Prince Edward Island
CANADA

Education and Early Childhood Development
English Curriculum

Prince Edward Island Mathematics Curriculum

CURRICULUM

Mathematics

Grade 3

Revised 2015
Prince Edward Island
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Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base. Developers have examined the curriculum proposed throughout Canada and secured the latest research in the teaching of mathematics, and the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for K-9 Mathematics* (2006) has been adopted as the basis for a revised mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories, and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP, and on the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➢ Essential Graduation Learnings

Essential graduation learnings (EGLs) are statements describing the knowledge, skills, and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills, and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work, and study today and in the future. Essential graduation learnings are cross curricular, and curriculum in all subject areas is focussed to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will demonstrate knowledge, skills, and attitudes expressed as essential graduation learnings, and will be expected to:

- respond with critical awareness to various forms of the arts, and be able to express themselves through the arts;
- assess social, cultural, economic, and environmental interdependence in a local and global context;
- use the listening, viewing, speaking, and writing modes of language(s), and mathematical and scientific concepts and symbols, to think, learn, and communicate effectively;
- continue to learn and to pursue an active, healthy lifestyle;
- use the strategies and processes needed to solve a wide variety of problems, including those requiring language and mathematical and scientific concepts;
- use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

➢ Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate to all educational partners high expectations for students in mathematics education. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to:

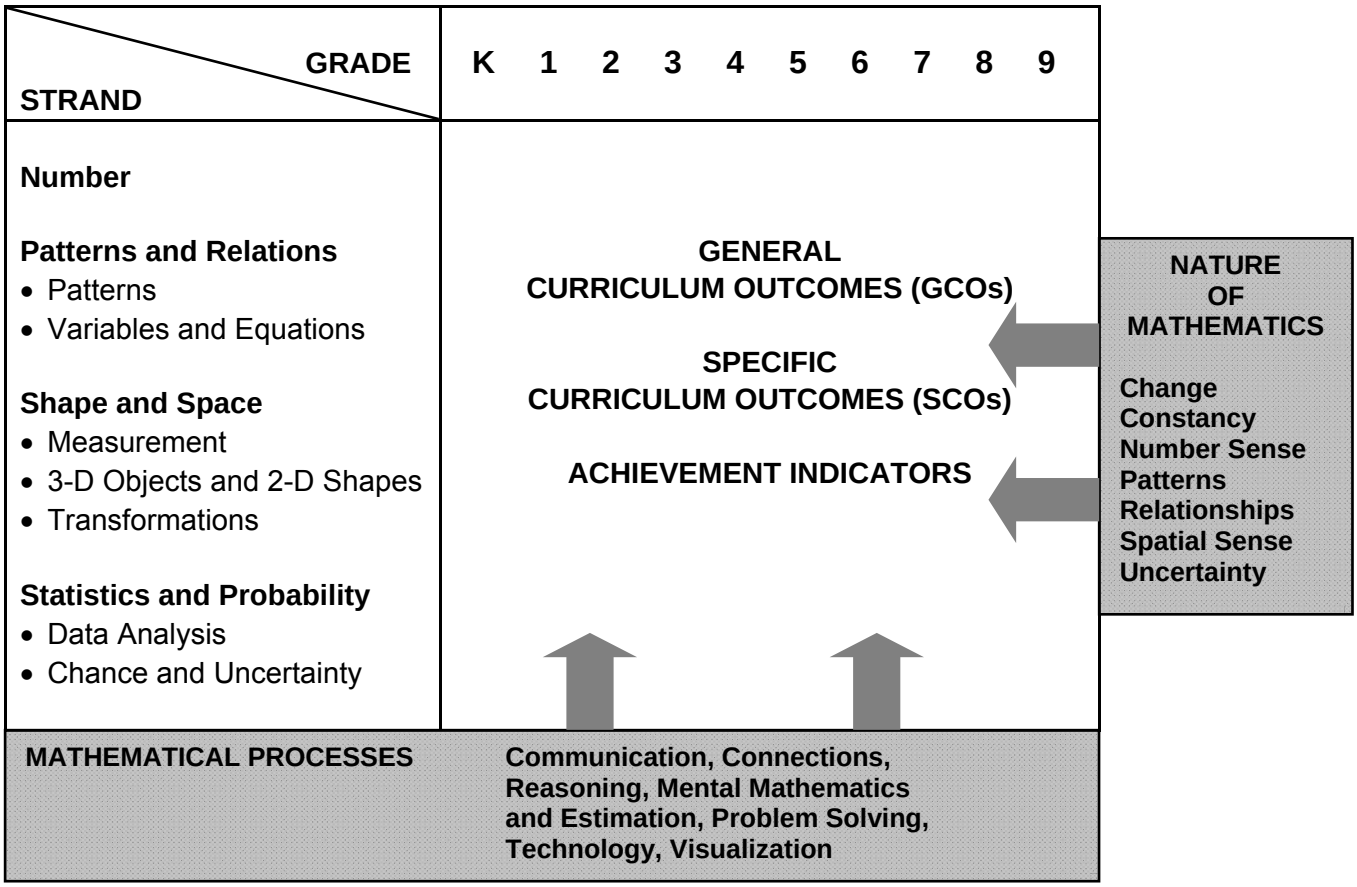
- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning;
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks;
- exhibit curiosity.

Conceptual Framework for K-9 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into four strands, namely **Number, Patterns and Relations, Shape and Space**, and **Statistics and Probability**. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connections among concepts both within and across strands. Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- Decreasing emphasis on rote calculation, drill, and practice, and the size of numbers used in paper and pencil calculations makes more time available for concept development.
- Problem solving, reasoning, and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➢ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to:

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]**
- develop visualization skills to assist in processing information, making connections, and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

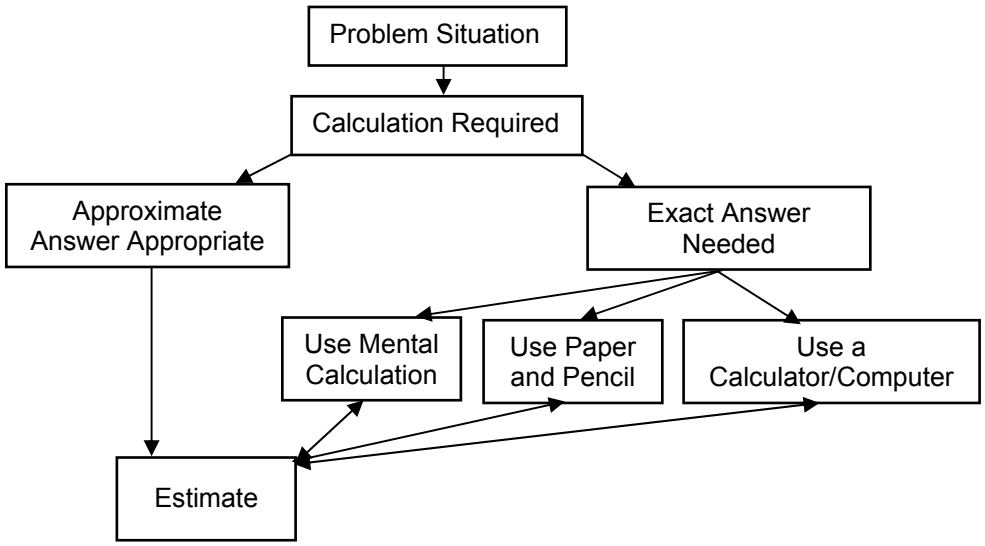
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves calculation without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need - more than ever before - with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001). Mental mathematics "provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers" (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know when to estimate, what strategy to use, and how to use it. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you. . . ?" or "How could you. . . ?" the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not

a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- use estimation
- guess and check
- look for a pattern
- make an organized list or table
- use a model
- work backwards
- use a formula
- use a graph, diagram, or flow chart
- solve a simpler problem
- use algebra.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations;
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K-3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform, and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and knowledge of several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

➢ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics which are woven throughout this document. These components include **change, constancy, number sense, patterns, relationships, spatial sense**, and **uncertainty**.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- skip counting by 2s, starting from 4;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180^0.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (*The Primary Program*, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, and flexible and intuitive with numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collecting and analysing of data, and describing relationships visually, symbolically, orally, or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to interpret representations of 2-D shapes and 3-D objects, and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 2-D shapes and 3-D objects.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to use dimensions and make predictions about the results of changing dimensions. For example:

- knowing the dimensions of an object enables students to communicate about the object and create representations;
- the volume of a rectangular solid can be calculated from given dimensions;
- doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of

probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Young children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Their mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Initial problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➢ Connections across the Curriculum

There are many possibilities for connecting Grade 3 mathematical learning with the learning occurring in other subject areas. Making connections between subject areas gives students experiences with transferring knowledge and provides rich contexts in which students are able to initiate, make sense of, and extend their learnings. When connections between subject areas are made, the possibilities for transdisciplinary inquiries and deeper understanding arise. When making such connections, however, teachers must be cautious not to lose the integrity of the learning in any of the subjects.

➢ Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding, and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should replace some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill-oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a teacher can give a parent clearer understanding of the mathematics curriculum and of the child's progress in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

➢ Diversity in Student Needs

Every class has students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters, but should be designed to help all students (whether strong, weak or average) to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson from which all students come away with a better understanding of what the solution to an equation really means.

➢ Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean not only that enrolments of students of both genders and various cultural backgrounds in public school mathematics courses should reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➢ Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English deficiencies and cultural differences must not be barriers to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and co-ordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication "as an essential part of mathematics and mathematics education (p.60)." The *Standards* elaborate that all students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading, and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers and will facilitate "communicating to learn mathematics and learning to communicate mathematically" (NCTM, p.60).

To this end:

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counsellors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not on other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated with appropriate language support to both students and parents;
- to verify that barriers have been removed, educators should monitor enrolment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➢ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development - such as poverty alleviation, human rights, health, environmental protection, and climate change - into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental, and economic perspective, and exploring how those factors are interrelated and interdependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database

Resources for Rethinking, found at **http://r4r.ca/en**. It provides teachers with access to materials that integrate ecological, social, and economic spheres through active, relevant, interdisciplinary learning.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, or teaching has been effective, or how best to address student learning needs. The quality of the assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others.

➢ Assessment

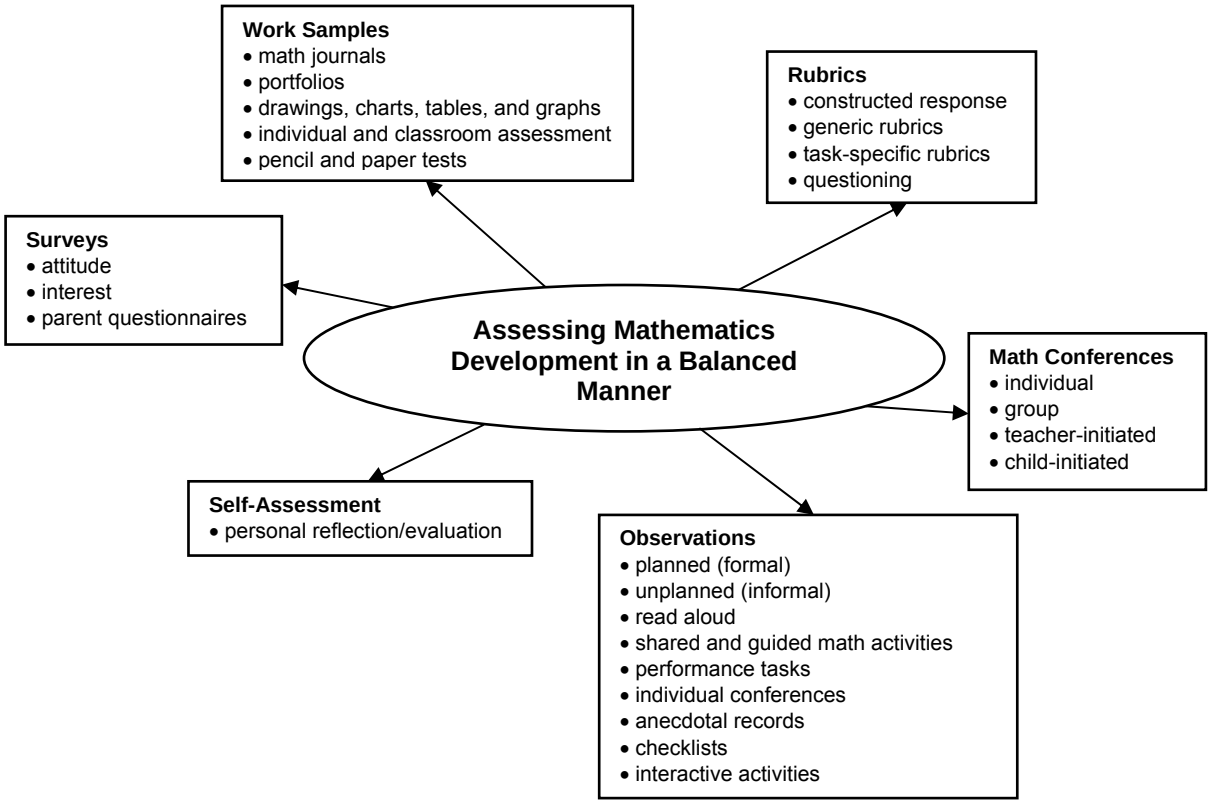
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as:

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children's learning;
- providing information to teachers on the effectiveness of their teaching, the program, and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources, including:

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment.

This balanced approach for assessing mathematics development is illustrated in the diagram below.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning, and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used:

- to engage students in their own learning and self-assessment;
- to help students understand what is important in the mathematical concepts and particular tasks they encounter;
- to develop effective habits of metacognition and self-coaching;
- to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used:

- to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
- to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
- to provide feedback to students about how they are doing and how they might improve;

- to provide feedback to other professionals and to parents about how to support students' learning.

Assessment *of* learning is used:

- to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
- to facilitate reporting;
- to provide the basis for sound decision-making about next steps in a student's learning.

➢ Evaluation

Evaluation is the process of analyzing, reflecting upon, and summarizing assessment information, and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analyzing and reflecting upon information about student learning gathered in a variety of ways.

This process requires:

- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information;
- using a high level of professional judgment in making decisions based upon that information.

➢ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes, and phone calls.

➢ Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration, and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles, as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes, or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.

- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.
- Assessment reports should be clear, accurate, and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that:

- the best interests of the student are paramount;
- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes;
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he/she knows and can do.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island mathematics curriculum are organized into four strands across the grades K-9. They are **Number, Patterns and Relations, Shape and Space**, and **Statistics and Probability**. These strands are further subdivided into sub-strands, which are the general curriculum outcomes (GCOs). They are overarching statements about what students are expected to learn in each strand or sub-strand from grades K-9.

Strand	General Curriculum Outcome (GCO)
Number (N)	**Number:** Develop number sense.
Patterns and Relations (PR)	**Patterns**: Use patterns to describe the world and solve problems.
	Variables and Equations: Represent algebraic expressions in multiple ways.
Shape and Space (SS)	**Measurement**: Use direct and indirect measure to solve problems.
	3-D Objects and 2-D Shapes: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.
	Transformations: Describe and analyze position and motion of objects and shapes.
Statistics and Probability (SP)	**Data Analysis**: Collect, display, and analyze data to solve problems.
	Chance and Uncertainty: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific outcome.

The first two pages for each outcome contain the following information:

- the corresponding **strand** and **General Curriculum Outcome;**
- the **Specific Curriculum Outcome**(s) and the mathematical **processes** which link this content to instructional methodology
- the **scope and sequence** of concept development related to this outcome(s) from grades 2 - 4;
- an **elaboration** of the outcome;
- a list of **achievement indicators**

  Students who have achieved a particular outcome should be able to demonstrate their understanding in the manner specified by the achievement indicators. It is important to remember, however, that these indicators are not intended to be an exhaustive list for each outcome. Teachers may choose to use additional indicators as evidence that the desired learning has been achieved.

The last two pages for each outcome contain lists of **instructional strategies** and **strategies for assessment**.

The primary use of this section of the guide is as an ***assessment for learning*** (formative assessment) tool to assist teachers in planning instruction to improve learning. However, teachers may also find the ideas and suggestions useful in gathering ***assessment of learning*** (summative assessment) data to provide information on student achievement.

Following the Specific Curriculum Outcomes for grade three, you will find the **Mental Math Guide** which outlines the **Fact Learning, Mental Computation and Estimation** strategies for this grade level. Included is an **Overview of the Thinking Strategies in Mental Math** for grades one to six complete with a description of each strategy as well as a scope and sequence table of the strategies for the elementary grades.

A **Glossary of Mathematical Models** (common manipulatives) is also provided in Appendix A followed by a one-page **List of Grade 3 Specific Curriculum Outcomes** in Appendix B. Then, there is a **correlation of our SCOs with the resource**, *Math Makes Sense 3,* in Appendix C. Finally, the last appendix is a **Table of Specifications** categorizing the SCOs into the four content strands of mathematics. The intent of the appendices is to provide mathematics teachers with practical references.

NUMBER

SCO: **N1: Say the number sequence forward and backward from 0 to 1000 by:**
- **5s, 10s or 100s using any starting point**
- **3s using starting points that are multiples of 3**
- **4s using starting points that are multiples of 4**
- **25s using starting points that are multiples of 25.**

[C, CN, ME]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math/Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Two	Grade Three	Grade Four
N1 Say the number sequence from 0 to 100 by: • 2s, 5s and 10s, forward and backward, using starting points that are multiples of 2, 5 and 10 respectively • 10s using starting points from 1 to 9 • 2s starting from 1.	**N1** Say the number sequence forward and backward from 0 to 1000 by: • 5s, 10s, or 100s, using any starting point • 3s using starting points that are multiples of 3 • 4s using starting points that are multiples of 4 • 25s, using starting points that are multiples of 25.	

Elaboration

In grade 3, students are continuing to develop an understanding of number and counting. A focus on skip counting in the early years helps them recognize and apply the patterns in our place value system and prepares them for later work involving money (Small, 2008, p. 86). Skip counting by 2s, 3s, 4s and 25s, is a cornerstone for later multiplicative reasoning.

This year, students counting experiences will include numbers to 1000. Extending the place value pattern beyond 100 may be initially challenging for many students. Some may be unaware that the tens sequence repeats within each hundred (1**10**, 1**20**, 1**30**, 1**40**…) and will be uncertain about the pattern when it bridges a decade (e.g., 98, 99**, 100,** 101, 102, 103, 104, 105, 106, 107, 108, 109**, 110**, 111…). Students will need many opportunities to construct this understanding of place value. Experiences with identifying and correcting errors and omissions in a given skip counting sequence, or recognizing and explaining the skip counting pattern for a given number sequence will help to reinforce the development of these concepts.

This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:
- Unit 1, Lesson 4, pp. 15–17
- Unit 1, Lesson 8, pp. 28–31
- Unit 2, Lesson 1, pp. 38–41
- Unit 2, Lesson 6, pp. 54–57
- Unit 2, Lesson 7, pp. 58–61
- Unit 2, Lesson 9, pp. 65–67
- Unit 2, Unit Problem, pp. 78, 79

Mental Math strategies will strengthen student understanding of this specific curriculum outcome. (Refer to pp. 115–158.)

SCO: **N1: Say the number sequence forward and backward from 0 to 1000 by:**
- **5s, 10s or 100s using any starting point**
- **3s using starting points that are multiples of 3**
- **4s using starting points that are multiples of 4**
- **25s using starting points that are multiples of 25.**

[C, CN, ME]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Extend a given skip counting sequence by 5s, 10s or 100s, forward and backward, using a given starting point.
- Extend a given skip counting sequence by 3s, forward and backward, starting at a given multiple of 3.
- Extend a given skip counting sequence by 4s, forward and backward, starting at a given multiple of 4.
- Extend a given skip counting sequence by 25s, forward and backward, starting at a given multiple of 25.
- Identify and correct errors and omissions in a given skip counting sequence.
- Determine the value of a given set of coins (nickels, dimes, quarters, loonies) by using skip counting.
- Identify and explain the skip counting pattern for a given number sequence.

SCO: **N1: Say the number sequence forward and backward from 0 to 1000 by:**
- **5s, 10s or 100s using any starting point**
- **3s using starting points that are multiples of 3**
- **4s using starting points that are multiples of 4**
- **25s using starting points that are multiples of 25.**

[C, CN, ME]

Instructional Strategies

Consider the following strategies when planning lessons:

- Give students frequent opportunities to count materials (large quantities) in a variety of ways.
- Highlight the numbers on a number line or hundred charts that occur when skip counting and ask students to describe the patterns they see. Ensure that the numbers extend beyond 100.
- Provide students with multiple experiences counting both forward and backward, with various starting points.
- Use the calculator constant feature to count by 3s, 4s, 5s, 10s, 25s, and 100s. Ask the student to predict what number will come next, before it appears on the display.
  Note: Many calculators have a constant feature which enables students to explore skip counting patterns. For example, if counting by 4s, enter the following sequence into the calculator: ***4 + = = =.*** *Each time the equal sign is pressed, a multiple of 4 will appear. The same feature can be used for backwards counting by 4s starting from a number which is a multiple of four, e.g.,* ***712 – 4 = = =.***

Suggested Activities

- Provide students with a hundred chart *(1-100, 101-200, etc.)* and have them colour in the pattern for a given skip counting sequence.
- Provide students with many number patterns to encourage skip counting; for example:

  25, 50, _, _, 125, _, _
  652, _, 452, _, 252, _
  95, 90, _, _, 75, _, _, _

- Have students identify and correct the error in a given skip counting sequence, such as:

  12, 16, 21, 24, 28, 32
  27, 30, 33, 35, 39, 42

- Provide coins for the students. Ask: Can you use 6 coins to make $1.00? Can you make a total of $1.45 with only 6 coins? What are the coins? This activity could be extended to use bills.
- Provide students with play coins. Tell them that you have, for example, 5 coins in your hand that total 81 cents. Ask: What coins am I holding? (This is a problem situation and may require time.)
- Have students count beans in a jar. Ask them how they grouped the beans (e.g., by 2s, 5s, 10s) for ease of counting.
- Use the repeat (constant) function (press 0, +, 25, =, =, =,…) on the calculator to skip count to a target number. For example, if you start at 0 and want to end at 400, by which number(s) could you skip count? (4, 5, 10, 25, 100) What if you started at a different point? What if you wanted to end at a different point?
- Play “What’s in the Can?” Tell the student that you are going to drop nickels (or dimes or quarters) into a can. Have the student listen as the coins drop and count to find the total. As an extension, tell the student that there is, for example, 45 cents in the can. Tell him/her that you are going to add nickels (or dimes) and ask him/her to keep track to find the total.

SCO: **N1: Say the number sequence forward and backward from 0 to 1000 by:**
- **5s, 10s or 100s using any starting point**
- **3s using starting points that are multiples of 3**
- **4s using starting points that are multiples of 4**
- **25s using starting points that are multiples of 25.**

[C, CN, ME]

Assessment Strategies

- Begin to count, “25, 50, 75, 100, 125, 150.” Ask the student to continue to count to 500 by 25s.
- Provide students with a number of beans (e.g., 60). Ask the student to separate the beans from the pile as he/she counts them by 3s, then by 4s.
- Ask: Why do you say fewer numbers when counting to 100 by 10s than when counting by 5s?
- Tell the student to decide which starting point, 6 or 7, is easier when counting by 3s. Ask the student to explain his/her choice.
- Write and say, “25, 50, 60, 65, 70.” Ask: What coins am I counting?
- Give the student 4 quarters, 3 dimes, 2 nickels and 6 pennies. Ask him/her to count the coins to find if an item costing $3 can be bought.
- Have students count by 5s until they reach sixty. Ask: What other numbers can you count by and still land on 60?
- Have students skip count following directions, such as:
  - Start at 95 and count forward by 5s to 140.
  - Start at 349 and count by 100s without going over 1000 but get as close as you can.
  - Start at 450 and count backward by 25s until you reach 250.

SCO: **N2: Represent and describe numbers to 1000, concretely, pictorially and symbolically.**
[C, CN, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Two	Grade Three	Grade Four
N4: Represent and describe numbers to 100, concretely, pictorially and symbolically.	**N2** Represent and describe numbers to 1000, concretely, pictorially and symbolically.	**N1** Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.

Elaboration

At this stage in their number sense development, it is not uncommon for many students to ignore 0 as a placeholder in multi-digit numbers. For example, some may record 67 for *six hundred seven*. It is, therefore, essential that students have many opportunities to represent numbers using a variety of concrete materials and models to emphasize the fact that each position in a number has a place-value name and that these names determine the value of each digit in a multi-digit whole number.

Students who have a deep understanding of numbers up to 1000 will be able to partition numbers in flexible ways. For example, they will know that 750 is the same as 700 + 25 + 25 or 500 + 200 + 30 + 20 etc. They will also know that each of these "parts" can be further broken down as required.

Students will also represent and describe numbers as expressions. An expression is a mathematical phrase made up of numbers connected by an operation. For example, 60 + 40 and 150 - 50 are both expressions to represent 100.

When reading multi-digit numbers, it is important to remember that the word "*and"* is used when reading *decimal numbers*. For example, beginning in grade 4, a number such as 1.3 is read as "*one and three tenths"*, not "*one decimal three"* or "*one point three".* A whole number such as 205 is read as "*two hundred five"*.

It should also be noted that when recording 4-digit numbers up to 9999, there is no space or comma between the thousands and hundreds place (e.g., 1000 not 1 000, nor 1,000). Although the focus in grade 3 is on numbers up to 1000, students in a *rich* classroom environment may explore numbers with five or more digits and should understand that these numbers are recorded with a space between the thousands and hundreds place (e.g., 10 000).

Students will also learn to write the number words for the multiples of ten and multiples of one hundred. However, while correct spelling of these words should be encouraged, it is not the ultimate intent of this outcome.

This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:

- Unit 2, Lesson 3, pp. 45–47
- Unit 2, Lesson 4, pp. 48, 49
- Unit 2, Lesson 8, pp. 62–64
- Unit 2, Lesson 11, pp. 72–74
- Unit 2, Unit Problem, pp. 78, 79
- Unit 7, Lesson 6, pp. 260, 261

SCO: **N2: Represent and describe numbers to 1000, concretely, pictorially and symbolically.**
[C, CN, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Read a given three-digit numeral without using the word "and," e.g., 321 is *three hundred twenty one*, not *three hundred AND twenty one*.
- Read a given number word (0 to 1000).
- Represent a given number as an expression, e.g., 300 – 50 for 250 or 230 + 20
- Represent a given number using manipulatives, such as base ten materials, in multiple ways.
- Represent a given number pictorially.
- Write number words for given multiples of ten to 90.
- Write number words for given multiples of hundred to 900.

SCO: **N2: Represent and describe numbers to 1000, concretely, pictorially and symbolically.** [C, CN, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Ensure students have many opportunities to use a variety of concrete materials.
- Have a math word wall available for students to assist with the correct spelling of number words.
- Provide students with frequent opportunities to represent numbers using words, pictures and symbols.
- Have students create different expressions for the same numbers.

Suggested Activities

- Invite students to create a "thousand" chart by writing the number sequence in ten blank hundred grids.
- Ask students to find examples from newspapers, magazines, internet, etc., where numbers up to 1000 are used and to represent these numbers pictorially.
- Have students rename a number less than 1000 as the sum of other numbers.
- Draw a line labeled 0 and 100 at opposite ends (or 200 and 400, 100 and 600 etc.). Mark a few different points on this line and ask students what number they think each point might be and why they think that.
- Have students create and solve number riddles such as "I have written a secret number between 600 and 800. It is an odd number. What might it be?"
- Use literature to provide a context for the number 1000. For example, "*How Much, How Many, How Far, How Heavy, How Long, How Tall, is 1000?*" by Helen Nolan.
- Ask students to record a series of numbers that are read to them. Include examples of numbers that contain a zero.
- Ask students to *read* a number represented by an *informal* arrangement of base ten materials.

SCO: **N2: Represent and describe numbers to 1000, concretely, pictorially and symbolically.**
[C, CN, V]

Assessment Strategies

- Ask students to rearrange the digits 1, 4 and 5 to create the number that is closest to 500.
- Provide students with a number line (0-1000) and have them place benchmarks numbers such 250, 500, 750.
- Ask students, “How many numbers can you make using 3 digits (for example 2, 3 and 4) if you only use each digit once in each number”.
- Ask students to record numbers read orally, both symbolically and with words, making sure to include numbers that have a zero.
- Have students work with a partner and record a number with words, exchange with their partner, record that number symbolically then say that number to their partner.
- Ask students, “Which of the following expressions represent 360?”

200+160	380-30	400-40
260+75+25	357+4	260+100

SCO: **N3: Compare and order numbers to 1000.**
[CN, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Two	Grade Three	Grade Four
N5 Compare and order numbers up to 100.	**N3** Compare and order numbers to 1000.	**N2** Compare and order numbers to 10 000.

Elaboration

Students should be able to read, write, compare and order two or more whole numbers, each less than 1000. Early instructional strategies will include situations involving hundreds charts and number lines, but then gradually progress towards the use of *place value positional names* in determining relative sizes. For example, to compare 667 and 607, students should notice that both numbers have 6 hundreds, but that the 667 is greater than 607 because it has more tens in the tens place. The numbers could also be compared by considering their relative position in the counting sequence: 667 comes after 607, so 667 is greater than 607.

Students should also be able to name numbers *greater than, less than*, or *between* given numbers and be able to arrange numbers in *ascending* and *descending* order.

In grade 3, the symbols “<” and “>” are introduced to represent the relative size of two numbers. The instructional emphasis, however, should be on using the phrases *greater than* and *less than* rather than these symbols when comparing numbers.

This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:

- Unit 2, Lesson 5, pp. 50–53

SCO: **N3: Compare and order numbers to 1000.**
[CN, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Place a given set of numbers in ascending or descending order and verify the result by using a hundred chart, e.g., a one hundred chart, a two hundred chart, a three hundred chart, a number line or by making references to place value.
- Create as many different 3-digit numerals as possible, given three different digits. Place the numbers in ascending or descending order.
- Identify errors in a given ordered sequence.
- Identify missing numbers in isolated parts of a given hundred chart (include charts beyond 100).
- Identify errors in a given hundred chart (include charts beyond 100).

SCO: **N3: Compare and order numbers to 1000.**
[CN, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Have students plot numbers on an open number line (horizontal and vertical) to show their relative positions. Have students share their thinking.
- Give the student some prepared cards with 3-digit numbers on them. Ask him/her to order the number cards in ascending and descending order.
- Show the students two numbers (e.g. 501 and 398) and ask which is greater. Have the students explain their answers. Encourage them to use base ten blocks and place value language in their explanations.
- Give students many opportunities to explore the magnitude of similar digits. Example: How are the digits in 777 similar? How are they different?

Suggested Activities

- Prepare a deck of number cards that contain both 2- and 3-digit numbers. Have the students deal all the cards face down to the players. Have each player turn the top card over; the one who has the greater (greatest) number "wins" both or all the cards in play. The winner is the one who has collected the most cards when all the cards are turned over.
- Provide a set of cards (10 to 15) with each card having a 2- or 3-digit number on it. Ask the student to order the number cards from least to greatest and to explain how he/she determined the relative number size.
- Have students use a prepared deck of 40 number cards (4 sets of 0 to 9). Have the student select three or four of the cards and arrange them to make the greatest possible number and the least possible number. Ask the student to model these numbers.
- As a class activity, repeatedly roll a die and have the students fill in the digits, one at a time, on a place-value chart. Alternate by having them try to make the greatest number or the least number. Model the task by placing your digits on an overhead chart or interactive whiteboard. Regularly ask questions such as, "What do you need? What don't you want me to roll?"
- Play "Guess My Number", in partners, with numbers less than 1000. Use *greater than*, *less than* in the response (e.g., "Is your number 489?" "No. My number is greater than that."). Continue the game until the number is guessed and then change roles and have the other partner guess.
- Give each of two students a spinner with 10 numbers that are in the hundreds (e.g., 345, 354, 381, 309, 608, 680, 853, 835, 903, and 930). Have them spin at the same time. The one who spins the higher number gets a token. The students play until someone has gathered 10 tokens. Select numbers according to the students' level of understanding.

SCO: **N3: Compare and order numbers to 1000.**
[CN, R, V]

Assessment Strategies

- Ask the student to use models to show why 243 is less than 324.
- Ask the student to find a number between 312 and 387 that can be represented using 8 base ten blocks.
- Ask: What do you do to compare the size of two numbers?
- Ask: Why are there more numbers greater than 123 than less than 123?
- Ask the student to write a number that is:
  - greater than 165 but less than 200
  - a little less than 300
  - between 463 and 474
  - greater than 348 but less than 360, etc.
- Ask the student to explain why a 3-digit number is always greater than a 2-digit number.
- Ask the student to select five numbers between 600 and 630, and to write them in increasing order.
- Provide number lines for students and ask them to estimate where some numbers might lie, and to give their reasoning; for example, 465.

SCO: **N4: Estimate quantities less than 1000 using referents.**
[ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Two	Grade Three	Grade Four
N6: Estimate quantities to 100 using referents.	**N4** Estimate quantities less than 1000 using referents.	

Elaboration

As students begin to develop a capacity for estimation, it is essential that they have opportunities to use *referents* when estimating the total of a larger group of objects. These experiences will lead to greater proportional understanding and reasoning.

A *referent* is any known sub-set of a larger amount and serves as a visual image to assist students in reasoning about the total. For example:

Knowing how much 10 stars is, helps estimate the larger group of stars.

Using their knowledge of one hundred, students can then use a referent to estimate larger quantities. A small bag of one hundred beans, for example, may be used to determine how many beans are in a much larger bag.

This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:

- Unit 2, Lesson 10, pp. 68–71
- Unit 2, Unit Problem, pp. 78, 79

Mental Math strategies will strengthen student understanding of this specific curriculum outcome. (Refer to pp. 115–158.)

SCO: **N4: Estimate quantities less than 1000 using referents.**
[ME, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Estimate the number of groups of ten in a given quantity using 10 as a referent (known quantity).
- Estimate the number of groups of a hundred in a given quantity using 100 as a referent.
- Estimate a given quantity by comparing it to a referent.
- Select an estimate for a given quantity by choosing among three possible choices.
- Select and justify a referent for determining an estimate for a given quantity.

SCO: **N4: Estimate quantities less than 1000 using referents.**
[ME, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students with many opportunities to count groups of 10 and 100 objects in order to develop a sense of the size of these benchmark numbers.
- Estimate a given quantity by comparing it to a *known* quantity such as 10 or 100.
- Select between three possible estimates for a given quantity and explain the choice.

Suggested Activities

- Show 100 paper-clips as a visual referent for the students. Next display a larger group of paper clips. Ask students to estimate how many paper clips there are. Have students explain their thinking.
- Present a variety of situations which require students to consider larger quantities. For example, ask, "*About how many* …
  - Candy bars would cover the floor of your room
  - Steps an ant would take to walk around the school building
  - Grains of rice would fill a cup
  - Quarters could be stacked in one stack floor to ceiling
  - Pennies can be laid side by side down an entire room or hallway
  - Pieces of notebook paper would cover the gym floor
  - Minutes you have lived?"

  For each scenario, help students identify an appropriate referent and discuss how this referent could be used to determine the total estimate.
- Show a quantity of objects such as linking cubes. Ask: "If this is equal to 10 linking cubes, what might 143 linking cubes look like?
- Say; 100 counters takes up this much space. How much space would 783 counters take up if you placed them flat on the table? If they were in a milk jug? Explain your thinking?

SCO: **N4: Estimate quantities less than 1000 using referents.**
[ME, PS, R, V]

Assessment Strategies

- Show students a group of items and ask them to choose between three given estimates. Have students explain their reasoning.
- Place a pile of objects on a desk (e.g., paper clips, linking cubes, base ten units, buttons). Ask students to estimate the number. Observe and interview students to determine if they are using a referent. Guiding questions should include, “How did you pick that number?”
- Ask students to describe a strategy used to find an estimate.
- Show students 25 buttons; all buttons touching sides. Say: Susan said; “643 buttons will fit on top of a desk.” Do you agree or disagree? Explain.

SCO: **N5: Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.**
[C, CN, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Two	Grade Three	Grade Four
N7 Illustrate, concretely and pictorially, the meaning of place value for numerals to 100.	**N5** Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.	**N1** Represent and describe whole numbers to 10 000, pictorially and symbolically.

Elaboration

In grade 3, students will come to understand that there is a constant multiplicative relationship between the place values in a multi-digit number; i.e., from R to L, the value increases by powers of 10.
As they develop a deeper understanding of numbers to 1000, students will be able to compose and decompose numbers in more flexible ways. For example, they will begin to recognize 842 as *84 tens and 2 ones* or *8 hundreds and 42 ones* or *800 + 40 + 2*, or *500 + 300 + 20 + 20 + 2*, etc.).

Many problems that children later encounter with place-value concepts are believed to stem from inadequate attention to place-value activities in the early grades. Students need many opportunities to explore the place value of the digits in a number using *proportional* models i.e., the model for 100, is *physically* ten times the size of the model for 10. Examples would include popsicle sticks and rubber bands, beans and cups, linking cubes etc. Students should proceed from this kind of *groupable* proportional model to a "pre-grouped" proportional model such as base ten blocks, math racks, ten frames and Cuisenaire® rods. An example of a *non-proportional* model would be money in which, for example, a "*Loonie"* is not ten times the size of a dime, even though it is ten times greater in value.

Many comprehensive reviews of the research into the use of mathematical models have concluded that student understanding and achievement is increased as a result of long term exposure to mathematical models. It is important to remember, however, that it depends on how the models are used in the classroom. In themselves, mathematical models *do not teach* but, in concert with good teaching, make a great deal of difference.

This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:

- Unit 2, Lesson 2, pp. 42–44
- Unit 7, Lesson 6, pp 260, 261

SCO: **N5: Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.**
[C, CN, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Record, in more than one way, the number represented by given proportional and non-proportional concrete materials.
- Represent a given number in different ways using proportional and non-proportional concrete materials and explain how they are equivalent, e.g., 351 can be represented as three 100s, five 10s and one 1s, or two 100s, fifteen 10s and one 1s, or three 100s, four 10s and eleven 1s.
- Explain, and show with counters, the meaning of each digit for a given 3-digit numeral with all digits the same, e.g., for the numeral 222, the first digit represents two hundreds (two hundred counters) the second digit represents two tens (twenty counters) and the third digit represents two ones (two counters).
- Record a number represented by base ten blocks arranged informally (not grouped L to R from highest to lowest values).

SCO: **N5: Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.** [C, CN, R, V]

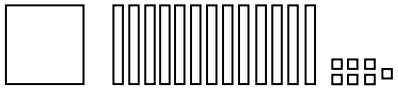
Instructional Strategies

Consider the following strategies when planning lessons:

- Ensure students have opportunities to use both proportional and non-proportional concrete materials.
- Have students represent the same number with different partitions. For example, 254 can be represented using 2 hundred dollar bills and 54 loonies or 1 hundred dollar bills, 15 ten dollar bills and 4 loonies.
- Provide multiple opportunities for students to show they understand that the position of a digit within a number determines its value.

Suggested Activities

- Ask student to record the number that is made up of 15 tens and 15 ones.
- Ask students to record the value of the base ten blocks shown:



- Ask students to build a model or draw a picture using base ten blocks. Ask: What is the value of the drawing or model?
- Have students create non-proportional concrete models and explain their values.
- Ask students to write as many numbers as possible with an 8 in the tens place.
- Give each group of students 12 base ten rods and 16 units. Have them record the number the blocks represent.
- Ask the students to model 507 in a variety of ways.
- Have students model numbers such as 421 and 139. Discuss which number has more tens, and how they know. Students should recognize that 421 has more tens, although has a smaller digit in the tens place.
- Ask the students to enter a certain number on a calculator (e.g., 235). Ask: How can you, without clearing the calculator, make the number 255? (35? 205? 261?).
- Ask students to record a specific 4-digit number, e.g., 3247 (or enter it into a calculator). Now, challenge them to remove one of the digits by performing only one operation. For example: to remove the value of the 2 from the number 3247, the student would need to subtract 200.

SCO: **N5: Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.**
[C, CN, R, V]

Assessment Strategies

- Ask the student to describe 324 without using the word “hundred” (e.g., thirty-two tens, four).
- Ask the student to show that 132 is 13 tens and 2 ones.
- Ask the student to describe 1000 in as many ways as they can.
- Ask the student to explain using words, numbers, and/or pictures how they know that 1000 is the same as 100 tens.
- Ask how 480 and 680 are the same and how they are different.

SCO: **N6: Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as:**
- **adding from left to right**
- **taking one addend to the nearest multiple of ten and then compensating**
- **using doubles.**

SCO: **N7: Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as:**
- **taking the subtrahend to the nearest multiple of ten and then compensating**
- **thinking of addition**
- **using doubles.**

[C, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Two	Grade Three	Grade Four
N10 Apply mental mathematics strategies, such as: using doubles; making 10; one more, one less; two more, two less; building on a known double; addition for subtraction to determine basic addition facts to 18 and related subtraction facts.	**N6** Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as: • adding from left to right • taking one addend to the nearest multiple of ten and then compensating • using doubles. **N7** Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as: • taking the subtrahend to the nearest multiple of ten and then compensating • thinking of addition • using doubles.	**N3** Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by: using personal strategies for adding and subtracting; estimating sums and differences; solving problems involving addition and subtraction.

Elaboration

When a problem requires an exact answer, students should first determine if they are able to calculate it mentally. This should become an automatic response. Situations must be regularly provided to ensure that students have sufficient practice with mental math strategies and that they use their skills as required. Using mental math will focus a student on the relationships between numbers and operations rather than relying on completing a traditional algorithm. For example, students might solve 49 + 99 mentally by adding 100 to 49, then subtracting 1. This method involves using benchmark numbers then compensating by adding or subtracting, whichever operation is necessary. Presenting appropriate practice items horizontally rather than vertically will encourage students to look to the numbers first and think about them in terms of their place values.

Students will develop, apply and describe mental math strategies to add and subtract two 2-digit numbers. Computational strategies that students should be introduced to include those listed in this outcome but are not limited to these alone. Some students may have already other, more sophisticated, thinking strategies that they use regularly.

SCO: **N6: Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as:**
- **adding from left to right**
- **taking one addend to the nearest multiple of ten and then compensating**
- **using doubles.**

SCO: **N7: Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as:**
- **taking the subtrahend to the nearest multiple of ten and then compensating**
- **thinking of addition**
- **using doubles.**

[C, ME, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

N6

- Add two given 2-digit numbers using a mental mathematics strategy and explain or illustrate the strategy.
- Explain how to use the "adding from left to right" strategy, e.g., to determine the sum of 23 + 46, think 20 + 40 and 3 + 6.
- Explain how to use the "taking one addend to the nearest multiple of ten" strategy, e.g., to determine the sum of 28 + 47, think 30 + 47 – 2 or 50 + 28 – 3.
- Explain how to use the "using doubles" strategy, e.g., to determine the sum of 24 + 26, think 25 + 25; to determine the sum of 25 + 26, think 25 + 25 + 1 or doubles plus 1.
- Apply a mental mathematics strategy for adding two given 2-digit numerals.

N7

- Subtract two given 2-digit numerals using a mental mathematics strategy and explain or model the strategy used.
- Explain how to use the "taking the subtrahend to the nearest multiple of ten" and then compensating strategy, e.g., to determine the difference of 48 – 19, think 48 – 20 + 1.
- Explain how to use the "thinking of addition" strategy, e.g., to determine the difference of 62 – 45, a student might think 45 + 5, then 50 + 12 and then 5 + 12. Using an open number line is helpful with this strategy.
- Explain how to use the "using doubles" strategy, e.g., to determine the difference of 24 – 12, think 12 + 12.
- Apply a mental mathematics strategy for subtracting two given 2-digit numerals.

N6: This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:
- Unit 3, Lesson 5, pp. 96–99
- Unit 3, Lesson 6, pp. 100, 101
- Unit 3, Lesson 13, pp. 124, 125
- Unit 3, Unit Problem, pp. 128, 129

N7: This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:
- Unit 3, Lesson 9, pp. 110-113
- Unit 3, Lesson 10, pp.114-115
- Unit 3, Unit Problem, pp. 128, 129

Mental Math strategies will strengthen student understanding of these specific curriculum outcomes. (Refer to pp. 115–158.)

SCO: **N6: Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as:**
- **adding from left to right**
- **taking one addend to the nearest multiple of ten and then compensating**
- **using doubles.**

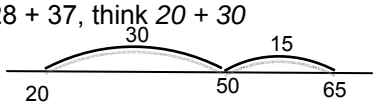
SCO: **N7: Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as:**
- **taking the subtrahend to the nearest multiple of ten and then compensating**
- **thinking of addition**
- **using doubles.**

[C, ME, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Ensure students recognize that mental math is an approach that they should use daily whenever they are required to calculate to solve a problem.
- Require that students know addition and subtraction facts to 18 with automaticity as described in outcome N10.
- Review "making a ten" with students. For example, for 28 + 4, one might think 28 and 2 make 30, and 2 more is 32. This can be extended to 2-digit numbers. For example, for 38 + 24, 38 plus 20 is 58, and 2 more is 60, plus 2 is 62.
- Have students add two 2-digit numbers using the front-end approach, and explain their thinking. For example, 24 + 31. Students might say, "20 and 30 makes 50. 54, 55 - the answer is 55."
- Relate addition to subtraction regularly, so students are better able to use this understanding to solve addition and subtraction problems and to check their work. Use missing addends to encourage this.
- Have students explain thinking using open number lines. For example, to solve 28 + 37, think *20 + 30 + 15* or *37 + 10 + 10 + 3 + 5* etc.



Suggested Activities

- Provide a set of computation practice items and ask students to circle those that they would like to solve mentally and describe the strategy they would use.
- Have students explain how they would use a calculator to solve 2-digit addition questions, such as 34 + ☐ = 69 or 39 + ☐ = 64.
- Have the student make a list of calculations involving 2-digit numbers which would be quicker to do mentally than on paper or with a calculator.
- Present calculations, such as the following, orally (or on an overhead), and ask the student to write only the answer. Allow only a few seconds for each question (e.g., 300 + 600, 200 - 40, 200 + 80 + 30, 220 - 40).
- Ask: How many different ways can you add 19 to 63 in your head?
- Ask: How many different ways can you subtract 19 from 63 in your head? Which way was easiest?
- Have the student list the doubles facts that might help him/her solve expressions such as 28 + 29 and 40 - 20 or 57 - 29.

SCO: **N6: Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as:**
- **adding from left to right**
- **taking one addend to the nearest multiple of ten and then compensating**
- **using doubles.**

SCO: **N7: Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as:**
- **taking the subtrahend to the nearest multiple of ten and then compensating**
- **thinking of addition**
- **using doubles.**

[C, ME, PS, R, V]

Assessment Strategies

- Ask the student to add mentally as you draw numbers from a bag, and to stop you when the sum has passed 40.
- Tell the student that to subtract 7 from 51, Jon said that he would rather subtract 6 from 50. Ask him/her if this works and why.
- Ask the student to describe a strategy for solving 48 - 9 (or 76 + 11) mentally using models, numbers, words, or pictures.
- Give the student this problem: 62 + 63 and say: Dave said he could use the doubles plus/minus one strategy to solve this problem. Explain Dave's thinking.
- Have students explain what is wrong with Lisa's method to solve 45 – 26. "Lisa said, 45 - 25 = 20 and 20 + 1 = 21.The answer is 21."
- Ask students how to solve 59 + 13 mentally and explain their thinking.

SCO: **N8: Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context.**
[C, ME, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Two	Grade Three	Grade Four
	N8 Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context.	

Elaboration

When asked to estimate, students often try to do the exact computation and then "round" their answer to produce an "estimate" that they think their teacher is looking for. Students need to understand that estimation is a valuable and useful "life skill", one that is used on a daily basis by many people.

Estimates can be very broad and general, or they can be quite close to the actual answer. It all depends on the reason for estimating in the first place, and these reasons can vary in context and according to the needs of the individual at the time.

In mathematics, it is essential that estimation strategies are used by students before attempting pencil/paper or calculator computations to help them determine whether or not their answers are reasonable. When teaching estimation strategies, it is important to use words and phrases such as *about, almost, between, approximately, a little more/less than, close to* and *near.*

Estimation strategies include *rounding to the nearest multiple of ten or 100*, *front-end estimation* or a combination of the two. It should also be noted that, in some cases, it can be just as easy and appropriate to get the actual answer as it is to estimate. This is often the case when employing a front-end strategy.

For any estimation, students need to consider the numbers and the operation involved to determine the best strategy. For example, it would be best to round both numbers up or down to get the answer to 84 – 27 but to add 84 +27 it would be better to round one up and one down. Exploring the proximity of the estimates to the exact answers and making comparisons to the numbers and operation used will enable students to become more efficient in their ability to estimate.

This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:

- Unit 3, Lesson 4, pp. 93–95
- Unit 3, Lesson 5, pp. 96-99
- Unit 3, Lesson 8, pp. 107-109
- Unit 3, Lesson 9, pp. 110-113

Mental Math strategies will strengthen student understanding of this specific curriculum outcome. (Refer to pp. 115–158)

SCO: **N8: Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context.**
[C, ME, PS, R]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Estimate the solution for a given story problem involving the sum of two 2-digit numerals, e.g., to estimate the sum of 43 + 56, use 40 + 50; the sum is close to 90.
- Estimate the solution for a given story problem involving the difference of two 2-digit numerals, e.g., to estimate the difference of 56 - 23, use 50 - 20; the difference is close to 30.

SCO: **N8: Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context.**
[C, ME, PS, R]

Instructional Strategies

Consider the following strategies when planning lessons:

- Ensure students recognize that estimation should be used daily whenever they are required to solve a problem, make predictions, or check answers.
- Use base ten blocks or a hundreds chart to help students as they begin estimating. For example, if the student thinks of the estimate of a two-digit number as a set of rods only, he/she might see that 37 (3 rods and 7 ones) is closer to 4 rods than to 3 rods. Eventually, students should realize that estimating can easily be performed without the base ten blocks.
- Help students to understand what it means to "round" numbers. Use number line models with multiples of 10 as increments (30, 40, 50, …; 240, 250, 260 etc.). Ask students to place other numbers on these lines and indicate which "ten" is closest.
- Introduce ***rounding*** in addition and subtraction. This strategy involves rounding each number to the nearest ten or multiple of ten, and then adding or subtracting the rounded numbers. Allow students to jot down the rounded numbers if they need to and then do the computation mentally. For now, avoid numbers which involve 5 or 50 in the rounding procedure.
- Introduce a ***front-end*** strategy for addition and subtraction. This strategy involves combining or finding the difference between only the highest place values to get a "ball park" estimate. Such estimates are adequate in many situations. For example, to estimate the sum of 43 + 54, think, "40 + 50 is 90." To estimate the difference between two numbers such as 534 - 254, think, "500 minus 200 is 300."

Suggested Activities

- Have students use estimation in story problem situations such as:
  - Tali baked 49 cookies and Miranda baked 58. Do they have enough to feed the hundred parents coming to Math Night?
- Tell the student that to estimate the sum of 36 and 29, Jake said, "30 and 20 are 50, and 6 and 9 are more than 10, so the answer is more than 60, but less than 70." Ask him/her to describe Jake's thinking.
- Play "A Fast Ten" with students. Students turn over two playing cards (a deck of cards numbered 1-9 only) to build a two-digit number. The student who determines which multiple of ten that number is closest to gets the cards. This game could be extended to add or subtract estimates of two pairs of cards.
- Tell the student that the sum of two numbers has been estimated to be about 120. Ask students to list four possible pairs of numbers that might have been added.

SCO: **N8: Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context.**
[C, ME, PS, R]

Assessment Strategies

- Ask students to identify situations where an exact answer would be required and those in which an estimate would be sufficient.
- Have the student explain two different ways to estimate the difference in 54 - 28.
- Tell the students that a number between 30 and 40 is added to a number between 40 and 50. Ask: What might be a good estimate for the answer? Why?
- Tell the student that José thinks she will need about 33 hot dogs for her party. Ask: If the hot dogs come in packages of ten, how many packages should José buy? Explain your answer.
- Tell the student that Jason knew there were 32 members in his Karate Club and about 28 members in the club in the neighbouring town. When asked to estimate the number of name tags to make for members of both clubs, Jason said, “I think I should make 60.” Ask: How do you think Jason estimated? Was it a good estimate?
- Have the student toss two dice and create a 2-digit number. Ask the student to estimate of how much should be added to each number to get a sum of about 100 or how much could be subtracted to get a difference of about 10?
- Tell the student that 4☐ + ☐8 is about 70. Ask what digits might go in the blanks.
- Ask the student which of the following solutions is close to 150, if rounding is the strategy used, and explain why.
  92 + 37 69 + 82 77 + 87
- Show the student the number of sports cards in James' collection.
  Baseball: 48 Football: 19 Hockey: 84
  Ask the student to estimate the total number of cards in the collection and to describe the strategy used.

SCO: **N9: Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2 and 3-digit numerals) by:**
- **using personal strategies for adding and subtracting with and without the support of manipulatives**
- **creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially and symbolically.**

[C, CN, ME, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Scope and Sequence

Grade Two	Grade Three	Grade Four
N9: Demonstrate an understanding of addition (limited to 1 and 2-digit numerals) with answers to 100 and the corresponding subtraction by: • using personal strategies for adding and subtracting with and without the support of manipulatives • creating and solving problems that involve addition and subtraction • explaining that the order in which numbers are added does not affect the sum • explaining that the order in which numbers are subtracted may affect the difference.	**N9:** Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2 and 3-digit numerals) by: • using personal strategies for adding and subtracting with and without the support manipulatives • creating and solving problems that involve addition and subtraction concretely, pictorially and symbolically	**N3** Demonstrate an understanding of addition of numbers with sums to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals) by • using personal strategies for adding and subtracting • estimating sums and differences • solving problems involving addition and subtraction.

Elaboration

Students are expected to apply what they know about the addition and subtraction of single digit numbers and the meanings of those operations to 2- and 3-digit numbers. As their understanding of these processes and of place value deepens, they will begin to record their work in ways that make sense to them. One of these ways might look very much like the traditional right to left, borrow/carry algorithm, but there are other methods that are just as acceptable. It is important for teachers to accept all of these personal procedures if they reflect understanding and correct reasoning. By looking at addition and subtraction conceptually, rather than procedurally, many students will have no more difficulty with problems involving regrouping than with those that do not. A traditional algorithm, once it is understood, should be considered as one more strategy to put in the class "tool box" of methods. Reinforce the idea that, like other strategies, it may be more useful in some instances than in others. Present problems in which a mental strategy is much more useful, such as 504 – 498. Discuss which method seems more reasonable to use.

In Grade 3, students are expected to be able to interpret word problems involving missing addends, minuends and subtrahends as well as missing sums, and write number sentences that represent these story situations. They are also expected to create their own word problems given an addition or subtraction number sentence.

SCO: **N9: Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2 and 3-digit numerals) by:**
- **using personal strategies for adding and subtracting with and without the support of manipulatives**
- **creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially and symbolically.**

[C, CN, ME, PS, R]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:
- Model the addition of two or more given numbers using concrete or visual representations and record the process symbolically.
- Model the subtraction of two given numbers using concrete or visual representations and record the process symbolically.
- Create an addition or subtraction story problem for a given solution.
- Determine the sum of two given numbers using a personal strategy, e.g., for 326 + 48, record 300 + 60 + 14.
- Determine the difference of two given numbers using a personal strategy, e.g., for 127 – 38, record 38 + 2 + 80 + 7 or 127 – 20 – 10 – 8.
- Solve a given problem involving the sum or difference of two given numbers.

This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:
- Unit 3, Lesson 1, pp. 82–85
- Unit 3, Lesson 2, pp. 86–88
- Unit 3, Lesson 5, pp. 96–99
- Unit 3, Lesson 6, pp. 100, 101
- Unit 3, Lesson 7, pp. 102–105
- Unit 3, Lesson 9, pp. 110–113
- Unit 3, Lesson 10, pp. 114, 115
- Unit 3, Lesson 11, pp. 116–119
- Unit 3, Lesson 12, pp. 120–123
- Unit 3, Lesson 13, pp. 124, 125
- Unit 3, Unit Problem, pp. 128, 129

Mental Math strategies will strengthen student understanding of this specific curriculum outcome. (Refer to pp. 115–158.)

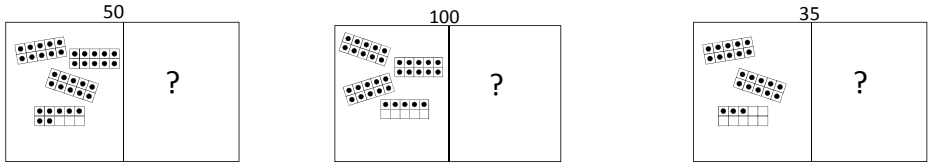
SCO: **N9: Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2 and 3-digit numerals) by:**
- **using personal strategies for adding and subtracting with and without the support of manipulatives**
- **creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially and symbolically.**

[C, CN, ME, PS, R]

Instructional Strategies

Consider the following strategies when planning lessons:

- Create sheets or overhead transparencies containing completely and partially filled ten-frames representing one part of a target number. Students apply strategies that make sense to them to determine the missing part. Since this is essentially a subtractive problem, many students will use "think addition" to work their way up to the target number.



- Help students extend their knowledge of addition and subtraction facts to addition and subtraction of 2- and 3-digit numbers with only one non-zero digit in each number. For example, 6 + 7 = 13, so 60 + 70 = 13 tens or 130; and 600 + 700 = 13 hundreds or 1300.

Suggested Activities

- Tell students that Fran had 25 cents. She spent 16 cents. How much change does she get back? Encourage the students to explain how they go about solving the problem; for instance: *"16 and 4 more are 20, plus 5 is 25. She gets 9 cents change*." Or, *"16 cents and 10 cents are 26 cents, so she gets only 9 cents back."*
- Set up a "store" within the classroom and have the students take turns being the cashier. Model for them how to "count on" when making change.
- Tell students that Paul has 78 cents, and his brother has 92 cents. How much more money does Paul's brother have?
- Use the following digits to create two, 2-digit numbers that have the greatest possible sum: 2, 3, 4, 5. Use the same digits to create the greatest difference. The diagrams below may be helpful to some students.

SCO: **N9: Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2 and 3-digit numerals) by:**
- **using personal strategies for adding and subtracting with and without the support of manipulatives**
- **creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially and symbolically.**

[C, CN, ME, PS, R]

Assessment Strategies

- Tell a student that someone told you that you do not have to learn to subtract if you know how to add. Ask: Do you agree? Why or why not?
- Observe the student as he/she adds 125 and 134 or subtracts 134 from 217 using base-ten materials.
- Provide the following addition or subtraction calculations for the student to complete. Ask the student to explain his/her strategy.
  - 38 + 97
  - 98 - 44
  - 400 - 255
- Show the student a number of addition and subtraction questions, some of which require regrouping and some of which do not. Ask him/her to circle the questions they could do quickly and explain why they made those choices.
- Have the student explain in writing why someone might first subtract 30 from 67 in order to calculate 674 - 26. Ask what would be done next.
- Display the numbers 124 and 75 with base ten blocks. Ask the student to describe the addition process as he/she manipulates the models.
- Tell the student that Sue was to add 36 + 59 and said, “36, 96, 95.” Have the student explain Sue’s thinking.
- Ask why someone might find it easier to subtract 123 - 99 than 123 - 87.
- Ask the student to prepare a display showing a variety of ways to calculate 57 - 18, indicating his/her preference and the reason for it.
- Ask the students to use a sales flyer to create some problems for his/her classmates. Have them record both problems and solutions.
- Using the numbers 62 and 25, create a subtraction problem that can be solved using addition. Solve.

N10: Apply mental mathematics strategies and number properties, such as:
- **using doubles**
- **making 10**
- **using the commutative property**
- **using the property of zero**
- **thinking addition for subtraction**

to recall basic addition facts to 18 and related subtraction facts.
[C, CN, ME, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Two	Grade Three	Grade Four
N10 Apply mental mathematics strategies, such as: • using doubles • making 10 • one/two more; one/two less • building on a known double • *think addition* for subtraction to determine basic addition facts to 18 and related subtraction facts.	**N10** Apply mental mathematics strategies and number properties, such as: • using doubles; making 10; • using the commutative property; • using the property of zero; • thinking addition for subtraction to determine answers for basic addition facts and to 18 related subtraction facts.	**N5** Describe and apply mental mathematics strategies such as: • skip counting from a known fact • using doubling or halving • using doubling or halving and adding or subtracting one more group • using patterns in the 9s facts • using repeated doubling to determine basic multiplication facts to 9 × 9 and related division facts.

Elaboration

Students develop and use thinking strategies to recall answers to basic facts, and are the foundation for the development of other mental calculation strategies. In Grade 3, they include using the *commutative* or "turn-around" property, the *zero property*, *doubles, make ten*, and *think addition for subtraction*. It is expected that by the end of Grade 3 students will have achieved fluency with the addition and subtraction facts to 18. It is important to provide regular and frequent opportunities to introduce, develop, reinforce and practice thinking strategies using games and meaningful contexts as much as possible. When recall of facts is automatic, students are no longer using strategies to retrieve them from memory.

Subtraction facts have traditionally been more difficult for students to master than addition. This is especially true when children have been taught subtraction through a "count three times" approach; for 9 – 5, count out 9, count off 5, count what's left. Research suggests that anyone who has mastered subtraction facts has not found this count-count-count approach very helpful. In fact, students learn very few, if any, subtraction facts without first mastering the corresponding addition facts. As students master groups of addition facts, it is appropriate to introduce the related subtraction facts as "think addition" so that they can apply their knowledge in a different way. For example, if students have mastered the "make ten" facts, they should be presented with subtraction facts such as 15 – 8 and encouraged to think, "8 plus what equals 15?"

In the *Grade 2 Mathematics Curriculum*, it is expected that Grade 2 students will be able to recall addition facts to 18 and the related subtraction facts. Mastery of basic facts (recall within 3 seconds) is expected at the end of that grade level.

N10: Apply mental mathematics strategies and number properties, such as:
- **using doubles**
- **making 10**
- **using the commutative property**
- **using the property of zero**
- **thinking addition for subtraction**

to recall basic addition facts to 18 and related subtraction facts.
[C, CN, ME, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Describe a mental mathematics strategy that could be used to determine a given basic fact, such as:
  - doubles, e.g., for 6 + 8, think 7 + 7
  - doubles plus one, e.g., for 6 + 7, think 6 + 6 + 1
  - doubles take away one, e.g., for 6 + 7, think 7 + 7 – 1
  - doubles plus two, e.g., for 6 + 8, think 6 + 6 + 2
  - doubles take away two, e.g., for 6 + 8, think 8 + 8 – 2
  - making 10, e.g., for 6 + 8, think 6 + 4 + 4 or 8 + 2 + 4
  - commutative property, e.g., for 3 + 9, think 9 + 3
  - addition to subtraction, e.g., for 13 – 7, think 7 + ? = 13.
- Provide a rule for determining answers for adding and subtracting zero.
- Recall basic addition facts to 18 and related subtraction facts to solve problems.

This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:
- Unit 3, Lesson 1, pp. 82–85
- Unit 3, Lesson 2, pp. 86–88

Mental Math strategies will strengthen student understanding of this specific curriculum outcome. (Refer to pp. 115–158.)

N10: Apply mental mathematics strategies and number properties, such as:
- **using doubles**
- **making 10**
- **using the commutative property**
- **using the property of zero**
- **thinking addition for subtraction**

to recall basic addition facts to 18 and related subtraction facts.
[C, CN, ME, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students with a variety of models to practice and help visualize the basic facts. Students can also use other strategies, such as drawing pictures and role playing various sums and differences in a problem solving context.
- Ensure students have the opportunity to share their strategies with others.
- Use practice and drills that are short with immediate feedback over an extended period of time. Practice items should be presented visually as well as orally.
- Use the addition table to explore patterns and help students identify the facts which they have mastered. The known facts can be shaded in with the goal of having the entire table shaded.

Suggested Activities

- Have students roll 2 number cubes (dice). They either add or subtract these values. For example: if a 5 and a 2 are rolled, they will work with either 5, 2, 7 or 5, 2, 3. Ask the students to make up a subtraction story based on these numbers, and write the corresponding number sentence. (Note: Ten-sided dice work well, as do prepared number cards.) If students are proficient with the addition facts, adapt this activity so that they must focus on subtraction.
- Play "Missing Part" game for two students to practice their fact recall. One student places a number of counters in front of them (e.g., 16) and then the student covers some of the counters with their hand. The other student must determine how many counters are hidden as quickly as possible.
- Use a "Looping Activity" where every student is given a card with a basic fact number sentence in which one of the numbers is missing; written as "Who has…?" (e.g., Who has 5 + __ = 11). The card also has the answer from someone else's card written as "I have…" Students take turns reading their cards in sequence by responding when their card answers someone else's.

"I have 12." "Who has 7 + 0?"	"I have 7." "Who has 8 + 6?"

- Provide students with cards with a subtraction number sentence (e.g., 13 – 7 =). Have students rewrite the sentence as a missing addend number sentence (e.g., 7 + □ = 13) and solve it.

N10: Apply mental mathematics strategies and number properties, such as:
- **using doubles**
- **making 10**
- **using the commutative property**
- **using the property of zero**
- **thinking addition for subtraction**

to recall basic addition facts to 18 and related subtraction facts.
[C, CN, ME, R, V]

Assessment Strategies

- Ask the student to explain, using a model, why he/she knows that 3 + 4 has to equal 4 + 3 even before finding the total.
- Ask: Why is it easy to add or subtract 0 to numbers? (interview student or write response in a journal)
- Ask: Why is it easy to add the numbers 5 + 5 + 6 + 4 + 8 + 2?
- Ask: How can you use addition to solve 16 - 7?
- Have students write all of the number sentences they can for a provided sum or difference (e.g., 6 as a difference: 6 - 0, 7 - 1, 8 - 2, 9 - 3, 10 - 4, 11 - 5, 12 - 6, 13 - 7, 14 - 8, 15 - 9).
- Ask: How does knowing 8 + 8 = 16, help you solve 58 + 8?
- Ask: How does knowing 13 – 6 = 7, help you solve 53 – 6?
- Ask students to describe as many different ways as possible to solve 8 + 9.

SCO: **N11: Demonstrate an understanding of multiplication to products of 36 with single digit factors by:**
- **representing and explaining multiplication using equal grouping and arrays**
- **creating and solving problems in context that involve multiplication**
- **modelling multiplication using concrete and visual representations, and recording the process symbolically**
- **relating multiplication to repeated addition**
- **relating multiplication to division.**

[C, CN, PS, R]

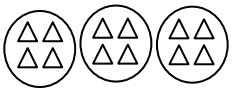
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

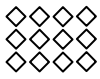
Grade Two	Grade Three	Grade Four
	N11 Demonstrate an understanding of multiplication to products of 36 with single digit factors by: • representing and explaining multiplication using equal grouping and arrays • creating and solving problems in context that involve multiplication • modelling multiplication using concrete and visual representations, and recording the process symbolically • relating multiplication to repeated addition • relating multiplication to division.	**N4** Explain the properties of 0 and 1 for multiplication and the property of 1 for division. **N5** Describe and apply mental mathematics strategies to determine basic multiplication facts to 9x9 and related division facts. **N6** Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems.

Elaboration

In multiplication, the numbers being multiplied are called *factors* and the answer is the *product*. It is important that students understand the "groups of" meaning for multiplication and to recognize that the product may be determined in a number of ways, including:
- repeated addition: for example, 4 + 4 + 4 can be written as 3 x 4 ;
- making sets of equal groups: for example, students can create equal-sized groups with actual items;
- the total number in an array.



3 groups of 4 make 12 (3 × 4 = 12)



3 rows of 4 make 12 (3 × 4 = 12)

Repeated addition is a helpful introduction to understanding multiplication, but students need to move beyond this strategy as their knowledge develops and other models become more efficient and appropriate. The array is a powerful tool to illustrate the order or commutative property in multiplication.

4 × 2 = 2 × 4

It is important that students come to understand the inverse relationship of multiplication and division and it is recommended that teachers "combine multiplication and division soon after multiplication has been introduced in order to help students see how they are related" (Van de Walle & Lovin, vol. 2, 2006, p. 60).

SCO: **N11: Demonstrate an understanding of multiplication to products of 36 with single digit factors by:**
- **representing and explaining multiplication using equal grouping and arrays**
- **creating and solving problems in context that involve multiplication**
- **modelling multiplication using concrete and visual representations, and recording the process symbolically**
- **relating multiplication to repeated addition**
- **relating multiplication to division.**

[C, CN, PS, R]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:
- Identify events from experience that can be described as multiplication.
- Represent a given story problem (orally, shared reading, written) using manipulatives or diagrams and record in a number sentence.
- Represent a given multiplication expression as repeated addition.
- Represent a given repeated addition as multiplication.
- Create and illustrate a story problem for a given number sentence, e.g., given 2 × 3, create and illustrate a story problem.
- Represent, concretely or pictorially, equal groups for a given number sentence.
- Represent a given multiplication expression using an array.
- Create an array to model the commutative property of multiplication.
- Relate multiplication to division by using arrays and writing related number sentences.
- Solve a given problem in context involving multiplication.

Note: It is not intended that students automatically recall the basic multiplication facts in grade 3, though many students will have mastered some by the end of the year. Teachers must help students become familiar with flexible ways to think about and work with numbers so that *products* can be determined. Thinking strategies should be introduced, practised, and reinforced on a regular basis in the classroom.

This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:
- Unit 8, Lesson 1, pp. 268–271
- Unit 8, Lesson 2, pp. 273-275
- Unit 8, Lesson 3, pp. 276-279
- Unit 8, Lesson 4, pp. 280-282
- Unit 8, Lesson 8, pp. 294-296
- Unit 8, Lesson 9, pp. 297–299
- Unit 8, Lesson 10, pp. 300, 301
- Unit 8, Unit Problem, pp. 304, 305

Mental Math strategies will strengthen student understanding of this specific curriculum outcome. (Refer to pp. 115–158.)

X	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

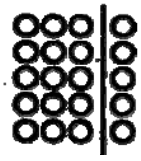
SCO: **N11: Demonstrate an understanding of multiplication to products of 36 with single digit factors by:**
- **representing and explaining multiplication using equal grouping and arrays**
- **creating and solving problems in context that involve multiplication**
- **modelling multiplication using concrete and visual representations, and recording the process symbolically**
- **relating multiplication to repeated addition**
- **relating multiplication to division.**

[C, CN, PS, R]

Instructional Strategies

Consider the following strategies when planning lessons:

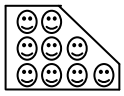
- Provide contexts involving multiplicative situations. For example, packaged foods - cans of pop (2 x 3), juice packs (1 x 3), hamburger buns (2 x 4).
- Play “Broken Calculator” to relate multiplication and addition. Students use the constant feature of the calculator to find various products without using the multiplication key. Challenge the students to model their product using counters.
- Explore the distributive property by displaying a 5 x 4 array of objects and place a ruler on the line as shown. Ask how this shows that 5 x 4 = 5 x 3 + 5 x 1. Then ask the student to move the ruler to show another way to find 5 x 4 and explain his/her thinking, and/or provide the student with a piece of paper upon which an array (5 x 5) has been drawn. Ask him/her to fold the paper to show different ways the multiplication can be expressed.



- Ask students to show multiple representations of a given multiplication fact.
- Ask the student to draw pictures showing various situations in which multiplication might be used.
- Give students many opportunities to solve missing factor problems. Example: It takes 4 toothpicks to build a square. How many of the same sized squares can be built with 16 toothpicks?

Suggested Activities

- Using counters, have students build as many arrays as possible for a given number, and write its corresponding equation.
- Create arrays on cards and cut off a corner so that some counters are missing but the intended number of rows and columns remains clear. Show cards to students and ask them how many counters the card had initially if all the rows and columns had the same number of counters.



- Have students investigate what happens when you multiply a number by 0, 1, 2, 3, 4, 5. Is there a pattern in the products?
- Invite a group of students to act out a skit modeling either a multiplication or division situation. Ask other students to suggest the number sentence being dramatized.
- Show students an array and have them provide the related multiplication and division sentences.
- Have the students create a realistic story problem to go with a given number sentence (e.g., 4 × 5) or describe a situation for which you might have to find the answer to 5 x 3.
- Ask students how they might use a hundred chart or a number line to find the product of 5 x 4.

SCO: **N11: Demonstrate an understanding of multiplication to products of 36 with single digit factors by:**
- **representing and explaining multiplication using equal grouping and arrays**
- **creating and solving problems in context that involve multiplication**
- **modelling multiplication using concrete and visual representations, and recording the process symbolically**
- **relating multiplication to repeated addition**
- **relating multiplication to division.**

[C, CN, PS, R]

Assessment Strategies

- Have students represent story problems using models or diagrams and record the corresponding number sentences.
- Have students create a real-life story problem that involve multiplication and solve it.
- Have students represent a given multiplication sentence, such as 5 x 3, using an array.
- Have students represent a given repeated addition as multiplication.
- Have students represent equal groups for a given number sentence concretely or pictorially.
- Ask students to model as many arrays as possible with 16 counters. Have them write the related multiplication and division facts for each array.
- Solve comparison problems such as: One tower is 8 blocks high. The other is 2 blocks high. How many times higher is the first tower?
- Ask students to put ten tiles, into rows of five. Ask how many rows are there.
- Ask students to model ten stamps in two rows. Ask how many stamps are in each row.

SCO: **N12: Demonstrate an understanding of division by:**
- **representing and explaining division using equal sharing and equal grouping**
- **creating and solving problems in context that involve equal sharing and equal grouping**
- **modeling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically**
- **relating division to repeated subtraction**
- **relating division to multiplication.**
  (limited to division related to multiplication facts up to products of 36 with single digit factors)

[C, CN, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Scope and Sequence

Grade Two	Grade Three	Grade Four
	N12 Demonstrate an understanding of division by: • representing and explaining division using equal sharing and equal grouping • creating and solving problems in context that involve equal sharing and equal grouping • modeling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically • relating division to repeated subtraction • relating division to multiplication. (limited to division related to multiplication facts up to products of 36 with single digit factors)	**N4** Explain the properties of 0 and 1 for multiplication and the property of 1 for division. **N5** Describe and apply mental mathematics strategies to determine basic multiplication facts to 9x9 and related division facts. **N7** Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems.

Elaboration

In division, the number representing the quantity we're starting with is called the *dividend.* The size or number of groups that this amount is being separated into is called the *divisor*. The final answer in a division computation is called the *quotient*. It is important that students see that division can mean:
- equal sharing: 16 ÷ 4 = 4 is the amount each person gets if 16 items are shared equally among 4 people
- equal grouping: 16 ÷ 4 = 4 is the number of equal groups of 4 you can make with 16 items
- repeated subtraction**:** 16 ÷ 4 = 4 is the number of times you can subtract 4 from 16 before you reach zero.

Multiplication and division are inverse operations so as students master clusters of multiplication facts, it is appropriate to have them learn the corresponding division facts as "think multiplication."
Problems should be worded in such a way as to develop this understanding. For example:

The ferry can hold 8 trucks. How many trips will it have to make to carry all 64 trucks across the river?

SCO: **N12: Demonstrate an understanding of division by:**
- **representing and explaining division using equal sharing and equal grouping**
- **creating and solving problems in context that involve equal sharing and equal grouping**
- **modeling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically**
- **relating division to repeated subtraction**
- **relating division to multiplication.**

(limited to division related to multiplication facts up to products of 36 with single digit factors)

[C, CN, PS, R]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Identify events from experience that can be described as equal sharing.
- Identify events from experience that can be described as equal grouping.
- Illustrate, with counters or a diagram, a given story problem involving equal sharing, presented orally or through shared reading and solve the problem.
- Illustrate, with counters or a diagram, a given story problem involving equal grouping, presented orally or through shared reading, and solve the problem.
- Listen to a story problem, represent the numbers using manipulatives or a sketch and record the problem with a number sentence.
- Create and illustrate with counters, a story problem for a given number sentence, e.g., given 6 ÷ 3, create and illustrate a story problem.
- Represent a given division expression as repeated subtraction.
- Represent a given repeated subtraction as a division expression.
- Relate division to multiplication by using arrays and writing related number sentences.

This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:

- Unit 8, Lesson 5, pp. 283–286
- Unit 8, Lesson 6, pp. 287-289
- Unit 8, Lesson 7, pp. 290-293
- Unit 8, Lesson 8, pp. 294-296
- Unit 8, Lesson 9, pp. 297–299
- Unit 8, Unit Problem, pp. 304, 305

Mental Math strategies will strengthen student understanding of this specific curriculum outcome. (Refer to pp. 115–158.)

SCO: **N12: Demonstrate an understanding of division by:**

- **representing and explaining division using equal sharing and equal grouping**
- **creating and solving problems in context that involve equal sharing and equal grouping**
- **modeling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically**
- **relating division to repeated subtraction**
- **relating division to multiplication.**
  (limited to division related to multiplication facts up to products of 36 with single digit factors)

[C, CN, PS, R]

Instructional Strategies

Consider the following strategies when planning lessons:

- Use concrete materials to help students understand the relationship between the meanings of division. Demonstrate that, in sharing 12 items among 3 people, for example, the actual giving of 1 item to each person is the same as creating a group of 3. In other words, sharing among 3 people is equivalent to finding how many groups of 3 can be formed.
- Play “Broken Calculator”. Students work in groups to find ways to use the calculator to solve division exercises without using the ÷ key.
- Provide problem solving situations in which solutions can be found using either multiplication or division.
- Explore various types of division problems: equal share, equal groups, and comparison.

Suggested Activities

- Provide the student with some toothpicks and ask him/her to use 12 to make 4 identical shapes. Ask the student what division and multiplication sentences could describe the creation of the shapes.
- Set up a 3 × 4 array and ask the student to give two multiplication and two division sentences that describe it by looking at the array from different perspectives.
- Invite a group of students to act out a skit modeling either a multiplication or division situation. Ask other students to guess the number sentence being dramatized.
- Ask the student to write problems in which one has to multiply or divide to find the answer. Have him/her illustrate the solutions and describe the multiplication/division relationship.
- Ask students to skip count backwards by 2’s, 4’s, 5’s from a given multiple of that number.
- Ask students to solve a division problem in as many ways as possible (including multiplication).

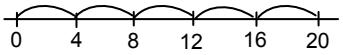
SCO: **N12: Demonstrate an understanding of division by:**

- **representing and explaining division using equal sharing and equal grouping**
- **creating and solving problems in context that involve equal sharing and equal grouping**
- **modeling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically**
- **relating division to repeated subtraction**
- **relating division to multiplication.**
  (limited to division related to multiplication facts up to products of 36 with single digit factors)

[C, CN, PS, R]

Assessment Strategies

- Ask the student to write a division story about 25 ÷ 5.
- Have the student draw a picture or use counters to show what 12 ÷ 3 means.
- Ask the student to describe a situation for which you might have to find the answer to 15 ÷ 3.
- Ask the student to draw pictures showing various situations in which either multiplication or division might be used.
- Show the following number line. Ask the student to record what multiplication and division sentences it might be showing.



- Show the student the multiplication sentence 5 × 4 = 20. Ask the student to write related division sentences.
- Tell the students that amusement park rides are priced as follows:
  - \$1 for the Ferris wheel,
  - \$2 for the Bullet, and
  - \$3 for the Twister.

  Ask: How many rides, and of which kind, can you have for \$13? Are there other possibilities?
- Show students an array of up to 25 counters. Ask students which multiplication and division family is shown by the array.

SCO: **N13: Demonstrate an understanding of fractions by:**
- **explaining that a fraction represents a part of a whole**
- **describing situations in which fractions are used**
- **comparing fractions of the same whole with like denominators.**

[C, CN, ME, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Two	Grade Three	Grade Four
	N13 Demonstrate an understanding of fractions by: • explaining that a fraction represents a part of a whole; • describing situations in which fractions are used; • comparing fractions of the same whole with like denominators.	**N8** Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to: name and record fractions for the parts of a whole or a set; compare and order fractions; model and explain that for different wholes, two identical fractions may not represent the same quantity; provide examples of where fractions are used.

Elaboration

The first goal in the development of fractions should be to help students construct the idea of *fractional parts of the whole* – the parts that result when the whole has been partitioned into *fair shares* (equal-sized portions). Sharing tasks are good places to begin the development of fractions. (Van de Walle & Lovin, vol. 1, 2006, p. 252). Early work with fractions focuses on two big ideas:

- A fraction is a number that describes a relationship between a part (represented by the numerator) and a whole (represented by the denominator) (Small, 2008, p. 196).
- A fraction can be read/written/modeled in different ways but still has the same meaning.

$$\textit{one fourth} = \textit{one quarter} = \frac{1}{4} = \textit{one of every four parts} = 25¢, \ldots$$

Representing fraction concepts with a variety of materials and drawings and within real life situations is essential to the development of fractional understanding so that fractions don't simply become associated with pieces of a chocolate bar or pizza. It is important that students develop visual images for fractions and be able to tell "about how much" a particular fraction represents. Is it close to almost nothing (zero), half, or one whole thing?

In Grade 3, the focus is on students developing a beginning understanding of fractions less than one, relating fractions to authentic situations, and comparing fractions with the same denominator.

This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:

- Unit 5, Lesson 1, pp. 182–184
- Unit 5, Lesson 2, pp. 185–188
- Unit 5, Lesson 3, pp. 189–192
- Unit 5, Lesson 4, pp. 193–195
- Unit 5, Lesson 5, pp. 197–199
- Unit 5, Lesson 6, pp. 200, 201
- Unit 5, Unit Problem, pp. 204, 205

SCO: **N13: Demonstrate an understanding of fractions by:**
- **explaining that a fraction represents a part of a whole**
- **describing situations in which fractions are used**
- **comparing fractions of the same whole with like denominators.**

[C, CN, ME, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Identify common characteristics of a given set of fractions.
- Describe everyday situations where fractions are used.
- Cut or fold a whole into equal parts, or draw a whole in equal parts; demonstrate that the parts are equal and name the parts.
- Sort a given set of diagrams of regions into those that represent equal parts and those that do not, and explain the sorting.
- Represent a given fraction concretely or pictorially.
- Name and record the fraction represented by the shaded and non-shaded parts of a given region.
- Compare given fractions with the same denominator using models.
- Identify the numerator and denominator for a given fraction.
- Model and explain the meaning of numerator and denominator.

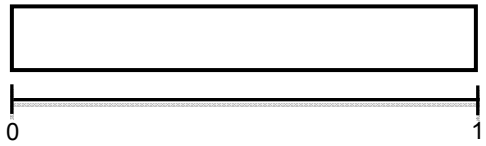
SCO: **N13: Demonstrate an understanding of fractions by:**
- **explaining that a fraction represents a part of a whole**
- **describing situations in which fractions are used**
- **comparing fractions of the same whole with like denominators.**

[C, CN, ME, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Have students explore various models for fractions: part of a region and part of a length.
- Ensure students develop an understanding that a fraction is not meaningful without knowing what the "whole" is.
- Provide students with rectangles and number lines that are the same length. Ask students to color half of a rectangle, and indicate where half is on the number line. Once students understand the concept of half, this activity could be extended to fourths (quarters) and thirds, etc.



Suggested Activities

- Ask students to fold strips of paper into equal parts (halves, fourths, eighths etc.)
- Show students three pictures of varying sizes of the same item, all items cut into the same number of pieces. Ask students which of the pieces they would like to have. Explain why they made that choice.
- Have students model a specific fraction using <u>at least</u> one of each of the pattern blocks. Ask students to rearrange the blocks and show the same fraction. How many different ways can this be done?
- Ask students to model a specific fraction using five pattern blocks. Draw their model on isometric grid paper and color the fractional part they have represented with their model.
- Have students model on a number line to 100 where $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}$ would be?

SCO: **N13: Demonstrate an understanding of fractions by:**
- **explaining that a fraction represents a part of a whole**
- **describing situations in which fractions are used**
- **comparing fractions of the same whole with like denominators.**

[C, CN, ME, R, V]

Assessment Strategies

- Ask students: “Is half a lot or a little?” Have them explain their thinking.
- Ask students to identify the numerator and denominator of a given fraction.
- Provide students with fractions with the same denominator and have them identify the larger (or smaller) and explain their reasoning using models.
- Ask students: “If you are really hungry and want a large piece of cake, would you cut the cake into thirds, fourths, or tenths?”
- Provide students with paper shapes. Have students cut or fold the shape into halves and/or fourths (quarters).
- Provide students with a five frame, and ask them to place a counter on $\frac{1}{5}$ of the squares.
- Have students place the following fractions on the number line below: $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}$.

PATTERNS AND RELATIONS

SCO: **PR1: Demonstrate an understanding of *increasing* patterns by describing**, **extending**, **comparing**, **and creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).**
SCO: **PR2: Demonstrate an understanding of *decreasing* patterns by describing**, **extending**, **comparing**, **and creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).**
[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Two	Grade Three	Grade Four
PR1 Demonstrate an understanding of repeating patterns (three to five elements) by: describing; extending; comparing; creating patterns using manipulatives, diagrams, sounds and actions. **PR2** Demonstrate an understanding of increasing patterns by: describing; reproducing; extending; creating patterns using manipulatives, diagrams, sounds and actions (numbers to 100).	**PR1** Demonstrate an understanding of increasing patterns; and **PR2** Demonstrate an understanding of decreasing patterns by: • describing • extending • comparing • creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).	**PR1** Identify and describe patterns found in tables and charts, including a multiplication chart. **PR2** Reproduce a pattern shown in a table or chart using concrete materials. **PR3** Represent and describe patterns and relationships using charts and tables to solve problems.

Elaboration

One of the key skills in learning mathematics is the ability to recognize, describe, and extend patterns, and to use them to solve problems. Number relationships, as well as concepts of equality, variable, and change are central to this work and are found in all strands of mathematics, including number, geometry, measurement and data. In grade 3, the work students do around “pattern rules” helps them recognize that patterns can be represented with numbers and with symbols, and this, in turn, leads to the development of algebraic thinking and the representation of this thinking with algebraic expressions.

Students will come to learn that a Pattern Rule has three aspects: starting point, direction and quantity.

PR1: This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:
- Unit 1, Lesson 1, pp. 6-8
- Unit 1, Lesson 2, pp. 9-11
- Unit 1, Lesson 3, pp. 12-14
- Unit 1, Lesson 4, pp. 15-17
- Unit 1, Lesson 5, pp. 18, 19
- Unit 1, Unit Problem, pp. 34, 35

PR2: This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:
- Unit 1, Lesson 6, pp. 21-24
- Unit 1, Lesson 7, pp. 25-27
- Unit 1, Lesson 8, pp. 28-31
- Unit 1, Unit Problem, pp. 34, 35

SCO: **PR1: Demonstrate an understanding of *increasing* patterns by describing**, **extending**, **comparing**, **and creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).**
SCO: **PR2: Demonstrate an understanding of *decreasing* patterns by describing**, **extending**, **comparing**, **and creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).**
[C, CN, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

PR1

- Describe a given increasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.
- Identify the pattern rule of a given increasing pattern and extend the pattern for the next three terms.
- Identify and explain errors in a given increasing pattern.
- Locate and describe various increasing patterns found on a hundred chart, such as horizontal, vertical and diagonal patterns.
- Compare numeric patterns of counting by 2s, 5s, 10s, 25s and 100s.
- Create a concrete, pictorial or symbolic representation of an increasing pattern for a given pattern rule.
- Create a concrete, pictorial or symbolic increasing pattern and describe the pattern rule.
- Solve a given problem using increasing patterns.
- Identify and describe increasing patterns in the environment.
- Identify and apply a pattern rule to determine missing elements for a given pattern.
- Describe the strategy used to determine missing elements in a given increasing pattern.

PR2

- Describe a given decreasing pattern by stating a pattern rule that includes the starting point and a description of how the pattern continues.
- Identify the pattern rule of a given decreasing pattern and extend the pattern for the next three terms.
- Identify and explain errors in a given decreasing pattern.
- Identify and describe various decreasing patterns found on a hundred chart, such as horizontal, vertical and diagonal patterns.
- Compare decreasing numeric patterns of counting backward by 2s, 5s, 10s, 25s and 100s.
- Create a concrete, pictorial or symbolic decreasing pattern for a given pattern rule.
- Create a concrete, pictorial or symbolic decreasing pattern and describe the pattern rule.
- Solve a given problem using decreasing patterns.
- Identify and describe decreasing patterns in the environment.
- Identify and apply a pattern rule to determine missing elements for a given pattern.
- Describe the strategy used to determine missing elements in a given decreasing pattern.

SCO: **PR1: Demonstrate an understanding of *increasing* patterns by describing**, **extending**, **comparing**, **and creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).**
SCO: **PR2: Demonstrate an understanding of *decreasing* patterns by describing**, **extending**, **comparing**, **and creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).**
[C, CN, PS, R, V]

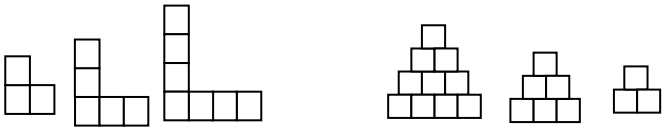
Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students with a variety of concrete and pictorial materials such as linking cubes, colour tiles or pattern blocks to create and extend increasing and decreasing patterns.
- Encourage students to discuss and write about how patterns increase or decrease, and how they might be extended.
- Invite students to identify the "pattern rules" of various increasing/decreasing patterns .
- Have students describe errors or missing elements within an increasing or decreasing pattern.
- Help students identify ways to represent the same pattern in a variety of ways including *concretely, pictorially, symbolically, orally, rhythmically,* and *physically.*

Suggested Activities

- Have students look for number patterns when exploring hundred charts to 1000 (*1-100, 101-200, 201-300*, etc.)
- Take students on a "Pattern Hunt" identifying increasing and decreasing patterns in their school environment. Have them use numbers, pictures and words to describe the patterns they discover.
- Provide students with the first 3 or 4 elements of a pattern. Have them use appropriate materials to extend and explain the pattern.



SCO: **PR1: Demonstrate an understanding of *increasing* patterns by describing**, **extending**, **comparing**, **and creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).**
SCO: **PR2: Demonstrate an understanding of *decreasing* patterns by describing**, **extending**, **comparing**, **and creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).**
[C, CN, PS, R, V]

Assessment Strategies

- Give students a diagram showing a square table with 4 chairs (one on each side). Tell students that if 2 tables were put together, it would seat 6. Ask: how many can we seat with 6 tables? 8? 10? What if we started with a table of 6? Have students explain their reasoning.
- Ask students to show you different ways these patterns could be extended.
  - 20, 40, ___, ___, ___
  - 1, 4, ___, ___
  - 1000, 500, ___, ____
- Tell students: "I am thinking of a pattern. I have landed on 50. What could I be counting by?" Accept any reasonable answer that includes an explanation.
- Ask the student to say a number that is 100 more (100 less, 10 more, 10 less) than a 2- or 3-digit number that is provided.
- Give students a pattern modeled with tiles and ask them to describe, recreate, and extend the pattern in another way.
- Have students identify the errors in the following patterns and correct them:
  - 3, 6, 9, 12, 15, 19, 21, 24, 28, 30
  - 40, 35, 29, 25, 20, 15, 10, 5

SCO: **PR3: Solve one-step addition and subtraction equations involving symbols representing an unknown number.**
[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Two	Grade Three	Grade Four
PR3 Demonstrate and explain the meaning of equality and inequality, concretely and pictorially. **PR4** Record equalities and inequalities symbolically, using the equal symbol or the not equal symbol.	**PR3** Solve one-step addition and subtraction equations involving symbols representing an unknown number.	**PR5** Express a given problem as an equation in which a symbol is used to represent an unknown number. **PR6** Solve one-step equations involving a symbol to represent an unknown number.

Elaboration

In Grade 2, students learned the concepts of equality and inequality and the meaning of the symbols = and ≠. This knowledge is extended in Grade 3 to solving equations that include symbols which represent *unknowns*. An equation is a mathematical statement that includes an equal sign and may have been called a "number sentence" in the earlier grades. It is important that students understand that unknowns can appear on either side of the equals sign, and that this sign indicates both sides of the equation are equal or *balanced.*

It is also important for students to recognize that the unknown value in an equation can be represented with a variety of symbols such as a square, a circle, a triangle, or a letter symbol. A variety of *combining* and *separating* story structures should be provided to help students understand that the unknown symbol can be in a different position in an equation depending on the context, but can have only one value. For example, *"Josh has some marbles and he bought 12 more. Now he has 33 marbles. How many marbles did he have at the start?"* This can be represented with an addition equation; △ + 12 = 33, or by a subtraction equation; 33 – 12 = △.

Problem contexts should vary to include the following equation forms:

a + b = △	(e.g., 6 + 3 = △)	or	(△ = a + b)
a + ○ = c	(e.g., 2 + ○ = 8)	or	(c = a + ○)
□ + b = c	(e.g., □ + 4 = 5)	or	(c = □ + b)
c - a = ◊	(e.g., 7 - 2 = ◊)	or	(◊ = c - a)
c - ▽ = b	(e.g., 4 - ▽ = 2)	or	(b = c - ▽)
☐ - a = b	(e.g., ☐ - 8 = 1)	or	(b = ☐ - a)

This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:

- Unit 3, Lesson 3, pp. 89–92
- Unit 3, Unit Problem, pp. 128, 129

SCO: **PR3: Solve one-step addition and subtraction equations involving symbols representing an unknown number.**
[C, CN, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Explain the purpose of the symbol, such as a triangle or a circle, in a given addition and in a given subtraction equation with one unknown.
- Create an addition or subtraction equation with one unknown to represent a given combination or separation action.
- Provide an alternative symbol for the unknown in a given addition or subtraction equation.
- Solve a given addition or subtraction equation that represents combining or separating actions with one unknown using manipulatives.
- Solve a given addition or subtraction equation with one unknown using a variety of strategies including guess and test.
- Explain why the unknown in a given addition or subtraction equation has only one value.

SCO: **PR3: Solve one-step addition and subtraction equations involving symbols representing an unknown number.**
[C, CN, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

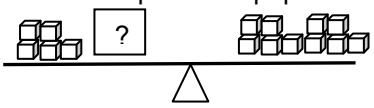
- Ensure students see and use a variety of symbols representing the unknown.
- Re-emphasize the part-part-whole relationship of addition and subtraction. For example, 8 + 4 = 12; "*If I put the two parts (8 & 4) together, I get the whole (12); If I take one of the parts away from the whole, I'm left with the other part;* OR *If I know one of the parts is 4 and the whole is 12, then I can figure out what the missing part is by counting on, adding, or subtracting."* Provide story structures that involve more than basic fact knowledge in the equations (e.g., △ + 15 = 36). Suggest that they explore the use of "guess and test" as a strategy to solve for the unknown.
- Use models such as counters to help students solve equations.

Suggested Activities

- Have students match equations with word problems where the unknown is in different locations. In the following examples, an *addition* OR a *subtraction* equation could be used to represent each problem.

Word Problem	Equation
Mia has 15 cherries and eats some. Now she has 6. How many did she eat?	15 – 6 = ○
Edmond has 6 hockey cards, but he would like to have 15. How many more does he need?	15 - △ = 6
Zane has 15 markers, but 6 of them no longer work. How many does he have that work?	6 + □ = 15
Some cookies are on a plate. Six cookies are in a jar making 15 cookies altogether. How many cookies are on the plate?	□ + 6 = 15

- Have students create problems to represent equations such as the following:
  4 + 7 = △ ▽ - 8 = 8 ▢ + 4 = 13
- Show the students a balance scale with linking cubes to represent an equation. Represent the unknown with a piece of paper with a question mark. Have the student write the equation and solve it. Students can replace the paper with linking cubes to help solve the equation or to check their answer.



SCO: **PR3: Solve one-step addition and subtraction equations involving symbols representing an unknown number.**
[C, CN, PS, R, V]

Assessment Strategies

- Ask the student to find the number that makes each equation true.
  5 + ☐ = 13
  16 - ☐ = 7
  ☐ = 6 + 4
  Have the student explain the strategy they used to solve it.
  Ask: Can there be more than one answer for each? Why or why not?
- Ask: How might you use counters to find the number to make this equation true?
  △+ 18 = 25
  Have the student write a story problem for this equation.
- Ask the student to create four different equations with unknowns for which the solution is 9. Ensure he/she uses the same fact and includes both addition and subtraction equations.
- Have students write the corresponding equation for a word problem and solve it. For example: "Gabrielle had some stickers and gave her friend 9. Now she has 8 left. How many did she have at the start?" (☆ - 9 = 8)

SHAPE AND SPACE

SCO: **SS1: Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years).**
[CN, ME, R]
SCO: **SS2: Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem-solving context.**
[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Two	Grade Three	Grade Four
SS1 Relate the number of days to a week and the number of months to a year in a problem-solving context.	**SS1** Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). **SS2** Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem solving context.	**SS1** Read and record time using digital and analog clocks, including 24-hour clocks.

Elaboration

Time is about the duration of an event from beginning to end and, as such, is a form of measurement. *Reading a clock* or *telling time* has **little to do** with actually measuring time. To measure time, students need to develop their own personal understanding of how long time units last. Personal referents for the passage of time can include such things as the time it takes to brush your teeth, the duration of a favourite television show, the amount of play time for recess, or the amount of time asleep each night. In grade 3, *passage of time* is measured using tools such as pendulums, sand timers, metronomes and calendars. In addition, it is important for students to have many opportunities to *estimate* and then *check* how many time units are needed to accomplish a variety of tasks. For example:

How high can you count in one minute?
How many times can you write your first name in one minute?
How many times will the pendulum swing back and forth in one minute?

As with all other types of measurement, the key understanding for students to construct is that the units must be uniform and appropriate and used consistently to ensure uniformity.

Although it is not an expectation that students use a digital or analog clock to tell time, these instruments may be referred to throughout the development of this outcome.

SS1: This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:
- Unit 4, Lesson 1, pp. 134–136
- Unit 4, Unit Problem, pp. 176, 177

SS2: This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:
- Unit 4, Lesson 2, pp. 137-140
- Unit 4, Lesson 3, pp. 141-144

SCO: **SS1: Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years).**
[CN, ME, R]
SCO: **SS2: Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem-solving context.**
[C, CN, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

SS1
- Select and use a non-standard unit of measure, such as television shows or pendulum swings, to measure the passage of time and explain the choice.
- Identify activities that can or cannot be accomplished in minutes, hours, days, months and years.
- Provide personal referents for minutes and hours.

SS2
- Determine the number of days in any given month using a calendar.
- Solve a given problem involving the number of minutes in an hour or the number of days in a given month.
- Create a calendar that includes days of the week, dates and personal events.

SCO: **SS1: Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years).**
[CN, ME, R]
SCO: **SS2: Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem-solving context.**
[C, CN, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Ask students to identify events that take exactly one minute. More than a minute? Less than a minute? This could be extended to other durations of time.
- Have students create their own non-standard unit timers to compare durations. (For examples, see Van de Walle, K-3, p. 242, Fig. 8.14)
- Discuss the duration of various school events occurring throughout the school day and year.
- Use children's literature such as *Counting Crocodiles* by Judy Sierra, *Time Flies* by Ellen Goodenow or *Time* by Nina Filipek to provide connections for students with this outcome.
- Provide a calendar for the year, and have the students figure out how many school days each month will have. How many Friday 13ths are there in the year? On what days do the birthdays of friends and family fall? Ask the students to write about their findings.

Suggested Activities

- Create time circles for days of the week and months of the year to demonstrate the cyclical nature of the passage of time (Small, p. 441).

- Ask the student to estimate how many times one can count to ten, while walking heel-to-toe across the classroom. Have the student verify his/her estimate. Ask why another student might get a different result.
- Have students work in pairs to predict which of two specified activities will take longer. One student times the other performing the two activities, then roles are reversed. Activities could include:
  - printing their name five times
  - walking the length of the classroom heel to toe
  - making a chain of 25 links or paperclips or linking cubes
  - completing 10 jumping jacks
  - singing "Happy Birthday"
- Show the student a calendar for the year. Ask him/her to point out the day's date and to find out what date it will be in six weeks. Seven weeks?
- Ask students to build a timeline reflecting the time and duration of a sequence of events, then make comparisons between the events.

SCO: **SS1: Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years).**
[CN, ME, R]
SCO: **SS2: Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem-solving context.**
[C, CN, PS, R, V]

Assessment Strategies

- Have students describe the duration of an event (e.g., physical education class) using their personal referent.
- Ask: How might you use a calendar to help subtract 14 from a number?
- Tell the student to find the 11^{th} day in a month on a calendar. Ask: how might you use it to add 16, and what is the date?
- Show the student a calendar for the year and ask him/her to identify ways in which months are the same and ways in which they differ.
- Ask: What is something you can do about 10 times in a minute? In an hour?
- Have students create a calendar and include their birthday and 3 other important dates for them.
- Ask students to tell how many:
  - seconds in a minute
  - minutes in an hour
  - days in a given month
- Give students a set of time cards (*minutes, hours, days, months, years*) and have students hold up the appropriate card to describe the duration of an event named by the teacher (e.g., recess – student holds up *minutes*).

SCO: **SS3: Demonstrate an understanding of measuring length (cm, m) by:**
- **selecting and justifying referents for the units cm and m**
- **modeling and describing the relationship between the units cm and m**
- **estimating length using referents**
- **measuring and recording length, width and height.**

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Two	Grade Three	Grade Four
SS2: Relate the size of a unit of measure to the number of units (limited to nonstandard units) used to measure length and mass (weight).	**SS3:** Demonstrate an understanding of measuring length (cm, m) by: • selecting and justifying referents for the units cm and m • modelling and describing the relationship between the units cm and m • estimating length using referents • measuring and recording length, width and height.	

Elaboration

Students have become familiar with measurement attributes using direct comparison and nonstandard units. Now, they will have the opportunity to explore why *standard* units are necessary to ensure consistency and uniformity when measuring and communicating measurements. Two *standard* units of length, the *centimetre (cm)* and *metre (m),* and the attributes of *width* and *height* will be introduced in grade 3.

Throughout this unit, students are encouraged to develop *referents* for metre and centimetre, and use these to estimate various lengths. For example, the thickness of a CD case is about 1 cm; the distance from the floor to the door knob is about 1 m. Using referents helps students think in a more concrete way about the actual size of each unit. Instead of just thinking that a centimetre is *really small* and a meter is *pretty big*, students can relate these measures to more familiar objects and visualize the actual length of a particular measure

This is the first year where students will begin to use a standard tool to measure length. If students actually make simple measuring instruments using unit models with which they are familiar, it is more likely they will understand how an instrument measures. For example:

By comparing their individual non-standard units with standard measurement tools *(eg. ruler, tape measure, meter stick)*, students will recognize how the formal instrument performs the same function.

SCO: **SS3: Demonstrate an understanding of measuring length (cm, m) by:**
- **selecting and justifying referents for the units cm and m**
- **modeling and describing the relationship between the units cm and m**
- **estimating length using referents**
- **measuring and recording length, width and height.**

[C, CN, ME, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:
- Provide a personal referent for one centimetre and explain the choice.
- Provide a personal referent for one metre and explain the choice.
- Match a given standard unit to a given referent.
- Show that 100 centimetres is equivalent to 1 metre by using concrete materials.
- Estimate the length of an object using personal referents.
- Determine and record the length and width of a given 2-D shape.
- Determine and record the length, width or height of a given 3-D object.
- Draw a line segment of a given length using a ruler.
- Sketch a line segment of a given length without using a ruler.

This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:
- Unit 4, Lesson 4, pp. 145-148
- Unit 4, Lesson 5, pp. 149–152
- Unit 4, Lesson 6, pp. 154-157
- Unit 4, Lesson 7, pp. 158, 159
- Unit 4, Unit Problem, pp. 176, 177
- Unit 7, Lesson 1, pp. 240-243

SCO: **SS3: Demonstrate an understanding of measuring length (cm, m) by:**
- **selecting and justifying referents for the units cm and m**
- **modeling and describing the relationship between the units cm and m**
- **estimating length using referents**
- **measuring and recording length, width and height.**

[C, CN, ME, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Provide opportunities for students to discover and share their personal referents for centimetres and metres. They should be able to explain their choices and recognize that there are many appropriate referents for each unit.
- Include measurement situations which are of interest to the students and that provide useful information, such as measuring book heights for a new bookcase or determining if a large piece of furniture can fit through the door.
- Have students create their own rulers. Initially numbers should not be included so students need to count the number of units, rather than looking at the number on the ruler. As they become more familiar with its use, numbers can be added.
- Present situations requiring students to choose the most appropriate unit of measure.

Suggested Activities

- Have students relate lengths to their own bodies. For example: “My legs are about half a metre long, my nose is 4 cm long, and 8 of my footprints would make a metre.”
- Compare connected 100 centimetre cubes (base ten blocks) to a metre.
- Have students use the same ruler to measure the same object in different ways (e.g., changing the start points or measuring different parts of the object and combining results (Van de Walle & Lovin, vol. 1, 2006, p. 233).
- Read the book, *How Big is a Foot?* by Rolf Myller, and relate the story to why standard units of measurement are valuable. As a follow up, discuss why it is not a good idea to tell someone how long a table is by using pieces of paper as a measurement unit.
- Have students develop a book on measurement that they can add to over time. This could include drawings of their personal referents, pictures of objects that they have estimated and measured, and descriptions of length, width and height.
- Give each student a metre-long piece of twine and ask them to use it to measure objects in their homes. Have them make lists of items that are almost a metre, one metre, or a little more than a metre. Have the students enter their findings in a table such as the one shown below.
-

almost a metre	one metre	more than a metre

How could one use the twine to identify objects that are about half a metre?

- Set up a mini-Olympics in which students compete in events such as a tissue kick, a penny thumb toss, and cotton ball puffing. Have students measure results to the nearest centimetre or metre, and then record and compare them.

SCO: **SS3: Demonstrate an understanding of measuring length (cm, m) by:**
- **selecting and justifying referents for the units cm and m**
- **modeling and describing the relationship between the units cm and m**
- **estimating length using referents**
- **measuring and recording length, width and height.**

[C, CN, ME, PS, R, V]

Assessment Strategies

- Ask students to estimate the length of a book using a personal referent for centimetres.
- Ask students to estimate the length of the classroom using a personal referent for metres.
- Ask students to cut a length of about 1 m from a ball of string. Have them verify their estimates.
- Have students draw a line segment that is about 7 cm long without using a ruler.
- Show students a line segment that is 95 cm and have students estimate its length and then measure it with a ruler.
- Have students use materials to show that a metre is the same as 100 centimetres.
- Provide students with a shoebox or other box and have them measure the length, width, and height.
- Have students use a ruler to measure the length of a pencil or other object without using zero as the starting point.
- Provide students with a photograph and have the student measure the length and width of the picture.

SCO: **SS4: Demonstrate an understanding of measuring mass (g, kg) by:**
- **selecting and justifying referents for the units g and kg**
- **modeling and describing the relationship between the units g and kg**
- **estimating mass using referents**
- **measuring and recording mass**.

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Two	Grade Three	Grade Four
SS2: Relate the size of a unit of measure to the number of units (limited to nonstandard units) used to measure length and mass (weight).	**SS4:** Demonstrate an understanding of measuring mass (g, kg) by: • selecting and justifying referents for the units g and kg • modelling and describing the relationship between the units g and kg • estimating mass using referents • measuring and recording mass.	

Elaboration

In previous grades, students have investigated mass using non-standard units. They will now begin to estimate and measure masses, using the gram (g) and kilogram (kg). Estimating mass is more difficult than estimating other measures, as the object's size and shape is not directly related to its mass. They should develop a sense of what a gram and kilogram "feel" like.

Students need to understand that grams are used to measure very light objects and kilograms are more appropriate units for heavier objects. It is also important for student to know that 1000 grams is equal to a kilogram. As with all measurement units, it is valuable for students to have a personal referent for:
- a gram (e.g. a raisin, paper clip, unit cube);
- 100 grams (e.g., individual size yogurt, 40 pennies, a 3-D base ten flat, a granola bar);
- a kilogram (e.g., a bag of sugar, 1L of water).

Measuring and comparing items with different masses will help students understand the necessity for using the same unit of measurement when comparing the amount of matter those objects contain. Having opportunities to compare objects that are alike will strengthen the understanding that the same object rearranged will maintain its original mass.

This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:
- Unit 4, Lesson 11, pp. 1169-170
- Unit 4, Lesson 12, pp. 171-173
- Unit 4, Unit Problem, pp. 176, 177

SCO: **SS4: Demonstrate an understanding of measuring mass (g, kg) by:**
- **selecting and justifying referents for the units g and kg**
- **modeling and describing the relationship between the units g and kg**
- **estimating mass using referents**
- **measuring and recording mass**.

[C, CN, ME, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Provide a personal referent for one gram and explain the choice.
- Provide a personal referent for one kilogram and explain the choice.
- Match a given standard unit to a given referent.
- Explain the relationship between 1000 grams and 1 kilogram using a model.
- Estimate the mass of a given object using personal referents.
- Determine and record the mass of a given 3-D object.
- Measure, using a scale, and record the mass of given everyday objects using the units g and kg.
- Provide examples of 3-D objects that have a mass of approximately 1 g, 100 g and 1 kg.
- Determine the mass of two given similar objects with different masses and explain the results.
- Determine the mass of an object, change its shape, re-measure its mass and explain the results.

SCO: **SS4: Demonstrate an understanding of measuring mass (g, kg) by:**
- **selecting and justifying referents for the units g and kg**
- **modeling and describing the relationship between the units g and kg**
- **estimating mass using referents**
- **measuring and recording mass**.

[C, CN, ME, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Have students compare the mass of objects to an established gram, 100 gram and 1 kg mass.
- Have students create masses of 1 g, 100 g, 1 kg (e.g., ask students to fill containers with various materials until they think a mass of 1 kg is reached).
- Have students find common items that are measured in grams and kilograms. Create a classroom display.
- Have students measure mass on a balance scale or other more accurate scales, as opposed to a bathroom scale with which mass is harder to interpret.
- Ensure students estimate and measure mass, using grams and kilograms as the units.
- Provide situations in which students make comparisons between the masses of two objects, one in grams; the other in kilograms.
- Provide opportunities for students to explore what happens to the mass of the same object if the shape of the object changes.

Suggested Activities

- Have students estimate and then measure the mass of different objects in the classroom.
- Ask the student to predict, from a collection of objects, which one has a mass of about 1 kilogram.
- Ask the student to choose a small item. Next have the student estimate and determine how many of the items would be required to make a mass of a kilogram.
- Ask the student to find something which has the same mass as two bags of marbles.
- Ask the student to find the number of potatoes in 2 kg. Ask: Will the number always be the same? Why or why not?
- Have students predict and measure the number of pennies needed for a mass of 100 grams. Repeat with other coins. Ask students how much 1 kg of that coin would be worth.
- Have students measure 20 g of un-popped popcorn. Have students predict, if the mass will be greater, the same, or less after it has been popped? Have students compare how much space is taken up by the popped versus un-popped popcorn.
- Investigate the number of kilograms students could comfortably carry in their backpack or the total number of kilograms of a group of books on a shelf, etc.
- Use balance scales to have students investigate the mass of different kinds of balls (e.g., ping pong ball vs. golf ball).
- Have students write what they know about the relationship between 1000 grams and a kilogram.
- Have the students select a personal referent for 1 g and 1 kg and explain their choice.

SCO: **SS4: Demonstrate an understanding of measuring mass (g, kg) by:**
- **selecting and justifying referents for the units g and kg**
- **modeling and describing the relationship between the units g and kg**
- **estimating mass using referents**
- **measuring and recording mass**.

[C, CN, ME, PS, R, V]

Assessment Strategies

- Ask: Could you eat a 1 kg cantaloupe? 1 kg of popcorn? Have students explain their thinking.
- Have students discuss which unit (g or kg) is more likely to be used in measuring:
  - a bag of potatoes
  - a box of paper clips
  - an apple
  - a bicycle
- Have students measure the mass of a ball of modeling clay. Have them use all of the clay to make a new object. Ask them to predict the mass of the new object and verify their prediction.
- Provide students with a collection of objects. Ask students to predict which have a mass of about 100 g and 1 kg. Have students explain their choices.

SCO: **SS5: Demonstrate an understanding of perimeter of regular and irregular shapes by:**
- **estimating perimeter using referents for centimetre or metre**
- **measuring and recording perimeter (cm, m)**
- **constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter.**

[C, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Two	Grade Three	Grade Four
SS4: Measure length to the nearest non-standard unit by: using multiple copies of a unit; using a single copy of a unit (iteration process). **SS5:** Demonstrate that changing the orientation of an object does not alter the measurements of its attributes.	**SS5** Demonstrate an understanding of perimeter of regular and irregular shapes by: • estimating perimeter, using referents for cm or m; • measuring and recording perimeter (cm, m); • constructing different shapes for a given perimeter (cm, m); to demonstrate that many shapes are possible for a perimeter.	**SS3** Demonstrate an understanding of area of regular and irregular 2-D shapes by: recognizing that area is measured in square units**;** selecting and justifying referents for the units cm^2 or m^2**;** estimating area by using referents for cm^2 or m^2**;** determining and recording area' constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.

Elaboration

Students extend their knowledge of measuring length to measuring a distance that is not a straight line. Perimeter is a linear measure of a distance that is a continuous line and is often referred to as the "distance around" an object. Constructing this meaning for perimeter will enable students to recognize the outside of any object as its perimeter. This is a foundation for a later understanding of dimensions, area, and the area model for multiplication. In grade 3, the standard units used to measure perimeter are centimetres and metres. Students should also use personal referents when estimating perimeter. Through estimation, students can verify whether their measurements are reasonable. Estimation may also be the only measurement necessary.

Teachers should provide investigations with a variety of concrete materials to help students develop strategies for finding perimeter. These strategies should include those involving direct and indirect measurement. Students should also be given opportunities to construct shapes with a given perimeter. The intent of this outcome is for students to explore their own methods for determining the perimeter of a shape and not developing or following a formula for perimeter.

This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:
- Unit 4, Lesson 8, pp. 160–163
- Unit 4, Lesson 9, pp. 164–166
- Unit 4, Lesson 10, pp. 167, 168
- Unit 4, Unit Problem, pp. 176, 177

SCO: **SS5: Demonstrate an understanding of perimeter of regular and irregular shapes by:**
- **estimating perimeter using referents for centimetre or metre**
- **measuring and recording perimeter (cm, m)**
- **constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter.**

[C, ME, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Measure and record the perimeter of a given regular shape, and explain the strategy used.
- Measure and record the perimeter of a given irregular shape, and explain the strategy used.
- Construct a shape for a given perimeter (cm, m).
- Construct or draw more than one shape for the same given perimeter.
- Estimate the perimeter of a given shape (cm, m) using personal referents.

SCO: **SS5: Demonstrate an understanding of perimeter of regular and irregular shapes by:**
- **estimating perimeter using referents for centimetre or metre**
- **measuring and recording perimeter (cm, m)**
- **constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter.**

[C, ME, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Ask students to predict the perimeter of a shape prior to making their measurements.
- Provide students with frequent opportunities to construct, measure and record perimeter of regular and irregular shapes.
- Ask students to construct or draw more than one shape for the same given perimeter.
- Use perimeter problem solving situations that provide a context for students (e.g., border around rooms or bulletin boards, frames, fences, trim, etc.).
- Provide many opportunities for students to measure the perimeter of irregular shapes using indirect measure with materials such as a string and ruler,
- Ask students to make comparisons between quantities of one object and the perimeter of another to make estimations concerning one fitting the dimensions of the other.

Suggested Activities

- Ask students, "*How can we find the distance around an object?"*(Provide regular and irregular shapes).
- Give each group a metre stick, tape measure and a 30 cm ruler, and string. Ask them to figure out how to find the perimeter of objects around the classroom. Discuss different results.
- Give students pieces of string (different lengths) and ask, "How many different objects can you find with a perimeter that is equal to the length of your string?"
- Ask students "How many objects can you find with a perimeter of 10 cm? 30 cm? 1 m? 3 m?"
- Provide students with geoboards or grid paper and ask "How many different shapes can you make with a given perimeter?"
- Say to students, *"I have drawn a shape in grid paper with a perimeter of 24 cm. What might my shape look like?"*
- Have students trace the outline of their bodies with sidewalk chalk and then estimate and measure the perimeter of their bodies.
- Use a trundle wheel to find the perimeter of the gym or playground.
- Ask students to estimate if a roll of crepe paper would be sufficient to go around the outside of their desk, a window, the door, the outside of the room...Explain why or why not?

SCO: **SS5: Demonstrate an understanding of perimeter of regular and irregular shapes by:**
- **estimating perimeter using referents for centimetre or metre**
- **measuring and recording perimeter (cm, m)**
- **constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter.**

[C, ME, PS, R, V]

Assessment Strategies

- Give students regular and irregular shapes and have them find the perimeter.
- Ask students to construct two different shapes with the same given perimeter.
- Ask students to estimate the perimeter of a given shape. Have them measure and record the actual length.
- Ask students to solve the following problem: "*Farmer Bill has 48 metres of fencing. How many different rectangular chicken coops can he make?*"

SCO: **SS6: Describe 3-D objects according to the shape of the faces, and the number of edges and vertices.**
[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

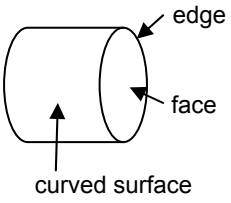
Grade Two	Grade Three	Grade Four
SS6: Sort 2-D shapes and 3-D objects using two attributes, and explain the sorting rule.	**SS6:** Describe 3-D objects according to the shape of the faces, and the number of edges and vertices.	**SS4** Describe and construct rectangular and triangular prisms.

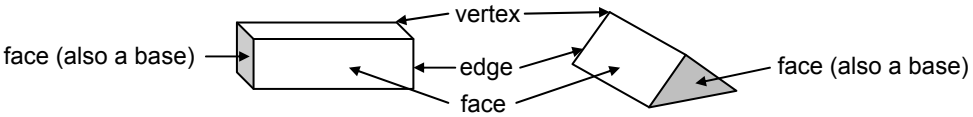
Elaboration

In Grade three, instruction is focused on describing and sorting 3-D objects according to their geometric attributes. Students will identify properties of objects using proper mathematical vocabulary.

The 3-D objects used in Grade 3 include:
- cubes
- spheres
- cones
- cylinders
- prisms (rectangular prisms and triangular prisms)
- pyramids

The geometric attributes of 3-D objects are:
- face: a 2-D shape that forms part of a 3-D object. It is a flat surface that can be traced. Both the shape of the face and the number of faces should be considered as an attribute.
- edge: occurs where two surfaces of a 3-D object join.
- vertex (vertices): a point where 3 or more edges meet. Note: on a cone, a vertex is the highest point above the base.
- curved surface: is a surface that is not flat, and not typically described as a face. These are found on cylinders, cones, and spheres.





Provide students with opportunities to explore these attributes through sorting and construction activities. As they become more familiar with identifying the attributes, students can determine the number of faces, edges, and vertices.

This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:
- Unit 6, Lesson 4, pp. 218–221
- Unit 6, Lesson 5, pp. 222–224
- Unit 6, Lesson 6, pp. 225-227
- Unit 6, Lesson 7, pp. 229-231
- Unit 6, Unit Problem, pp. 234,235

SCO: **SS6: Describe 3-D objects according to the shape of the faces, and the number of edges and vertices.**
[C, CN, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Identify the faces, edges and vertices of given 3-D objects, including cubes, spheres, cones, cylinders, pyramids and prisms.
- Identify the shape of the faces of a given 3-D object.
- Determine the number of faces, edges and vertices of a given 3-D object.
- Construct a skeleton of a given 3-D object and describe how the skeleton relates to the 3-D object.
- Sort a given set of 3-D objects according to the number of faces, edges or vertices.

SCO: **SS6: Describe 3-D objects according to the shape of the faces, and the number of edges and vertices.**
[C, CN, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students with concrete models of given 3-D objects (geometric solids or other objects), including cubes, spheres, cones, cylinders, pyramids, and prisms.
- Identify and sort 3-D objects according to the number of faces, edges and vertices.
- Construct and describe skeletons of 3-D objects.
- Use cross-curricular opportunities to explore 3-D objects in Art and Science classes.
- Ask students to identify particular 3-D objects in their environment and in pictures and to justify their answers.

Suggested Activities

- Ask students “In a bag I have an object that has flat faces, and straight edges. What might this object be?” (Other attributes should be used to extend this activity.)
- Tell students, “The object behind my back is able to roll.” Ask what might it be? (Other attributes should be used to extend this activity.)
- Have the students create a mini book about 3-D objects that includes a picture of each and its attributes.
- Ask students to build a wall using 3-D objects. Discuss what 3-D objects could and could not be used.
- Ask students, “What can you tell me about a pyramid? A cone? A prism?” Have them focus on the attributes.
- Have students build skeletons of 3-D objects using toothpicks and marshmallows, the toothpicks are the edges, the marshmallows are the vertices. Have students describe their objects.
- Give each group a collection of 3-D objects. Have them sort the objects according to the geometric attributes and provide the sorting rule.
- Have students choose two different 3-D shapes. Have them write and illustrate three ways they are the same, and different.
- Read *Sir Cumference and the Sword in the Cone* by Cindy Neuschwander. Discuss.
- Ask students to play a game of “Name That 3-D Shape”. Students must determine the object from the clues given.
- Cut out and label pictures from magazines to build a collage and to identify 3-D objects in the environment.

SCO: **SS6: Describe 3-D objects according to the shape of the faces, and the number of edges and vertices.**
[C, CN, PS, R, V]

Assessment Strategies

- Ask students to describe objects according to their attributes, making sure correct mathematical terms are used for names of objects and faces, edges and vertices.
- Have 3-D objects sorted by attribute, and ask students for the sorting rule.
- Have students make “Wanted” posters for 3-D objects, describing number of faces, edges and vertices, and shapes of faces.
- Have students construct a skeleton of a 3-D object and describe it using correct terminology.
- Have students sort a group of objects according to the number of faces, edges and vertices.
- Have students identify the shape of the faces of a given 3-D object.

SCO: **SS7: Sort regular and irregular polygons, including:**
- **triangles**
- **quadrilaterals**
- **pentagons**
- **hexagons**
- **octagons**

according to the number of sides.
[C, CN, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

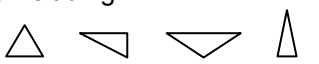
Scope and Sequence

Grade Two	Grade Three	Grade Four
SS7 Describe, compare and construct 3-D objects, including: cubes; spheres; cones; cylinders' pyramids. **SS8** Describe, compare and construct 2-D shapes, including: triangles; squares; rectangles; circles.	**SS7** Sort regular and irregular polygons, including: • triangles • quadrilaterals • pentagons • hexagons • octagons according to the number of sides.	

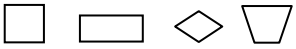
Elaboration

Prior to grade 3, students have had many opportunities to explore 2-D shapes through sorting, patterning, and building activities. Their previous experiences with describing and comparing *regular* polygons included squares, triangles, and rectangles and probably the shapes in a set of "pattern blocks". Some students may form the misconceptions for example, that a triangle must be equilateral, or a hexagon must be the same shape as the one in the pattern blocks set. When sorting or classifying polygons, students should focus on the number of sides as the key attribute and they should come to know the names of a number of different kinds of polygons, including:

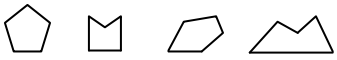
3 straight sides: triangles



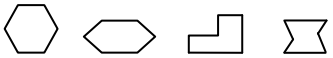
4 straight sides: quadrilaterals



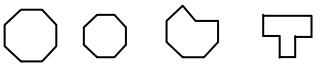
5 straight sides: pentagons



6 straight sides: hexagons



8 straight sides: octagons



Varying the orientation, form, configuration, and size of 2-D shapes will help students focus on the number of sides as the key attribute in naming polygons and lead them to construct the understanding that side lengths in regular polygons are *equal*, but in irregular polygons, they are not.
This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:
- Unit 6, Lesson 1, pp. 208-211
- Unit 6, Lesson 2, pp. 212-215
- Unit 6, Lesson 3, pp. 216, 217
- Unit 6, Unit Problem, pp. 234, 235

SCO: **SS7: Sort regular and irregular polygons, including:**
- **triangles**
- **quadrilaterals**
- **pentagons**
- **hexagons**
- **octagons**

according to the number of sides.
[C, CN, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:
- Classify a given set of regular and irregular polygons according to the number of sides.
- Identify given regular and irregular polygons having different dimensions.
- Identify given regular and irregular polygons having different orientations.

SCO: **SS7: Sort regular and irregular polygons, including:**
- **triangles**
- **quadrilaterals**
- **pentagons**
- **hexagons**
- **octagons**

according to the number of sides.
[C, CN, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Use questioning to assist student focusing on the attributes of polygons. For example: What other shapes look like this one? In what way are the shapes alike? In which ways are they different?
- Have students create different polygons on geoboards or dot paper. Challenge the students to create different types of triangles (quadrilaterals, pentagons, etc.).
- Have students create a book of polygon shapes. Include a variety of examples (regular and irregular) for each type of polygon included in this outcome. This could be extended to include other types of polygons.
- Use geo-strips or strips of paper of different lengths to create various polygons.
- Use children's literature, such as *The Greedy Triangle* by Marilyn Burns and *The Warlord's Puzzle* by Virginia Walton Pilegard to further explore the attributes of polygons.
- Integrate art activities using these shapes. For example, create a piece of art using only a single 2-D shape, but changing the other attributes (size, orientation, length of sides, colour, etc.).

Suggested Activities

- Ask the students to make a triangle on a geoboard that has 2 pegs inside, then one that has three. Ask: What is the greatest number of pegs that can be inside a triangle on a geoboard? Repeat this activity with other shapes.
- Have students sort a collection of pattern blocks by the type of polygon.
- Provide students with sets of tangrams and pentominoes. Have them sort the shapes into triangles, quadrilaterals, pentagons, hexagons, and octagons. Note: there are no pentagons and there are some shapes that have more than 8 sides.
- Have groups of students create a "path of polygons" using sidewalk chalk and drawing a sequence of different polygons.

SCO: **SS7: Sort regular and irregular polygons, including:**
- **triangles**
- **quadrilaterals**
- **pentagons**
- **hexagons**
- **octagons**

according to the number of sides.
[C, CN, R, V]

Assessment Strategies

- Provide students with a sheet that includes a number of different polygons (regular and irregular) that are different sizes, forms, configurations, and/or orientations. Have the students sort and name the polygons.
- Have students create two different pentagons (or other polygons) on a geoboard.
- Have students explain how an octagon and a hexagon (or other shapes) look similar and different.
- Ask: if you draw a pentagon (or other shape) and your friend draws a pentagon, will the two shapes look exactly the same? Why or why not?

STATISTICS AND PROBABILITY

SCO: **SP1: Collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions.**
[C, CN, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Two	Grade Three	Grade Four
SP1 Gather and record data about self and others to answer questions.	**SP1** Collect first-hand data and organize it using: • tally marks • line plots • charts • lists to answer questions.	**SP1** Demonstrate an understanding of many-to-one correspondence.

Elaboration

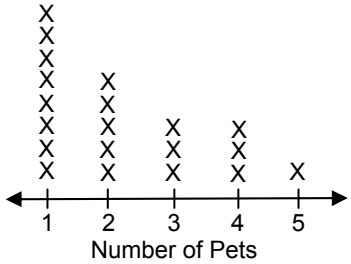
Students should develop strategies to collect and record information. This first-hand data (data that students have collected themselves) should concern the students themselves, their school or community, and/or other topics that are meaningful to them. Attention should be paid as to how best to ask questions (if necessary) to gather information.

It is important to use real-world contexts in order to establish a purpose for collecting data, and to use this data to solve problems. Students can compare and analyze data then make predictions by arranging the data graphically. The expectation is that students will communicate their understanding by recording data in an organized manner and by writing, asking and answering questions concerning data.

Students should be encouraged to organize and record their data using a tally system, line plots, charts and lists to solve problems. A line plot is a graph that uses a number line or words as its base and shows each piece of data with an "x". "Line plots are useful counts of things along a numeric scale. One advantage of a line plot is that every piece of data is shown on the graph" (Van de Walle & Lovin, vol. 2, 2006, p. 333).

Number of Pets	Number of Students
1	~~𝍸~~ III
2	~~𝍸~~
3	III
4	III
5	I

Tally system



Line plot

Number of Pets	Number of Students
1	8
2	5
3	3
4	3
5	1

Chart/List

This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:

- Unit 7, Lesson 1, pp. 240-243
- Unit 7, Lesson 2, pp. 244-247
- Unit 7, Lesson 5, pp. 256-258
- Unit 7, Lesson 6, pp. 260, 261
- Unit 7, Unit Problem, pp. 264, 265

SCO: **SP1: Collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions.**
[C, CN, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Record the number of objects in a given set using tally marks.
- Determine the common attributes of line plots by comparing line plots in a given set.
- Organize a given set of data using tally marks, line plots, charts or lists.
- Collect and organize data using tally marks, line plots, charts and lists.
- Answer questions arising from a given line plot, chart or list.
- Answer questions using collected data.

SCO: **SP1: Collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions.**
[C, CN, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Have pairs of students decide on the procedure they will use to collect and display data that will show interesting information about class members.
- Have students plan and conduct an in-class survey about a favourite _____ (e.g., toy, television program, hockey player). Ask them to present the results of the survey in an organized form.
- Have students conduct a survey to find out what types of things 8- and 9-year-olds like to collect. They will need to decide who to survey and how to organize and present their data.
- Ask small groups of students to brainstorm an interesting list of questions for a possible survey.
- Use a variety of objects to represent a tally (e.g., paper clips chains, linking cubes, clothespins, craft sticks).

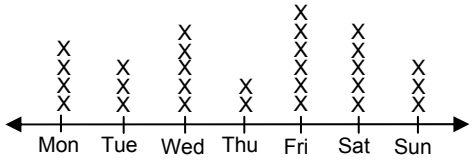
Suggested Activities

- Ask students why it is easier to count the "yes" responses when they are shown like this,  , rather than like this: 
- Have students collect, record and organize data in a line plot, chart, or list to describe the favourite books of their classmates (or other relevant topic).
- Model recording the same data in a line plot, list and chart format and discuss the advantages and disadvantages of each type of data display.
- Collect and display data that represent:
  - accomplishments of favourite sport figures or friends (e.g., the number of goals, hits, points)
  - the distance class members can throw a ball
  - prices of board games
  - mass of various fruits or vegetables
  - mass of subject textbooks
  - mass of different breeds of dogs
- Give students a list of questions and have them identify the questions that might be used for a particular graph or set of data.
- Ask students to build a list of criteria they would expect to find in every "well built" line plot.
- Give students a graph that has no labels nor name and ask them to label and name it then analyze the data.

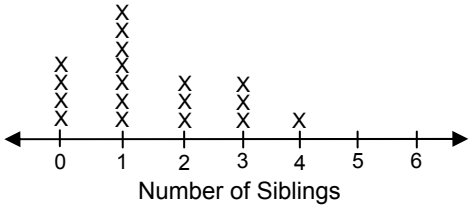
SCO: **SP1: Collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions.**
[C, CN, V]

Assessment Strategies

- Ask students to select a topic, survey family members and/or neighbours, and present their findings to the class in an organized way.
- Ask the students to keep track of weather conditions over the period of one month and to design a way to present the information in an organized manner.
- Show students a line plot such as the one below and ask what it may represent.



- Ask students how they would represent the children in their class who play soccer and those who do not.
- Show the student the following line plot and ask questions such as:
  What is the most common number of siblings (brothers and sisters)?
  How many students have two siblings or less? How many students have 4 siblings?



SCO: **SP2: Construct, label and interpret bar graphs to solve problems.**
[PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

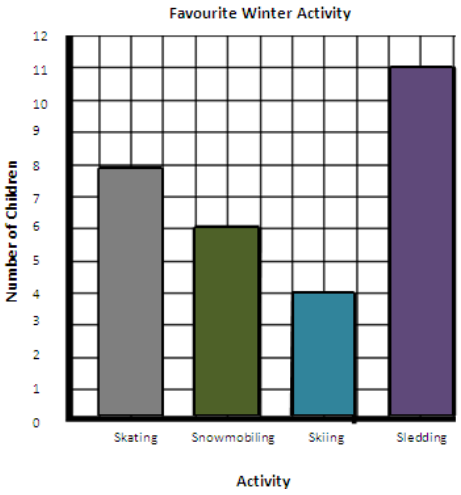
Scope and Sequence

Grade Two	Grade Three	Grade Four
SP2 Construct and interpret concrete graphs and pictographs to solve problems.	**SP2** Construct, label and interpret bar graphs to solve problems.	**SP2** Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.

Elaboration

Statistical literacy is a life skill that effective citizens use to read, question, and interpret data in our world. In elementary school, students are expected to develop an understanding of graphs and how graphs depict information. Students should come to learn that the primary purpose of data, either in graphical form or in numeric form, is to answer questions about the population from which the data are drawn.

Bar graphs use the lengths or heights of bars to represent quantities. This is an extension of the Grade 2 outcome, where students created concrete graphs using models such as linking cubes. In Grade 3 it is helpful for students to work on grid paper to ensure the squares are all equal in size. Students will learn that their bar graphs require a **title**, **labelled categories** (including units if needed), **labelled axes**, an **appropriate scale** and **correctly plotted points**. Bar graphs in Grade 3 can be either *horizontal* or *vertical displays* and should be limited to a *one-to-one correspondence* (i.e., the number scale uses 1, 2, 3, … and not multiples of 2, 5, 10, etc.). Units are not a common occurrence at this stage, but should be included if required in a context that has meaning for the students (i.e. cm is required when labelling the category for the the height of plants).



Questioning should be ongoing whenever students use graphs to encourage students to interpret the data presented and to draw inferences. It is important to ask questions that go beyond simplistic reading of a graph. Both literal questions and inferential questions should be posed. For example:

- What can you tell about……by looking at this graph?
- How many more/less than….?
- Based on the information presented in the graph, what other conclusions can you make?
- Why do you think . . . ?

This specific curriculum outcome is addressed in *Math Makes Sense 3* in the following units:

- Unit 7, Lesson 3, pp. 248-251
- Unit 7, Lesson 4, pp. 252-255
- Unit 7, Lesson 5, pp. 256, 258

SCO: **SP2: Construct, label and interpret bar graphs to solve problems.**
[PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Determine the common attributes, title and axes, of bar graphs by comparing bar graphs in a given set.
- Create bar graphs from a given set of data including labelling the title and axes.
- Draw conclusions from a given bar graph to solve problems.
- Solve problems by constructing and interpreting a bar graph.

SCO: **SP2: Construct, label and interpret bar graphs to solve problems.**
[PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Emphasize using real data when constructing graphs.
- Use grid paper to ensure bar graphs are as accurate as possible.
- Determine common attributes of bar graphs by examining examples from various sources.
- Make use of opportunities to integrate graphing concepts in other areas, such as science, morning message, social studies, etc.

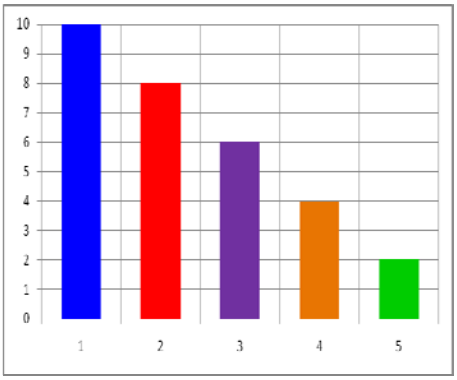
Suggested Activities

- Ask students to create a bar graph to show the kinds of pets students in the class have at home. Have them write two questions about their graph.
- Provide several bar graphs. Have students compare and determine the common attributes, making sure title, and axes and labels are included.
- Provide several bar graphs. Have students draw conclusions and answer questions about the graphs.
- Provide students with a real-life problems to solve such as “What game should we play in Phys Ed?” or “What special activity should be at the Celebration Assembly?” or “What book should be read during Literacy time?” Create a bar graph from collected data, and use it to make decisions or solve problems.

SCO: **SP2: Construct, label and interpret bar graphs to solve problems.**
[PS, R, V]

Assessment Strategies

- Show students a bar graph on a topic of interest to students. Have them answer questions about the graphs and have them make up questions about the graph.
- Provide students with data. Have them construct a bar graph on grid paper.
- Have students answer the following: “This is a graph of a survey I did with my Grade 3 class. What might the survey be about? Label the graph, make up a title, and then, make up 3 questions that could be answered with this graph.”



MENTAL MATH

Fact Learning
Mental Computation
Estimation

Mental Math in the Elementary Mathematics Curriculum

Mental math in this guide refers to fact learning, mental computation, and computational estimation. The Prince Edward Island Mathematics Curriculum supports the acquisition of these skills through the development of thinking strategies across grade levels.

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Pre-Operational Skills

Many children begin school with a limited understanding of number and number relationships. Counting skills, which are essential for ordering and comparing numbers, are an important component in the development of number sense. Counting on, counting back, concepts of more and less, and the ability to recognize patterned sets, all mark advances in children's development of number ideas.

Basic facts are mathematical operations for which some students may not be conceptually prepared.

Basic facts are mathematical operations for which some students may not be conceptually prepared. As a minimum, the following skills should be in place before children are expected to acquire basic facts.

- Students can immediately name the number that comes after a given number from 0-9, or before a given number from 2-10.
- When shown a familiar arrangement of dots ≤ 10 on ten frames, dice, or dot cards, students can quickly identify the number without counting.
- For numbers ≤ 10 students can quickly name the number that is one-more, one-less; two-more, two-less. (The concept of less tends to be more problematic for children and is related to strategies for the subtraction facts.)

Fact learning is a mental exercise with an oral and/or visual prompt; the focus is oral, rather than paper-and pencil; drills should be short with immediate feedback over an extended period of time.

Curriculum Outcomes	Thinking Strategies
Grade 1 **N1**- Say the number sequence, 1 to 100 by: • 1s forward between any two given numbers • 2s to 20, forward starting at 0 • 5s and 10s to 100, forward starting at 0 **N2**- Recognize, at a glance, and name familiar arrangements of 1 to 10 objects or dots (subitize). **N3-** Demonstrate an understanding of counting by: • indicating that the last number said identifies "how many" • showing that any set has only one count • using the counting on strategy • using parts or equal groups to count sets **N5**- Compare sets containing up to 20 elements to solve problems using: • referents • one-to-one correspondence **N6**- Estimate quantities to 20 by using referents. **N8**- Identify the number, up to 20, that is one more, two more, one less and two less than a given number. **N9**- Demonstrate an understanding of addition of numbers with answers to 20 and their corresponding subtraction facts, concretely, pictorially and symbolically by: • using familiar and mathematical language to describe additive and subtractive actions from their experience • creating and solving problems in context that involve addition and subtraction • modeling addition and subtraction using a variety of concrete and visual representations, and recording the process symbolically. **N10**- Describe and use mental mathematics strategies (memorization not intended), such as: • counting on and counting back • making 10 • doubles • using addition to subtract to determine the basic addition facts to 18 and related subtraction facts	**Pre-Operation** • Patterned Set Recognition • Part-Part-Whole Relationships • Counting On and Back • Next Number • Ten-Frame Visualization for Numbers 1-10 • One More / One Less, Two More/Two Less Relationships **Addition Facts With Answers to 20** • Doubles • Plus 1 Facts • Plus 2 Facts • Plus 3 Facts **Corresponding Subtraction Facts** • Think-Addition • Ten Frame Visualization • Counting Back **Adding 10 to a Number**
Grade 2 **N1**- Say the number sequence, 0 to 100 by: • 2s, 5s and 10s, forward and backward, using starting points that are multiples of 2, 5 and 10 respectively • 10s using starting points 1 to 9 • 2s starting from 1. **N6**- Estimate quantities to 100 using referents. **N9**- Demonstrate an understanding of addition (limited to 1 and 2-digit numerals) with answers to 100 and the corresponding subtraction by: • using personal strategies for adding and subtracting with and without the support of manipulatives • creating and solving problems that involve addition and subtraction • explaining that the order in which numbers are added does not affect the sum	**Addition Facts With Answers to 20** • Near Doubles • 2-Aparts • Plus zero • Make 10 **Corresponding Subtraction Facts** • Up Through 10 • Back Down Through 10 **Addition Facts Extended to Numbers in the 10s** …continued

• explaining that the order in which numbers are subtracted may affect the difference. **N10**- Apply mental mathematics strategies, such as: • using doubles • making 10 • one more, one less • two more, two less • building on a known double • addition for subtraction to determine basic addition facts to 18 and related subtraction facts.	**Front-End Addition Finding** **Compatibles Compensation** **Rounding in Addition and Subtraction** (5 or 50 not involved in rounding process until grade 4)

Curriculum Outcomes	Thinking Strategies
Grade 3 **N1**- Say the number sequence forward and backward from 0 to 1 000 by: • 5s, 10s or 100s using any starting point • 3s using starting points that are multiples of 3 • 4s using starting points that are multiples of 4 • 25s using starting points that are multiples of 25. **N4**- Estimate quantities less that 1 000 using referents. **N6**- Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as: • adding from the left to right • taking one addend to the nearest multiple of ten and then compensating • using doubles **N7**- Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as: • taking the subtrahend to the nearest multiple of ten and then compensating • thinking of addition • using doubles **N8**- Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context. **N9**- Demonstrate an understanding of addition and subtraction of numbers with answers to 1 000 (limited to 1, 2 and 3-digit numerals) by: • using personal strategies for adding and subtracting with and without the support of manipulatives • creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially and symbolically. **N10**- Apply mental mathematics strategies and number properties, such as: • using doubles • making 10 • using the commutative property • using the property of zero • thinking addition for subtraction to recall basic addition facts to 18 & related subtraction facts. …continued	**Multiplication Facts** • x 2 Facts • Fives • Ones • Tricky Zeros • Fours • Threes **Break Up and Bridge** **Front-End Estimation for Addition and Subtraction** **Adjusted Front-End** **Estimation for Addition and Subtraction**

N11- Demonstrate an understanding of multiplication to products of 36 with single digit factors by: • representing and explaining multiplication using equal grouping and arrays • creating and solving problems in context that involve multiplication • modeling multiplication using concrete and visual representation, and recording the process symbolically • relating multiplication to repeated addition • relating multiplication to division **N12-** Demonstrate an understanding of division by: • representing and explaining division using equal sharing and equal grouping • creating and solving problems in context that involve equal sharing and equal grouping • modeling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically • relating division to repeated subtraction • relating division to multiplication (limited to division related to multiplication facts up to products of 36 with single digit factors)	
Grade 4 **N3** - Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by: • using personal strategies for adding and subtracting • estimating sums and differences • solving problems involving addition and subtraction. **N5**- Describe and apply mental mathematics strategies, such as: • skip counting from a known fact • using doubling or halving • using doubling or halving and adding or subtracting one more group • using patterns in the 9s facts to determine basic multiplication facts to 9x9 and related division facts. **N6**- Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by: • using personal strategies for multiplication with and without concrete materials • using arrays to represent multiplication • connecting concrete representations to symbolic representations • estimating products. **N7**- Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by: • using personal strategies for dividing with and without concrete materials • estimating quotients • relating division to multiplication. **N11**- Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by: • using compatible numbers • estimating sums and differences • using mental math strategies to solve problems.	**Make 10s, 100s, 1 000s for Addition** **Subtraction Facts Extended to Numbers in the 10s, 100s, and 1000s** **Compensation** (new for subtraction) **Break Up and Bridge** (new for subtraction) **Multiplication Facts to 9 x 9** • Doubles / x 2 Facts • Fives / Clock Facts • Ones • Tricky Zeros • Fours • Threes • Nifty Nines • Last Six Facts **Multiply by 10 and 100 using a place-value-change strategy**

Mental mathematics must be a consistent part of instruction in computation from primary through the elementary and middle grades.

Curriculum Outcomes	Thinking Strategies
Grade 5 **N2**- Use estimation strategies including: • front-end rounding • compensation • compatible umbers in problem-solving contexts. **N3**- Apply mental mathematics strategies and number properties, such as: • skip counting from a known fact • using doubling or halving • using patterns in the 9s facts • using repeated doubling or halving to determine answers for basic multiplication facts to 81 and related division facts. **N4**- Apply mental mathematics strategies for multiplication, such as: • annexing then adding zero • halving and doubling • using the distributive property. ***By grade 5, students should possess a variety of strategies to compute mentally. It is important to recognize that these strategies develop and improve over the years with regular practice.***	**Balancing for a Constant Difference** **Multiply by 0.1, 0.01, 0.001 using a place-value-change strategy** **Front-End Multiplication** (Distributive Principle) **Compensation in Multiplication** **Rounding in Multiplication** **Divide by 10, 100, 1000 using a place-value-change strategy** **Related Division Facts** • "Think multiplication"
Grade 6 **N2**- Solve problems involving large numbers, using technology. **N8**- Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).	**Divide by 0.1, 0.01, 0.001 using a place-value-change strategy** **Finding Compatible Factors** (Associative Property) **Halving and Doubling** **Using division facts for 10's, 100's, 1000's** **Partitioning the Dividend** (Distributive Property)

By grade 5, students should possess a variety of strategies to compute mentally. It is important to recognize that these strategies develop and improve over the years with regular practice.

Definitions and Connections

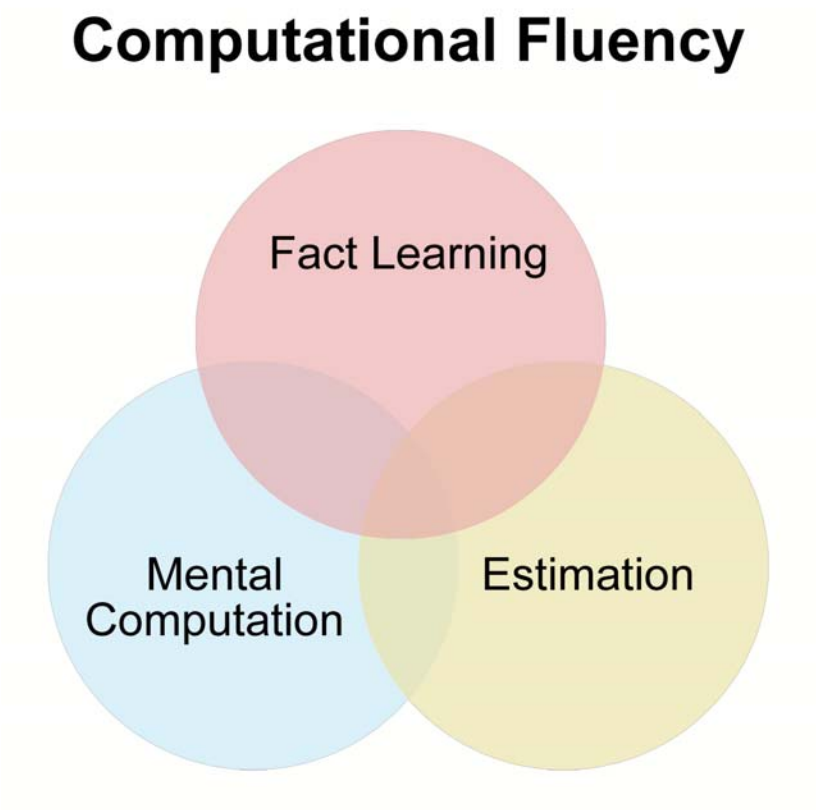
Fact learning refers to the acquisition of the 100 number facts relating to the single digits 0-9 in each of the four operations. Mastery is defined by a correct response in 3 seconds or less.

Mental computation refers to using strategies to get exact answers by doing most of the calculations in one's head. Depending on the number of steps involved, the process may be assisted by quick jottings of sub-steps to support short term memory.

Computational estimation refers to using strategies to get approximate answers by doing calculations mentally.

Students develop and use thinking strategies to recall answers to basic facts. These are the foundation for the development of other mental calculation strategies. When facts are automatic, students are no longer using strategies to retrieve them from memory.

Basic facts and mental calculation strategies are the foundations for estimation. Attempts at estimation are often thwarted by the lack of knowledge of the related facts and mental math strategies.



Rationale

In modern society, the development of mental computation skills needs to be a goal of any mathematical program for two important reasons. First of all, in their day-to-day activities, most people's calculation needs can be met by having well developed mental computational processes. Secondly, while technology has replaced paper-and-pencil as the major tool for complex computations, people still need to have well developed mental strategies to be alert to the reasonableness of answers generated by technology.

In modern society, the development of mental computation skills needs to be a goal of any mathematics program.

Besides being the foundation of the development of number and operation sense, fact learning is critical to the overall development of mathematics. Mathematics is about patterns and relationships and many of these are numerical. Without a command of the basic facts, it is very difficult to detect these patterns and relationships. As well, nothing empowers students more with confidence, and a level of independence in mathematics, than a command of the number facts.

...nothing empowers students more with confidence, and a level of independence in mathematics, than a command of the number facts.

Teaching Mental Computation Strategies

The development of mental math skills in the classroom should go beyond drill and practice by providing exercises that are meaningful in a mathematical sense. All of the strategies presented in this guide emphasize learning based on an understanding of the underlying logic of mathematics.

While learning addition, subtraction, multiplication and division facts, for instance, students learn about the properties of these operations to facilitate mastery. They apply the commutative property of addition and multiplication, for example, when they discover that 3 + 7 is the same as 7 + 3 or that 3 x 7 = 7 x 3. Knowing this greatly reduces the number of facts that need to be memorized. They use the distributive property when they learn that 12 x 7 is the same as (10 + 2) x 7 = (7 x 10) + (2 x 7) which is equal to 70 + 14 = 84.

Understanding our base ten system of numeration is key to developing computational fluency. At all grades, beginning with single digit addition, the special place of the number 10 and its multiples is stressed.

Understanding our base ten system of numeration is key to developing computational fluency. At all grades, beginning with single digit addition, the special place of the number 10 and its multiples is stressed. In addition, students are encouraged to add to make 10 first, and then add beyond the ten. Addition of ten and multiples of ten is emphasized, as well as multiplication by 10 and its multiples.

Connections between numbers and the relationship between number facts should be used to facilitate learning. The more connections that are established, and the greater the understanding, the easier it is to master facts. In multiplication, for instance, students learn that they can get to 6 x 7 if they know 5 x 7, because 6 x 7 is one more group of 7.

Introducing Thinking Strategies to Students

In general, a strategy should be introduced in isolation from other strategies. A variety of practice should then be provided until it is mastered, and then it should be combined with other previously learned strategies. Knowing the name of a strategy is not as important as knowing how it works. That being said, however, knowing the names of the strategies certainly aids in classroom communication. In the mental math guides for each grade, strategies are consistently named; however, in some other resources, you may find the same strategy called by a different name.

When introducing a new strategy, use the chalkboard, overhead or LCD projector, to provide students with an example of a computation for which the strategy works. Are there any students in the class who already have a strategy for doing the computation in their heads? If so, encourage them to explain the strategy to the class with your help. If not, you could share the strategy yourself.

Explaining the strategy should include anything that will help students see its pattern, logic, and simplicity. That might be concrete materials, diagrams, charts, or other visuals.

In the initial activities involving a strategy, you should expect to have students do the computation the way you modeled it. Later, however, you may find that some students employ their own variation of the strategy. If it is logical and efficient for them, so much the better. Your goal is to help students broaden their repertoire of thinking strategies and become more flexible thinkers; it is not to prescribe what they must use.

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You may find that there are some students who have already mastered the simple addition, subtraction, multiplication and division facts with single-digit numbers. Once a student has mastered these facts, there is no need to learn new strategies for them. In other words, it is not necessary to re-teach a skill that has been learned in a different way.

On the other hand, most students can benefit from the more difficult problems even if they know how to use the written algorithm to solve them. The emphasis here is on mental computation and on understanding the place-value logic involved in the algorithms. In other cases, as in multiplication by 5 (multiply by 10 and divide by 2), the skills involved are useful for numbers of all sizes.

Practice and Reinforcement

In general, it is the frequency rather than the length of practice that fosters retention. Thus daily, brief practices of 5-10 minutes are most likely to lead to success.

In general, it is the frequency rather than the length of practice that fosters retention. Thus daily, brief practices of 5-10 minutes are most likely to lead to success. Once a strategy has been taught, it is important to reinforce it. The reinforcement or practice exercises should be varied in type, and focus as much on the discussion of how students obtained their answers as on the answers themselves.

The selection of appropriate exercises for the reinforcement of each strategy is critical. The numbers should be ones for which the strategy being practiced most aptly applies and, in addition to lists of number expressions, the practice items should often include applications in contexts such as money, measurements and data displays. Exercises should be presented with both visual and oral prompts and the oral prompts that you give should expose students to a variety of linguistic descriptions for the operations. For example, 5 + 4 could be described as:

- the sum of 5 and 4
- 4 added to 5
- 5 add 4
- 5 plus 4
- 4 more than 5
- 5 and 4 etc.

Response Time

- **Basic Facts**

In the curriculum guide, fact mastery is described as a correct response in 3 seconds or less and is an indication that the student has committed the facts to memory. This 3-second-response goal is a guideline for teachers and does not need to be shared with students if it will cause undue anxiety. Initially, you would allow students more time than this as they learn to apply new strategies, and reduce the time as they become more proficient.

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- **Mental Computation Strategies**

With other mental computation strategies, you should allow 5 to 10 seconds, depending on the complexity of the mental activity required. Again, in the initial stages, you would allow more time, and gradually decrease the wait time until students attain a reasonable time frame. While doing calculations in one's head is the principal focus of mental computation strategies, sometimes in order to keep track, students may need to record some sub-steps in the process. This is particularly true in computational estimation when the numbers may be rounded. Students may need to record the rounded numbers and then do the calculations mentally for these rounded numbers.

In mental math activities it is reasonable for the teacher to present a mental math problem to students, ask for a show of hands, and then call on individual students for a response. In other situations, it may be more effective when all students participate simultaneously and the teacher has a way of checking everyone's answers at the same time. Individual response boards or student dry-erase boards are tools which can be used to achieve this goal.

Struggling Students and Differentiated Instruction

It is imperative that teachers identify the best way to maximize the participation of all students in mental math activities.

It is imperative that teachers identify the best way to maximize the participation of all students in mental math activities. Undoubtedly there will be some students who experience considerable difficulty with the strategies assigned to their grade and who require special consideration. You may decide to provide these students with alternative questions to the ones you are expecting the others to do, perhaps involving smaller or more manageable numbers. Alternatively, you may just have the student complete fewer questions or provide more time.

The more senses you can involve when introducing the facts, the greater the likelihood of success for all students, but especially for students experiencing difficulty.

There may be students in the upper grades who do not have command of the basic facts. For the teacher, that may mean going back to strategies at a lower grade level to build success, and accelerating them vertically to help students catch up. For example, if the students are in grade 6 and they don't yet know the addition facts, you can find the strategies for teaching them in the grade 2 mathematics curriculum guide in the mental math section. The students, however, are more intellectually mature, so you can immediately apply those same strategies to tens, hundreds, and thousands, and to estimation of whole numbers and decimal sums.

The more senses you can involve when introducing the facts, the greater the likelihood of success for all students, but especially for students experiencing difficulty.

Many of the thinking strategies supported by research and outlined in the curriculum advocate for a variety of learning modalities.
For example:

- **Visual** (images for the addition doubles; hands on a clock for the "times-five" facts)
- **Auditory** (silly sayings and rhymes: "6 times 6 means dirty tricks; so 6 x 6 = 36")
- **Patterns in Number** (the product of an even number multiplied by 5 ends in 0 and the tens digit is half of the number being multiplied)
- **Tacticle** (ten frames, base ten blocks)
- **Helping Facts** (8 x 9 = 72, so 7 x 9 is one less group of 9; 72 – 9 = 63)

Whatever differentiation you make, it should be to facilitate the student's development in mental computation, and this differentiation should be documented and examined periodically to be sure it is still necessary.

Combined Grade Classrooms

What you do in these situations may vary from one strategy to another. Sometimes the students may be all doing the same strategy, sometimes with the same size or type of number, sometimes with different numbers. For example, in a combined grade 2/3 class, students might be working on the "make ten" strategy for addition. The teacher would ask the grade 2 students questions such as 9 + 6 or 5 + 8, while the grade 3 students would be given questions such as 25 + 8 or 39 + 6; the same strategy is applied, but at different levels of difficulty.

Other times, you may decide to introduce different strategies at different times on the first day, but conduct the reinforcements at the same time on subsequent days using the appropriate exercises for each grade level.

It is important to remember that there will be students in the lower grade who can master some, or all, the strategies expected for the higher grade, and some students in the higher grade who will benefit from the reinforcement of the strategies from the lower grade.

Assessment

Your assessment of mental computation should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the practice sessions. You should also ask students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student's thinking, especially in situations where paper-and-pencil responses are weak.

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Timed Tests of Basic Facts

Some of the former approaches to fact learning were based on stimulus-response; that is, the belief that students would automatically give the correct answer if they heard the fact over-and-over again. No doubt, many of us learned our facts this way. These approaches often used a whole series of timed tests of 50 to 100 items to reach the goal.

…the thinking strategy approach prescribed by our curriculum is to teach students strategies that can be applied to a group of facts with mastery being defined as a correct response in 3 seconds or less.

In contrast, the thinking strategy approach prescribed by our curriculum is to teach students strategies that can be applied to a group of facts with mastery being defined as a correct response in 3 seconds or less. The traditional timed test would have limited use in assessing this goal. To be sure, if you gave your class 50 number facts to be answered in 3 minutes and some students completed all, or most, of them correctly, you would expect that these students know their facts. However, if other students only completed some of these facts and got many of those correct, you wouldn't know how long they spent on each question and you wouldn't have the information you need to assess the outcome. You could use these sheets in alternative ways, however.

For example:

- Ask students to quickly circle the facts which they think are "hard" for them and just complete the others. This type of self assessment can provide teachers with valuable information about each student's level of confidence and perceived mastery.
- Ask students to circle and complete only the facts for which a specific strategy would be useful. For example, circle and complete all the "double-plus-1" facts.
- Ask them to circle all the "make ten" facts and draw a box around all "two-apart" facts. This type of activity provides students with the important practice in strategy selection and allows the teacher to assess whether or not students recognize situations for which a particular strategy works.

Parents and Guardians:
Partners in Developing Mental Math Skills

Parents and guardians are valuable partners in reinforcing the strategies you are developing in school. You should help parents understand the importance of these strategies in the overall development of their children's mathematical thinking, and encourage them to have their children do mental computation in natural situations at home and out in the community. Through various forms of communication, you should keep parents abreast of the strategies you are teaching and the types of mental computations they should expect their children to be able to do.

MENTAL MATH

Fact Learning

Fact Learning – Addition

• Reviewing Addition Facts and Fact Learning Strategies

Mastery of the addition facts with sums to 18 is the expectation in Grade 2. At the beginning of grade 3, it is important to ensure that students review these facts and the fact learning strategies from the previous grades.

Addition Facts to 18

Doubles
1 + 1
2 + 2
3 + 3
4 + 4
5 + 5
6 + 6
7 + 7
8 + 8
9 + 9

Near Doubles
2 + 3 3 + 2
3 + 4 4 + 3
4 + 5 5 + 4
5 + 6 6 + 5
6 + 7 7 + 6
7 + 8 8 + 7
8 + 9 9 + 8

Plus 1 Facts
2 + 1 1 + 2
3 + 1 1 + 3
4 + 1 1 + 4
5 + 1 1 + 5
6 + 1 1 + 6
7 + 1 1 + 7
8 + 1 1 + 8
9 + 1 1 + 9

Plus 2 Facts
3 + 2 2 + 3
4 + 2 2 + 4
5 + 2 2 + 5
6 + 2 2 + 6
7 + 2 2 + 7
8 + 2 2 + 8
9 + 2 2 + 9

Plus 3 Facts
4 + 3 3 + 4
5 + 3 3 + 5
6 + 3 3 + 6
7 + 3 3 + 7
8 + 3 3 + 8
9 + 3 3 + 9

2-Apart Facts
1 + 3 3 + 1
2 + 4 4 + 2
3 + 5 5 + 3
4 + 6 6 + 4
5 + 7 7 + 5
6 + 8 8 + 6
7 + 9 9 + 7

Plus or Minus 0
Have students model simple word problems using counters and a two-part mat.

For example, "*Mark found 4 golf balls on Saturday. (Student puts 4 counters on one side of the mat.) He didn't find any balls on Sunday. How many balls did Mark find altogether? (Student is unable to put any counters on the other part of the mat, so the total answer remains 4.)*

Make 10 Facts
2 + 8 8 + 2
3 + 8 8 + 3
4 + 8 8 + 4
5 + 8 8 + 5
6 + 8 8 + 6
7 + 8 8 + 7
9 + 8 8 + 9
2 + 9 9 + 2
3 + 9 9 + 3
4 + 9 9 + 4
5 + 9 9 + 5
6 + 9 9 + 6
7 + 9 9 + 7
7 + 3 3 + 7
4 + 7 7 + 4
5 + 7 7 + 5
6 + 7 7 + 6
7 + 7

A mental strategy is a way of thinking that helps complete a fact quickly. It must be done mentally and it must be efficient. Students who have mastered the number facts no longer rely on thinking strategies to recall them.

Addition Facts Extended to 10s and 100s

The following are the addition fact strategies with examples, and examples of the same facts applied to 10s, and 100s:

- **Doubles Facts**: 4 + 4, 40 + 40, 400 + 400
- **Plus One Facts**: (next number) 5 + 1, 50 + 10, 500 + 100
- **Plus Two Facts**: (2-more-than facts) 7 + 2, 70 + 20, 700 + 200
- **Plus Three Facts**: 6 + 3, 60 + 30, 600 + 300
- **Near Doubles Facts / 1-Aparts**: 3 + 4, 30 + 40, 300 + 400
- **Plus Zero Facts / No-Change**: 8 + 0, 80 + 0, 800 + 0
- **Doubles Plus 2 Facts / Double In-Between / 2-Aparts**: 5 + 3, 50 + 30, 500 + 300
- **Make 10 Facts**: 9 + 6, 90 + 60, 900 + 600; 8 + 4, 80 + 40, 800 + 400; 7 + 4, 70 + 40, 700 + 400

Doubles Facts

There are only ten doubles from 0 + 0 to 9 + 9 and most students learn them quickly. The doubles posters, which have been specially created for classroom use, provide a visual context for these facts. Grade 3 teachers should briefly review the addition doubles and then extend to the 10s and 100s.

Practice Items

40 + 40 =	20 + 20 =	200 + 200 =
70 + 70 =	10 + 10 =	400 + 400 =
90 + 90 =	80 + 80 =	500 + 500 =
800 + 800 =	30 + 30 =	900 + 900 =
100 + 100 =	300 + 300 =	700 + 700 =

Near-Doubles Facts / 1-Aparts

The near-doubles facts are also called the “1-Aparts” or the “doubles plus one” facts and include all combinations where one addend is one more than the other. The strategy is to double the smaller number and add one.

For example: 6 + 7 is double 6, plus 1.

Review these near-doubles facts and then extend to 10s and 100s.

Practice Items

Numbers in the 10s

30 + 40 =	30 + 20 =	70 + 80 =
50 + 60 =	20 + 30 =	10 + 20 =
80 + 90 =	60 + 70 =	50 + 40 =

Numbers in the 100s

400 + 500 =	700 + 800 =	800 + 900 =
400 + 300 =	900 + 800 =	300 + 200 =
200 + 100 =	700 + 600 =	600 + 700 =
600 + 500 =	300 + 400 =	

Doubles Plus 2 Facts / Double In-Between / 2-Aparts

These facts include facts whose addends differ by 2, such as 3 + 5, 4 + 6, or 5 + 7. There are two possible strategies each depending on knowledge of the doubles.

- Double the smaller number and add 2. For example, 4 + 6 is double 4 plus 2 more.
- Double the number "in between". For example, for 5 + 7, you can double 6 which is doubling the number between 5 and 7, and that makes 12.

Review the 2-Apart facts and then extend to the 10s and 100s.

Examples

- 30 + 50 = double 30 plus 20 = 60 + 20 = 80
- 70 + 90 = (70 + 10) + (90 – 10) or 80 + 80 = 160

Practice Items

Numbers in the 10s

40 + 60 =	30 + 10 =
60 + 80 =	20 + 40 =
90 + 70 =	50 + 30 =
10 + 30 =	40 + 20 =
30 + 50 =	60 + 40 =
50 + 70 =	70 + 50 =
80 + 60 =	70 + 90 =

Numbers in the 100s

100 + 300 =	700 + 900 =
700 + 500 =	800 + 600 =
300 + 500 =	600 + 400 =
200 + 400 =	300 + 100 =
400 + 600 =	400 + 200 =
500 + 700 =	600 + 800 =
900 + 700 =	500 + 300 =

Plus Zero Facts / No-Change

Zero is known as the *Identity Element for Addition* because when you add 0 to any number, you end up with that number. Students need to have a good understanding of the meaning of zero and of addition (7 + 0 = 7, 0 + 7 = 7) and recognize that adding 0 makes "no change".

Practice Items

5 + 0 =	90 + 0 =	800 + 0 =
0 + 9 =	50 + 0 =	300 + 0 =
6 + 0 =	0 + 40 =	0 + 400 =

Make 10 or 100

Make Ten is a thinking strategy introduced in grade 2 for addition facts which have an 8 or a 9 as one of the addends and can even be extended to facts which involve a 7. To help develop this strategy, students use two ten frames and counters to model these number facts and then rearrange the counters so that the facts read as "10 plus some more".

For example, students model the "make 10" fact 8 + 6 with 8 counters on one ten frame and 6 on the other. Then they move 2 counters from the 6 and give them to the 8 to make 10 + 4. Students should understand that the purpose of this strategy is to get a 10 which is easy to add. In order for this to be an effective strategy, students must be able to immediately recognize all the numbers between 10 and 20 as the "teen" numbers and to know, for example, that 10 + 6 = 16 without hesitation. *Considerable work with ten frames is required to help students understand the relationship before they are expected to perform the process mentally.*

In order for the "Make-10" strategy to be effective, students must be able to immediately recognize all the numbers between 10 and 20 as the "teen" numbers and to know, for example, that 10 + 6 = 16 without hesitation.

In Grade 3, students apply a "make 10s" strategy to sums involving single-digit numbers added to 2-digit numbers, and the addition of two 2-digit numbers which are both multiplies of 10.

Examples
For 28 + 6, think, "28 + 2 (from the 6) is 30, and 30 + 4 (the rest of the 6) is 34."
For 80 + 40, think, "80 + 20 (from the 40) is 100, and 100 + 20 (the rest of the 40) equals 120."

Practice Items

4 + 18 =	18 + 8 =	70 + 30 =
19 + 8 =	19 + 4 =	80 + 70 =
17 + 5 =	30 + 90 =	30 + 80 =
4 + 18 =	80 + 30 =	80 + 40 =
19 + 6 =	50 + 80 =	70 + 70 =
6 + 18 =	90 + 30 =	70 + 60 =
19 + 5 =	60 + 80 =	40 + 90 =
8 + 19 =	40 + 70 =	50 + 90 =
17 + 6 =	60 + 90 =	

Add your own practice items

Plus One, Plus Two, Plus Three Facts

The thinking strategy for addition facts which have 1 or 2 as one of the addends is the *one-more-than, two-more-than* relationship. If students have not yet developed these relationships, then *counting* can be used. Counting on can also be used for facts that have 3 as one of the addends.

A common error occurs when students include the starting number as one of their "counts". For example, for 7 + 3, some students will think, "*seven, eight, nine*". The use of a number line in the development of this counting strategy will help correct this difficulty.

Teachers should exercise caution when introducing *counting* as a thinking strategy. Many students will try to extend it to other number facts and not realize how slow and inefficient the strategy can be. Teachers must help students to recognize when it is appropriate and inappropriate to use counting as a strategy for number facts.

Examples
For 70 + 20, think, "**70**: 80…90"
For 500 + 300, think, "**500**: 600…700…800"

Practice Items

70 + 20 =	60 + 20 =	30 + 20 =
40 + 20 =	50 + 20 =	10 + 20 =
80 + 20 =	200 + 700 =	600 + 200 =
300 + 200 =	200 + 400 =	800 + 200 =
500 + 200 =	50 + 30 =	90 + 30 =
80 + 30 =	30 + 60 =	30 + 10 =
30 + 70 =	20 + 30 =	40 + 30 =
400 + 300 =	600 + 300 =	700 + 300 =
100 + 300 =	900 + 300 =	300 + 800 =
400 + 500 =	200 + 300 =	300 + 500 =

Add your own practice items

Fact Learning – Subtraction

After students have mastered each cluster of addition facts, it is appropriate to have them learn the corresponding subtraction facts. Many students will apply a "think-addition" strategy for all subtraction facts.

- ### Subtraction Facts and Fact Learning Strategies

The subtraction facts are first introduced in grade 1, but are emphasized more in grades 2 and 3. They are directly related to the addition facts with sums to 18 and should be completed using a "think addition" strategy. As students master groups of addition facts, it is appropriate to introduce the related subtraction facts so that they can apply their knowledge in a different way. For example, if students have mastered the addition *doubles*, they should be presented with subtraction facts such as 12 – 6 = and think, "6 *plus what equals 12? Double six is 12, therefore 12 – 6 = 6."*

There are other thinking strategies, as well, that will help students master the subtraction facts. For example:

- ### Up Through 10:

This strategy involves counting the difference between the two numbers by starting with the smaller number, keeping track of the distance to ten, and the adding this amount to the rest of the distance to the greater number.

Examples

a) For 12 – 7, think, "Starting at 7, it's 3 to get to 10 and then 2 more to get to 12, so that's 5 altogether".
b) For 16 – 9, think,"It's 1 from 9 to get to 10, and then 6 more to 16, so that's 7 altogether".

- **Back Down Through 10**

With this strategy, you start with the larger number and subtract part of the subtrahend to get to 10, and then subtract the rest of the subtrahend.

Examples

a) For 15 – 8, think, "15 subtract 5 (one part of the 8) gets me to 10, and then 3 more (the rest of the 8) takes me to 7".
b) For 13 – 4, think, "13 subtract 3 is 10, and then 1 more takes me to 9".

Subtraction Facts (Related Subtraction Facts for Addition Facts to 18)

Doubles
2 – 1 12 - 6
4 – 2 14 – 7
6 – 3 16 – 8
8 – 4 18 – 9
10 – 5

Near Doubles
5 – 2 5 – 3
7 – 3 7 – 4
9 – 4 9 – 5
11 – 5 11 – 6
13 – 6 13 – 7
15 – 7 15 – 8
17 – 8 15 – 9

Plus 1 Facts
3 – 1 3 – 2
4 – 1 4 – 3
5 – 1 5 – 4
6 – 1 6 – 5
7 – 1 7 – 6
8 – 1 8 – 7
9 – 1 9 – 8
10 – 1 19 – 9

Plus 2 Facts
5 – 2 5 – 3
6 – 2 6 – 4
7 – 2 7 – 5
8 – 2 8 – 6
9 – 2 9 – 7
10 – 2 10 – 8
11 – 2 11 – 9

Plus 3 Facts
7 – 3 7 – 4
8 – 3 8 – 5
9 – 3 9 – 6
10 – 3 10 – 7
11 – 3 11 – 8
12 – 3 12 – 9

2-Apart Facts
4 – 3 4 – 1
6 – 4 6 – 2
8 – 5 8 – 3
10 – 4 10 – 6
12 – 5 12 – 7
14 – 6 14 – 8
16 – 7 16 – 9

Make 10 Facts
10 – 2 10 – 8
11 – 3 11 – 8
12 – 4 12 – 8
13 – 5 13 – 8
14 – 6 14 – 8
15 – 7 15 – 8
17 – 9 17 – 8
11 – 2 11 – 9
12 – 3 12 – 9
13 – 4 13 – 9
14 – 5 14 – 9
15 – 6 15 – 9
16 – 7 16 – 9
10 – 3 10 – 7
11 – 4 11 – 7
12 – 5 12 – 7
13 – 6 13 – 7
14 – 7

- **Subtraction Facts Extended to 10s and 100s**

Doubles and Near Doubles

60 – 30 =
100 – 50 =
20 – 10 =
40 – 20 =
180 – 90 =
140 – 70 =
160 – 80 =
120 – 60 =
50 – 20 =
1600 – 800 =
800 – 400 =
1800 – 900 =
1000 – 500 =
1200 – 600 =
70 – 30 =
50 – 20 =
130 – 60 =
150 – 70 =
900 – 400 =
1700 – 800 =
500 – 200 =
1100 – 500 =
1300 – 600 =
1500 – 700 =
200 – 100 =

Make 10s and 100s

110 – 30 =
130 – 80 =
130 – 60 =
120 – 30 =
100 – 80 =
150 – 90 =
140 – 60 =
160 – 90 =
120 – 70 =
150 – 70 =
1500 – 800 =
1400 – 500 =
1100 – 400 =
1100 – 900 =
1300 – 600 =
1400 – 600 =
10000 – 400 =
1100 – 800 =

Fact Learning – Multiplication

• Multiplication Fact Learning Strategies

The concept of multiplication and its relationship to division is introduced in grade 3. It is not intended that students automatically recall the basic multiplication facts in grade 3, though many students will have mastered some by the end of the year. Teachers must help students become familiar with flexible ways to think about and work with numbers so that *products* can be determined. Thinking strategies should be introduced, practiced, and reinforced on a regular basis in the classroom. (PEI Gr. 3 Curriculum Guide, p. 57)

Multiplication Facts

Facts With 2 (Addition Doubles) 2 x 1 1 x 2 2 x 2 2 x 3 3 x 2 2 x 4 4 x 2 2 x 5 5 x 2 2 x 6 6 x 2 2 x 7 7 x 2 2 x 8 8 x 2 2 x 9 9 x 2 **Facts With 5** (Clock Facts) 5 x 1 1 x 5 5 x 2 2 x 5 5 x 3 3 x 5 5 x 4 4 x 5 5 x 5 5 x 6 6 x 5 5 x 7 7 x 5	**Facts With 1** (No-Change Facts) 1 x 1 1 x 2 2 x 1 1 x 3 3 x 1 1 x 4 4 x 1 1 x 5 5 x 1 1 x 6 6 x 1 1 x 7 7 x 1 1 x 8 8 x 1 1 x 9 9 x 1 **Facts With 0** (Facts with 0 have products of 0) 0 x 0 0 x 1 1 x 0 0 x 2 2 x 0 0 x 3 3 x 0 0 x 4 4 x 0 0 x 5 5 x 0 0 x 6 6 x 0 0 x 7 7 x 0 0 x 8 8 x 0 0 x 9 9 x 0	**Square Facts** (These facts, and others like them, form square arrays.) 1 x 1 2 x 2 3 x 3 4 x 4 5 x 5 6 x 6 **Facts With 4** (Double-Double) 4 x 1 1 x 4 4 x 2 2 x 4 4 x 3 3 x 4 4 x 4 4 x 5 5 x 4 4 x 6 6 x 4 4 x 7 7 x 4 4 x 8 8 x 4 4 x 9 9 x 4 **Facts With 3** (Double + 1 more set) 3 x 6 6 x 3 3 x 7 7 x 3 3 x 8 8 x 3 3 x 9 9 x 3

Thinking Strategies

• x2 Facts (with turnarounds): 2 x 2, 2 x 3, 2 x 4, 2 x 5, 2 x 6, 2 x 7, 2 x 8, 2 x 9

These facts are directly related to the addition doubles. The *doubles posters* for grades 3 and 4 help to make this connection clear.

For example, for 4 x 2, and 2 x 4, students should think, “double 4” and recall the “Spider Legs Double” visual 4 + 4 = 8. So 4 x 2 = 8 and 2 x 4 = 8.

• Fives Facts (with turnarounds): 5 x 1, 5 x 2, 5 x 3, 5 x 4, 5 x 5, 5 x 6, 5 x 7

It is easy to make the connection to the multiplication facts involving 5s using an analog clock. For example, if the minute hand is on the 3 and the students know that means 15 minutes after the hour, then the connection to 3 x 5 = 15 can be made. This is why you may see the Five Facts referred to as the “clock facts”. This would be the best strategy for students who know how to tell time on an analog clock.

You should also introduce the two patterns that result when numbers are multiplied by 5:

1. For even numbers multiplied by 5, the answer always ends in zero, and the digit in the tens place is half the other number. So, 4 x 5 = 20.
2. For odd numbers multiplied by 5, the product always ends in 5, and the digit in the tens place is half of the number that comes before the other number. So 5 x 5 = 25.

One of the values of patterns in mathematics is that they help us do seemingly difficult things quite easily. The Fives Facts pattern illustrates clearly one of the values of pattern and regularity in mathematics.

- **Ones Facts (with turnarounds): 1 x 1, 1 x 2, 1 x 3, 1 x 4, 1 x 5, 1 x 6, 1 x 7, 1 x 8, 1 x 9**

While the ones facts are the "no change" facts, it is important that students understand why there is no change. Many students get these facts confused with the addition facts involving 1. For example 6 x 1 means 6 groups of 1 *or* 1 + 1 + 1 + 1 + 1 + 1 *and* 1 x 6 means one group of 6. It is important to avoid teaching arbitrary rules such as "any number multiplied by one is that number". Students will come to this rule on their own given opportunities to develop understanding.

- **The Tricky Zeros Facts**

As with the ones facts, students need to understand why these facts all result in zero: they are easily confused with the addition facts involving zero. Teachers must help students understand the meaning of the number sentence.

For example, 6 x 0 means "six 0's" *or* "six sets of nothing". Ask students to use counters or blocks to build two sets of 6, then 1 set of 6 and finally zero sets of 6 where they don't use any counters or blocks. They will quickly realize why zero is the product. Similar to the previous strategy for teaching the ones facts, it is important not to teach a rule such as "any number multiplied by zero is zero". Students will come to this rule on their own, given opportunities to develop understanding.

- **Fours Facts (with turnarounds): 4 x 4, 4 x 6, 4 x 7, 4 x 8, 4 x 9**

One strategy that works for any number multiplied by 4 is "double-double".

For example, for 6 x 4, you would double the 6 (12) and then double again (24)

Another strategy that works for any time one (or both) of the factors is even, is to divide the even number in half, then multiply, and then double your answer. So, for 7 x 4, you could multiply 7 x 2 (14) and then double that to get 28. For 16 x 9, think 8 x 9 (72) and 72 + 72 = 70 + 70 (140) plus 4 = 144.

- **Threes Facts (with turnarounds): 3 x 3, 3 x 4, 3 x 6, 3 x 7, 3 x 8, 3 x 9**

The strategy here is for students to think "times 2, plus another group". So for 7 x 3 or 3 x 7, the student should think "7 times 2 is 14, plus 7 more is 21.

MENTAL MATH

Mental Computation

Mental Computation – Addition

- **Front-End Addition** (continued from Grade 2)

This strategy is first introduced in grade 2 and involves adding the highest place values and then adding the sums of the next place value(s). Start by modelling the addition of two 2-digit numbers using base ten blocks. For 24 + 35, you would use 2 rods and 4 unit cubes for 24, and 3 rods, 5 unit cubes for 35. Join these two amounts by combining the rods first and then the unit cubes. Students should also be given the opportunity to model addition in this manner.

Examples
For 37 + 26, think: "3 tens plus 2 tens is 5 tens, or 50; and 2 ones plus 6 ones is 8 ones (8); So 50 + 8 = 58."

For 42 + 17, think, "40 and 10 make 50 and 2 plus 7 equals 9, so 50 plus 9 is 59."

For 24 + 12, think, "20 + 10 + 30 = 60, and 4 + 2 + 1 =7; 60 plus 7 equals 67."

Practice Items

27 + 31 =	74 + 19 =
16 + 32 =	32 + 28 =
32 + 65 =	45 + 35 =
25 + 63 =	37 + 44 =
72 + 26 =	56 + 36 =
63 + 33 =	34 + 27 =

Add your own practice items

Your goal for teaching mental computation should be to show students a wide variety of mental methods, provide opportunities where each method can be employed, and encourage students to use mental methods regularly to improve their skills.

- **Break Up and Bridge** (New)

This strategy is similar to front-end addition except that you begin with all of the first number and then add on parts of the second number beginning with the largest place value. Again, you should start by modelling the addition of two 2-digit numbers using base ten blocks. For 24 + 35, you would use 2 rods and 4 unit cubes for 24, and 3 rods, 5 unit cubes for 35. Join these two amounts by combining the 2 rods and 4 units with just the 3 rods in the second number for a sum of 54. Now, add on the remaining 5 unit cubes for a total of 59.

Students should also be given the opportunity to model addition in this manner.

Examples
For 45 + 36, think, "45 and 30 (from the 36) is 75, and 75 plus 6 (the rest of the 36) is 81."
For 26 + 34, think, "26 plus 30 is 56 and 56 plus 4 equals 60."

Practice Items

37 + 45 =	72 + 28 =	25 + 76 =
38 + 43 =	59 + 15 +	66 + 27 =
31 + 25 +	74 + 16 =	37 + 24 =
62 + 24 =	31 + 24 =	51 + 36 =

Add your own practice items

Situations must be regularly provided to ensure that students have sufficient practice with mental math strategies and that they use their skills as required. It is recommended that regular, maybe daily, practice be provided.

- **Finding Compatibles** (Extension)

Compatible numbers are sometimes referred to as friendly numbers or nice numbers in other professional resources. Some examples of common compatible numbers include 1 and 9; 40 and 60; 75 and 25 and 300 and 700.

This strategy for addition involves looking for pairs of numbers that combine to make a sum that will be easy to work with. In grade 3, the focus is on numbers that add up to 10 and 100.

Practice Items

6 + 9 + 4 + 5 + 1 =
2 + 4 + 3 + 8 + 6 =
4 + 6 + 2 + 3 + 8 =
7 + 1 + 3 + 9 + 5 =
4 + 5 + 6 + 2 + 5 =
60 + 30 + 40 =
75 + 95 + 25 =

5 + 3 +5 + 7 + 4 =
9 + 5 + 8 + 1 + 5 =
2 + 7 + 6 + 3 + 8 =
9 + 4 + 6 + 5 + 1 =
30 + 20 + 70 + 80 =
50 + 15 + 25 + 5 =
25 + 20 + 75 + 40 =

Add your own practice items

- **Compensation** (Extension)

This strategy involves changing one number in an addition question to the nearest ten to make the calculation easier, carrying out the addition, and then adjusting the answer to compensate for the original change. Students may have already used this strategy when learning their addition facts involving 9s in Grade 2. For example, for 9 + 7, they may have added 10 + 7 and then subtracted 1.

Example

For 52 + 39, think, “52 plus 40 is 92, but I added 1 too many to take me to the next 10, so I subtract one from my answer to get 91.”

Practice Items

43 + 9 =
45 + 8 =
44 + 27 =

56 + 8 =
65 + 29 =
14 + 58 =

72 + 9 =
13 + 48 =
21 + 48 =

Add your own practice items

Mental Computation – Subtraction

- **Back Down Through 10s** (Extension)

This strategy extends one of the strategies students learned for fact learning. It involves subtracting a part of the subtrahend to get to the nearest ten or hundred, and then subtracting the rest of the subtrahend.

Examples
For 15 – 8, think: "15 subtract 5 (one part of the 8) is 10, and 10 subtract 3 (the other part of the 8) is 7."
For 74 – 6, think: "74 subtract 4 (one part of the 6) is 70 and 70 subtract 2 (the other part of 6) is 68."

Practice Items

15 – 6 =	42 – 7 =	34 – 7 =
13 – 4 =	61 – 5 =	82 – 6 =
13 – 6 =	15 – 7 =	14 – 6 =
74 – 7 =	97 – 8 =	53 – 5 =

Add your own practice items

- **Up Through 10s** (Extension)

This strategy is an extension of the "Up through 10" strategy that students learned to help master the subtraction facts.

To apply this strategy, you start with the smaller number (the subtrahend) and keep track of the distance to the next 10, and then add this amount to the rest of the distance to the greater number (the minuend).

Examples
For 12 – 9, think, "It's 1 from 9 to 10 and 2 from 10 to 12; so the difference is 1 plus 2, or 3."
For 84 – 77, think, "It's 3 from 77 to 80 (the next ten) and 4 more to get to 84; so that's a difference of 7."

Practice Items

15 – 8 =	14 – 9 =	16 – 9 =
11 – 7 =	17 – 8 =	13 – 6 =
12 – 8 =	15 – 6 =	16 – 7 =
95 – 86 =	67 – 59 =	46 – 38 =
58 – 49 =	34 – 27 =	71 – 63 =
88 – 79 =	62 – 55 =	42 – 36 =

Add your own practice items

MENTAL MATH

Estimation

Estimation – Addition and Subtraction

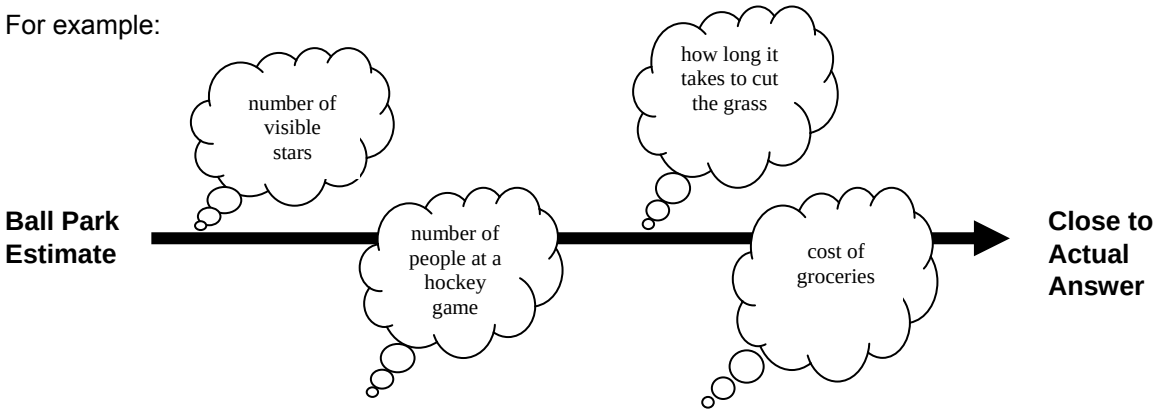
When asked to estimate, students often try to do the exact computation and then "round" their answer to produce an estimate that they think their teacher is looking for. Students need to see that estimation is a valuable and useful skill, one that is used on a daily basis by many people.

Students need to see that estimation is a valuable and useful skill, one that is used on a daily basis by many people.

Estimates can be very broad and general, or they can be quite close to the actual answer. It all depends on the reason for estimating in the first place, and these reasons can vary in context and according to the needs of the individual at the time.

Help students identify situations outside of school where they would estimate distances, number, temperature, length of time and discuss how accurate their estimates needed to be. Place these situations on an estimation continuum with broad, ball-park estimates at one end and estimates that are very close to the actual answer at the other.

For example:



In mathematics, it is essential that estimation strategies are used by students before attempting pencil/paper or calculator computations to help them determine whether or not their answers are reasonable.

When teaching estimation strategies, it is important to use words and phrases such as, about, almost, between, approximately, a little more than, a little less than, close to and near.

- **Front End Addition and Subtraction** (New)

This strategy involves combining or finding the difference between only the values in the highest place value to get a "ball-park" estimate. Such estimates are adequate in many situations.

Examples

To estimate 43 + 54, think, "40 + 50 is 90."
To estimate 92 – 53, think, "90 subtract 50 is 40"
To estimate 437 + 541, think, "400 plus 500 is 900."
To estimate 534 – 254, think, "500 subtract 200 is 300."

Practice Items

62 + 31 =
21 + 43 =
44 + 23 =
13 + 82 =
73 + 12 =

34 + 42 =
54 + 33 =
12 + 51 =
71 + 14 =
24 + 73 =

93 – 62 =
91 – 42 =
64 – 23 =
43 – 12 =
81 – 54 =

32 – 23 =
72 – 33 =
84 – 61 =
54 – 21 =
73 – 44 =

234 + 432 =
741 + 138 =
341 + 610 +
647 + 312 =
632 + 207 =

703 + 241 =
423 + 443 =
816 + 111 =
512 + 224 =
534 + 423 =

327 – 142 =
928 – 741 =
804 – 537 =

516 – 234 =
639 – 426 =

Add your own practice items

Ongoing practice in computational estimation is key to developing understanding of numbers and number operations and increasing mental process skills.

• Rounding in Addition and Subtraction (Extension)

This strategy for addition and subtraction, first introduced in grade 2, involves rounding the highest place value in each number and then adding or subtracting the rounded numbers. To help support short-term memory, it will be necessary for most students to first jot down the rounded numbers and then do the computation mentally.

At this grade level, numbers which involve 5 or 50 in the rounding procedure are not included in the practice items. This is introduced in grade 4.

Examples

To estimate 27 + 31, think, "27 rounds to 30 and 31 rounds to 30, so 30 plus 30 is 60."

To estimate 348 + 230, think, "348 rounds to 300 and 230 rounds to 200, so 300 plus 200 is 500."

To estimate 87 - 32, think, "87 rounds to 90 and 32 rounds to 30, so 90 subtract 30 is 60."

To estimate 594 - 203, think, "594 rounds to 600 and 203 rounds to 200, so 600 subtract 200 is 400."

Practice Items

48 + 23 =	34 + 59 =	61 + 48 =
18 + 22 =	97 + 12 =	14 + 32 =
28 + 57 =	41 + 34 =	57 – 14 =
84 – 9 =	82 – 59 =	36 – 22 =
43 – 8 =	54 – 18 =	68 – 34 =
99 – 47 =	93 – 12 =	326 + 590 =
218 + 411 =	290 + 570 =	520 + 679 =
680 + 124 =	530 + 360 =	420 – 198 =
840 – 715 =	970 – 430 =	830 – 580 =
870 – 399 =	940 – 642 =	260 – 98 =
594 – 301 =	780 – 270 =	324 – 176 =

Add your own practice items

• Adjusted Front End (New)

This strategy begins with a front-end estimate and then making an adjustment by considering some or all the values in the other place values. This will result in a more accurate estimate. Sometimes the numbers in the ones do not account for another ten and therefore, do not affect the estimation.

Examples

To estimate 23 + 48, think, "20 plus 40 is 60 and 3 plus 8 would account for about another 10; therefore, the adjusted estimate is 60 + 10 or 70."

To estimate 72 - 38, think, "70 subtract 30 is 40; the 8 would account for about another 10; so the adjusted estimate is 30."

To estimate 31 + 22, think, "30 plus 20 is 50; the numbers in the ones place would not account for another 10, so the best estimate is still 50."

To estimate 82 – 31, think, "80 subtract 30 is 50; and 1 would not account for another 10, so the best estimate is still 50"

Practice Items

28 + 33 =	47 + 31 =
76 + 13 =	62 + 29 =
39 + 64 =	38 + 34 =
48 + 25 =	82 + 17 =
54 + 28 =	29 + 53 =

Add your own practice items

Computational estimation is a mental activity; therefore, regular oral practice, accompanied by the sharing of strategies must be provided.

OVERVIEW OF THINKING STRATEGIES IN MENTAL MATH

Thinking Strategies in Mental Math

Mental math proficiency represents one important dimension of mathematical knowledge. Not all individuals will develop rapid mental number skills to the same degree. Some will find their strength in mathematics through other avenues, such as visual or graphic representations or creativity in solving problems. But mental math has a clear place in school mathematics. It is an area where many parents and families feel comfortable offering support and assistance to their children.

The following table identifies all of the thinking strategies in Mental Math: Fact Learning, Mental Computation and Estimation and the grade level in which they are first introduced. These strategies are then extended and developed in subsequent years.

For example, Front End Addition involving 2-digit numbers is first introduced in grade 2, continued in grade 3, extended to 3-digit numbers in grade 4, and to decimal tenths, hundredths, and thousandths in grades 5 and 6. The Mental Math section found in each grade level's mathematics curriculum guide contains a complete description of each strategy with examples and practice items.

Strategy	Description
Grade 1	
Pre-Operation • Patterned Set Recognition	• Students are able to identify common configuration sets of numbers such as the dots on a standard die, dominoes and dot cards without counting.
• Part-Part-Whole Relationships	• Recognition of two parts in a whole. Leads to the understanding that numbers can be decomposed into component parts.
• Counting On and Back	• Students can count on and back from a given number 0-9.
• Next Number	• Students are able to immediately state the number that comes after any given number from 0-9.
• Ten-Frame Visualization for Numbers 0-10	• Students can visualize the standard ten-frame representation of numbers and answer questions from their visual memories.
• One More/One Less, Two More/Two Less Relationships	• Students are presented with a number and asked for the number that is one more, one less, two more, or two less than the number.
Addition Facts to 10 • Doubles • Plus 1 Facts • Plus 2 Facts • Plus 3 Facts	• Doubles posters created as visual images • Next number facts • Ten-frame, skip counting, 2-more-than relationship, counting on • Ten-frame, 2-more-than plus 1, counting on
Subtraction Facts With Minuends to 10 • Think-Addition • Ten Frame Visualization • Counting Back	• For 9 - 3, think, "3 plus what equals 9?" • Visualize the minuend on a ten-frame, remove the subtrahend, to determine the difference. • For -1, -2, -3 facts
Adding 10 to a Number	For numbers 11-20

Grade 2	
Addition Facts to 18 • Near Doubles • 2-Aparts • Plus zero • Make 10	 • Double the smaller number and add 1 • Double the number in between • No change facts • For facts with 8 or 9 as addends. Eg. 7 + 9 is the same as 10 + 6
Subtraction Facts With Minuends to 18 • Up Through 10 • Back Down Through 10	• For 13 - 8, think, “From 8 up to 10 is 2, and then 3 more is 5.” • For 14 - 6, think, “14 - 4 gets me to 10, and then 2 more brings meto 8.”
Addition facts extended to numbers in the 10's	2-Apart Facts: 3 + 5 is double 4, so 30 + 50 is double 40.
Front-end Addition	Highest place values are totaled first and then added to the sum of the remaining place values.
Finding Compatibles	Looking for pairs of numbers that add easily, particularly, num bers that add to 10.
Compensation	One or both numbers are changed to make the addition easier and the answer adjusted to compensate for the change.
Rounding in Addition and Subtraction (5 or 50 not involved in rounding process until grade 4)	Round to nearest 10.

Grade 3	
Multiplication Facts to Products of 36 with single digit factors • x 2 facts • Fives • Ones • Tricky Zeros • Fours • Threes	 • Related to the addition doubles • Clock facts, patterns • No change facts • Groups of zero • Double-double • Double plus 1 more set
Break Up and Bridge	With this front-end strategy, you start with all of the first num ber and add it to the highest place value in the other number, and then add on the rest.
Front-End Estimation for Addition and Subtraction	Add or subtract just the largest place values in each number to produce a “ball park” estimate.
Adjusted Front-End Estimation for Addition and Subtraction	Same as above, except the other place values are considered for a more accurate estimate.

Grade 4	
Make 10's, 100's, 1000's for addition	48 + 36 is the same as 50 + 34 which is 84
Multiplication Facts to 9 x 9 • Nifty Nines • Last Six Facts	Patterns, helping fact For facts not already covered by previous thinking strategies
Subtraction facts extended to num bers in the 10's, 100's 100's	Only 1 non-zero digit in each number eg., 600 - 400 =
Compensation (new for subtraction)	For 17-9, think, “17 - 10 is 7, but I subtracted 1 too m any, so the answer is 8."
Break Up and Bridge (new for subtraction)	For 92 - 26, think, “92 - 20 is 72 and then 6 more is 66."
Multiply by 10 and 100 using a place- value- change strategy	The place values for a number multiplied by 100 increase 2 places. Eg. 34 x 100; The 4 ones becomes 4 hundreds and the 3 tens becomes 3 thousand; 3000 + 400 = 3400

Grade 5	
Multiplication Facts to 81 and Related Division Facts • “Think-Multiplication”	Mastery by year-end For 36 ÷ 6, think “6 times what equals 36?”
Balancing for a Constant Difference	Involves changing both number in a subtraction sentence by the same amount to make it easier to complete. The difference between the two numbers remains the same. Eg. for 27 - 16, add 3 to each number and think, “30 - 19 = 11"
Multiply by 0.1, 0.01, 0.001 using a place- value-change strategy	The place values for a number multiplied by 0.1 decrease 1 place. Eg. 34 x 0.1; The 4 ones becomes 4 tenths and the 3 tens becomes 3 ones; 3 and 4 tenths, or 3.4.
Front-End Multiplication (Distributive Principle)	Involves finding the product of the single-digit factor and the digit in the highest place value of the second factor, and adding to this product a second sub-product. 706 x 2 = (700 x 2) + (6 x 2) = 1412
Compensation in Multiplication	Involves changing one factor to a 10 or 100, carrying out the multiplication, and then adjusting the product to compensate for the change. 7 x 198 = 7 x 200 (1400) subtract 14 = 1386
Divide by 10, 100, 1000 using a place-value- change strategy.	The place values for a number divided by 10 decrease 1 place. Eg. 34 ÷ 10; The 4 ones becomes 4 tenths and the 3 tens becomes 3 ones; 3 and 4 tenths, or 3.4.
Rounding in Multiplication	Highest place values of factors are rounded and multiplied. W hen both num bers are close to 5 or 50, one num ber rounds up and the other down.

Grade 6	
Divide by 0.1, 0.01, 0.001 using a place-value- change strategy	The place values for a number divided by 0.01 increase 2 places. Eg. 34 ÷ 0.01; The 4 ones becomes 4 hundreds and the 3 tens becomes 3 thousand; 3000 + 400 = 3400
Finding Compatible Factors (Associative Property)	Involves looking for pairs of factors, whose product is easy to work with, usually multiples of 10. For example, for 2 x 75 x 500, think, “2 x 500 = 1000 and 1000 x 75 is 75 000.
Halving and Doubling	One factor is halved and the other is doubled to make the multiplication easier. Students would need to record sub-steps. For example, 500 x 88 = 1000 x 44 = 44 000.
Using division facts for 10's, 100's 1000's	Dividends in the 10's, 100's, and 1000's are divided by single digit divisors. The quotients would have only one digit that wasn’t a zero. For example, for 12 000 ÷ 4, think single digit division facts: 12 ÷ 4 = 3, and thousands divided by ones is thousands, so the answer is 3000.
Partitioning the Dividend (Distributive Property)	The dividend is broken up into two parts that are more easily divided by the divisor. For example, for 372 ÷ 6, think, “(360 + 12) ÷ 6, so 60 + 2 is 62.”

MENTAL MATH: FACT LEARNING SCOPE AND SEQUENCE

GRADE 1 FACT LEARNING

Pre-operation Strategies

- Patterned Set Recognition for numbers 1-6 (not dependent on counting)
- Part-Part-Whole Relationships
- Counting On, Counting Back
- Next Number
- Ten Frame Recognition and Visualization for Numbers 0-10
- One More/ One Less and Two More/Two Less Relationships

Addition Facts With Sums to 10 Thinking Strategies

- Doubles
- Plus 1 Facts
- Plus 2 Facts
- Plus 3 Facts
- Ten Frame Facts

Subtraction Facts With Minuends to 10 Thinking Strategies

- Think-Addition
- Ten Frame Facts
- Counting Back

GRADE 2 FACT LEARNING

Addition and Subtraction Facts

- Mastery of facts with sums and minuends to 10 by mid-year
- Mastery of facts with sums and minuends to 18 by year end

New Thinking Strategies for Addition

- Near Doubles/Doubles Plus One/1-Aparts
- 2-Apart Facts
- Plus 0 Facts
- Make 10 Facts

New Thinking Strategies for Subtraction Facts

- Up Through 10
- Back Down Through 10

GRADE 3 FACT LEARNING

Addition

- Review and reinforce facts with sums to 18 and thinking strategies
- Addition facts extended to 2-digit numbers: Think *single-digit* addition facts and apply the appropriate place value.

Subtraction

- Review and reinforce facts with minuends to 18 and thinking strategies.
- Subtraction facts extended to 2-digit numbers. Think *single-digit* subtraction facts and apply the appropriate place value.

Multiplication Facts Thinking Strategies (focus on products to 36 with single digit factors)

- x2 Facts (related to addition doubles)
- x5 Facts (clock facts, patterns)
- x1 Facts (“no-change” facts)
- x0 Facts (products of zero)
- x4 Facts (double-double)
- x3 Facts (double plus 1 set)

GRADE 4 FACT LEARNING

Addition and Subtraction

Review and reinforce thinking strategies for addition and subtraction facts with sums/minuends to 18

Multiplication Thinking Strategies (focus 9x9)

- x2 Facts (related to addition doubles)
- x10 Facts (patterns)
- x5 Facts (clock facts, patterns)
- x1 Facts (“no-change” facts)
- x0 Facts (products of zero)
- x4 Facts (double-double)
- x3 Facts(double plus 1 set)
- Last Six Facts (new: various strategies)

MENTAL MATH: FACT LEARNING SCOPE AND SEQUENCE (continued)

GRADE 5 FACT LEARNING

Addition and Subtraction

Review and reinforce thinking strategies for addition and subtraction facts with sums/minuends to 18

Multiplication

- Review and reinforce thinking strategies for multiplication facts (focus 9x9)
- Mastery by year end

Division

- Review and reinforce thinking strategies for division facts with dividends to 81 (related facts to 9x9) using a "Think-Multiplication" strategy
- Mastery by year end

GRADE 6 FACT LEARNING

- Review Addition, Subtraction, Multiplication and Division Facts
- Reintroduce thinking strategies to struggling students
- See the Mental Math section in each grade level's mathematics curriculum guide for a complete description of each strategy with examples and practice items. Mental Math sections are part of the mathematics curriculum guide for each grade from grade one to six inclusively.

MENTAL MATH: MENTAL COMPUTATION SCOPE AND SEQUENCE

GRADE 1 MENTAL COMPUTATION

Addition

- Adding 10 to a number without counting

GRADE 2 MENTAL COMPUTATION

Addition

- Addition facts extended to 2-digit numbers. Think *single-digit addition* facts and apply the appropriate place value. (New)
- Front End Addition (2-digit numbers)
- Finding Compatibles (single-digit number combinations that make 10)
- Compensation (single-digit numbers)

Subtraction

- *Think-Addition* (extended to 2-digit numbers)

GRADE 3 MENTAL COMPUTATION

Addition

- Front End Addition (continued from Grade 2)
- Break Up and Bridge (New)
- Finding Compatibles (single digit numbers that add up to 10, 2-digit numbers that add up to 100)
- Compensation (extended to 2-digit numbers)

Subtraction

- Back Down Through 10s (extended to subtraction of a single digit from a 2-digit number)
- Up Through 10s (extended to 2-digit numbers)

GRADE 4 MENTAL COMPUTATION

Addition

- Facts Extended to Addition of Numbers in 10s, 100s, and 1000s
- Front End Addition (extended to numbers in 1000s)
- Break Up and Bridge (extended to numbers in 100s)
- Compensation (extended to numbers in 100s)
- Make 10s, 100s, 1000s (Extension)

Subtraction

- Facts Extended to Subtraction of Numbers in 10s, 100s, and 1000s
- Back Down Through 10s (extended to numbers in 100s)
- Up Through 10s (extended to numbers in the 100s)
- Compensation (New for Subtraction)
- Break Up and Bridge (New for Subtraction)

Multiplication

- Multiplying to 10 and 100 using a “place-value-change” strategy rather than an “attach zeros” strategy

GRADE 5 MENTAL COMPUTATION

Addition

- Front End Addition (extended to decimal 10^{th} and 100^{th})
- Break Up and Bridge (extended to numbers in 1000s and to decimal 10^{th}s and 100^{th}s)
- Finding Compatible (extended to 1000s and to decimal 10^{th}s and 100^{th}s)
- Compensation (extended to numbers in 1000s and to decimal 10^{th}s and 100^{th}s)
- Make 10s, 100s, 1000s (continued from Grade 4)

Subtraction

- Back Down Through 10s, 100s, 1000s (Extension)
- Up Through 10s (extended to numbers in the 1000s and to decimal 10^{th}s and 100^{th}s)
- Compensation (extended to numbers in 1000s)
- Break Up and Bridge (extended to numbers in 1000s)

Multiplication

- Facts Extended to 10s, 100s and 1000s
- Multiplying by 10, 100, 1000 using a “Place-Value-Change” strategy, rather than an “attach zeros” strategy (continued from Grade 4)
- Multiplying by 0.1, 0.01, and 0.001 using a place-value-change strategy (New)
- Front End Multiplication (New)
- Compensation (New for Multiplication)

MENTAL MATH: MENTAL COMPUTATION SCOPE AND SEQUENCE

GRADE 6 MENTAL COMPUTATION

Addition

Practice items provided for review of mental computation strategies for addition.

- Front End
- Break Up and Bridge
- Finding Compatibles
- Compensation
- Make 10s, 100s, 1000s

Subtraction

- Back Down Through 10s, 100s, 1000s
- Up Through 10s, 100s, 1000s
- Compensation
- Balancing for a Constant Difference (continued from Grade 5)
- Break Up and Bridge (extended to numbers in 10 000s)

Multiplication and Division

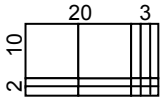
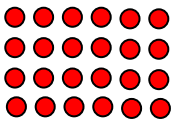
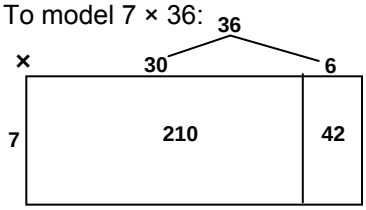
- Multiplying and Dividing by 10, 100, 1000 using a “Place-Value-Change” strategy
- Multiplying by 0.1, 0.01, and 0.001 (continued from Grade 5)
- Dividing by 0.1, 0.01, 0.001 using a “Place-Value-Change” strategy (New)
- Front End Multiplication (continued from Grade 5)
- Compensation (continued from Grade 5)
- Finding Compatible Factors (New)
- Halving and Doubling (New)
- Using Division Facts for 10s, 100s, 1000s (New)
- Dividends of 10s, 100s, 1000s divided by single-digit divisors
- Partitioning The Dividend (New)

MENTAL MATH: ESTIMATION SCOPE AND SEQUENCE

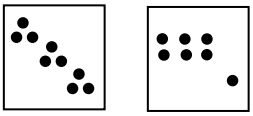
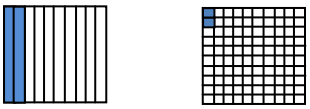
GRADE 1 ESTIMATION
Due to the focus on Pre-Operational Skills being reinforced in Grade 1, there are no estimation thinking strategies outlined at this level. However, students are asked to estimate quantities to 20 by using referents (see Number Outcome N6).
GRADE 2 ESTIMATION
• Rounding in Addition and Subtraction (2-digit numbers; 5 is not involved in the rounding procedure until Grade 4)
GRADE 3 ESTIMATION
• Front End Addition and Subtraction (New) • Rounding in Addition and Subtraction (extended to 3-digit numbers; 5 or 50 not involved in the rounding procedure until Grade 4) • Adjusted Front End in Addition and Subtraction (new)
GRADE 4 ESTIMATION
• Rounding in Addition and Subtraction (extended to 4-digit numbers and involving 5, 50 and 500 in the rounding procedure) • Adjusted Front End in Addition and Subtraction (extended to numbers in 1000s)
GRADE 5 ESTIMATION
• Rounding in Addition and Subtraction (continued from Grade 4) • Rounding in Multiplication (2-or-3- digit factor by single digit factor; 2-digit by 2-digit) • Adjusted Front End in Addition and Subtraction (extended to decimal 10^{th}s and 100^{th}s)
GRADE 6 ESTIMATION
• Rounding in Addition and Subtraction (continued from Grade 5) • Rounding in Multiplication (extended from Grade 5 to include 3-digits by 2-digits) • Rounding in Division (New)

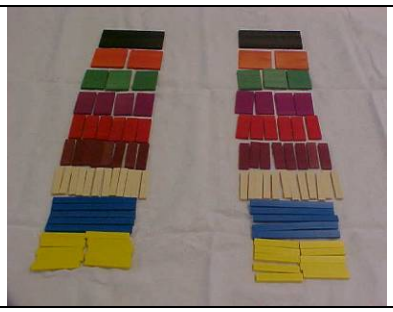
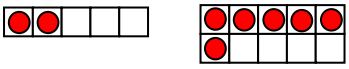
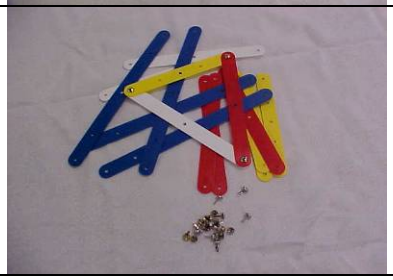
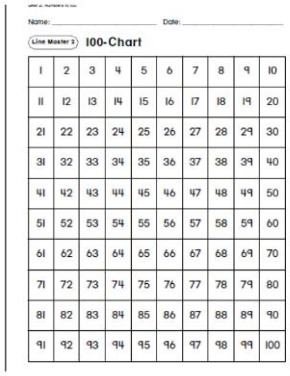
GLOSSARY OF MODELS

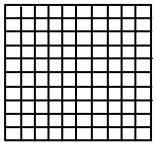
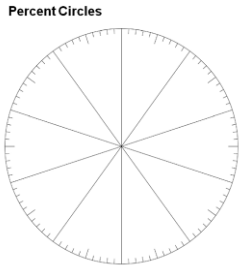
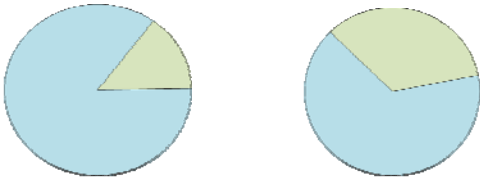
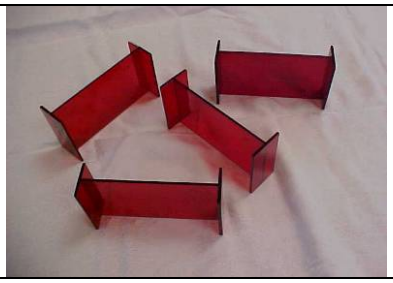
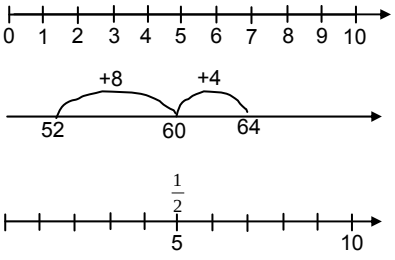
Mathematical models, often referred to as “manipulatives”, have a variety of uses at different grade levels and are referenced throughout the curriculum and in many resources. Many comprehensive reviews of the research into the use of mathematical models have concluded that student achievement is increased as a result of long term exposure to mathematical models. It is important to remember, however, that it depends on how the models are used in the classroom. In themselves, mathematical models *do not teach* but, in concert with good teaching, make a great deal of difference. The purpose of this glossary is to provide a visual reference for each model and a brief description of it. It is the responsibility of individual schools to maintain and enhance their inventory of available mathematical models.

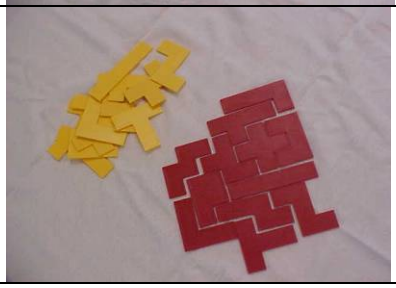
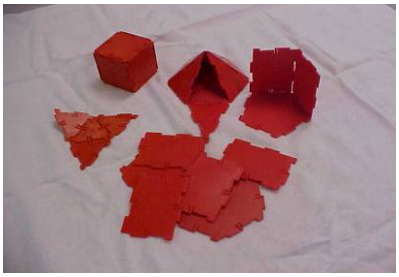
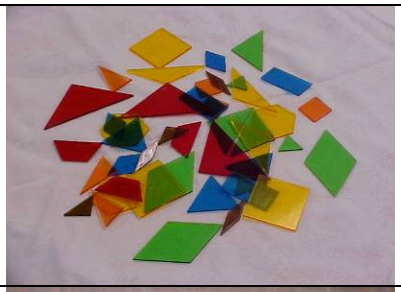
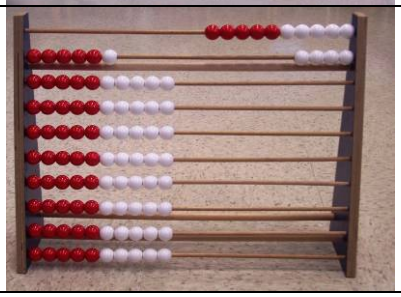
Name	Picture	Description
Area Model	To model 12 × 23: 	• Use base ten blocks to represent the parts of each number that is being multiplied. • To find the answer for the example shown, students can add the various parts of the model: 200 + 30 + 40 + 6 = 276. • This model can also be used for fraction multiplication.
Arrays and Open Arrays	To model 4 × 6:  To model 7 × 36: 	• Use counters arranged in equal rows or columns or a Blackline Master with rows and columns of dots. • Helpful in developing understanding of multiplication facts. • Grids can also be used to model arrays. • Open arrays allows students to think in amounts that are comfortable for them and does not lock them into thinking using a specific amount. These arrays help visualize repeated addition and partitioning and ultimately using the distributive property.
Attribute Blocks		• Sets of blocks that vary in their attributes: o 5 shapes circle, triangle, square, hexagon, rectangle o 2 thicknesses o 2 sizes o 3 colours
Balance (pan or beam) scales		• Available in a variety of styles and precision. • Pan balances have a pan or platform on each side to compare two unknown amounts or represent equality. Weights can be used on one side to measure in standard units. • Beam balances have parallel beams with a piece that is moved on each beam to determine the mass of the object on the scale. Offer greater accuracy than a pan balance.

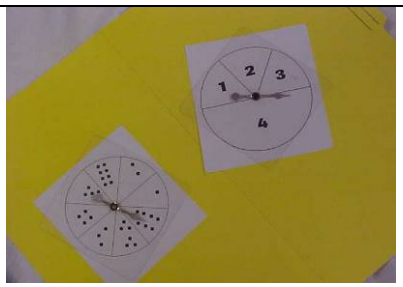
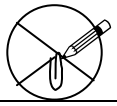
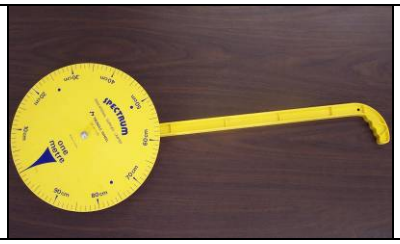
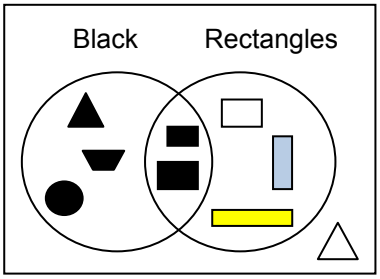
Base Ten Blocks		• Include unit cubes, rods, flats, and large cubes. • Available in a variety of colours and materials (plastic, wood, foam). • Usually 3-D.
Carroll Diagram	<table><tr><td></td><td>African</td><td>Asian</td></tr><tr><td>F</td><td>3600 kg</td><td>2720 kg</td></tr><tr><td>M</td><td>5500 kg</td><td>4990 kg</td></tr></table>	• Used for classification of different attributes. • The table shows the four possible combinations for the two attributes. • Similar to a Venn Diagram
Colour Tiles		• Square tiles in 4 colours (red, yellow, green, blue). • Available in a variety of materials (plastic, wood, foam).
Counters (two colour)		• Counters have a different colour on each side. • Available in a variety of colour combinations, but usually are red & white or red & yellow. • Available in different shapes (circles, squares, beans).
Cubes (Linking)		• Set of interlocking 2 cm cubes. • Most connect on all sides. • Available in a wide variety of colours (usually 10 colours in each set). • Brand names include: Multilink, Hex-a-Link, Cube-A-Link. • Some types only connect on two sides (brand name example: Unifix).
Cuisenaire Rods®		• Set includes 10 different colours of rods. • Each colour represents a different length and can represent different number values or units of measurement. • Usual set includes 74 rods (22 white, 12 red, 10 light green, 6 purple, 4 yellow, 4 dark green, 4 black, 4 brown, 4 blue, 4 orange). • Available in plastic or wood.

Dice (Number Cubes)		• Standard type is a cube with numbers or dots from 1 to 6 (number cubes). • Cubes can have different symbols or words. • Also available in: o 4-sided (tetrahedral dice) o 8-sided (octahedral dice) o 10-sided (decahedra dice) o 12-sided, 20-sided, and higher o Place value dice
Dominoes		• Rectangular tiles divided in two-halves. • Each half shows a number of dots: 0 to 6 or 0 to 9. • Sets include tiles with all the possible number combinations for that set. • Double-six sets include 28 dominoes. • Double-nine sets include 56 dominoes.
Dot Cards		• Sets of cards that display different number of dots (1 to 10) in a variety of arrangements. • Available as free Blackline Master online on the "Teaching Student-Centered Mathematics K-3" website (BLM 3-8).
Decimal Squares®		• Tenths and hundredths grids that are manufactured with parts of the grids shaded. • Can substitute a Blackline Master and create your own class set.
Fraction Blocks		• Also known as Fraction Pattern blocks. • 4 types available: pink "double hexagon", black chevron, brown trapezoid, and purple triangle. • Use with basic pattern blocks to help study a wider range of denominators and fraction computation.
Fraction Circles		• Sets can include these fraction pieces: $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}$ • Each fraction graduation has its own colour. • It is helpful to use ones without the fractions marked on the pieces for greater flexibility (using different piece to represent 1 whole).

Fraction Pieces		• Rectangular pieces that can be used to represent the following fractions: $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}$ • Offers more flexibility as different pieces can be used to represent 1 whole. • Each fraction graduation has its own colour. • Sets available in different quantities of pieces.
Five Frames **Ten Frames**		• Available as a Blackline Master in many resources or you can create your own. • Use with any type of counter to fill in the frame as needed.
Geoboards		• Available in a variety of sizes and styles. o 5 × 5 pins o 11 × 11 pins o Circular 24 pin o Isometric • Clear plastic models can be used by teachers and students on an overhead. • Some models can be linked to increase the size of the grid.
Geometric Solids		• Sets typically include a variety of prisms, pyramids, cones, cylinders, and spheres. • The number of pieces in a set will vary. • Available in different materials (wood, plastic, foam) and different sizes.
Geo-strips		• Plastic strips that can be fastened together with brass fasteners to form a variety of angles and geometric shapes. • Strips come in 5 different lengths. Each length is a different colour.
Hundred Chart	 Name: _____ Date: _____ Line Master 3 **100-Chart** 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100	• 10 × 10 grid filled in with numbers 1-100 or 0 - 99. • Available as a Blackline Master in many resources or you can create your own. • Also available as wall charts or “Pocket” charts where cards with the numbers can be inserted or removed.

Hundred Grid		• 10 × 10 grid. • Available as Blackline Master in many resources.
Hundredths Circle		• Also known as “percent circles”. • Two circles can be cut out on different coloured card stock and overlapped to represent tenths and hundredths. 
Mira®		• Clear red plastic with a bevelled edge that projects reflected image on the other side. • Other brand names include: Reflect-View and Math-Vu™.
Number Lines (standard, open, and double)		• Number lines can begin at 0 or extend in both directions. • Open number lines do not include pre-marked numbers or divisions. Students place these as needed. • Double number lines have numbers written above and below the line to show equivalence.

Pattern Blocks		• Standard set includes: Yellow hexagons, red trapezoids, blue parallelograms, green triangles, orange squares, beige parallelograms. • Available in a variety of materials (wood, plastic, foam).
Pentominoes		• Set includes 12 unique polygons. • Each is composed of 5 squares which share at least one side. • Available in 2-D and 3-D in a variety of colours.
Polydron		• Geometric pieces snap together to build various geometric solids as well as their nets. • Pieces are available in a variety of shapes, colours, and sizes: Equilateral triangles, isosceles triangles, right-angle triangles, squares, rectangles, pentagons, hexagons • Also available as Frameworks (open centres) that work with Polydrons and another brand called G-O-Frames™.
Power Polygons™		• Set includes the 6 basic pattern block shapes plus 9 related shapes. • Shapes are identified by letter and colour.
Math Rack (Rekenrek®)		• Counting frame that has 10 beads on each bar: 5 white and 5 red. • Available with different number of bars (1, 2, or 10).

Spinners		• Create your own or use manufactured ones that are available in a wide variety: o number of sections; o colours or numbers; o different size sections; o blank. • Simple and effective version can be made with a pencil held at the centre of the spinner with a paperclip as the part that spins. 
Tangrams		• Set of 7 shapes (commonly plastic): o 2 large right-angle triangles o 1 medium right-angle triangle o 2 small right-angle triangles o 1 parallelogram o 1 square • 7-pieces form a square as well as a number of other shapes. • Templates also available to make sets.
Trundle Wheel		• Tool for measuring longer distances. • Each revolution equals 1 metre usually noted with a click.
Venn Diagram		• Used for classification of different attributes. • Can be one, two, or three circles depending on the number of attributes being considered. • Attributes that are common to each group are placed in the interlocking section. • Attributes that don't belong are placed outside of the circle(s), but inside the rectangle. • Be sure to draw a rectangle around the circle(s) to show the "universe" of all items being sorted. • Similar to a Carroll Diagram.

List of Grade 3 Specific Curriculum Outcomes

Number (N)

N1 Say the number sequence forward and backward from 0 to 1000 by: 5s, 10s, or 100s, using any starting point; 3s using starting points that are multiples of 3; 4s using starting points that are multiples of 4; 25s, using starting points that are multiples of 25.
N2 Represent and describe numbers to 1000, concretely, pictorially and symbolically.
N3 Compare and order numbers to 1000.
N4 Estimate quantities less than 1000 using referents.
N5 Illustrate, concretely & pictorially, the meaning of place value for numerals to 1000
N6 Describe and apply mental mathematics strategies for adding two 2-digit numerals.
N7 Describe and apply mental mathematics strategies for subtracting two 2-digit numerals.
N8 Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context.
N9 Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2 and 3-digit numerals).
N10 Apply mental mathematics strategies and number properties, such as: using doubles; making 10; using the commutative property; using the property of zero; thinking addition for subtraction to determine answers for basic addition facts and related subtraction facts (to 18).
N11 Demonstrate an understanding of multiplication to products of 36 with single digit factors.
N12 Demonstrate an understanding of division (related multiplication to products of 36 with single digit factors).
N13 Demonstrate an understanding of fractions by: explaining that a fraction represents a part of a whole; describing situations in which fractions are used; comparing fractions of the same whole with like denominators.

Patterns & Relations (PR)

(Patterns)

PR1 Demonstrate an understanding of increasing patterns by: describing, extending, comparing, creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).
PR2 Demonstrate an understanding of decreasing patterns by: describing, extending, comparing, creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).

(Variables and Equations)

PR3 Solve one-step addition and subtraction equations involving symbols representing an unknown number.

Shape and Space (SS)

(Measurement)

SS1 Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years).
SS2 Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem solving context.
SS3 Demonstrate an understanding of measuring length (cm, m) by: selecting and justifying referents for the units cm and m; modeling and describing the relationship between the units cm and m; estimating length using referents; measuring and recording length, width and height.
SS4 Demonstrate an understanding of measuring mass (g, kg).

(3-D Objects and 2-D Shapes)

SS5 Demonstrate an understanding of perimeter of regular and irregular shapes.
SS6 Describe 3-D objects according to the shape of the faces, and the number of edges and vertices.
SS7 Sort regular and irregular polygons, including: triangles, quadrilaterals, pentagons, hexagons, octagons, according to the number of sides.

Statistics and Probability (SP)

(Data Anaylsis)

SP1 Collect first-hand data and organize it using: tally marks, line plots, charts, lists to answer questions.
SP2 Construct, label and interpret bar graphs to solve problems.

Correlation of Grade 3 SCOs to *Math Makes Sense 3*

Number

General Outcome: Develop number sense

Grade 3 Specific Curriculum Outcomes	*Mathematics Makes Sense 3*
N1. Say the number sequence forward and backward from 0 to 1000 by: • 5s, 10s or 100s using any starting point • 3s using starting points that are multiples of 3 • 4s using starting points that are multiples of 4 • 25s using starting points that are multiples of 25.	Unit 1, Lesson 4, pp. 15–17 Unit 1, Lesson 8, pp. 28–31 Unit 2, Lesson 1, pp. 38–41 Unit 2, Lesson 6, pp. 54–57 Unit 2, Lesson 7, pp. 58–61 Unit 2, Lesson 9, pp. 65–67 Unit 2, Unit Problem, pp. 78, 79
N2. Represent and describe numbers to 1000, concretely, pictorially and symbolically.	Unit 2, Lesson 3, pp. 45–47 Unit 2, Lesson 4, pp. 48, 49 Unit 2, Lesson 8, pp. 62–64 Unit 2, Lesson 11, pp. 72–74 Unit 2, Unit Problem, pp. 78, 79 Unit 7, Lesson 6, pp. 260, 261
N3. Compare and order numbers to 1000.	Unit 2, Lesson 5, pp. 50–53
N4. Estimate quantities less than 1000 using referents.	Unit 2, Lesson 10, pp. 68–71 Unit 2, Unit Problem, pp. 78, 79
N5. Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.	Unit 2, Lesson 2, pp. 42–44 Unit 7, Lesson 6, pp. 260, 261
N6. Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as: • adding from left to right • taking one addend to the nearest multiple of ten and then compensating • using doubles.	Unit 3, Lesson 5, pp. 96–99 Unit 3, Lesson 6, pp. 100, 101 Unit 3, Lesson 13, pp. 124, 125 Unit 3, Unit Problem, pp. 128, 129
N7. Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as: • taking the subtrahend to the nearest multiple of ten and then compensating • thinking of addition • using doubles.	Unit 3, Lesson 9, pp. 110–113 Unit 3, Lesson 10, pp. 114–115 Unit 3, Unit Problem, pp. 128, 129
N8. Applying estimation strategies to predict sums and differences of two 2-digit numerals in a problem-solving context.	Unit 3, Lesson 4, pp. 93–95 Unit 3, Lesson 5, pp. 96–99 Unit 3, Lesson 8, pp. 107–109 Unit 3, Lesson 9, pp. 110–113
N9. Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2, and 3-digit numerals) by: • using personal strategies for adding and subtracting with and without the support of manipulatives • creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially and symbolically.	Unit 3, Lesson 1, pp. 82–85 Unit 3, Lesson 2, pp. 86–88 Unit 3, Lesson 5, pp. 96–99 Unit 3, Lesson 6, pp. 100, 101 Unit 3, Lesson 7, pp. 102–105 Unit 3, Lesson 9, pp. 110–113 Unit 3, Lesson 10, pp. 114, 115 Unit 3, Lesson 11, pp. 116–119 Unit 3, Lesson 12, pp. 120–123 Unit 3, Lesson 13, pp. 124, 125 Unit 3, Unit Problem, pp. 128, 129

Grade 3 Specific Curriculum Outcomes	*Mathematics Makes Sense 3*
N10. Apply mental mathematics strategies and number properties, such as: • using doubles • making 10 • using the commutative property • using the property of zero • thinking addition for subtraction to recall basic addition facts to 18 and related subtraction facts.	Unit 3, Lesson 1, pp. 82–85 Unit 3, Lesson 2, pp. 86–88
N11. Demonstrate an understanding of multiplication to products of 36 with single digit factors by: • representing and explaining multiplication using equal grouping and arrays • creating and solving problems in context that involve multiplication • modelling multiplication using concrete and visual representations, and recording the process symbolically • relating multiplication to repeated addition • relating multiplication to division	Unit 8, Lesson 1, pp. 268–271 Unit 8, Lesson 2, pp. 273–275 Unit 8, Lesson 3, pp. 276–279 Unit 8, Lesson 4, pp. 280–282 Unit 8, Lesson 8, pp. 294–296 Unit 8, Lesson 9, pp. 297–299 Unit 8, Lesson 10, pp. 300, 301 Unit 8, Unit Problem, pp. 304, 305
N12. Demonstrate an understanding of division by: • representing and explaining division using equal sharing and equal grouping • creating and solving problems in context that involve equal sharing and equal grouping • modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically • relating division to repeated subtraction • relating division to multiplication. (limited to division related to multiplication facts up to products of 36 with single digit factors)	Unit 8, Lesson 5, pp. 283–286 Unit 8, Lesson 6, pp. 287–289 Unit 8, Lesson 7, pp. 290–293 Unit 8, Lesson 8, pp. 294–296 Unit 8, Lesson 9, pp. 297–299 Unit 8, Unit Problem, pp. 304, 305
N13. Demonstrate and understanding of fractions by: • explaining that a fraction represents a part of a whole • describing situations in which fractions are used • comparing fractions of the same whole with like denominators.	Unit 5, Lesson 1, pp. 182–184 Unit 5, Lesson 2, pp. 185–188 Unit 5, Lesson 3, pp. 189–192 Unit 5, Lesson 4, pp. 193–195 Unit 5, Lesson 5, pp. 197–199 Unit 5, Lesson 6, pp. 200, 201 Unit 5, Unit Problem, pp. 204, 205

Patterns and Relations (Patterns)

General Outcome: Use patterns to describe the world and solve problems.

Grade 3 Specific Curriculum Outcomes	*Mathematics Makes Sense 3*
PR1. Demonstrate an understanding of increasing patterns by: • describing • extending • comparing • creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).	Unit 1, Lesson 1, pp. 6–8 Unit 1, Lesson 2, pp. 9–11 Unit 1, Lesson 3, pp. 12–14 Unit 1, Lesson 4 , pp. 15–17 Unit 1, Lesson 5, pp. 18, 19 Unit 1, Unit Problem, pp. 34, 35

Grade 3 Specific Curriculum Outcomes	*Mathematics Makes Sense 3*
PR2. Demonstrate an understanding of decreasing patterns by: • describing • extending • comparing • creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).	Unit 1, Lesson 6, pp. 21-24 Unit 1, Lesson 7, pp. 25–27 Unit 1, Lesson 8, pp. 28–31 Unit 1, Unit Problem , pp. 34, 35

Patterns and Relations (Variables and Equations)

General Outcome: Represent algebraic expressions in multiple ways.

Grade 3 Specific Curriculum Outcomes	*Mathematics Makes Sense 3*
PR3. Solve one-step addition and subtraction equations involving symbols representing an unknown number.	Unit 3, Lesson 3, pp. 89-92 Unit 3, Unit Problem, pp. 128–129

Shape and Space (Measurement)

General Outcome: Use direct or indirect measurement to solve problems.

Grade 3 Specific Curriculum Outcomes	*Mathematics Makes Sense 3*
SS1. Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years).	Unit 4, Lesson 1, pp. 134–136 Unit 4, Unit Problem, pp. 176,177
SS2. Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem-solving context.	Unit 4, Lesson 2, pp. 137–140 Unit 4, Lesson 3, pp. 141-144
SS3. Demonstrate an understanding of measuring length (cm, m) by: • selecting and justifying referents for the units cm and m • modelling and describing the relationship between the units cm and m • estimating length using referents • measuring and recording length, width and height.	Unit 4, Lesson 4, pp. 145–148 Unit 4, Lesson 5, pp. 149–152 Unit 4, Lesson 6, pp. 154–157 Unit 4, Lesson 7, pp. 158, 159 Unit 4, Unit Problem, pp. 176, 177 Unit 7, Lesson 1, pp. 240–243
SS4. Demonstrate an understanding of measuring mass (g, kg) by: • selecting and justifying referents for the units g and kg • modelling and describing the relationship between the units g and kg • estimating mass using referents • measuring and recording mass.	Unit 4, Lesson 11, pp. 169–170 Unit 4, Lesson 12, pp. 171-173 Unit 4, Unit Problem, pp. 176, 177
SS5. Demonstrate an understanding of perimeter of regular and irregular shapes by: • estimating perimeter using referents for centimetre or metre • measuring and recording perimeter (cm, m) • constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter.	Unit 4, Lesson 8, pp. 160-163 Unit 4, Lesson 9, pp. 164-166 Unit 4, Lesson 10, pp. 167, 168 Unit 4, Unit Problem, pp. 176, 177

Shape and Space (3-D Objects and 2-D Shapes)

General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Grade 3 Specific Curriculum Outcomes	*Mathematics Makes Sense 3*
SS6. Describe 3-D objects according to the shape of the faces, and the number of edges and vertices.	Unit 6, Lesson 4, pp. 218–221 Unit 6, Lesson 5, pp. 222-224 Unit 6, Lesson 6, pp. 225-227 Unit 6, Lesson 7, pp. 229-231 Unit 6, Unit Problem, pp. 234, 235
SS7. Sort regular and irregular polygons, including: • triangles • quadrilaterals • pentagons • hexagons • octagons according to the number of sides.	Unit 6, Lesson 1, pp. 208-211 Unit 6, Lesson 2, pp. 212-215 Unit 6, Lesson 3, pp. 216, 217 Unit 6, Unit Problem, pp. 234, 235

Statistics and Probability (Data Analysis)

General Outcome: Collect, display and analyze data to solve problems.

Grade 3 Specific Curriculum Outcomes	*Mathematics Makes Sense 3*
SP1. Collect first-hand data and organize it using: • tally marks • line plots • charts • lists to answer questions.	Unit 7, Lesson 1, pp. 240–243 Unit 7, Lesson 2, pp. 244-247 Unit 7, Lesson 5, pp. 256-258 Unit 7, Lesson 6, pp. 260-261 Unit 7, Unit Problem, pp. 264, 265
SP2. Construct, label and interpret bar graphs to solve problems.	Unit 7, Lesson 3, pp. 248-251 Unit 7, Lesson 4, pp. 252-255 Unit 7, Lesson 5, pp. 256, 258

Grade 3 Specific Curriculum Outcomes: Table of Specifications (DRAFT)

Content Strands	Level 1	Level 2	Level 3
Number Strand – 48%			
Number	N1, N2, N3, N6, N7	N1, N2, N5, N6, N7, N8, N9, N11, N12, N13	N4, N9, N11, N13
Patterns and Relations Strand- 15%			
Patterns		PR1, PR2	
Variables and Equations	PR3		
Shape and Space Strand – 30%			
Measurement	SS5	SS1, SS3, SS4, SS5	
3-D Objects and 2-D Shapes	SS6	SS6, SS7	
Statistics and Probability Strand – 7%			
Data Analysis	SP1	SP2	

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