32. They agreed to share the debt equally. What is each person's share of the debt? 	a. $9 \div 3 = \square$	b. $(-9) \div (-3) = \square$	$6 \div 3 = \square$	$(-6) \div (-3) = \square$	$3 \div 3 = \square$	$(-3) \div (-3) = \square$	$0 \div 3 = \square$	$0 \div (-3) = \square$	$-3 \div 3 = \square$	$3 \div (-3) = \square$
a. $9 \div 3 = \square$	b. $(-9) \div (-3) = \square$										
$6 \div 3 = \square$	$(-6) \div (-3) = \square$										
$3 \div 3 = \square$	$(-3) \div (-3) = \square$										
$0 \div 3 = \square$	$0 \div (-3) = \square$										
$-3 \div 3 = \square$	$3 \div (-3) = \square$										

Section 8.4 – Dividing Integers (pp. 306-311)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> N7 (E F G H) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> determine integer quotients using a number line apply a sign rule when dividing integers <p>After this lesson, students should understand the following concept:</p> <ul style="list-style-type: none"> sign rule (for division) – the quotient of two integers with the same sign is positive; the quotient of two integers with different signs is negative <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> pp. 309-311: #1-20, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Some students may need assistance generalizing how to use a number line to divide integers and how to use signs. Have these students model their understanding with integer chips or a number line and verbalize the similarities and differences between the two methods. They could then apply their results to the sign rules. Students could write the sign rules in chart form and use the chart as a quick reference. It may be beneficial to show students how to check their work using multiplication. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Write a division or multiplication sentence to solve each problem. <ul style="list-style-type: none"> a. $\square \div (-3) = -9$ b. $57 \div \square = (-3)$ Name as many division problems as possible, based on each of the following multiplication sentences, using the fact that multiplication and division are inverse operations. <ul style="list-style-type: none"> a. $-5 \times (-4) = 20$ b. $6 \times (-3) = -18$ Write a problem that can be solved using the number sentence $-12 \div 3 = -4$

Section 8.5 – Applying Integer Operations (pp. 312-317)

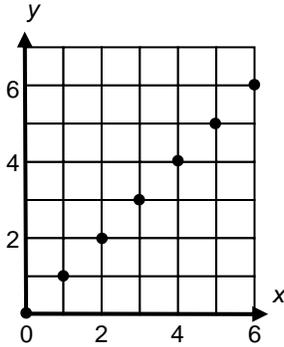
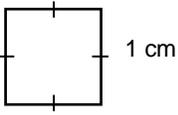
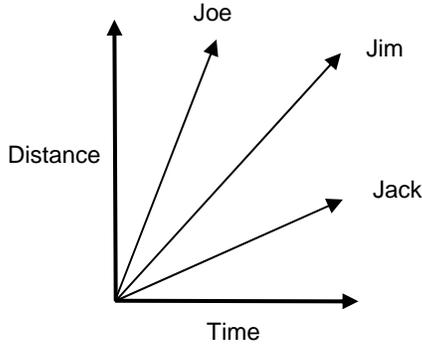
ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> N7 (I) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> apply the order of operations to solve problems involving integers <p>Literacy Links:</p> <ul style="list-style-type: none"> Omitting Positive Signs or Brackets – A positive integer can be written without the positive sign or brackets. For example, $(+3) \times (+4)$ can be written as 3×4. Negative numbers must include the negative sign. The brackets can be omitted from a negative integer that does not follow an operation symbol. For example, $(-9) \div (-3)$ can be written as $-9 \div (-3)$. Understanding the Mean – The mean of a set of integers is found by dividing their sum by the number of integers. For example, the mean of -4, $+8$ and -10 is $\frac{(-4) + (+8) + (-10)}{3}$, which equals -2. <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> pp. 314-317: #1-21 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Ensure that students understand why the order of operations is important in solving problems. For some students, an acronym, such as BDMAS, may help in remembering the order of operations. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Simplify each of the following problems. <ol style="list-style-type: none"> $-4 \times 27 + (-4) \times 73$ $-6 \times 73 + (-6) \times 27$ A list of temperatures for seven days was prepared, but the ink became wet and one temperature was unreadable. The mean temperature was -3°C, and the six known temperatures were 2°C, -4°C, -6°C, -2°C, 4°C and -5°C. Find the missing temperature. To win a free trip, contestants had to answer the following skill-testing question correctly: $-3 \times (-4) + (-18) \div 6 - (-5)$ The contest organizers said that the answer was $+4$. Explain why there is a problem with the solution.

CHAPTER 9
LINEAR RELATIONS

SUGGESTED TIME

15 classes

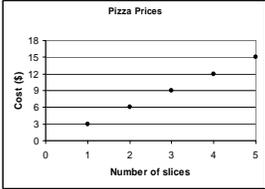
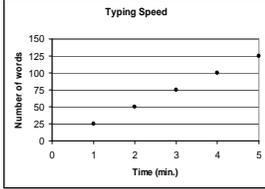
Section 9.1 – Analysing Graphs of Linear Relations (pp. 332-341)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> PR1 (A B D) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> describe patterns on the graph of a linear relation create a table of values using the points on the graph <p>Literacy Links:</p> <ul style="list-style-type: none"> Relationship – A relationship is a pattern formed by two sets of numbers Table of Values – A table of values is a chart showing two sets of related numbers. Linear Relation – A linear relation is a pattern made by a set of points that lie in a straight line.  <ul style="list-style-type: none"> Perimeter of a Square $P = 1+1+1+1$ $P = 4$ The perimeter is 4 cm.  <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> pp. 337-341: #1-13, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Students should feel comfortable with all three formats of expressing linear relations, namely graphs, tables and models. Some students may benefit from verbalizing what is different about each format. Encourage alternate wordings for describing the patterns formed on a graph. Ensure that all students use the correct labels for rows and columns in their tables of values and that the labelling is based on the axes of the graph. Some students may find it beneficial to verbalize the pattern in a linear graph. Ask them questions such as “How do you get from one point to the next?” Drawing in the vertical and horizontal moves may help some learners to identify the pattern. As they draw the moves, have them verbalize their thinking. Have students arrange ordered pairs in ascending order based on the x-coordinate to establish a pattern to determine the missing value in a set. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> According to the graph below, who ran fastest? Explain.  <ul style="list-style-type: none"> Determine the missing values in the following set of ordered pairs, if each set represents a linear relation. <ol style="list-style-type: none"> $\{(0,0), (1,12), (2,24), (3,\square)\}$ $\{(-4,\square), (-2,-6), (0,2), (2,10), (\square,18)\}$

Section 9.2 – Patterns in a Table of Values (pp. 342-351)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES														
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • PR1 (B C D) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • identify relationships in a table of values • decide if a table of values represents a linear relation • graph points represented in a table <p>Literacy Links:</p> <ul style="list-style-type: none"> • Variable – A variable is a letter that represents an unknown quantity. For example, in $3a - 5$, the variable is a. It can be helpful to choose variables that are meaningful. For example, t for time and s for score. • Expression – An expression is any single number or variable, or a combination of operations (+, −, ×, ÷) involving numbers and variables. An expression does not include an equal sign. The following are examples of expressions: 5, r, $8t$, $x + 9$, and $2y - 7$. • Linear Relationships and Tables – If consecutive values for the first variable do not have the same difference, it is difficult to tell from the table whether the relationship is linear. You may be able to tell by drawing a graph. <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> • pp. 348-351: #1-16, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Explain to students the different ways of representing a pattern, namely, a diagram, a table of values, words, a graph and an expression. • Ensure that all students label the axes on their graphs correctly. They should use the variable from the first value of the ordered pair (x) for the horizontal axis and the variable from the second value of the ordered pair (y) for the vertical axis. • Some students may benefit from drawing the horizontal and vertical distances on the graph in order to use patterning to identify the differences in the values of x and the values of y. • Some learners may benefit from verbalizing the pattern they see before attempting to write it as an expression. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • For the given tile pattern: <ol style="list-style-type: none"> a. Make a table of values, and describe the pattern in words. <div style="text-align: center; margin: 10px 0;"> </div> <ol style="list-style-type: none"> b. Use the pattern and description to write a mathematical equation identifying what the variable(s) represent. c. Use the equation to help determine the tenth entry in the table. <ul style="list-style-type: none"> • A certain rectangle has a width which is $\frac{1}{2}$ of the length. <ol style="list-style-type: none"> a. Make a table showing the relationship between the length and the perimeter. b. Describe the relationship between the length and the perimeter in words. c. Write a mathematical rule to relate length and perimeter, identifying what the variable(s) stand for. d. Use the rule to find the perimeter when the length is 99 m. • Complete the following table of values for a linear relation. <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <tbody> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">−3</td> <td style="padding: 5px;">−2</td> <td style="padding: 5px;">−1</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">2</td> </tr> <tr> <td style="padding: 5px;">y</td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;">7</td> <td style="padding: 5px;">9</td> <td style="padding: 5px;">11</td> </tr> </tbody> </table>	x	−3	−2	−1	0	1	2	y				7	9	11
x	−3	−2	−1	0	1	2									
y				7	9	11									

Section 9.3 – Linear Relationships (pp. 352-359)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> PR1 (B C D) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> graph from a formula graph from an equation using integers solve problems using the graph of a linear relation <p>Literacy Links:</p> <ul style="list-style-type: none"> Formula – A formula is a mathematical statement that represents the relationship between two, or more, specific quantities. An example is $C = \pi d$, where C is the circumference and d is the diameter of the circle. Equation – An equation is a mathematical statement with two expressions that have the same value. The two expressions are separated by an equal sign. For example, $x + 2 = 3$, $y - 7 = -4$, $3a - 2 = a + 2$, and $b = 4$. <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> pp. 356-359: #1-12, 14-17 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Discuss with students that when selecting values for x, it is beneficial to choose values that differ by equal increments (e.g., 1, 2, 3, ..., or 10, 20, 30, ..., depending on the question). This will make it easier for them to see a pattern in the values of y. Some students may not know how to decide what values of x to use when negative values are allowed. Consider getting them to generate a standard set of numbers such as $\{-3, -2, -1, 0, 1, 2, 3\}$. It is important for all students to realize that if a point lies on the x-axis, its y-coordinate is zero. Similarly, if a point lies on the y-axis, its x-coordinate is zero. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Make stacks or rods as shown and find the surface area and volume when there are 1, 2, 3, 4, ..., 10 rods. Organize this information in a table. Each rod is 6 units long and its ends are 1 unit square. Each rod is placed 1 cm from the end of the one before it. <div style="text-align: center;">  </div> <ol style="list-style-type: none"> Determine an equation for the surface area and volume of n rods. Graph the two sets of data and discuss the shapes of the graphs. <ul style="list-style-type: none"> Graph each of the following equations, using a table of values, computer software or a graphing calculator. $y = 2x + 1 \quad y = 2x + 3 \quad y = 2x + 5$ <ol style="list-style-type: none"> How are the graphs alike? How are the graphs different? What conclusions can you draw? How would the graph of $y = 2x + 2$ compare with the graphs of the above three. Which one of the following two graphs would have the points connected? <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div>

CHAPTER 10
SOLVING LINEAR EQUATIONS

SUGGESTED TIME

15 classes

Section 10.1 – Modelling and Solving One-Step Equations: $ax = b$, $\frac{x}{a} = b$

(pp. 370-379)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> PR2 (A B C D E G) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> model problems with linear equations solve linear equations and show how they worked out the answer <p>After this lesson, students should understand the following concept:</p> <ul style="list-style-type: none"> linear equation – an equation that, when graphed, results in points that lie along a straight line; examples are $y = 4x$, $d = \frac{c}{2}$, $5w + 1 = t$ <p>Literacy Links:</p> <ul style="list-style-type: none"> Equation – An equation is a mathematical statement with two expressions that have the same value. The two expressions are separated by an equal sign. For example, $2x = 3$, $\frac{a}{3} = 5$, and $b = 4$. In the equation $4y - 7 = -3$, the numerical coefficient is 4, the variable is y and the constants are 7 and -3. Opposite Operations – An opposite operation “undoes” another operation. Examples of opposite operations are addition-subtraction, and multiplication-division. You may sometimes hear opposite operations called <i>inverse operations</i>. Isolating the Variable – Isolate the variable means to get the variable by itself on one side of the equation. Notation – $\frac{1}{3}t$ is the same as $\frac{t}{3}$ or $t \div 3$. <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> pp. 376-379: #1-23, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Some students may have difficulty describing the process they used to solve an equation. As students demonstrate the process, verbalize what they are doing. Next, have them verbalize what they did, and record their explanation using a diagram and words. Using an overhead with algebra tiles, integer chips or diagrams provides an effective visual for modelling equations and performing opposite operations. You may wish to have students follow along with their own concrete materials or diagrams. Continue to use the appropriate vocabulary when isolating the variable and identifying the opposite operations used to solve an equation. Observe that students find the opposite operation and not the inverse of the number. For example, the opposite of dividing by -12 is multiplying by -12, not multiplying by $+12$. As a class, you may wish to develop criteria for assessing a solution to an equation. For example, <ul style="list-style-type: none"> ➤ Correctly identify the variable, numerical coefficient and operations present in the question. ➤ Use the correct opposite operations in the proper order. ➤ Check a solution by substituting it into the original equation. The criteria may assist students in guiding them through the process. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Use the opposite operation to solve each equation. Check your answer. <ol style="list-style-type: none"> $64 = 8d$ $-44 = \frac{p}{-4}$ $\frac{e}{7} = -16$ $-6y = -72$ The length of a skateboard is 4 times its width. The length of Mika’s skateboard is 78 cm. What is its width?

Section 10.2 – Modelling and Solving Two-Step Equations: $ax + b = c$ (pp. 380-387)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES												
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> PR2 (A B C D E G) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> model problems with two-step linear equations solve two-step linear equations and show how they worked out the answer <p>Literacy Links:</p> <ul style="list-style-type: none"> Translating Words into Symbols – To solve a problem, you sometimes need to translate words into an equation. For example, <i>two more</i> means you need to add 2, and <i>three times</i> means you need to multiply by 3. Order of Operations – When substituting a value into an equation, make sure to use the correct order of operations: <ul style="list-style-type: none"> ➤ First, multiply and divide in order from left to right. ➤ Finally, add and subtract in order from left to right. Reverse Order of Operations – When isolating a variable, follow the reverse order of operations: <ul style="list-style-type: none"> ➤ add and/or subtract ➤ multiply and/or divide <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> pp. 384-387: #1-17, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Some students will require help working with the reverse order of operations when solving equations. Have them write out the order of operations. Having this information available when solving equations may help them to visualize the order in reverse and make appropriate selections for the process. All students will benefit from a discussion of what happens when there is a subtraction sign in front of the variable. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Solve each of the following equations. <ol style="list-style-type: none"> $x + 2 = 10$ $4x + 8 = 40$ Write the symbolism to match each step in the solution shown explain what is happening in each step. <table border="1" data-bbox="870 932 1373 1381"> <tbody> <tr> <td data-bbox="870 932 1049 1087"></td> <td data-bbox="1052 932 1263 1087"></td> <td data-bbox="1266 932 1373 1087">Step 1</td> </tr> <tr> <td data-bbox="870 1092 1049 1205"></td> <td data-bbox="1052 1092 1263 1205"></td> <td data-bbox="1266 1092 1373 1205">Step 2</td> </tr> <tr> <td data-bbox="870 1209 1049 1289"></td> <td data-bbox="1052 1209 1263 1289"></td> <td data-bbox="1266 1209 1373 1289">Step 3</td> </tr> <tr> <td data-bbox="870 1293 1049 1381"></td> <td data-bbox="1052 1293 1263 1381"></td> <td data-bbox="1266 1293 1373 1381">Step 4</td> </tr> </tbody> </table> <ul style="list-style-type: none"> A taxicab company charges a basic rate of \$2.00 plus \$1.50 for every kilometre driven. If the total bill was \$21.50, use algebra to find how far the cab ride was. The grade eight students had a Christmas dance. The disc jockey charged \$150 for setting up the music plus \$3 per student who attended the dance. The disc jockey was paid \$375. How many students attended the dance? Some cows and some chickens live on a farm. If the total number of legs is 38 and the total number of heads is 16, use algebra to determine how many cows and how many chickens live on the farm. (<i>Hint</i>. If there are x cows, there are $16 - x$ chickens.) 			Step 1			Step 2			Step 3			Step 4
		Step 1											
		Step 2											
		Step 3											
		Step 4											

Section 10.3 – Modelling and Solving Two-Step Equations: $\frac{x}{a} + b = c$ (pp. 388-393)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> PR2 (A B C D E G) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> model problems with two-step linear equations solve two-step linear equations and show how they worked out the answer <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> pp. 391-393: #1-15, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Create a chart on the board listing the similarities and differences in operations to help highlight the pairs of reverse operations. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Solve each equation. Check your answer. <ol style="list-style-type: none"> $\frac{x}{2} + 1 = 5$ $-3 = \frac{n}{7} - 7$ $2 + \frac{a}{-8} = -4$ Half of Xien's age added to 2 equals the age of her sister, Airah, who is 11. How old is Xien? Alex is working on the equation $4x - 5 = 7$. The first thing he does is divide the whole equation by 4. He writes $\frac{4x}{4} - \frac{5}{4} = \frac{7}{4}$. He thinks he may have done something wrong. Has he? Justify your answer.

Section 10.4 – Modelling and Solving Two-Step Equations: $a(x + b) = c$

(pp. 394-399)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> PR2 (A B C D E F G) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> model problems with two-step linear equations solve two-step linear equations and show how they worked out the answer <p>Literacy Links:</p> <ul style="list-style-type: none"> Order of Operations – When substituting a value into an equation, be sure to use the correct order of operations: <ul style="list-style-type: none"> ➤ brackets ➤ multiply and divide in order from left to right ➤ add and subtract in order from left to right Distributive Property – The distributive property states that $a(b + c)$ equals $a \cdot b + a \cdot c$. <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> pp. 397-399: #1-13, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Discuss with the class how to use the distributive property when a negative integer is involved. Use an area model to expand expressions to explain the distributive property. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Solve each equation. Verify your answers. <ol style="list-style-type: none"> $42 = 7(y + 4)$ $-4(c - 10) = 40$ $-1(r + 8) = 0$ $-18 = 6(j - 5)$ If you take the number of points the Panthers football team scored in their first game, add the 21 points they scored in their second game, and double the total, you will get 62 total points. How many points did they score in their first game? During a school fundraiser, Room 19 raised triple the amount of money that Rooms 16 and 17 raised together. Room 19 brought in \$1095. Room 16 brought in \$165. What was the total amount of money raised by Room 17? Kim used the distributive property to solve the equation $12(x - 3) = 72$. Check her work to see if her solution is correct. If there is an error, correct it. $12(x - 3) = 72$ $12x - 36 = 72$ $12x - 36 - 36 = 72 - 36$ $12x = 36$ $\frac{12}{12}x = \frac{36}{12}$ $x = 3$ Which of the following equations produces the smallest value for d? <ol style="list-style-type: none"> $7d = 42$ $\frac{d}{5} = -2$ $3d + 4 = -5$ $\frac{d}{4} + 12 = 36$ $5(d + 4) = -15$

CHAPTER 11
PROBABILITY

SUGGESTED TIME
12 classes

Section 11.1 – Determining Probabilities Using Tree Diagrams and Tables

(pp. 410-418)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • SP2 (A) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • determine the sample space of a probability experiment with two independent events • represent the sample space in the form of a tree diagram or table • express the probability of an event as a fraction, a decimal and a percent <p>After this lesson, students should understand the following concepts:</p> <ul style="list-style-type: none"> • independent events – results for which the outcome of one event has no effect on the outcome of another event • probability – the likelihood or chance of an event occurring • sample space – all possible outcomes of a probability experiment • favourable outcome – a successful result in a probability experiment <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> • pp. 416-418: #1-10, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Ensure that students understand the difference between all possible outcomes and favourable outcomes. Remind students that favourable outcomes cannot be greater than the total number of possible outcomes. • Review the approaches for determining a sample space (tree diagrams and tables) and extend this to establish the fact that the space can be determined by simply multiplying the outcomes together. For example: A menu offers a lunch special of either a hot dog or a hamburger with a choice of an apple, orange or banana for dessert. As a result, 2×3, or 6 meal different combinations could be ordered. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • A box contains 3 red balls and 2 green balls. <ul style="list-style-type: none"> a. You remove two balls from the box; the second one is removed without replacing the first. <ul style="list-style-type: none"> ➤ Draw a tree diagram to show all the possible outcomes for this situation. ➤ What is the probability of removing 2 green balls? b. You remove two balls from the box; the first one is replaced before removing the second. <ul style="list-style-type: none"> ➤ Draw a tree diagram to show all the possible outcomes for this situation. ➤ What is the probability of removing 2 green balls? • Keith wrote a different number from 1 to 10 on each of ten small pieces of paper and put them in a basket. He drew one number from the bag. At the same time, he tossed a coin. Determine, and list, the total number of possible outcomes.

Section 11.2 – Outcomes of Independent Events (pp. 419-425)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • SP2 (A) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • determine the outcomes of two or more independent events • verify the total number of possible outcomes using a different strategy <p>Literacy Link:</p> <ul style="list-style-type: none"> • Combination – The order is not important in a combination. For example, {juice, cookie} is the same as {cookie, juice}. <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> • Essential: #1-12, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Help students realize that the product of the possible outcomes for each independent event represents the total number of possible outcomes in a probability experiment. • Help students understand that the multiplication method is useful for determining the total number of possible outcomes. Tables and tree diagrams are useful organizers for determining favourable outcomes as well as total possible outcomes. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • The following describes events A and B. Decide whether the events are dependent or independent and explain your thinking. <ul style="list-style-type: none"> a. A: Mrs. Brown's first child was a boy. B: Mrs. Brown's second child will be a boy. b. A: It snowed last night. B: Jon will be late for school this morning. c. A: Leif swam 2 hours every day for the last ten months. B: Leif's swimming times have improved. d. A: Allison got an A in her last math test. B: Allison studied for 3 hours the night before. e. A: Matthew flipped heads in his last coin toss. B: Matthew will flip heads in his next coin toss.

Section 11.3 – Determining Probabilities Using Fractions (pp. 426-435)

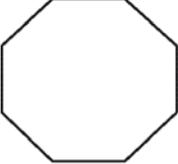
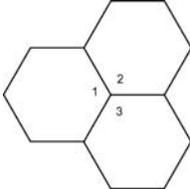
ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • SP2 (A B C) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • solve probability problems • verify your answers using a different method <p>Literacy Links:</p> <ul style="list-style-type: none"> • Simulation – In a simulation, you model a real situation using an experiment. • Types of Probability – An experimental probability is the probability of an event occurring based on experimental results. A theoretical probability is the calculated probability of an event occurring. <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> • pp. 432-435: #1-14, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Encourage students to use multiplication to determine the number of outcomes, and then to verify their answer using a table or a tree diagram. • Encourage students to verbalize their understanding of the differences between experimental probability and theoretical probability. Clarify any misunderstandings. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> • Two black counters and 4 red counters are put in a bag. <ul style="list-style-type: none"> a. Find the probability of drawing <ul style="list-style-type: none"> ➤ two black counters; ➤ two red counters; ➤ first a red and a then a black counter from the bag when the first one is replaced before drawing the second. b. Find the probability of drawing <ul style="list-style-type: none"> ➤ two black counters; ➤ two red counters; ➤ first a red and a then a black counter from the bag when the first one is not replaced before drawing the second. • Sue places two green and two red cubes in a bag. Find the probability of drawing two green cubes if the first one is not returned before drawing the second. • A city survey found that 50% of high school students have a part-time job. The same survey found that 60% plan to attend university. If a student is chosen at random, what is the probability that the student has a part-time job and plans to attend university. • A large basket of fruit contains 3 oranges, 2 apples and 5 bananas. If a piece of fruit is chosen at random, what is the probability of getting an orange or a banana? • At the cafeteria, you can choose milk, pop or juice to drink, have a ham or turkey sandwich, and apple, cherry or pumpkin pie for dessert. What is the probability that a student will have a turkey sandwich with milk and cherry pie?

CHAPTER 12
TESSELLATIONS

SUGGESTED TIME

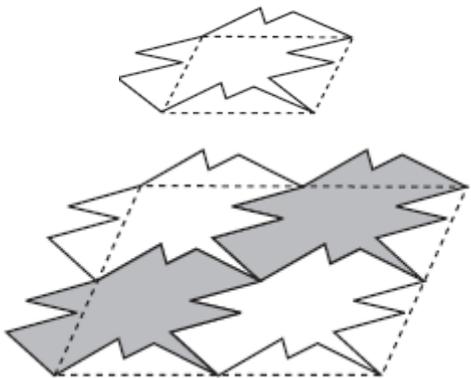
16 classes

Section 12.1 – Tessellations (pp. 446-451)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> SS6 (A B E G) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> identify regular and irregular polygons that can be used to create tessellations describe why certain regular and irregular polygons can be used to tessellate the plane create simple tessellating patterns using polygons <p>After this lesson, students should understand the following concepts:</p> <ul style="list-style-type: none"> tiling pattern – a pattern that covers an area of plane without overlapping or leaving gaps; also called a <i>tessellation</i> tiling the plane – using repeated congruent shapes to cover an area without leaving gaps or overlapping; also called <i>tessellating the plane</i> <p>Literacy Link:</p> <ul style="list-style-type: none"> Plane – The term <i>plane</i> means a two-dimensional flat surface that extends in all directions. <p>Suggested Problems in MathLinks 8:</p> <ul style="list-style-type: none"> p. 449-451: #1-10, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> Reinforce the difference between tiling and not tiling the plane. Encourage students to check the measures of the interior angles to determine if they add up to 360° where they meet. You may wish to provide additional examples and non-examples of regular and irregular polygons that tile the plane. Use an overhead with different shapes in different colours of acetate to provide an effective visual for showing which shapes can or cannot tile the plane. Some students may benefit from using tracing paper to trace the shape of the tile and use it to transform the tiling piece. Some students may benefit from using tiling manipulatives. Make a list of tiling and non-tiling polygons. <p>Possible Assessment Strategies:</p> <ul style="list-style-type: none"> Determine whether each polygon is regular or irregular. <ol style="list-style-type: none">    Consider the figure at the right. <ol style="list-style-type: none"> What is the measurement of angle 1? angle 2? angle 3? What is the sum of the three angles? Can you tile the plane with the hexagon? 

Section 12.2 – Constructing Tessellations Using Translations and Reflections

(pp. 452-456)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • SS6 (C E F) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • identify how translations and reflections can be used to create a tessellation • create tessellating patterns using two or more polygons <p>After this lesson, students should understand the following concept:</p> <ul style="list-style-type: none"> • transformation – a change in a figure that results in a different position or orientation <p>Literacy Link:</p> <ul style="list-style-type: none"> • Dodecagon – A dodecagon is a 12-sided polygon. <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> • pp. 454-456: #1-4, 6-8, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Reinforce the similarities and differences between tiling patterns created using translations versus reflections. • Encourage students to draw lines on a paper to use as a line of reflection. This may assist visual learners. • You may wish to provide additional examples of shape combinations and transformations that can be used to create the same tiling pattern. Using an overhead with different shapes in different colours of acetate provides an effective visual for which shapes can be combined and which transformations result in the desired tiling pattern. • Provide students with manipulatives to re-create given patterns so students can move them around and see what effects the changes have on the pattern or reflection. • Ensure students understand why some designs or shapes tessellate the plane and others do not. • Encourage students to use diagrams or tiles when creating combinations of polygons to use to make a pattern. • Ask students to construct a shape on the bottom and translate it to the top, as shown below. Translate the new polygon to create tessellations as shown below. This can be done using pencil and paper techniques, or by using software. Please note that in this example, the new polygon was created with translations so the tessellation will only work by translating the polygon. <div style="text-align: center;">  </div>

Section 12.3 – Constructing Tessellations Using Rotations (pp. 457-460)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • SS6 (C E F G) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • identify how rotations can be used to create a tessellation • create tessellating patterns using two or more polygons <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> • pp. 459-460: #1-4, 5 or 6, 7, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Focus on the idea that the shapes must have interior angles that have a sum of 360° at the point where the vertices meet, which is the point of rotation. • Encourage students to use diagrams or polygon tiles when rotating polygons (regular or irregular) to make a pattern. • Ask students to use wallpaper store discards or discontinued wallpaper books to explain the transformations evident in wallpaper designs.

Section 12.4 – Creating Escher-Style Tessellations (pp. 461-465)

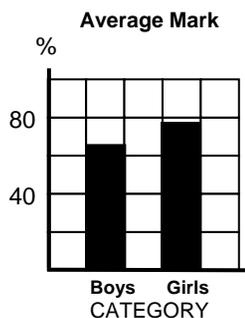
ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
<p>Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:</p> <ul style="list-style-type: none"> • SS6 (C D E F) <p>After this lesson, students will be expected to:</p> <ul style="list-style-type: none"> • create tessellations from combinations of regular and irregular polygons • describe the tessellations in terms of the transformation used to create them <p>Suggested Problems in <i>MathLinks 8</i>:</p> <ul style="list-style-type: none"> • pp. 464-465: #1-8, 9 or 10, Math Link 	<p>Possible Instructional Strategies:</p> <ul style="list-style-type: none"> • Students may benefit from observing while you cut off a piece of a polygon and reattach it, to confirm that area is maintained. • Encourage students to design something that has meaning to them. • After students have had some opportunity to review some of M.E. Escher's work, ask them to create their own design, using a similar technique.



GLOSSARY OF MATHEMATICAL TERMS

B

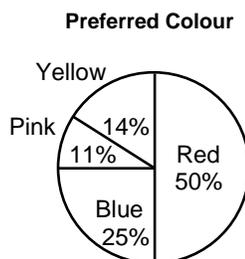
- **bar graph** – a graph that uses horizontal or vertical bars to represent data visually



- **base (of a prism or cylinder)** – any face of a prism that shows the named shape of the prism; the base of a rectangular prism is any face; the base of a triangular prism is a triangular face; the base of a cylinder is a circular face

C

- **circle graph** – a graph that represents data using sections of a circle; the sum of the percents in a circle graph is 100%



- **combined percents** – adding individual percents together
- **commutative property** – the order of adding or multiplying quantities does not affect the result:

$$a + b = b + a$$

$$a \cdot b = b \cdot a$$
- **constant** – a number that does not change

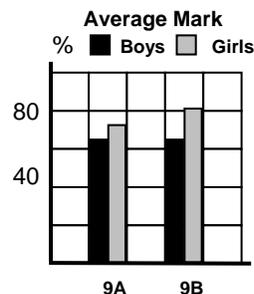
- **cylinder** – a three-dimensional object with two parallel and congruent circular bases



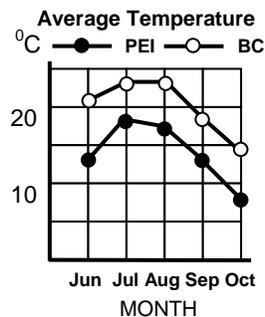
D

- **distort** – to change the appearance or twist the meaning of something in a way that is misleading
- **distributive property** – multiplication of each term inside the brackets of an expression by the term outside the brackets:

$$a(b + c) = a \cdot b + a \cdot c = ab + ac$$
- **double bar graph** – a graph that uses two sets of horizontal or vertical bars to compare two sets of data across categories

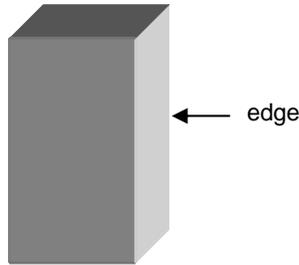


- **double line graph** – a graph that uses two lines to represent changes of two sets of data over time



E

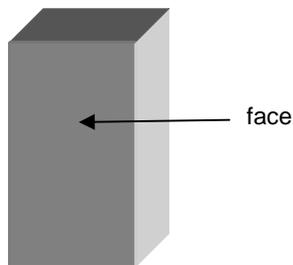
- **edge** – a line segment where two faces meet



- **equation** – a mathematical statement with two expressions that have the same value; the two expressions are separated by an equal sign; $3a - 2 = 4$ is an equation
- **expression** – any single number, single variable or combination of operations (+, −, ×, ÷) involving numbers and variables; an expression does not include an equal sign; $x + 9$, $2y - 7$, $8t$ and 5 are expressions

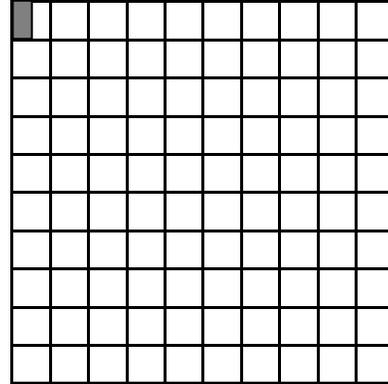
F

- **face** – a flat or curved surface



- **favourable outcome** – a successful result in a probability experiment
- **formula** – a mathematical statement that represents the relationship between specific quantities; an example is $C = \pi d$, where C is the circumference and d is the diameter of a circle

- **fractional percent** – a percent that includes a portion of a percent, such as $\frac{1}{2}\%$, 0.42% , $7\frac{3}{8}\%$, $125\frac{3}{4}\%$, 4.5%

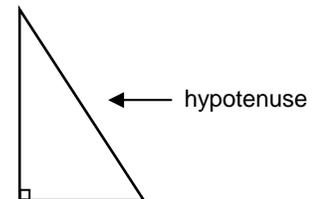


H

- **height (of a parallelogram)** – the perpendicular distance from the base to the opposite side; common symbol is h



- **height (of a prism or a cylinder)** – the perpendicular distance between the two bases of a prism or cylinder
- **hypotenuse** – the longest side of a right triangle; the side opposite the right angle



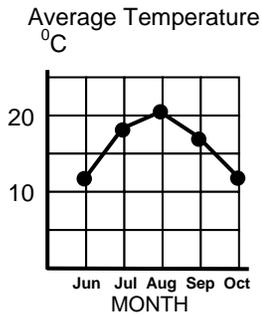
I

- **improper fraction** – a fraction in which the numerator is greater than the denominator; $\frac{4}{3}$ is an improper fraction
- **independent events** – results for which the outcome of one event has no effect on the outcome of another event

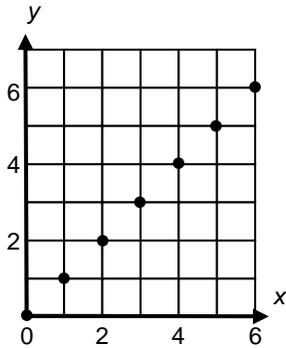
- **integer** – any of the numbers ..., -3, -2, -1, 0, +1, +2, +3, ...
- **interval** – the spread between the smallest and the largest numbers in a range of numbers

L

- **line graph** – a graph that uses a line to represent changes in data over time



- **linear equation** – an equation that, when graphed, results in points that lie along a straight line; examples are $y = 4x$, $d = \frac{C}{2}$, $5w + 1 = t$
- **linear relation** – a pattern made by a set of points that lie in a straight line

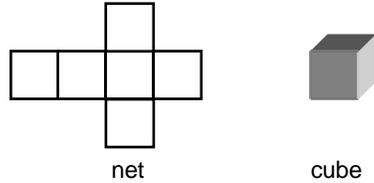


M

- **mixed number** – a number made up of a whole number and a fraction, such as $3\frac{1}{2}$

N

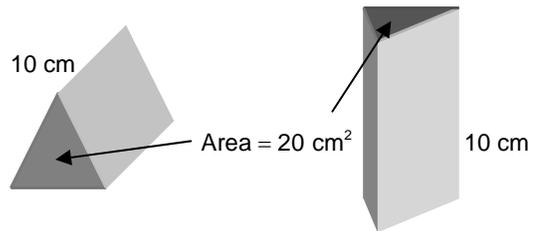
- **net** – a two-dimensional shape that, when folded, encloses a 3-D object



- **numerical coefficient** – a number that multiplies the variable; in $2x + 4$, the number 2 is the numerical coefficient

O

- **opposite operation** – a mathematical operation that undoes another operation; subtraction and addition are opposite operations; multiplication and division are opposite operations; also called inverse operations
- **order of operations** – the correct sequence of steps for a calculation: brackets first, then multiply and divide from left to right, and then add and subtract in order from left to right
- **orientation** – the different position of an object formed by translating, rotating or reflecting the object

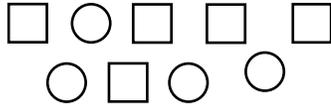


P

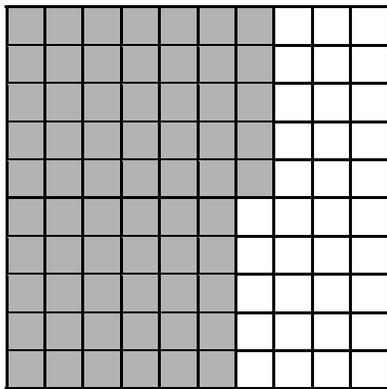
- **part-to-part ratio** – compares different parts of a group to each other; 5 : 4 is the part-to-part ratio of squares to circles



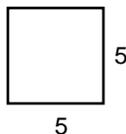
- **part-to-whole ratio** – compares one part of a group to the whole group; 5 : 9 is the part-to-whole ratio of squares to the total number of shapes; can also be written as $\frac{5}{9}$



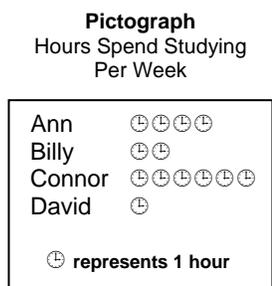
- **percent** – means out of 100; another name for hundredths; 65% means 65 out of 100 or $\frac{65}{100}$ or 0.65



- **perfect square** – a number that is the product of the same two factors; has only an even number of prime factors in its prime factorization; $5 \times 5 = 25$, so 25 is a perfect square

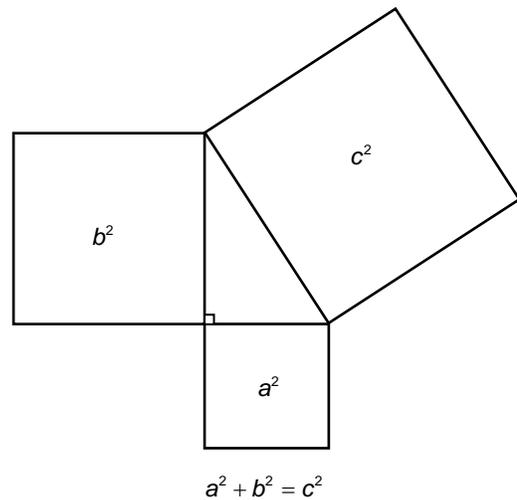


- **pictograph** – a graph that illustrates data using pictures and symbols



- **plane** – a two-dimensional flat surface that extends in all directions

- **prime factorization** – a number written as a product of its prime factors; the prime factorization of 6 is 2×3
- **probability** – the likelihood or chance of an event occurring; probability can be expressed as a ratio, a fraction or a percent
- **proper fraction** – a fraction in which the denominator is greater than the numerator; $\frac{5}{8}$ is a proper fraction
- **proportion** – a relationship that says that two ratios or two rates are equal; can be written in fraction form, $\frac{2}{3} = \frac{6}{9}$ or $\frac{2 \text{ km}}{3 \text{ h}} = \frac{6 \text{ km}}{9 \text{ h}}$; it can also be written in ratio form, 1 : 4 = 4 : 16
- **Pythagorean relationship** – the relationship among the lengths of the sides of a right triangle; the sum of the areas of the squares attached to the legs of a right triangle equals the area of the square attached to the hypotenuse



R

- **rate** – compares two quantities measured in different units; \$1.69 per 100 g or $\$1.69/100 \text{ g}$ is a rate for purchasing bulk food; 72 beats per minute or 72 beats/min is a heart rate
- **reciprocal** – the multiplier of a number to give a product of 1; $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$, because $\frac{3}{2} \times \frac{2}{3} = 1$; the result of switching the numerator and denominator in a fraction

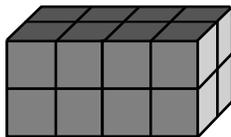
- **rectangular prism** – a prism whose bases are congruent rectangles



- **right prism** – a prism that has sides that are perpendicular to the base of the prism



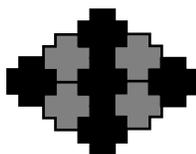
- **sample space** – all possible outcomes of a probability experiment
- **sign rule (for division)** – the quotient of two integers with the same sign is positive; the quotient of two integers with different signs is negative
- **sign rule (for multiplication)** – the product of two integers with the same sign is positive; the product of two integers with different signs is negative
- **square root** – a number that when multiplied by itself equals a given value; the symbol is $\sqrt{\quad}$; 6 is the square root of 36 because $6 \times 6 = 36$
- **surface area** – the number of square units needed to cover a 3-D object; the sum of the areas of all the faces of an object



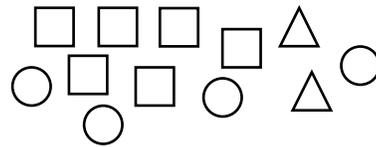
- **table of values** – a chart showing two sets of related numbers

x	1	2	3	4
y	5	10	15	20

- **tessellation** – a pattern that covers an area or plane without overlapping or leaving gaps; also called a tiling pattern



- **three-term ratio** – compares three quantities measured in the same units; written as $a : b : c$ or a to b to c

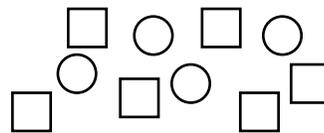


squares : circles : triangles is 6 : 4 : 2

- **tiling pattern** – a pattern that covers an area or plane without overlapping or leaving gaps; also called a tessellation
- **tiling the plane** – using repeated congruent shapes to cover an area without leaving gaps or overlapping; also called tessellating the plane
- **transformation** – a change in a figure that results in a different position or orientation; examples are translations, reflections and rotations
- **trend** – the general direction in which a line graph is going
- **triangular prism** – a prism with two triangular bases, each the same size and shape



- **two-term ratio** – compares two quantities measured in the same units; written as $a : b$ or a to b



squares : circles is 6 : 4

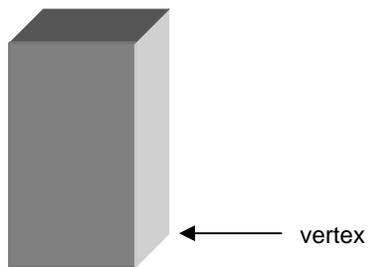


- **unit price** – a unit rate used when shopping; often shown per 100 g or per 100 mL; for example, \$5.00 per 100 g is a unit price; makes it easier for shoppers to compare costs of similar items
- **unit rate** – a rate in which the second term is one; for example, 20 km/h and 64 beats/min



- **variable** – a letter that represents an unknown number; in $3a - 5$, the variable is a

- **vertex** – the point where three or more edges of a figure meet; the plural is vertices



- **volume** – the amount of space an object occupies; measured in cubic units

Z

- **zero pair** – a pair of integer chips, with one chip representing +1 and one chip representing -1; the pair represents zero because $(+1) + (-1) = 0$; any whole number of zero pairs represents zero



SOLUTIONS TO POSSIBLE ASSESSMENT STRATEGIES

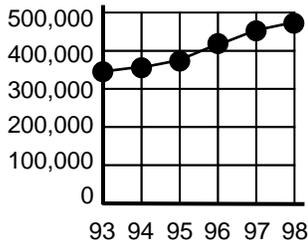
SECTION 1.1

- a. Bar graph – best graph to compare two different categories
- b. Line graph – best graph to display data over time

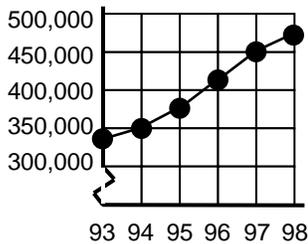
SECTION 1.2

- Answers may vary. Possible solutions are:

a.



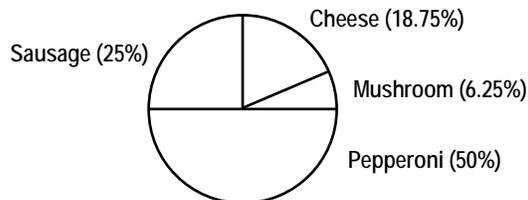
b.



- Kendra should not be worried about her fourth quarter mark because the vertical axis begins at the relatively high mark of 86, making the intervals seem larger than they are.

SECTION 1.3

- a. As a percentage, the defence budget decreased from 6.0% in 1980 to approximately 5.8% the following year.
- b. As a dollar amount, the defence budget increased from 30 million dollars in 1980 to 35 million the following year.
- Answers may vary. One possible solution is the following circle graph:



SECTION 2.1

- a. 15 cats
- b. 30 dogs
- 1: 25,000,000
- $\frac{2}{5}$
- No, because 17 is not a whole number multiple of 3. In fact, the value of \square is $\frac{136}{3}$, or approximately 45.3.
- The ratio 1 : 20,000,000 means that 1 unit on a map represents 20,000,000 units on the earth. If we use centimetres as units, we get 1 cm on a map representing 20,000,000 cm on the earth. Converting cm to km, we get 1 cm on a map representing 200 km on the earth.
- Answers may vary. Possible solutions are:
 - 4 red and 3 blue
 - 2 red and 1 blue
 - 1 red and 3 blue
 - 2 red, 3 blue and 5 green

SECTION 2.2

- 656 km; We assumed that the speed of 80 km/h was constant for all of that time.
- 168 bags
- \$0.75
- The girls would get the bigger portion of pizza, since $\frac{4}{9}$ is larger than $\frac{3}{7}$. We assume that all boys get a slice that is the same size and all girls get a slice that is the same size.
- The rate 50 mL/h cannot be expressed as a percentage because the compared units are different.

SECTION 2.3

- 2000 mL
- \$17.94; Twelve cans of pop cost \$2.99. Since she needs 72 cans, which is six times as many cans, she can multiply the price by six to get the answer.
- a. The situations are not proportional because the ratios of $\frac{5}{6}$ and $\frac{4}{5}$ are not equal.

- b. The first situation should be more flavourful since the ratio $\frac{5}{6}$ is greater than $\frac{4}{5}$. We assumed that both ratios were exact.
- This is not a proportion problem because all people age at exactly the same rate, regardless of their age.
- Proportion (a) has the largest answer.
 - a. $a = 12$
 - b. $b = 6.75$
 - c. $c = 11$
- 1.2 L of orange juice for \$2.00 is the better buy because it has a unit price of \$1.67/L, compared to \$1.87/L for 0.75 L of juice.
- 80 mm; 240 mm; We make the assumption that the rain continues to fall at the same rate.
- 1.5 L of grapes, 1 L of melon, 2 L of pineapple

SECTION 3.1

- 6; 7; 9
- a. $9 \text{ m} \times 9 \text{ m}$
- b. $7 \text{ m} \times 7 \text{ m} \times 7 \text{ m}$
- a. 80
- b. 110
- c. 30
- d. 26
- This expression cannot be a whole number because all squares must end in one of the following digits: 0, 1, 4, 5, 6, 9.
- a. 15
- b. 10
- Since 7569 has an odd number of factors, the middle factor when written in ascending order, 87, will be the square root.
- 18
- 1764; 42

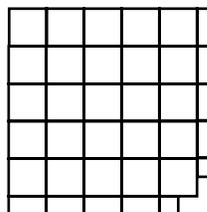
SECTION 3.2

- Since $6^2 + 8^2 = 100 = 10^2$, 6-8-10 is also a Pythagorean triple. Any whole number multiple of 3-4-5 is a Pythagorean triple.
- If we apply the Pythagorean theorem, then in order for this to be a right triangle, the longest side, 15, would have to be the hypotenuse, so 15^2 must equal $7^2 + 11^2$. However, we get values of 225 and 170 for these expressions. Since they are not equal, this is not a right triangle.

- a. Corey's work is incorrect because in his relationship, r is indicated as the hypotenuse. In fact, his formula should be $p^2 = q^2 + r^2$.
- b. Mia's work is incorrect because her triangle is not a right triangle.

SECTION 3.3

- a. Answers may vary. The square root of 20 is approximately 4.5.
- b. 4.47
- c. Jan's diagram is not quite a perfect square with side length of 4.5, which means that the actual answer must be slightly less.
- d. Answers may vary. The square root is 5.48.



- Any whole number from 37 to 48 will work.
- a. 3.7
- b. 5.9
- a. 3.3 is reasonable, as $\sqrt{11}$ is less than halfway between $\sqrt{9}$ and $\sqrt{16}$.
- b. 5.9 is not reasonable, as $\sqrt{27}$ is much closer to $\sqrt{25}$ than $\sqrt{36}$.
- The rug will not fit into her room, because the dimensions of the rug are approximately 4.1 m by 4.1 m.

SECTION 3.4

- Draw a triangle with legs of length 1 and 1 to get $\sqrt{2}$. Draw a triangle with legs of length 2 and 2 to get $\sqrt{8}$.
- a. 3 and 5
- b. 1 and 6
- c. 2 and 8

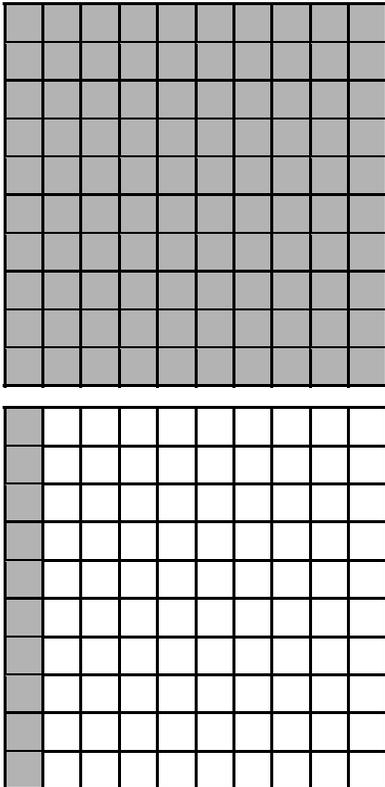
SECTION 3.5

- a. Yes – Using the Pythagorean theorem, the base of the ladder will be approximately 3.9 m from the wall, which is at least one-third of the length of the ladder (2.7 m).
- b. 12.7 m
- a. 6 m

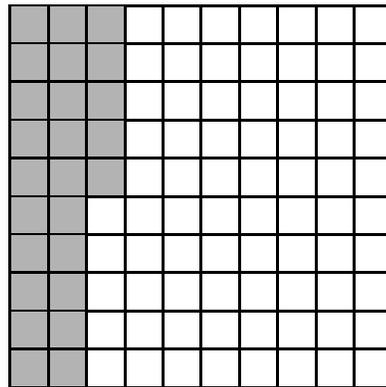
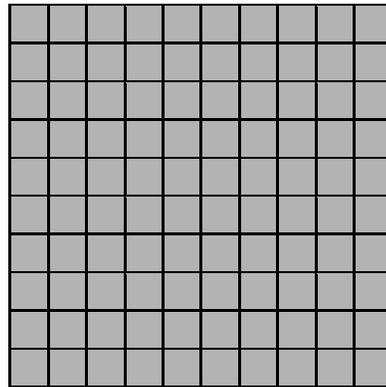
- b. 1.5 m^2
- a. The total area is 148.32 m^2 . This will require 2966 shingles to cover the roof.
- b. We assume that the roof is rectangular and that the shingles all have exactly the same exposed area.

SECTION 4.1

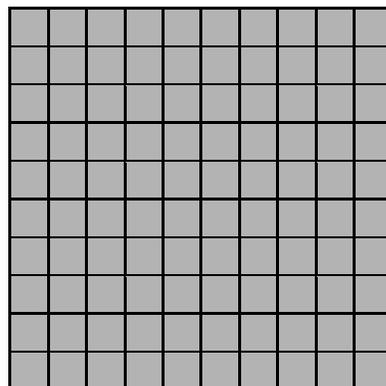
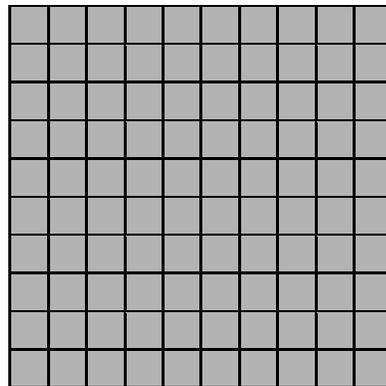
- a.



- b.



- c.



- Sarah, because she will earn 6% simple interest, which is greater than $5\frac{3}{4}\%$.
- a. Answers may vary. Jane may have interpreted the percentage incorrectly as simply 0.9, which is the same as 90%.
- b. Jack interpreted the percentage correctly as a percentage less than 1%.

SECTION 4.2

- Answers may vary. The percentages, to one decimal place, are:
 - 147.1%
 - 104.2%
 - 0.9%

SECTION 4.3

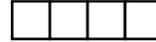
- \$80
- 23.1%
- 3575 votes
- The number is close to 40. The actual number is 37.5.
- The number is between 15 and 150.
- Since 30 is less than half, or 50% of 70, 60% is not a good estimate.
- Answers may vary. The exact answer is 800%.
- a. 200
- b. 4.6
- 22.2%

SECTION 4.4

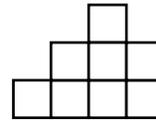
- a. \$117.94
- b. 36.2%
- c. \$135.63
- d. It does not produce an accurate result, because the percentages are not based on the same initial amount.
- a. Sarah took off 20% from the amount left after each year. Her friend took off 60% from the initial amount.
- b. Sarah's answer is correct, because the depreciation is calculated on the value of the car remaining after each year.
- a. \$95.63; This is not the same as a 40% discount, since the discounts of 45% and 15% are not calculated on the same initial amount. The difference is \$5.63.
- b. The 25% changes are not calculated on the same initial amount. (Calculated \$234.38)

SECTION 5.1

- Top:



- Front:



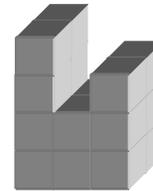
- Side:



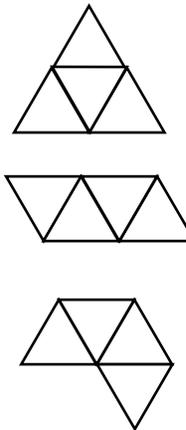
- B

SECTION 5.2

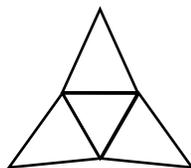
- Answers many vary. One possible solution is:



- Triangular pyramid with equilateral faces:



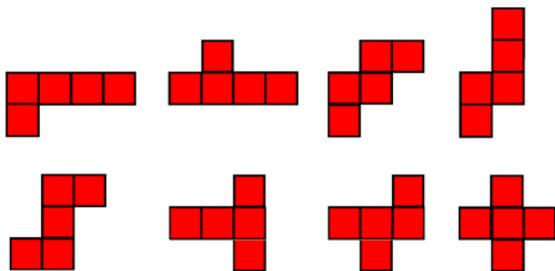
Triangular pyramid with isosceles faces:



There are more triangular pyramid with equilateral faces because all of the faces are congruent.

SECTION 5.3

- 132 cm²
- The triangular prism is the better box, with a surface area of 312 cm², as compared with 340 cm² for the rectangular prism.



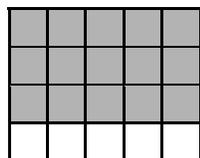
- The surface area of a 3-D object is equal to the sum of the areas of the shapes in its net.
- 582.1 cm²
- The surface area of the box is 2592 cm², which is equivalent to 0.2592 m², so she has enough paper.

SECTION 5.4

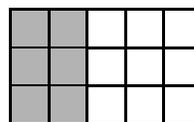
- The first container has approximately 3126 cm² of plastic and the second container has approximately 2758 cm² of plastic. Therefore, the first container requires more plastic to make.
- 172.7 cm²
- 63.8 cm²
- 11,200 cm²

SECTION 6.1

- a.



- b.



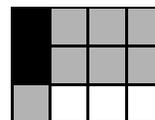
- a. 250 children
- b. \$1900
- Since 21 is slightly larger than 20, a multiple of 4, the answer will be slightly larger than three-quarters of 20, or 15. (Calculated: $15\frac{3}{4}$)
- $48 \div 6$
- 16; One-fifth of 20 is 4, therefore four-fifths of 20 is four times as much, or 16.
- a. 20 questions
- b. 18 students

SECTION 6.2

- a. $\frac{1}{6}$
- b. $\frac{1}{12}$
- c. $\frac{1}{9}$
- d. $\frac{1}{4}$
- $\frac{5}{24}$

SECTION 6.3

- $\frac{1}{4} \times \frac{2}{3}$ or $\frac{1}{3} \times \frac{2}{4}$
- $\frac{3}{5}$



- $\frac{1}{4}$

SECTION 6.4

- Answers may vary. The exact solutions are:
 - a. 50
 - b. $32\frac{3}{4}$ or $\frac{131}{4}$

- a. $33\frac{1}{2}$ or $\frac{67}{2}$
- b. 42
- c. 9
- a. Answers may vary. One possible solution is $\frac{29}{3} \times \frac{3}{2}$.
- b. Answers may vary. One possible solution is $\frac{5}{3} \times \frac{1}{4}$.

SECTION 6.5

- a. $9 \div 3 = 3$; $9 \div 1 = 9$; $9 \div \frac{1}{3} = 27$; $9 \div \frac{1}{9} = 81$

The answers increase by a factor of 3.

- b. $2 \div \frac{1}{2} = 4$; $1 \div \frac{1}{2} = 2$; $\frac{1}{2} \div \frac{1}{2} = 1$; $\frac{1}{4} \div \frac{1}{2} = \frac{1}{2}$

The answers decrease by a factor of $\frac{1}{2}$.

- a. $10\frac{10}{23}$ or $\frac{240}{23}$
- b. $5\frac{11}{17}$ or $\frac{96}{17}$
- a. 16 guests
- b. $\frac{3}{4}$ m

SECTION 6.6

- a. 96 muffins
- b. 75 cups
- a. $\left(\frac{1}{2} + \frac{1}{4}\right) \times \frac{2}{3} = \frac{1}{2}$
- b. $\frac{3}{4} \times \left(\frac{1}{5} + \frac{2}{3}\right) \times \frac{5}{3} = 1\frac{1}{12}$

SECTION 7.1

- a. 9 cm
- b. 5 cm
- 12.5 cm

SECTION 7.2

- 64 cm^3
- 1080 cm^3

- The triangular block of cheese has a volume of 245 cm^3 and the rectangular block of cheese has a volume of 240 cm^3 . The triangular block is the better buy.

SECTION 7.3

- The cylinder with a height of 27 cm will have an approximate volume of 3978 cm^3 . The cylinder with a height of 43 cm will have an approximate volume of 2497 cm^3 . Therefore, the height should be 27 cm to get the greater volume.
- Cylinder A has an approximate volume of 137 cm^3 and Cylinder B has an approximate volume of 192 cm^3 , so Cylinder B holds more water.

SECTION 7.4

- 10 cookies
- Yes, the volume is 128 cm^3 .

SECTION 8.1

- a. $0 - [4 \times (-2)] = 8$
- b. $0 + [3 \times (-3)] = -9$
- a. $4 \times (-3) = -12$
- b. $3 \times (-5) = -15$
- $-16 = 1 \times (-16)$; $-16 = 2 \times (-8)$; $-16 = 4 \times (-4)$;
 $-16 = 8 \times (-2)$; $-16 = 16 \times (-1)$; $-16 = -1 \times 16$;
 $-16 = -2 \times 8$; $-16 = -4 \times 4$; $-16 = -8 \times 2$;
 $-16 = -16 \times 1$
 $16 = 1 \times 16$; $16 = 2 \times 8$; $16 = 4 \times 4$; $16 = 8 \times 2$;
 $16 = 16 \times 1$; $16 = -1 \times (-16)$; $16 = -2 \times (-8)$;
 $16 = -4 \times (-4)$; $16 = -8 \times (-2)$; $16 = -16 \times (-1)$

Both numbers have 10 integer pairs.

- Her net worth increased by \$16, since she will not have to pay that debt back.

SECTION 8.2

- a. -6
- b. -4
- 10,200; Since $25 \times 4 = 100$, it can be easily seen that $100 \times (-102) = -10,200$.
- 0; The product of any factors and 0 is always zero.
- a. -406
- b. 584
- a. $-6 \div 3 = -2$

b. $9 \div 3 = 3$

- Answers may vary. Possible solutions are:
 - Tom owes \$5 to each of his four brothers. What is his total net worth?
 - Jill borrowed \$6 from each of five friends. What will Jill's net worth be if her friends decide to forgive the debt?

SECTION 8.3

- a. $9 \div 3 = 3$; $6 \div 3 = 2$; $3 \div 3 = 1$; $0 \div 3 = 0$;
 $-3 \div 3 = -1$
The answers decrease by 1.
- b. $(-9) \div (-3) = 3$; $(-6) \div (-3) = 2$;
 $(-3) \div (-3) = 1$; $0 \div (-3) = 0$; $3 \div (-3) = -1$
The answers decrease by 1.
- $-32 \div 4 = -8$; Each person's share of the debt is \$8.

SECTION 8.4

- a. $-9 \times (-3) = 27$
- b. $57 \div (-3) = -19$
- a. $20 \div (-5) = -4$; $20 \div (-4) = -5$
- b. $-18 \div 6 = -3$; $-18 \div (-3) = 6$
- Answers may vary. One possible solution is: Melissa owes a total of \$12 to three people equally. How much does she owe to each person?

SECTION 8.5

- a. -400
- b. -600
- -10^0C
- The correct order of operations was not followed. The correct answer is 14.

SECTION 9.1

- Joe ran the fastest because he covered the greatest distance in the shortest time.
- a. 36
- b. -14; 4

SECTION 9.2

- a.

BLACK SQUARES	WHITE SQUARES
1	5
2	7
3	9

The number of white squares in a figure is equal to three more than two times the number of black squares,

- b. $y = 2x + 3$
- c. 10 black squares and 23 white squares
- a.

LENGTH	PERIMETER
1	3
2	6
3	9
4	12
5	15

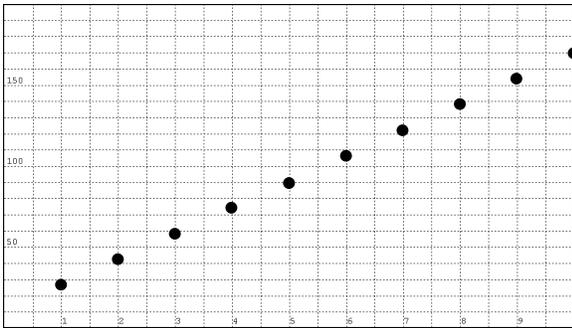
- b. The perimeter is equal to three times the length.
- c. $P = 3l$, where P is the perimeter and l is the length
- d. 297 m

x	-3	-2	-1	0	1	2
y	1	3	5	7	9	11

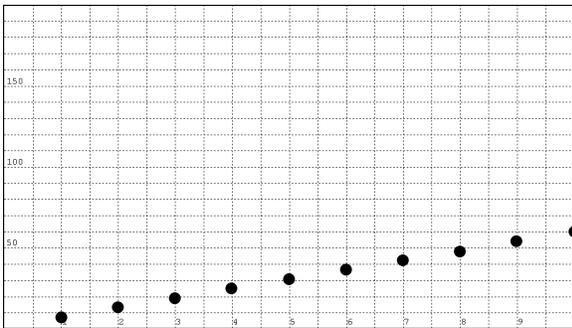
SECTION 9.3

RODS	SURFACE AREA	VOLUME
1	26	6
2	42	12
3	58	18
4	74	24
5	90	30
6	106	36
7	122	42
8	138	48
9	154	54
10	170	60

$S = 16n + 10$

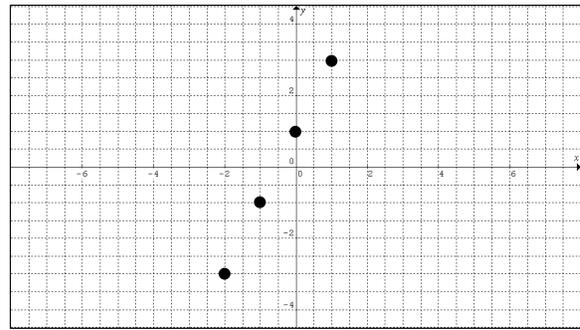


$V = 6n$

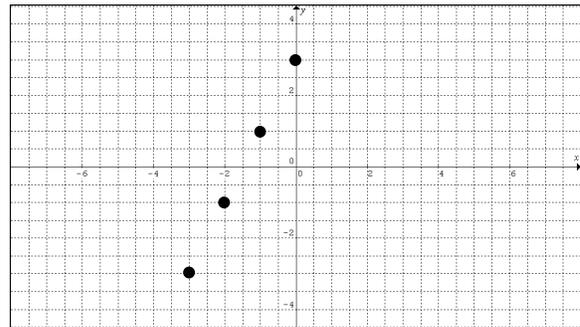


Both graphs are linear.

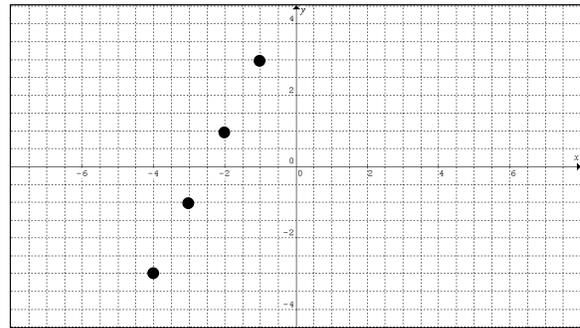
$y = 2x + 1$



$y = 2x + 3$



$y = 2x + 5$



- a. All three lines have the same steepness.
 - b. The lines cross the x - and y -axes at different points.
 - c. The lines are all parallel to each other.
 - d. It would be parallel to all three lines.
- The Typing Speed graph would be connected because time is a continuous variable while the number of pizza slices is a discrete variable.

SECTION 10.1

- a. $d = 8$
- b. $p = 176$
- c. $e = -112$
- d. $y = 12$
- 19.5 cm

SECTION 10.2

- a. $x = 8$
- b. $x = 8$
- Step 1: $-2x - 2 = 6$
- Step 2: $-2x = 8$
- Step 3: $-x = 4$
- Step 4: $x = -4$
- 13 km
- 75 students
- 3 cows and 13 chickens

SECTION 10.3

- a. $x = 8$
- b. $n = 28$
- c. $a = 48$
- 18
- Alex's method is not wrong but it is less efficient. He should have first added 5 to both sides of the equation, otherwise he will have to work with fractions if he divides first.

SECTION 10.4

- a. $y = 2$
- b. $c = 0$
- c. $r = -8$
- d. $j = 2$
- 10 points
- \$200
- The corrected work should be:

$$12(x - 3) = 72$$

$$12x - 36 = 72$$

$$12x - 36 + 36 = 72 + 36$$

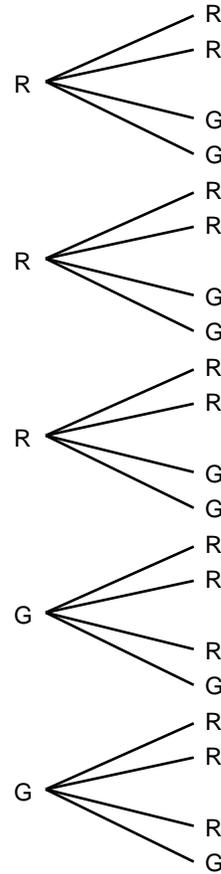
$$12x = 108$$

$$\frac{12}{12}x = \frac{108}{12}$$

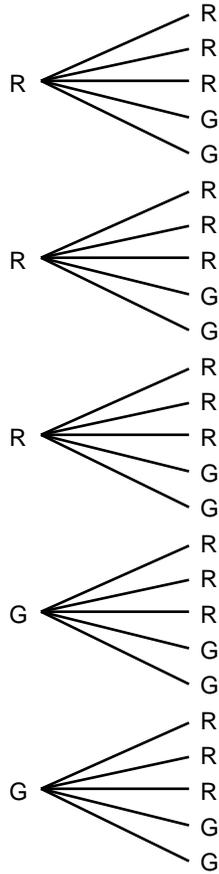
$$x = 9$$
- Equation (b) produces the smallest value:
 - $d = 6$
 - $d = -10$
 - $d = -3$
 - $d = 96$
 - $d = -7$

SECTION 11.1

- a. $\frac{1}{10}$



b. $\frac{4}{25}$



- 20 outcomes

	H	T
1	1,H	1,T
2	2,H	2,T
3	3,H	3,T
4	4,H	4,T
5	5,H	5,T
6	6,H	6,T
7	7,H	7,T
8	8,H	8,T
9	9,H	9,T
10	10,H	10,T

SECTION 11.2

- a. independent
- b. dependent
- c. dependent
- d. dependent
- e. independent

SECTION 11.3

- a. $\frac{1}{9}; \frac{4}{9}; \frac{2}{9}$
- b. $\frac{1}{15}; \frac{2}{5}; \frac{4}{15}$
- $\frac{1}{6}$
- $\frac{3}{10}$ or 30%
- $\frac{4}{5}$
- $\frac{1}{18}$

SECTION 12.1

- a. irregular
- b. irregular
- c. regular
- a. $\angle 1 = 120^\circ, \angle 2 = 120^\circ, \angle 3 = 120^\circ$
- b. 360°
- c. yes

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