



Prince Edward Island Mathematics Curriculum

Education and Early
Childhood Development
English Programs

Mathematics

Grade 2

CURRICULUM



2009

Prince Edward Island
Department of Education and
Early Childhood Development
PO Box 2000
Charlottetown, Prince Edward Island
Canada, C1A 7N8
Tel. (902) 368-4600
Fax. (902) 368-4622
www.gov.pe.ca/educ/

Acknowledgements

The Prince Edward Island Department of Education and Early Childhood Development gratefully acknowledges the contributions of the following groups and individuals toward the development of the *Prince Edward Island Grade 2 Mathematics Curriculum Guide*:

- The following specialists from the Prince Edward Island Department of Education and Early Childhood Development:

Eric Arseneault,
Sec. French Math/Science Specialist,
Department of Education and
Early Childhood Development

Jonathan Hayes,
Secondary Science Specialist,
Department of Education and
Early Childhood Development

J. Blaine Bernard,
Secondary Mathematics Specialist,
Department of Education and
Early Childhood Development

Bill MacIntyre,
Elementary Mathematics Specialist,
Department of Education and
Early Childhood Development

- The Western and Northern Canadian Protocol (WNCP) for Collaboration in Education
- The Alberta Department of Education
- The New Brunswick Department of Education

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Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base. Developers have examined the curriculum proposed throughout Canada and secured the latest research in the teaching of mathematics, and the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for K-9 Mathematics* (2006) has been adopted as the basis for a revised mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories, and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP, and on the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➤ Essential Graduation Learnings

Essential graduation learnings (EGLs) are statements describing the knowledge, skills, and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills, and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work, and study today and in the future. Essential graduation learnings are cross curricular, and curriculum in all subject areas is focussed to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will demonstrate knowledge, skills, and attitudes expressed as essential graduation learnings, and will be expected to

- respond with critical awareness to various forms of the arts, and be able to express themselves through the arts;
- assess social, cultural, economic, and environmental interdependence in a local and global context;
- use the listening, viewing, speaking, and writing modes of language(s), and mathematical and scientific concepts and symbols, to think, learn, and communicate effectively;
- continue to learn and to pursue an active, healthy lifestyle;
- use the strategies and processes needed to solve a wide variety of problems, including those requiring language and mathematical and scientific concepts;
- use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

➤ Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate to all educational partners high expectations for students in mathematics education. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to

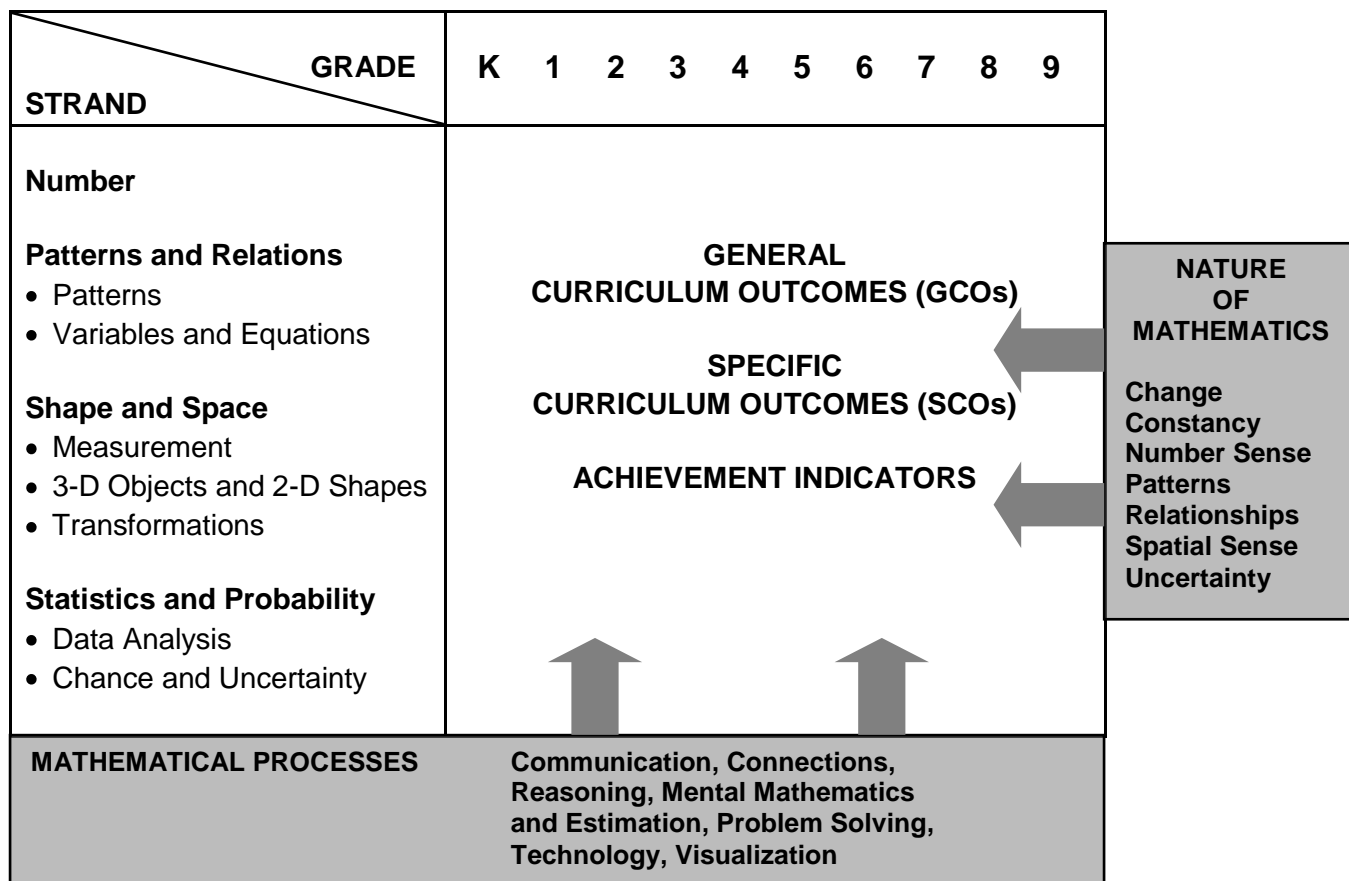
- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning;
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks;
- exhibit curiosity.

Conceptual Framework for K-9 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into four strands, namely Number, Patterns and Relations, Shape and Space, and Statistics and Probability. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connections among concepts both within and across strands. Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- Decreasing emphasis on rote calculation, drill, and practice, and the size of numbers used in paper and pencil calculations makes more time available for concept development.
- Problem solving, reasoning, and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➤ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]**
- develop visualization skills to assist in processing information, making connections, and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

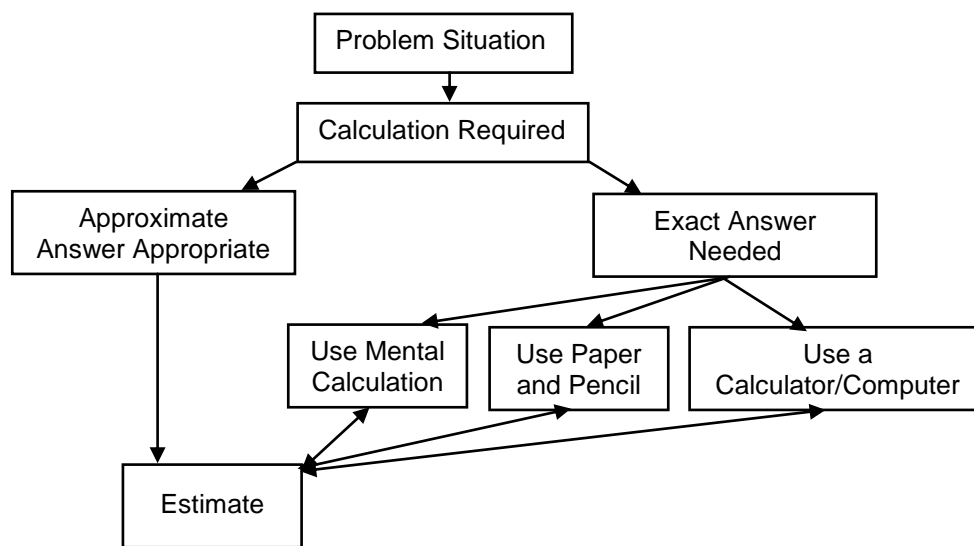
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves calculation without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need - more than ever before - with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein, 2001). Mental mathematics “provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers” (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know when to estimate, what strategy to use, and how to use it. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you. . . ?” or “How could you. . . ?” the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not

a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- using estimation
- guessing and checking
- looking for a pattern
- making an organized list or table
- using a model
- working backwards
- using a formula
- using a graph, diagram, or flow chart
- solving a simpler problem
- using algebra.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations;
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K-3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform, and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and knowledge of several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

➤ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics which are woven throughout this document. These components include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, ... can be described as

- skip counting by 2s, starting from 4;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180° .
- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (*The Primary Program*, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, and flexible and intuitive with numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collecting and analysing of data, and describing relationships visually, symbolically, orally, or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to interpret representations of 2-D shapes and 3-D objects, and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 2-D shapes and 3-D objects.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to use dimensions and make predictions about the results of changing dimensions.

- Knowing the dimensions of an object enables students to communicate about the object and create representations.
- The volume of a rectangular solid can be calculated from given dimensions.
- Doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of

probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Young children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Their mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Initial problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➤ **Connections across the Curriculum**

There are many possibilities for connecting Grade 2 mathematical learning with the learning occurring in other subject areas. Making connections between subject areas gives students experiences with transferring knowledge and provides rich contexts in which students are able to initiate, make sense of, and extend their learnings. When connections between subject areas are made, the possibilities for transdisciplinary inquiries and deeper understanding arise. When making such connections, however, teachers must be cautious not to lose the integrity of the learning in any of the subjects.

➤ **Homework**

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding, and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should replace some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill-oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a teacher can give a parent clearer understanding of the mathematics curriculum and of the child's progress in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

➤ **Diversity in Student Needs**

Every class has students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters, but should be designed to help all students (whether strong, weak or average) to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson from which all students come away with a better understanding of what the solution to an equation really means.

➤ Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean not only that enrolments of students of both genders and various cultural backgrounds in public school mathematics courses should reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➤ Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English deficiencies and cultural differences must not be barriers to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and co-ordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication “as an essential part of mathematics and mathematics education (p.60).” The *Standards* elaborate that all students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading, and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers and will facilitate “communicating to learn mathematics and learning to communicate mathematically” (NCTM, p.60).

To this end,

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counsellors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not on other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated with appropriate language support to both students and parents;
- to verify that barriers have been removed, educators should monitor enrolment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➤ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development - such as poverty alleviation, human rights, health, environmental protection, and climate change - into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental, and economic perspective, and exploring how those factors are interrelated and interdependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database

Resources for Rethinking, found at <http://r4r.ca/en>. It provides teachers with access to materials that integrate ecological, social, and economic spheres through active, relevant, interdisciplinary learning.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, or teaching has been effective, or how best to address student learning needs. The quality of the assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others.

➤ Assessment

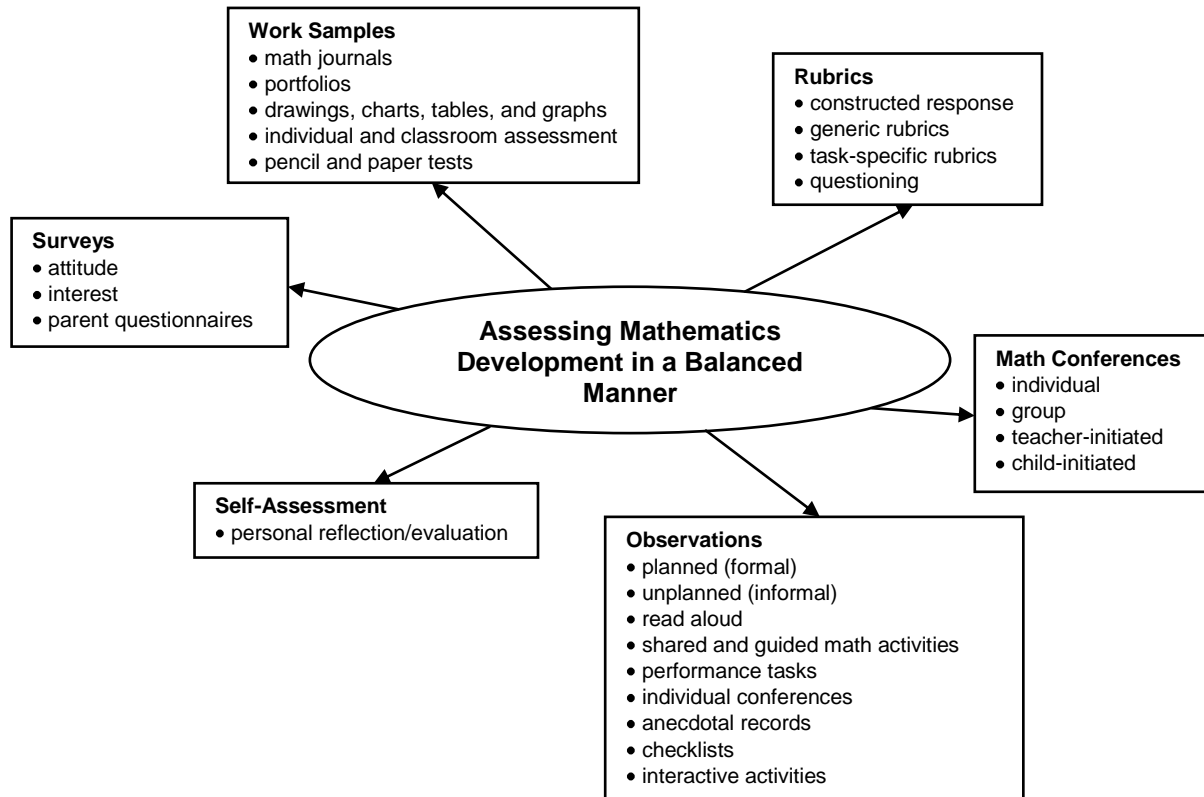
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children's learning;
- providing information to teachers on the effectiveness of their teaching, the program, and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources, including

- | | |
|------------------------------------|------------------------------|
| • formal and informal observations | • portfolios |
| • work samples | • learning journals |
| • anecdotal records | • questioning |
| • conferences | • performance assessment |
| • teacher-made and other tests | • peer- and self-assessment. |

This balanced approach for assessing mathematics development is illustrated in the diagram below.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning, and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used

- to engage students in their own learning and self-assessment;
- to help students understand what is important in the mathematical concepts and particular tasks they encounter;
- to develop effective habits of metacognition and self-coaching;
- to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used

- to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
- to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
- to provide feedback to students about how they are doing and how they might improve;

- to provide feedback to other professionals and to parents about how to support students' learning.

Assessment of learning is used

- to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
- to facilitate reporting;
- to provide the basis for sound decision-making about next steps in a student's learning.

➤ Evaluation

Evaluation is the process of analysing, reflecting upon, and summarizing assessment information, and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires

- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information;
- using a high level of professional judgment in making decisions based upon that information.

➤ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes, and phone calls.

➤ Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration, and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles, as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes, or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.

- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.
- Assessment reports should be clear, accurate, and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that

- the best interests of the student are paramount;
- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes;
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he/she knows and can do.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island mathematics curriculum are organized into four strands across the grades K-9. They are Number, Patterns and Relations, Shape and Space, and Statistics and Probability. These strands are further subdivided into sub-strands, which are the general curriculum outcomes (GCOs). They are overarching statements about what students are expected to learn in each strand or sub-strand from grades K-9.

Strand	General Curriculum Outcome (GCO)
Number (N)	Number: Develop number sense.
Patterns and Relations (PR)	Patterns: Use patterns to describe the world and solve problems.
	Variables and Equations: Represent algebraic expressions in multiple ways.
Shape and Space (SS)	Measurement: Use direct and indirect measure to solve problems.
	3-D Objects and 2-D Shapes: Describe the characteristics of 3-D objects and 2-D shapes, and analyse the relationships among them.
	Transformations: Describe and analyse position and motion of objects and shapes.
Statistics and Probability (SP)	Data Analysis: Collect, display, and analyse data to solve problems.
	Chance and Uncertainty: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific outcome.

The first two pages for each outcome contain the following information:

- the corresponding **strand** and **general curriculum outcome**
- the **Specific Curriculum Outcome(s)** and the mathematical **processes** which link this content to instructional methodology
- the **scope and sequence** of concept development related to this outcome(s) from K - 2
- an **elaboration** of the outcome
- a list of **achievement indicators**

Students who have achieved a particular outcome should be able to demonstrate their understanding in the manner specified by the achievement indicators. It is important to remember, however, that these indicators are not intended to be an exhaustive list for each outcome. Teachers may choose to use additional indicators as evidence that the desired learning has been achieved.

The last two pages for each outcome contain lists of **instructional strategies** and **strategies for assessment**.

The primary use of this section of the guide is as an **assessment for learning** (formative assessment) tool to assist teachers in planning instruction to improve learning. However, teachers may also find the ideas and suggestions useful in gathering **assessment of learning** (summative assessment) data to provide information on student achievement.

NUMBER

SCO: **N1: Say the number sequence, 0 to 100, by:**

- 2s, 5s and 10s, forward and backward, using starting points that are multiples of 2, 5 and 10 respectively
- 10s using starting points from 1 to 9
- 2s starting from 1.

[C, CN, ME, R]

<p>[C] Communication [T] Technology</p>	<p>[PS] Problem Solving [V] Visualization</p>	<p>[CN] Connections [R] Reasoning</p>	<p>[ME] Mental Math/Estimation</p>
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Scope and Sequence

Grade One	Grade Two	Grade Three
<p>N1 Say the number sequence, 0 to 100, by:</p> <ul style="list-style-type: none"> • 1s forward and backward between any two given numbers • 2s to 20, forward starting at 0 • 5s and 10s to 100, forward starting at 0 	<p>N1 Say the number sequence from 0 to 100 by:</p> <ul style="list-style-type: none"> • 2s, 5s and 10s, forward and backward, using starting points that are multiples of 2, 5 and 10 respectively • 10s using starting points from 1 to 9 • 2s starting from 1. 	<p>N1 Say the number sequence forward and backward from 0 to 1000 by:</p> <ul style="list-style-type: none"> • 5s, 10s, or 100s, using any starting point • 3s using starting points that are multiples of 3 • 4s using starting points that are multiples of 4 • 25s, using starting points that are multiples of 25

Elaboration

Students are continuing to develop an understanding of **number** and **counting**. A wide range of activities, presented regularly throughout the year, will foster student development of number sense and number concepts. Students will extend their experience with **counting patterns** to 100.

Students will develop the following **skip counting** skills:

- counting by 2s, 5s, 10s backwards and forwards (starting from multiples of 2, 5, and 10)
- counting using coins (pennies, nickels, dimes)
- counting by 10s using starting points from 1 to 9
- counting by 2s starting from 1

Students will be able to count a collection of coins (e.g., for 2 dimes, 3 nickels, and a penny count, “10, 20, 25, 30, 35, 36”). Before students are able to do this, they need sufficient experience counting pennies, nickels, and dimes separately. As well, they should come to recognize that to find the total for a collection of coins, sorting them and counting the larger coins first is easiest.

SCO: N1: Say the number sequence, 0 to 100, by:

- 2s, 5s and 10s, forward and backward, using starting points that are multiples of 2, 5 and 10 respectively
- 10s using starting points from 1 to 9
- 2s starting from 1.

[C, CN, ME, R]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Extend a given skip counting sequence (by 2s, 5s or 10s) forward and backward.
- Skip count by 10s, given any number from 1 to 9 as a starting point.
- Identify and correct errors and omissions in a given skip counting sequence.
- Count a given sum of money with pennies, nickels or dimes (to 100¢).
- Count quantity using groups of 2s, 5s or 10s and counting on.

SCO: N1: Say the number sequence, 0 to 100, by:

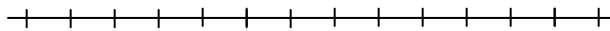
- 2s, 5s and 10s, forward and backward, using starting points that are multiples of 2, 5 and 10 respectively
- 10s using starting points from 1 to 9
- 2s starting from 1.

[C, CN, ME, R]

Instructional Strategies

Consider the following strategies when planning lessons:

- Include rhythmic skip counting activities such as clapping, marching, drumming with hands on the desk, and striking instruments.
- Use a walk-on number line or an open number line to experience skip counting.



- Use a hundred chart as a tool to explore counting patterns.
- Use the repeat (constant) function (press 0, +, 2, =, =, =, ...) on the calculator to skip count to a target number. For example, if you start at 0 and want to end at 40, by which number(s) could you skip count? (2, 5, 10) What if you started at a different point? What if you wanted to end at a different point?
- Use a variety of problems including open-ended (more than one solution). For example, tell students that you have some coins that total 61 cents. Ask: What could they be?
- Provide students with a variety of authentic activities that require students to recognize when it is more efficient to skip count quantities.

Suggested Activities

- Have students use their fingers to move counters in pairs as they count by 2s.
- Use number patterns to encourage skip counting; for example:
 - 25, 35, _ , _ , 65, _ , _
 - 65, _ , 55, _ , 45, _
 - 1, 3, _ , 7, _ , _ , 13, _ , _
- Have students correct the following counting by 2s sequence: 2, 4, 6, 7, 8, 10, 11
- Begin to count, “10, 20, 30, 40.” Ask the student to continue to count to 100 by 10s.
- Give the student 3 dimes, 2 nickels and 6 pennies. Ask him/her to count the coins. Do they have enough money to buy an item costing 90¢?
- Provide coins for the student. Ask: Can you use 6 of these coins to make 43 cents?
- Provide a student with a number of beans (60, for example). Ask the student to separate the beans from the pile as he/she counts them by groups. Have them discuss which method is the most efficient way of counting the beans.
- Use the calculator repeat function to count by 5s. Ask the student to predict each number before it appears on the display.

SCO: N1: Say the number sequence, 0 to 100, by:

- **2s, 5s and 10s, forward and backward, using starting points that are multiples of 2, 5 and 10 respectively**
- **10s using starting points from 1 to 9**
- **2s starting from 1.**

[C, CN, ME, R]

Assessment Strategies

- Have students count a collection of beans (or other counters). Ask them to explain their method of counting (e.g., by 2s, 5s, 10s).
- Ask students to count by 2s (5s or 10s) as you clap. Have students tell you or record the final number when you finish clapping.
- Show students a sequence with an error or missing number. Have students identify and correct the sequence.
- Ask the student to count backwards by 2s starting at 40.
- Ask the student: "If you count by 10's starting at 7, will you say 35?" Have them explain why or why not.
- Tell students that you have some coins in your hand that total 84 cents. Ask them to record what combinations of coins are possible.

SCO: **N2: Demonstrate if a number (up to 100) is even or odd.**
 [C, CN, PS, R]

[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation
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Scope and Sequence

Grade One	Grade Two	Grade Three
	N2 Demonstrate if a number (up to 100) is even or odd.	

Elaboration

Students learn that **even numbers** are the numbers they say when they count by 2s from 0. **Odd numbers** are the numbers they say when they count by 2s from 1.

It is important to provide students with a variety of concrete and pictorial representations of even and odd numbers to give students the experience they need to understand **“even-ness”** and **“odd-ness”**.

Students will know that:

- even numbers can be used to make groups of two (or two equal groups) with none leftover;
- odd numbers can be used to make groups of two (or two equal groups) with one left over.

SCO: **N2: Demonstrate if a number (up to 100) is even or odd.**
[C, CN, PS, R]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

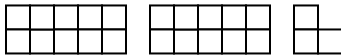
- Use concrete materials or pictorial representations to determine if a given number is even or odd.
- Identify even and odd numbers in a given sequence, such as in a hundred chart.
- Sort a given set of numbers into even and odd.

SCO: N2: Demonstrate if a number (up to 100) is even or odd.
[C, CN, PS, R]

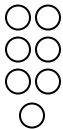
Instructional Strategies

Consider the following strategies when planning lessons:

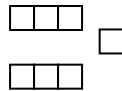
- Use hundred charts and number lines to show even and odd number patterns.
- Use children's literature such as, *Even Stephen, Odd Todd* by K. Cristaldi and H. Morehouse
- Include discussions as part of the daily routine that relate to even and odd numbers (e.g., Do we have an even number of people here today?)
- Clarify misconceptions regarding numbers with both an even and odd digit (e.g., 23) by representing the number with ten frames or square tiles arranged as shown below.



- Use concrete models to represent numbers to demonstrate their evenness or oddness.



7 (groups of 2 with one left over)



7 (2 equal groups with one left over)

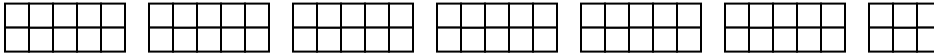
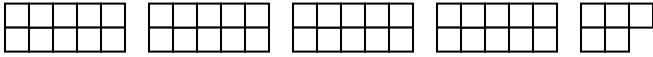
Suggested Activities

- Ask students to use sets of colored tiles to determine whether the number of tiles is even or odd.
- Show a set of counters on the overhead. Ask students to use ten frames to determine whether the set of counters shown is odd or even by placing counters on their ten frame. Have students explain their thinking.
- Work as a class to represent numbers with counters to determine their evenness or oddness. Then color the corresponding numeral on the hundred chart to begin creating a pattern (red for even, blue for odd). Ask students to continue discovering the even/odd pattern.
- Ask students to sort the following numbers into even and odd: 9, 24, 30, 51, 77, 86.
- Ask students to solve problems, such as: Dave's class has 23 students. He is planning snacks for snack time and wants to know if he will need an even or odd number of snacks if he gives each student 1 snack? 2 snacks? 3 snacks?

SCO: N2: Demonstrate if a number (up to 100) is even or odd.
[C, CN, PS, R]

Assessment Strategies

- Ask students to tell you if the following representations are even or odd and explain why.



- Have students use a hundred chart to explain if the following numbers are even or odd:
3, 18, 37, 55, 71.
- Ask students to sort the following numbers into even and odd by using linking cubes or other materials: 11, 23, 39, 40, 48.

<p>SCO: N3: Describe order or relative position using ordinal numbers (up to tenth). [C, CN, R]</p> <p>SCO: N5: Compare and order numbers up to 100. [C, CN, R, V]</p>			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Math and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Scope and Sequence

Grade One	Grade Two	Grade Three
<p>N5: Compare sets containing up to 20 elements to solve problems using: referents; one-to-one correspondence.</p>	<p>N3: Describe order or relative position using ordinal numbers (up to tenth). N5: Compare and order numbers up to 100.</p>	<p>N3: Compare and order numbers to 1000.</p>

Elaboration

N3

Students are familiar with ordinal numbers through everyday experiences. They should be regularly used in context. The most important ideas for students to realize are that:

- position matters
- for every **cardinal number** (e.g., 9), there is an **ordinal number** (9th)
- “first” is not always fixed. It depends on point of view. For example, the circle below can be described as the first from the left and the last from the right:



N5

Students should encounter a variety of numbers in context. These contexts help them develop an understanding of number size. Students will be able to order a set of numbers in **ascending** and **descending** order. They will be able to justify their solutions using benchmarks, hundred charts, number lines, ten frames and/or place value. Visual models encourage **reasoning**, as students consider how to compare and order numbers. As with all concepts, begin with concrete models. Initially use groupable materials (e.g., beans or popsicle sticks) and then move to pre-grouped materials (e.g., base ten blocks or ten frames). Students should be given many experiences with materials before moving to more pictorial and symbolic representations.

One strategy for comparing numbers is to use **benchmarks** that are familiar to the student. For example, 48 is less than 95 since 95 is closer to 100; 37 is more than 27 since 37 is more than 30 and 27 is less than 30. An open number line (no marked increments) provides opportunity for students to refine their knowledge of number relationships.

Another strategy for comparing numbers is to consider place value. Students may refer to the number of tens when comparing numbers. For example, 47 is more than 21 since 47 is more than 4 tens, but 21 is only a bit more than 2 tens. Students should focus on the fact that the digit “4” in 47 is 40 and the digit “2” in 21 is 20.

SCO: **N3: Describe order or relative position using ordinal numbers (up to tenth).**

[C, CN, R]

SCO: **N5: Compare and order numbers up to 100.**

[C, CN, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

N3

- Indicate a position of a specific object in a sequence by using ordinal numbers up to tenth.
- Compare the ordinal position of a specific object in two different given sequences.

N5

- Order a given set of numbers in ascending or descending order and verify the result using a hundred chart, number line, ten frames or by making references to place value.
- Identify missing numbers in a given hundred chart or a given sequence.
- Identify errors in a given ordered sequence (i.e., hundred chart).
- Identify errors in a given hundred chart.

SCO: **N3: Describe order or relative position using ordinal numbers (up to tenth).**

[C, CN, R]

SCO: **N5: Compare and order numbers up to 100.**

[C, CN, R, V]

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Use calendars to provide a useful context for ordinal numbers.
- Have students observe position in line-ups. “Who is third?” “If there are 6 people in front of you, where are you in the line?” Note: Students not lined up can see these relationships more clearly than students in the line.
- Build student’s awareness of the relative size of numbers (e.g., eight is a large number when thinking of number of children in a family, but it is small when thinking of a class).
- Remember to move from concrete to pictorial to symbolic representations.

Suggested Activities

N3

- Ask a child to place a set of objects in a line by following directions given on cards. For example:

horse 3 rd

cat 1 st

cow 5 th

- Ask students to use counters to create a pattern in which the amount in the 4th position is less than the amount in the 3rd.
- Ask the student to use pattern blocks to make a row of five different shapes in which the first shape is a triangle and the third is a square. “What would you have to do to make the square fourth?”

N5

- Provide cards with 2-digit numbers, such as:

34	43	17	21
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- Ask students to arrange them in ascending order (concrete or pictorial models may be used).
- Provide 9 base ten rods and 9 unit cubes. Have the student create two different amounts, each using exactly 5 of the objects provided. Ask: “Which amount is greater than the other? How is it possible to use the same number of objects, but still have one amount greater than the other?” Student responses will indicate their level of understanding.
- Provide a teacher-made “flyer” in which prices are less than a dollar. Ask the student to circle the item that costs the most (the least, more than 50 cents, etc.).
- Ask the student to identify the errors in the following descending sequence:
95, 81, 69, 72, 46, 27, 31, 19, 10
- Have students work in small groups to list some situations in which they would rather have 22 than 28 (e.g., bowling score, race time, brussel sprouts).

SCO: **N3: Describe order or relative position using ordinal numbers (up to tenth).**

[C, CN, R]

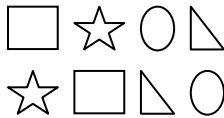
SCO: **N5: Compare and order numbers up to 100.**

[C, CN, R, V]

Assessment Strategies

N3

- Ask the student to make a “train” of 10 linking cubes in which the 3rd and 7th cars from the front of the train are different colours from the rest of the train. Ask: “What position are these cars if you are standing at the other end of the train?”
- Ask the student to name the position of the star in each row of shapes (starting from the right).



N5

- Ask students to explain why 42 is greater than 29 (may use concrete or pictorial representations).
- Have students fill in a hundred chart with missing numbers or correct a chart with errors.
- Ask: Is a number with a 7 in it always greater than a number with a 6 in it? Explain.
- Ask the students to identify the errors in the following ascending sequence and to order the numbers correctly.

7, 13, 20, 32, 28, 56, 69, 71, 44

SCO: **N4: Represent and describe numbers to 100, concretely, pictorially and symbolically.**
[C, CN, V]

[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation
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Scope and Sequence

Grade One	Grade Two	Grade Three
N4: Represent and describe numbers to 20 concretely, pictorially and symbolically.	N4: Represent and describe numbers to 100, concretely, pictorially and symbolically.	N2: Represent and describe numbers to 1000, concretely, pictorially and symbolically.

Elaboration

It is important that students have many and varied experiences with materials that they can count and group in a variety of ways. These activities should be purposeful and presented throughout the year as students build their sense of number. A deep understanding of number gives students a firm foundation for later work with place value.

Students need to represent and describe numbers in many ways. Their representations should include other concrete materials before base ten blocks (e.g., Popsicle sticks, counters, ten frames, coins) and pictorial representations (ten frames, drawings of materials, tallies). These lead to symbolic representations such as **expressions** (e.g., $38 + 4$, $50 - 10$, $12 + 12$).

Students need to use accurate language when reading and writing number symbols or words. It is important to provide students with multiple representations of numbers to help them make connections. Students may be able to write the symbols, but may not be able to connect those symbols to the words.

SCO: **N4: Represent and describe numbers to 100, concretely, pictorially and symbolically.**
[C, CN, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Represent a given number using concrete materials, such as ten frames and base ten materials.
- Represent a given number using coins (pennies, nickels, dimes and quarters).
- Represent a given number using tallies.
- Represent a given number pictorially.
- Represent a given number using expressions, e.g., $24 + 6$, $15 + 15$, $40 - 10$.
- Read a given number (0–100) in symbolic or word form.
- Record a given number (0–20) in words.

SCO: **N4: Represent and describe numbers to 100, concretely, pictorially and symbolically.**
[C, CN, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Use various representations to explore numbers. Different representations of the same number can be compared.
- Have available a class or individual chart/word wall that matches pictorial, and/or symbolic representations with the words to support students as they learn number words.
- Compare benchmark quantities of the same item (e.g., 50 counters and 10 counters) to provide a visual of the relative size of numbers.

Suggested Activities

- Give each student a different number (e.g., 25, 36, 42, 48 . . .) of counters. Ask students to arrange their counters in groups to make it easier for a classmate to count them; for example, 25 might be grouped as 5 groups of 5. Have students rotate around the room determining the number of counters each classmate has.
- Begin with the first day of school in September, and do “Number of the Day” activities. Have students express the number in as many ways as they can. For example, day 26, probably sometime in October, may be expressed as $5 + 5 + 5 + 5 + 5 + 1$; $20 + 6$; $10 + 10 + 6$; $10 + 16$; 26 ones; 2 tens and 6 ones; 1 quarter, 1 penny, etc. It is not unreasonable to expect students to express a number in many different ways.
- Have students choose a price card and represent that amount with coins (create context by setting up a class ‘store’ center or provide shopping flyers). “How many ways can you pay for one item?”
- Have students find something that comes in a set amount and then ask, “What would five packages or sets of those items look like?” This would encourage students to count by that number and reinforce outcome N1.

SCO: N4: Represent and describe numbers to 100, concretely, pictorially and symbolically.
[C, CN, V]

Assessment Strategies

- Ask students to represent 52 (or any 2-digit number) with:
 - base ten blocks
 - ten frames
 - tallies
 - coins
 - a picture
 - an expression
- Ask students, “Which of the expressions in the box represents 36?”
Have students create two more of their own representations that would also equal 36.

$$\begin{array}{cc} 30 + 6 & 28 + 8 \\ & 3 + 6 \quad 40 - 4 \\ & 20 + 26 \\ 66 - 30 & 35 + 2 \end{array}$$

- Ask students to describe a number in at least three different ways (e.g., *36 is four less than forty*).
- Ask students to use numbers, pictures, and/or physical materials to represent the number of children in their class in different ways.

SCO: **N6: Estimate quantities to 100 using referents.**
[C, ME, PS, R]

[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation
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Scope and Sequence

Grade One	Grade Two	Grade Three
N6: Estimate quantities to 20 by using referents.	N6: Estimate quantities to 100 using referents.	N4: Estimate quantities less than 1000 using referents.

Elaboration

Estimation helps students develop flexible, intuitive ideas about numbers, further developing number sense. Estimating is difficult for students; therefore, time must be spent developing an understanding of “about” without expecting a specific number.

For example:

- *More or Less than ___? “Will it be more or less than 10 steps?”*
- *Closer to ___ or to ___? “Will it be closer to 5 steps or 30 steps?”*
- *About ___? “About how many steps will it take to get to the office?”*

Referents (a known quantity) are valuable when estimating. Students use referents to determine the amount of a large group of similar objects. For example, “if I know what a group of 5 people looks like, I can estimate the number of people in a classroom.” Dot cards or similar models are helpful in providing visual referents and build estimation skills. Subitizing activities worked on in kindergarten and grade one should be continued.

Students need a strong sense of “10-ness” in order to use ten as a referent. Provide opportunities for students to see ten in a variety of different contexts and arrangements (10 people, 10 chairs, 10 counters).

SCO: N6: Estimate quantities to 100 using referents. [C, ME, PS, R]
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Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Estimate a given quantity by comparing it to a referent (known quantity).
- Estimate the number of groups of ten in a given quantity using 10 as a referent.
- Select between two possible estimates for a given quantity and explain the choice

SCO: **N6: Estimate quantities to 100 using referents.**
[C, ME, PS, R]

Instructional Strategies

Consider the following strategies when planning lessons:

- Use the same unit (e.g., footsteps) to estimate and check the attributes of a variety of things. If students are able to explore these types of tasks in succession with the same unit, they refine their estimating skills.
- Use children’s literature with an estimation focus, such as *Counting on Frank* by Rod Clement.

Suggested Activities

- Ask students to estimate how many footsteps:
 - from the door to the window
 - from the door to the top of the stairs
 - from the door to your desk
- Show 10 paperclips on the overhead for the students to have a visual reference. Then display a larger group of paperclips. Ask students to estimate how many paperclips there are. Ask, “Why do you think that?”
- Give a small group of students a series of baggies with a number of bread ties. In each baggie is also a card asking “Is it closer to __ or __?” (For example, is it closer to 20 or 50?). Bags are considered one at a time by the group, with students explaining their choice. The group may then count the quantity to determine the closer value.
- Challenge the students to guess how many times they can print their name in 1 minute. Individual considerations include: length of name or speed of printing.
- Have students draw a card that indicates how many marbles they need to get from a bucket (less than 20, between 30 and 50, about 20). They must choose which size scoop to use to get that many items. Students count to check. Variation: Have only one scoop and several buckets of different sized objects. Students have to decide which object to scoop to get their target range.

SCO: N6: Estimate quantities to 100 using referents.
[C, ME, PS, R]

Assessment Strategies

- Show students a group of items and ask them to choose between two given estimates. Have them explain their reasoning.
- Place a pile of paperclips on a desk. Ask students to estimate the number of paperclips. Observe\interview students to determine if they are using a referent. Guiding questions should include, “How did you pick that number?”
- Give the student a “train” of four linking cubes. Ask them to estimate the number of cubes in a “train” at the front of the room. Have students explain their thinking.

SCO: **N7: Illustrate, concretely and pictorially, the meaning of place value for numerals to 100.**
[C, CN, R, V]

[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation
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Scope and Sequence

Grade One	Grade Two	Grade Three
	N7: Illustrate, concretely and pictorially, the meaning of place value for numerals to 100.	N5: Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.

Elaboration

The foundation for the development of the concept of place value centers around grouping activities and developing children’s understanding of **unitizing**, the concept that ten ones equal one ten. Proportional, groupable models should be used first, that is, materials which can be put together or taken apart to make (or unmake) tens which are ten times the size of the ones. Suggested materials include Popsicle sticks which can be placed together by rubber bands in groups of 10, linking cubes which can be connected to make strips of 10, or 10 beans which can be bagged or placed in cups. **It is important that this stage not be rushed. Many problems that children later encounter with place-value concepts are believed to stem from inadequate attention to early place-value activities.** When students make that important connection between all that they know about counting by ones and the concept of grouping by tens, they should notice how much easier it is to count.

Students should proceed from the "groupable" model to a "pre-grouped" proportional model. The size of the ten model continues to be equivalent to 10 of the ones models; the difference is that the ten cannot be separated into individual ones. Examples pre-grouped models are 10 beans glued to a stick, orange Cuisenaire® rods, Rekenrek®, ten frames or base-ten rods.

Note that students easily attach words (i.e., “tens”) to both materials and groups without realizing what the materials or symbols represent if they begin working with pre-grouped models prematurely. **The big idea must be to understand the counting of groups and the units within the groups of ten as the foundation of place value.** When students understand place value they are able to think of a number, such as 37, not only as 3 tens and 7 ones, but also 2 tens and 17 ones.

SCO: **N7: Illustrate, concretely and pictorially, the meaning of place value for numerals to 100.**
[C, CN, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Explain and show with counters the meaning of each digit for a given 2-digit numeral with both digits the same, e.g., for the numeral 22, the first digit represents two tens (twenty counters) and the second digit represents two ones (two counters).
- Count the number of objects in a given set using groups of 10s and 1s, and record the result as a 2-digit numeral under the headings of 10s and 1s.
- Describe a given 2-digit numeral in at least two ways, e.g., 24 as two 10s and four 1s, twenty and four, two groups of ten and four left over, one ten and 14 ones and twenty four ones.
- Illustrate using ten frames and diagrams that a given numeral consists of a certain number of groups of ten and a certain number of ones.
- Illustrate using proportional base 10 materials that a given numeral consists of a certain number of tens and a certain number of ones.
- Explain why the value of a digit depends on its placement within a numeral.
- Represent one unit if shown a pre-grouped model representing ten (e.g., what would one look like if this is ten?).

SCO: **N7: Illustrate, concretely and pictorially, the meaning of place value for numerals to 100.**
 [C, CN, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Ensure that although physical models play a key role, they are not the concept. Students must construct the concept and relate it to the model. Using a variety of materials allows students to construct a deep understanding of place value concepts.
- Precede activities with pre-grouped models with activities using groupable models so students will know that a ten rod and ten units are the same amount. The best base ten models are proportional and can be classified as “groupable” or “pre-grouped”.

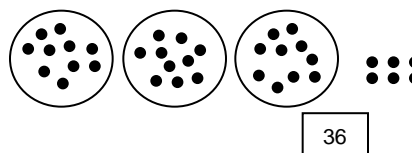
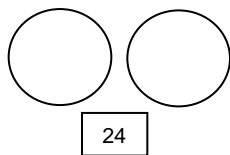
Suggested Activities

- Give the student some coffee stir sticks. Ask him/her to bundle them in groups of ten. Guiding questions could include: How many sticks are there? How is this easier than counting by ones? How do you know there are that many sticks? (Note how the student responds. Is he/she counting by groups of ten?)
- Give students many opportunities to count and sort large numbers of objects within a relevant context (e.g., take inventory of classroom objects).
- Ask the students to record the letters of the alphabet, or the names of some of their classmates, in ten-frames without leaving any spaces, as shown below. Ask: How many letters are there in all?

S	A	N	D	Y
M	A	R	K	O

A	M	Y	J	E
F	F	A	N	N

- Give students a pre-grouped model and ask if this is “ten”, then what would “one” look like (what would 3 or 17 or 85 look like)?
- Set out a number of arrangements of beans grouped by tens (in cups, on sticks, on plates, ten frames, etc.) and individual units. Also have some empty “stations” as shown below. Provide the students with a set of prepared number cards. Direct them to place the matching card in front of the appropriate display and to build the arrangements for the other cards. Use a large collection of numbers, for example, 13, 16, 18, 24, 26, 28, 33, 36, 38, and 40.

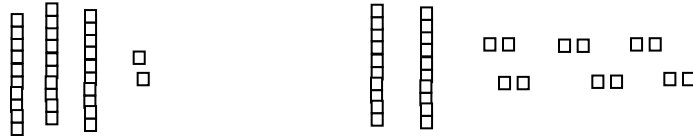


- Show students on an overhead projector a number of counters. Have the students count them. Get them to close their eyes while you change the amount by one or two. Ask the students to open their eyes and tell you how the group of counters has changed.

SCO: N7: Illustrate, concretely and pictorially, the meaning of place value for numerals to 100.
[C, CN, R, V]

Assessment Strategies

- Show a number of base-ten unit cubes (34, for example). Beside these show 5 rods and 6 unit cubes. Ask: Which has more? Is one easier to count than the other? Explain.
- Give students 2 representations, point to one and ask, “Is this number more, less or are they the same?” Have students explain their thinking.



- Ask students to pick up a handful of counters and represent the total using ten frames. Ask: “How else could you show this number?”
- Show the students a 2-digit number with both digits the same (e.g., 44). Have students model the value of each digit. Ask students to explain why these digits do not represent the same value.
- Have students illustrate their strategies for counting a large number of objects (less than 100). Observe whether students grouped by tens or used other efficient strategies.
- Give students a place value chart and a group of objects. Have them determine how many and record their results in the chart.

tens	ones

SCO: **N8: Demonstrate and explain the effect of adding zero to or subtracting zero from any number.**
[C, R]

[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation
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Scope and Sequence

Grade One	Grade Two	Grade Three
<p>N8: Identify the number, up to 20, that is one more, two more, one less and two less than a given number.</p>	<p>N8: Demonstrate and explain the effect of adding zero to or subtracting zero from any number.</p>	

Elaboration

The idea of **zero** is conceptually different from any other number. Zero cannot be connected to an actual object or represented by a concrete item. Zero indicates an absence of quantity or the quantity before the count begins. Therefore addition and subtraction with zero do not change the original value.

Children do not understand the combination of the place value columns and the use of zero because it is a digit and is used as a place “holder” rather than as a number.

SCO: N8: Demonstrate and explain the effect of adding zero to or subtracting zero from any number.
[C, R]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Add zero to a given number and explain why the sum is the same as the addend.
- Subtract zero from a given number and explain why the difference is the same as the given number.

SCO: N8: Demonstrate and explain the effect of adding zero to or subtracting zero from any number.
[C, R]

Instructional Strategies

Consider the following strategies when planning lessons:

- Develop story problems involving zero to help students understand zero in addition and subtraction.
- Model the addition and subtraction of zero frequently to develop student's understanding that addition does not always increase the quantity and subtraction does not always decrease the quantity.
- Role play activities where students receive/give away zero.

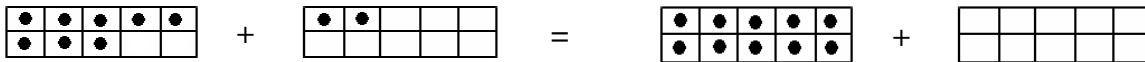
Suggested Activities

- Read children's literature such as, *A Place for Zero* by Angeline Sparagna LoPresti and discuss with students.
- Build a walk-on number line on the floor and have students take zero steps forward and zero steps backward and discuss.
- Have students predict the answer when zero is added to a number. Use the constant function on the calculator and add zero several times to a number to show the constancy of the number. Repeat with different numbers. Repeat using the subtract function. Discuss the pattern that results.
- Show students a number for a few seconds using dot patterns or your fingers. Tell students that a number has been added. Flash the sum (same as original value) and ask students to name that addend (0).
- Tell students: "After I subtracted zero I have this many (illustrate value with dot cards, fingers, etc.). How many did I start with?"
- Use a pan balance with equal amounts on each side and have the students explore what needs to be added/subtracted to keep the balance?

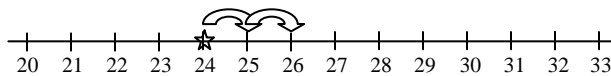
SCO: N8: Demonstrate and explain the effect of adding zero to or subtracting zero from any number.
[C, R]

Assessment Strategies

- Use ten frames to construct addition and subtraction situations for the students that require them to use a blank ten frame to complete the operation. How many counters will I need to add to make the two sides equal?



- Give students a prepared number line with a start point (e.g., 24). Ask students to show the *jumps* on the number line as you dictate to them. For example: Add two, subtract zero. Where are you now? Why?



- Have students create a problem with 0 and 36 in it.

SCO: N9: Demonstrate an understanding of addition (limited to 1 and 2-digit numerals) with answers to 100 and the corresponding subtraction by:

- **using personal strategies for adding and subtracting with and without the support of manipulatives**
- **creating and solving problems that involve addition and subtraction**
- **explaining that the order in which numbers are added does not affect the sum**
- **explaining that the order in which numbers are subtracted may affect the difference.**

[C, CN, ME, PS, R, V]

[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation
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Scope and Sequence

Grade One	Grade Two	Grade Three
<p>N9: Demonstrate an understanding of addition of numbers with answers to 20 and their corresponding subtraction facts, concretely, pictorially and symbolically by:</p> <ul style="list-style-type: none"> • <i>using familiar and mathematical language to describe additive and subtractive actions from their experience</i> • <i>creating and solving problems in context that involve addition and subtraction</i> • <i>modeling addition and subtraction using a variety of concrete and visual representations, and recording the process symbolically.</i> 	<p>N9: Demonstrate an understanding of addition (limited to 1 and 2-digit numerals) with answers to 100 and the corresponding subtraction by:</p> <ul style="list-style-type: none"> • <i>using personal strategies for adding and subtracting with and without the support of manipulatives</i> • <i>creating and solving problems that involve addition and subtraction</i> • <i>explaining that the order in which numbers are added does not affect the sum</i> • <i>explaining that the order in which numbers are subtracted may affect the difference.</i> 	<p>N8: Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context.</p> <p>N9: Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2 and 3-digit numerals) by:</p> <ul style="list-style-type: none"> • <i>using personal strategies for adding and subtracting with and without the support of manipulatives;</i> • <i>creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially and symbolically.</i>

Elaboration

The meanings and relationship of addition and subtraction are developed using situations, which are first modeled concretely, then pictorially and lastly with symbols. As students are introduced to addition and subtraction situations, they will gradually begin to use number sentences to describe those situations. The connection between situations, models, diagrams, and symbols should be constructed by students. The use of proper vocabulary is important to development of mathematical understanding: “minus”, “subtract”, and “difference” are words students should hear when describing subtraction situations.

It is essential that students understand how each part of the number sentence relates to the situation. Students should also have practice creating a word problem when a model and/or the equations are provided. Creating and solving problems requires students to think about the meaning of the operations more deeply than does simply solving problems.

Because students are able to compose and decompose numbers in different ways, they will develop personal strategies for computation. Invented strategies should be shared, but students should not use a strategy they do not understand. Students should be aware that every time they encounter either an addition or a subtraction situation, the other operation is implicit.

SCO: N9: Demonstrate an understanding of addition (limited to 1 and 2-digit numerals) with answers to 100 and the corresponding subtraction by:

- using personal strategies for adding and subtracting with and without the support of manipulatives
- creating and solving problems that involve addition and subtraction
- explaining that the order in which numbers are added does not affect the sum
- explaining that the order in which numbers are subtracted may affect the difference.

[C, CN, ME, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Model addition and subtraction using concrete materials or visual representations and record the process symbolically.
- Create an addition or a subtraction number sentence and a story problem for a given solution.
- Solve a given problem involving a missing addend and describe the strategy used.
- Solve a given problem involving a missing minuend or subtrahend and describe the strategy used.
- Match a number sentence to a given missing addend problem.
- Match a number sentence to a given missing subtrahend or minuend problem.
- Add a given set of numbers in two different ways, and explain why the sum is the same, e.g., $2 + 5 + 3 + 8 = (2 + 3) + 5 + 8$ or $5 + 3 + (8 + 2)$.

SCO: **N9: Demonstrate an understanding of addition (limited to 1 and 2-digit numerals) with answers to 100 and the corresponding subtraction by:**

- using personal strategies for adding and subtracting with and without the support of manipulatives
- creating and solving problems that involve addition and subtraction
- explaining that the order in which numbers are added does not affect the sum
- explaining that the order in which numbers are subtracted may affect the difference.

[C, CN, ME, PS, R, V]

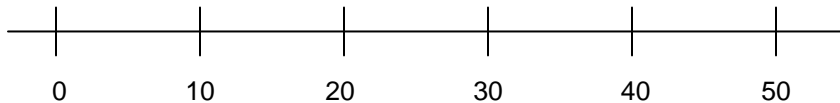
Instructional Strategies

Consider the following strategies when planning lessons:

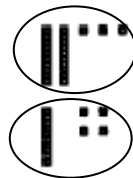
- Have students explore that a word problem could be represented with an addition or a subtraction number sentence. For example: $2 + ? = 7$ describes the same situation as $7 - 2 = ?$ Either of these is acceptable.
- Continue to use models and other representations as long as students find them helpful.
- Write number sentences horizontally to encourage more divergent thinking and development of personal strategies. Students using personal strategies, find solving problems with numbers requiring “regrouping” as easy as those problems that do not require “regrouping”.
- Encourage students to create problems involving all three meanings of subtraction: take away, comparison and missing addend.

Suggested Activities

- Tell the student that Janet read 18 books and Fred read 42. Have them record a number sentence and then explain how to find the difference using a number line (or other representation).



- Tell a student that someone told you that you do not have to learn to subtract if you know how to add. Ask: Do you agree? Why or why not?
- Ask the student to use Popsicle sticks to show why $42 + 26 = 68$.
- Let students choose a favourite story and create addition and subtraction problems related to the story. These could be shared through dramatizations, pictures, or writing.
- Show two numbers modeled with base ten blocks. After showing the students one pre-grouped model (top model on the right) and the other pre-grouped model (model below the other), ask the students what addition and/or subtraction sentences these represent and to explain their thinking (e.g., $37 - 23 = 14$).



SCO: N9: Demonstrate an understanding of addition (limited to 1 and 2-digit numerals) with answers to 100 and the corresponding subtraction by:

- using personal strategies for adding and subtracting with and without the support of manipulatives
- creating and solving problems that involve addition and subtraction
- explaining that the order in which numbers are added does not affect the sum
- explaining that the order in which numbers are subtracted may affect the difference.

[C, CN, ME, PS, R, V]

Assessment Strategies

- Ask students to write the addition sentence that would help them solve:
 $? = 16 - 8$
 $18 - 9 = ?$
 $50 - ? = 20$
- Have students add the following equations two different ways and explain why the sum is the same regardless of the order that they used.
 $65 + 28 = ?$
 $7 + 4 + 3 + 6 = ?$
- Have students solve problems, such as the following.
My dad made 43 chocolate chip cookies and some peanut butter cookies. There were 92 cookies on the cupboard. How many were peanut butter? Solve and explain your thinking.
My mom used 28 screws to make a birdhouse. There are 55 screws left in the box. How many were in the box before she started? Solve and explain your thinking.
- Tell students, that the answer is 31 balloons. Have students make up a story problem and give the number sentence that matches this answer.
- Give students an addition and/or subtraction number sentence, and have them show different strategies to solve it. Encourage them to show as many different ways as they can.

SCO: N10: Apply mental mathematics strategies, such as:

- using doubles
- making 10
- one more, one less
- two more, two less
- building on a known double
- addition for subtraction

to determine basic addition facts to 18 and related subtraction facts.
[C, CN, ME, R, V]

[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation
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Scope and Sequence

Grade One	Grade Two	Grade Three
<p>N10: Describe and use mental mathematics strategies (memorization not intended), such as:</p> <ul style="list-style-type: none"> • <i>counting on and back</i> • <i>making 10</i> • <i>doubles</i> • <i>using addition to subtract</i> <p>for the basic addition and subtraction facts to 18.</p>	<p>N10: Apply mental mathematics strategies, such as:</p> <ul style="list-style-type: none"> • <i>using doubles</i> • <i>making 10</i> • <i>one more, one less</i> • <i>two more, two less</i> • <i>building on a known double</i> • <i>addition for subtraction</i> <p>to determine basic addition facts to 18 and related subtraction facts.</p>	<p>N10: Apply mental mathematics strategies and number properties, such as:</p> <ul style="list-style-type: none"> • <i>using doubles</i> • <i>making 10</i> • <i>using the commutative property</i> • <i>using the property of zero</i> • <i>thinking addition for subtraction</i> <p>to determine answers for basic addition facts and related subtraction facts (to 18).</p>

To achieve **computational fluency**, students need to develop efficient tools to help master **addition and subtraction facts to 18**. In order for students to develop these, they first must have a strong understanding of number relationships. They should be encouraged to use their knowledge of how numbers are related to 5 and 10 to help master facts such as $8 + 6$ using ten frames. Students need to be given many opportunities to explore the strategies using concrete and visual materials. Once they have established a strategy, provide opportunities to practice them in a problem solving context.

Students will build on their **mental mathematics strategies** learned in grade one. Students should further develop their knowledge of doubles facts (e.g., $7 + 7$) to use them to find “near doubles” ($7 + 8$). They should also extend counting on and counting back skills to determine “one-more-than” and “two-more-than” facts and “one-less-than” and “two-less-than facts”. To solve subtraction problems, “think addition” is the most effective strategy for students.

Developing and applying strategies significantly reduces the number of facts students have to memorize independently. It is very important that students have the opportunity to discuss their strategies with others. It is also important that students are encouraged to use their strategies in meaningful contexts when solving a problem that requires a computation. By year end, it is expected that students will be able to recall addition facts to 18 and the related subtraction facts. Ultimately, each child may be “doing their facts” in a unique way; this could be a combination of visualization, quick strategy application and memory. It really does not matter which strategy children use as long as recall is immediate (i.e., within about three seconds). Although some students may require more practice, drill is only effective when strategies have been mastered.

SCO: N10: Apply mental mathematics strategies, such as:

- using doubles
 - making 10
 - one more, one less
 - two more, two less
 - building on a known double
 - addition for subtraction
- to determine basic addition facts to 18 and related subtraction facts.
[C, CN, ME, R, V]

Elaboration

- Apply a thinking strategy for sums to 18 and corresponding subtraction such as:

double the number in between two numbers (for $4 + 6$, think, “ $5 + 5 = 10$ ”)

doubles plus one (for $4 + 5$, think, “Double four is 8, plus 1 equals 9”)

doubles minus one (for $4 + 5$, think, “Double five is 10, minus 1 equals 9”)

doubles plus two (for $4 + 6$, think, “Double four is 8, plus 2 equals 10”)

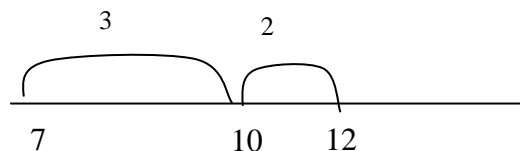
doubles minus two (for $4 + 6$, think, “Double six is 12, minus 2 equals 10”)

make 10 (for $8 + 5$, think, “ $10 + 3$ ”)

think-addition for subtraction (for $7 - 3$, think, “Three plus what equals 7?”)

back down through ten for subtraction (for $14 - 6$ think, “ $14 - 4$ (part of the subtrahend) puts me at 10, and then 2 more puts me at 8.”)

up through ten for subtraction (for $12 - 7$, start with the smaller number and think, “Three more puts me at 10, and then it’s two more to get to 12, so that’s 5 altogether.”) using an open number line helps support students in the development of this strategy and illustrates subtraction as the *difference* between two numbers. For a fact such as $12 - 7 =$ (or for any subtraction), draw a line on the chalkboard and write the numbers 7 and 12 in the appropriate position. Starting with 7, draw a “hop” of 3 on the line to land at 10 (write the number 10 on the line) and then take a hop of 2 (or 1 and 1) to get to 12. Add the size of the hops together (5). $12 - 7 = 5$. Help children practice using an open number line in this way. The emphasis is on thinking about number relationships and there are many different size “hops” that a student might take depending on how they are reasoning with the numbers.



SCO: N10: Apply mental mathematics strategies, such as:

- using doubles
 - making 10
 - one more, one less
 - two more, two less
 - building on a known double
 - addition for subtraction
- to determine basic addition facts to 18 and related subtraction facts.
[C, CN, ME, R, V]

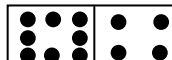
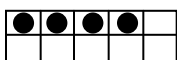
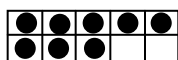
Instructional Strategies

Consider the following strategies when planning lessons:

- Ask students to use as many representations as possible for determining sums and differences, including dramatization, drawing pictures, verbally explaining their ideas, using concrete materials and writing number sentences.
- Facilitate the learning addition and subtraction facts by having students solve problems with familiar contexts. Encourage students to create their own problems.
- Encourage ongoing discussion and exploration of the most efficient ways to add and subtract numbers. The discussion should focus more on how students obtained their answer rather than the actual sum or difference. The strategies will vary depending on the problem and individual students.
- Use “missing part” (e.g., $6 + \square = 8$) and “join type” problems to develop the connection between addition and subtraction. (e.g., Sally had some marbles. Sandra gave her 5 more. Now she has 11 marbles.)
- Provide opportunities for strategy practice, using games and meaningful context as much as possible, rather than only requiring the memorization of facts in isolation. Board games in which students are required to find the sum of 2 dice to determine how far to move are good examples.
- Use ten frames or Rekenreks[®] to develop the concept of bridging through 10 (adding and subtracting).

Suggested Activities

- Use ten frames to develop number relationships. For example, to solve $9 + 4$, students can use the ten frame to see that 9 is one less than ten, and they can take one from the 4 to fill the ten frame. They can then see that the sum is 10 and 3 more or 13.
- Use dot cards and dominoes to practice addition facts. For example, show one with “4” and ask the students to say the ten fact that goes with it ($4 + 6 = 10$).
- Use multiple representations of numbers to reinforce number relationships.



- Have students work in pairs to sort addition facts into groups of facts which are related. Give them time to share with others and to explain their sortings.

SCO: N10: Apply mental mathematics strategies, such as:

- **using doubles**
 - **making 10**
 - **one more, one less**
 - **two more, two less**
 - **building on a known double**
 - **addition for subtraction**
- to determine basic addition facts to 18 and related subtraction facts.**
[C, CN, ME, R, V]

Assessment Strategies

- Ask: *What other facts could $4 + 4 = 8$ help you with?*
- Have the student describe in as many ways as he/she can why $15 - 8 = 7$.
- Have students list all the subtraction questions they can for which both numbers are less than ten and their difference is 3 (or other similar problems).
- Ask the students to list 3 other facts (addition or subtraction) that would be easier to remember if they know that $6 + 5 = 11$.
- Ask the student to tell why $\square + 5$ has to be 2 greater than $\square + 3$.
- Record observations of student's explanations of their strategies that they use daily to solve computational problems. This can also be done through individual interviews, which can provide insights into a student's thinking and help identify groups of students that can all benefit from the same kind of instruction and practice.

PATTERNS AND RELATIONS

SCO: **PR1: Demonstrate an understanding of repeating patterns (three to five elements) by:**
PR2: Demonstrate an understanding of increasing patterns by:

- describing
- extending
- comparing
- creating

patterns using manipulatives, diagrams, sounds and actions (numbers to 100)
 [C, CN, PS, R, V, ME]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Math and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Scope and Sequence

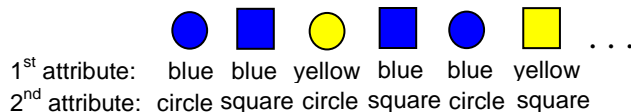
Grade One	Grade Two	Grade Three
PR1 Demonstrate an understanding of repeating patterns (two to four elements) by: describing; reproducing; extending; creating patterns using manipulatives, diagrams, sounds and actions.	PR1 Demonstrate an understanding of repeating patterns (three to five elements) by: PR2 Demonstrate an understanding of increasing patterns by: <ul style="list-style-type: none"> • describing • reproducing • extending • creating patterns using manipulatives, diagrams, sounds and actions (PR2: numbers to 100).	PR1 Demonstrate an understanding of increasing patterns by: describing; extending; comparing; creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000). PR2 Demonstrate an understanding of decreasing patterns by: describing; extending; comparing; creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).

Elaboration

The foundation of algebraic thinking is investigating patterns and their representations. Contextual, open ended and situation specific problem solving should be an integral part of everyday mathematics instruction, activities and assessment. Students need to recognize and **extend** many different forms of the same pattern, including those constructed or in their environment. They must identify the **core** or **sequence**, and be able to predict an **element** in **repeating** and **increasing patterns** using many strategies within a variety of contexts (for example, hundred charts, calendars, number lines, tiling patterns). These predictions should be verified by extending the pattern concretely, pictorially and symbolically.

Students must be able to explain the **rule** (in both **words** and **symbols**) used to create a given repeating or increasing non-numerical or numerical pattern and represent the pattern in another mode (materials, actions, sounds, etc.). Because students find it easier to demonstrate rather than articulate patterns, they must learn to describe the pattern rule including both the repeating/increasing elements and the first or beginning elements. With this understanding, students are able to identify errors and missing elements within patterns. Students move their thinking from single to **double attribute** patterns. The challenge for students will be to identify patterns with multiple changing attributes of different core lengths.

For example:



SCO: **PR1: Demonstrate an understanding of repeating patterns (three to five elements) by:**
PR2: Demonstrate an understanding of increasing patterns by:

- describing
- extending
- comparing
- creating

patterns using manipulatives, diagrams, sounds and actions (numbers to 100)
[C, CN, PS, R, V, ME]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

PR1

- Identify the core of a given repeating pattern.
- Describe and extend a given double attribute pattern.
- Explain the rule used to create a given repeating non-numerical pattern.
- Predict an element in a given repeating pattern using a variety of strategies.
- Predict an element of a given repeating pattern and extend the pattern to verify the prediction.

PR2

- Identify and describe increasing patterns in a variety of given contexts, e.g., hundred chart, number line, addition tables, calendar, a tiling pattern or drawings.
- Represent a given increasing pattern concretely and pictorially.
- Identify errors in a given increasing pattern.
- Explain the rule used to create a given increasing pattern.
- Create an increasing pattern and explain the pattern rule.
- Represent a given increasing pattern using another mode, e.g., colour to shape.
- Solve a given problem using increasing patterns.
- Identify and describe increasing patterns in the environment, e.g., house/room numbers, flower petals, book pages, calendar, pine cones, leap years.
- Determine missing elements in a given concrete, pictorial or symbolic increasing pattern and explain the reasoning.

SCO: **PR1: Demonstrate an understanding of repeating patterns (three to five elements) by:**
PR2: Demonstrate an understanding of increasing patterns by:

- describing
- extending
- comparing
- creating

patterns using manipulatives, diagrams, sounds and actions (numbers to 100)
 [C, CN, PS, R, V, ME]

Instructional Strategies

Consider the following strategies when planning lessons:

- Include concrete materials in patterning activities.
- Integrate patterns in physical education, music, art, science and other subject areas to provide a context.
- Encourage students to see that patterns may continue in many different ways unless a pattern rule is defined. For example, a pattern that begins 1, 2, 3, might continue
 - 1, 2, 3, 1, 2, 3, 1, 2, 3, ... (repeating 1,2,3)
 - 1, 2, 3, 4, 5, 6, 7, ... (growing)
 - 1, 2, 3, 5, 8, 13, ... (pattern rule: each number after 2 being the sum of the two preceding ones)
- Provide opportunities for students to make predictions about patterns using a variety of materials.
- Use concrete materials for students to build double attribute patterns that can be described in more than one way.
- Expect students to communicate their thinking about patterns, verbally and written.

Suggested Activities

- Provide the start of a pattern, using base-ten blocks. Ask the student to continue the pattern in more than one way, and to describe the pattern rule in each case.
- Ask the students to create two patterns which are similar, but not exactly the same. Ask them to comment on the similarities and differences.
- Tell a student that the first two numbers in a pattern are 5 and 10. Ask for several different ways in which the pattern might be continued.
- Tell the student that another child continued the pattern 1, 2, 3, 4, by saying 2, 3. Ask: Do you think that the other child is incorrect or is there a rule which might explain continuing the pattern in this way?
- Provide students with a repeating pattern to extend. Before students begin to extend the pattern, have them predict exactly what element will be in a specific position (e.g., what would be in the sixth position?). Have students provide a reason for their prediction before extending their pattern to check their prediction. If their prediction is incorrect, have them examine their reasoning and try to figure out why the prediction was off.
- Provide students with a repeating pattern and have them predict the position of a specific element (e.g., where would the fourth triangle be in the sequence?).

SCO: **PR1: Demonstrate an understanding of repeating patterns (three to five elements) by:**
PR2: Demonstrate an understanding of increasing patterns by:

- describing
- extending
- comparing
- creating

patterns using manipulatives, diagrams, sounds and actions (numbers to 100)
 [C, CN, PS, R, V, ME]

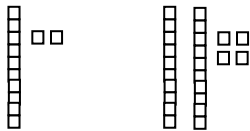
Assessment Strategies

- Have students use three colours of tiles to create a pattern.
- Have students create a repeating pattern that has a core of three elements.
- Have students create a repeating pattern where a specific element is to be identified; e.g., 4th element is green.
- Have students create a growing pattern where a specific element is identified; e.g., 10th element is 100.
- Have students create a pattern that grows by varying amounts each time; e.g., 1, 2, 4, 8, ...
- Present the following pattern:

-| || -| || -| ||

Ask students to translate a pattern like this to a different mode (sounds, shapes, etc.).

- Present the following base ten blocks:



Ask the student what would be next. Explain why.

- Have students determine the missing element in a given pattern.



<p>SCO: PR3: Demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100). [C, CN, R, V]</p> <p>PR4: Record equalities and inequalities symbolically using the equal symbol or the not equal symbol. [C, CN, R, V]</p>

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Math and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Scope and Sequence

Grade One	Grade Two	Grade Three
<p>PR3 Describe equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20).</p> <p>PR4 Record equalities, using the equal symbol.</p>	<p>PR3 Demonstrate and explain the meaning of equality and inequality, concretely and pictorially.</p> <p>PR4 Record equalities and inequalities symbolically, using the equal symbol or the not equal symbol.</p>	<p>PR3 Solve one-step addition and subtraction equations involving symbols representing an unknown number.</p>

Elaboration

Students need to understand that the **equal (=) symbol** represents a relationship between elements. It tells us the quantity on the left is the same as the quantity on the right (balanced). Conversely the **not equal (≠) symbol** tells us the quantity on the left is not the same as the quantity on the right (imbalanced). They must be able to write these symbols to describe the relationship between quantities with or without an operation (e.g., $72 = 72$; $7 \neq 5$; $21 = 18 + 3$; $10 - 6 \neq 3 + 3$).

By using a balance scale, students will understand the concept of equating the two sides (start with 2 different quantities and adjust them to make them equal). To build the foundation for solving symbolic equations, students must experience working with balance scale problems, concretely and pictorially. The expectation is that students will explain their use of a symbol and justify their answer. Informal work with **inequalities** and **equalities** leads students to discover that there may be more than one possible answer to satisfy the situation (e.g., $5 + ? + ? = 9$). Students should explore these concepts with numbers from zero to one hundred.

It is important to encourage **relational thinking**. When the focus is on the number relationships, students become more flexible in their thinking and will find more efficient ways to solve problems. Encourage students to use their number sense and “eyeball” the numbers to compare numbers and simplify computations. For example, in the sentence $8 + 7 - 7 = \underline{\quad}$, students might recognize that adding and then subtracting 7 will leave 8 unaffected, therefore avoiding computation. Other equations, such as $8 + 4 = \underline{\quad} + 5$, can be solved only if students have a broad understanding of the equals sign. Students can solve this sentence using relational thinking by noticing that 5 is 1 more than 4, so the unknown number has to be one less than 8.

SCO: **PR3: Demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100).**

[C, CN, R, V]

PR4: Record equalities and inequalities symbolically using the equal symbol or the not equal symbol.

[C, CN, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

PR3

- Determine whether two given quantities of the same object (same shape and mass) are equal by using a balance scale.
- Construct and draw two unequal sets using the same object (same shape and mass) and explain the reasoning.
- Demonstrate how to change two given sets, equal in number, to create inequality.
- Choose from three or more given sets the one that does not have a quantity equal to the others and explain why.

PR4

- Determine whether two sides of a given number sentence are equal (=) or not equal (\neq). Write the appropriate symbol and justify the answer.
- Model equalities using a variety of concrete representations and record the equality.
- Model inequalities using a variety of concrete representations and record the inequality.

SCO: PR3: **Demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100).**
 [C, CN, R, V]
 PR4: **Record equalities and inequalities symbolically using the equal symbol or the not equal symbol.**
 [C, CN, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:




- Introduce students to a variety of equation types. For example: $6+7=13$ or $68=61+7$ or $18+5=10+13$.
- Use of the words “the same as” for the equal sign and “not the same as” for the unequal sign will help students see that the symbols represent a relationship. Students should become familiar with the terminology that “inequality” means “is greater than” or “is less than” and “equality” means “the same as”.
- Give students opportunities to develop their own equations and inequalities.
- Give students many opportunities to construct equal and unequal sets and to translate their findings to diagrams and then to number sentences.





Suggested Activities




- Challenge students to find different ways to express a particular number encouraging the use of different operations or relationships. For example: $10 \neq 10+3$, $10 = 2 + 8$, 10 is less than 24. Include examples with more than 2 numbers on the same side. For example: $10 = 4 + 1 + 5$, $17 - 2 - 5 = 10$.
- Ask: Which mystery bag is the heaviest? How do you know?



- Use “True or False” activities. Present various relationships, such as $8 + 4 = 12 + 5$ or $13 = 13$ or $16 = 25 - 9$. Have students tell whether it is true or false and justify their answers.
- Ask students to write true sentences in several different forms.
- For example, $_ + _ = _ + _ ;$ or $_ - _ = _ - _ ,$ or $_ + _ = _ - _ ,$ or $_ + _ \neq _ - _ .$
- Ask students to find dominoes that satisfy relationships such as the following:



 “is the same as”
 

 

 =
 



 “does not equal”
 


- Have students compare three or more given sets to determine which are equal/not equal and explain their reasoning.
- Ask students to write a problem that involves the number 12 as shown below in the number sentence (the blanks are numbers): $_ (+ \text{ or } -) 12 = _ .$

SCO: PR3: Demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100).

[C, CN, R, V]

PR4: Record equalities and inequalities symbolically using the equal symbol or the not equal symbol.

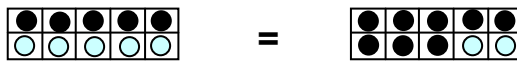
[C, CN, R, V]

Assessment Strategies

- Complete the number sentence with a number that makes it true. For example: $5 + 1 = __ + 2$, or $4 + __ = 2 + 2 + 2$; or $__ + 0 = 30 - 1$; or $16 + 5 \neq __ - 7$.
- Provide students with a number sentence representing an equality and ask students to translate the number sentence to a pan balance example and to explain their diagram.
- Have students build an equal or an unequal relationship using Cuisenaire® rods and explain their thinking to a partner.



- Ask a student to write a number sentence that is true and one that is untrue and then explain their reasoning.
- Ask students to build number sentences to demonstrate an understanding of equality using ten frames. Place a ten frame on either side of the equal sign and have students place different coloured counters to show more than one combination. For example: $5 + 5 = 8 + 2$ or $10 = 2 + 2 + 6$ or $6 + 4 = 7 + 3$.



SHAPE AND SPACE

SCO: **SS1** : Relate the number of days to a week and the number of months to a year in a problem-solving context.
[C, CN, PS, R]

[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation
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Scope and Sequence

Grade One	Grade Two	Grade Three
	<p>SS1 Relate the number of days to a week and the number of months to a year in a problem-solving context.</p>	<p>SS1 Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). SS2 Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem solving context.</p>

Elaboration

The **calendar** provides rich opportunities to explore not only knowledge of time, but also number concepts.

By the end of grade 2, students should know the **days of the week** and the **months of the year**. Students may have more difficulty with the months for which they have less experience in their own lives. Ask questions about the months regularly, such as, “It is March. Which month comes next?”

Using calendars throughout the school year strengthens the students’ sense of time. Each month brings a new calendar to explore.

SCO: SS1 : Relate the number of days to a week and the number of months to a year in a problem-solving context.
[C, CN, PS, R]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Read a date on a calendar.
- Name and order the days of the week.
- Identify the day of the week and the month of the year for an identified calendar date.
- Communicate that there are seven days in a week and twelve months in a year.
- Determine whether a given set of days is more or less than a week.
- Identify yesterday's/tomorrow's date.
- Identify the month that comes before and the month that comes after a given month.
- Name and order the months of the year.
- Solve a given problem involving time which is limited to the number of days in a week and the number of months in a year.

SCO: **SS1** : Relate the number of days to a week and the number of months to a year in a problem-solving context.
[C, CN, PS, R]

Instructional Strategies

Consider the following strategies when planning lessons:

- Use a calendar daily to explore days and months.
- Give students problematic situations to work on, such as pointing out the date on the calendar and asking how many days/weeks to a specific holiday.
- Provide students with experiences involving comparisons between events of differing durations to help them understand time. For example: How many days in a school week as compared to a calendar week? How many months to your friend's birthday compared to your birthday?
- Use school activity calendars and lunch menus to explore days and months.

Suggested Activities

- List the days of the week in order along a seven section number line. Attach the ends of the number line to complete a circle. This unit (one week) demonstrates the cyclical property of how seven days equals one week (unitizing). This activity can be extended to include several weeks. A similar activity can be developed to show the months of the year.
- Show the student a calendar for the year. Ask him/her to point out the day's date and to find out what date it will be in six weeks.
- Show the student a calendar for the year and ask him/her to identify ways in which months are the same and ways in which they differ.
- Read children's literature such as *Chicken Soup with Rice* by Maurice Sendak to your students. Provide students with a random set of the sequence of months from the poem and have them order the months. This activity could be duplicated with the days of the week and a poem such as *Monday's Child* (Mother Goose nursery rhyme).
- Have students solve problems where they must determine what day it would be 3 days from now. This activity could be varied by using months instead of days and changing the number.
- Have students prepare a schedule on a blank calendar starting on a particular day/date, for a specific job (e.g., recycling pick up), that is repeated regularly (e.g., biweekly).

SCO: **SS1** : Relate the number of days to a week and the number of months to a year in a problem-solving context.
[C, CN, PS, R]

Assessment Strategies

- Have students solve problems involving time, such as:
Grandma takes a pill every second day for three weeks. How many pills does she need?
A teacher told her students that there are 6 more days before the school field trip. What might be some possible days for the school trip? Explain your thinking.
- Ask students: How might you use a calendar to help subtract 14 from a number?
- Tell the student that it is the 11th of the month. Ask: How might you use a calendar to add 16, and what is the date?
- Sequence a set of events occurring in a familiar story involving days or months.

SCO: **SS2: Relate the size of a unit of measure to the number of units (limited to nonstandard units) used to measure length and mass (weight).**
SS3: Compare and order objects by length, height, distance around and mass (weight) using nonstandard units, and make statements of comparison.
 [C, CN, ME, R, V]

[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation
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Scope and Sequence

Grade One	Grade Two	Grade Three
<p>SS1: Demonstrate an understanding of measurement as a process of comparing by: identifying attributes that can be compared; ordering objects; making statements of comparison; filling, covering or matching.</p>	<p>SS2: Relate the size of a unit of measure to the number of units (limited to nonstandard units) used to measure length and mass (weight). SS3: Compare and order objects by length, height, distance around and mass (weight) using nonstandard units, and make statements of comparison.</p>	<p>SS3: Demonstrate an understanding of measuring length (cm, m) by: selecting and justifying referents for the units cm and m; modeling and describing the relationship between the units cm and m; estimating length using referents; measuring and recording length, width and height. SS4: Demonstrate an understanding of measuring mass (g, kg) by: selecting and justifying referents for the units g and kg; modeling and describing the relationship between the units g and kg; estimating mass using referents; measuring and recording mass.</p>

Elaboration

The **attributes** of an object being measured must be understood to measure anything meaningfully. Once the attribute is understood, a **unit of measure** with a similar attribute can be chosen to measure that object. Informal units allow students to consider the attributes of the object being measured rather than a focus on the unit of measure. Estimation is a key component to the measurement process. Prior to actually measuring an object, students should be encouraged to estimate.

Children should recognize that **length** tells about the extent of an object along one dimension. Initially, they would compare lengths informally by simply viewing the two lengths. Later, they should investigate strategies to compare the lengths of two or more objects in situations involving both **direct** and **indirect measurement**. Direct measurement consists of comparing lengths by lining up items side by side, beginning at a common base. Students should be led to see why a common starting point is important. Indirect measurement consists of comparing lengths when it is not possible to physically line up the objects. For example, to compare length of hand to wrist size, students might cut pieces of string the length of their hands, and then wind the strings around their wrists for comparison.

Students should recognize that **mass** tells about the “heaviness” of an object. They should explore methods to compare and order masses, including situations involving both direct and indirect measurement. Direct measurement involves, for instance, placing two objects on a balance simultaneously and comparing the mass of one with that of the other. Indirect measurement involves comparing the masses of two objects by using another object as a referent.

SCO: **SS2: Relate the size of a unit of measure to the number of units (limited to nonstandard units) used to measure length and mass (weight).**

SS3: Compare and order objects by length, height, distance around and mass (weight) using nonstandard units, and make statements of comparison.

[C, CN, ME, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

SS2

- Explain why one of two given non-standard units may be a better choice for measuring the length of an object.
- Explain why one of two given non-standard units may be a better choice for measuring the mass of an object.
- Select a non-standard unit for measuring the length or mass of an object and explain why it was chosen.
- Estimate the number of non-standard units needed for a given measurement task.
- Explain why the number of units of a measurement will vary depending upon the unit of measure used.
- Explain why overlapping or leaving gaps does not result in accurate measures.
- Count the number of non-standard units required to measure the length of a given object using a single copy or multiple copies of a unit.

SS3

- Estimate and measure a given object using multiple copies of a non-standard unit and using a single copy of the same unit many times, and explain the results.
- Estimate and measure, using non-standard units, a given length that is not a straight line.

SCO: **SS2: Relate the size of a unit of measure to the number of units (limited to nonstandard units) used to measure length and mass (weight).**

SS3: Compare and order objects by length, height, distance around and mass (weight) using nonstandard units, and make statements of comparison.

[C, CN, ME, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students with experiences using a broad measurement vocabulary. For example, they should hear and use words such as - heavier, lighter, has the same mass; length, including “longer,” “taller,” “wider,” etc.
- Focus should be on answering, in every measurement situation, questions that focus on the attributes such as, “Which book is the tallest? How do you know?”
- Some of these results should be recorded for comparison and ordinality.
- Have students order objects from longest to shortest. Include situations in which students are dealing with other attributes, such as objects which are not straight or which are also wide or thick.
- Encourage students to recognize that the mass of objects is not related to its size (e.g., a small iron ball can be heavy).
- Have students estimate and measure objects in non-standard units for the purpose of answering relevant and practical questions; for example, do people with longer legs usually jump farther?
- Use the same non-standard unit occasionally to measure a variety of items in order to compare them; for example, how many wooden blocks would balance a sneaker? a book? a grapefruit?
- Provide students the opportunity to make their own simple measuring instruments using an object such as paperclips. The units are lined up and marked off on a length of paper. This will link the understanding that it is the space and not the mark that indicates a measure of length on a ruler.
- Give students many opportunities to use a variety of non standard units to measure an object and then to discuss which is most appropriate and why?

Suggested Activities

- Have students search for items that are a given number of paper clips long or that have a mass of two bags of marbles. Provide student with a length and have them estimate how many paper clips long or provide student with an object and have them determine how many marbles to match the weight.
- Have students develop a book on measurement that they can add to over time.
- Have students prepare a set of ribbons for first, second, and third places in a race, so that the faster runner gets a longer ribbon.
- Have students compare the masses of two objects by using their hands as the pan balance.
- Create an activity centre where students compare and sort various objects as longer, shorter or about the same length as a specified object. The comparison for mass would be heavier, lighter or about the same mass.

SCO: **SS2: Relate the size of a unit of measure to the number of units (limited to nonstandard units) used to measure length and mass (weight).**
SS3: Compare and order objects by length, height, distance around and mass (weight) using nonstandard units, and make statements of comparison.
 [C, CN, ME, R, V]

Assessment Strategies

- Have students compare two or more crooked *paths*, made with skipping ropes, lengths of rope, string, etc., and determine which path is longest and shortest.
- Ask students how they would compare the height of a garbage can to the distance around the top of the can and explain.
- Ask students: What unit might you use to measure the mass of a watermelon?
- Have students choose one of two nonstandard units to measure a particular length/mass and to defend their choice. (e.g., linking cube and a straw to measure the length of a table)
- Ask students to use several different non standard units to measure the same object. Which would give you the closest measure? How do you know? (e.g., if students are given paper clips, pencils, and linking cubes, they may identify the cubes as being the most accurate if they fit almost exactly on the item they are measuring.)

Ask students:



what does that tell us about the mass of the cylinder and/or the cube?

- Give students an object to hold in one hand. Ask each student to show how many of another non-standard unit would have the same mass and to explain their thinking.

<p>SCO: SS4: Measure length to the nearest non-standard unit by:</p> <ul style="list-style-type: none"> • using multiple copies of a unit • using a single copy of a unit (iteration process). <p>[C, ME, R, V]</p> <p>SS5: Demonstrate that changing the orientation of an object does not alter the measurements of its attributes.</p> <p>[C, R, V]</p>
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[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Math and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Scope and Sequence

Grade One	Grade Two	Grade Three
<p>SS1: Demonstrate an understanding of measurement as a process of comparing by: identifying attributes that can be compared; ordering objects; making statements of comparison; filling, covering or matching.</p>	<p>SS4: Measure length to the nearest non-standard unit by:</p> <ul style="list-style-type: none"> • using multiple copies of a unit • using a single copy of a unit (iteration process). <p>SS5: Demonstrate that changing the orientation of an object does not alter the measurements of its attributes.</p>	<p>SS3: Demonstrate an understanding of measuring length (cm, m) by: selecting and justifying referents for the units cm and m; modeling and describing the relationship between the units cm and m; estimating length using referents; measuring and recording length, width and height.</p>

Elaboration

In order to measure, a series of uniform units must be used, or a single unit must be used repeatedly. The outcome of the measure is dependent upon; the type of unit used, the placement of the units, the uniformity of the unit and the unit’s orientation.

It is essential that students understand, that when measuring, the units must border each other and the orientation must be consistent. When using multiples of non-standard units the units must be congruent.

SCO: **SS4: Measure length to the nearest non-standard unit by:**

- using multiple copies of a unit
- using a single copy of a unit (iteration process).

[C, ME, R, V]

SS5: Demonstrate that changing the orientation of an object does not alter the measurements of its attributes.

[C, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

SS4

- Explain why overlapping or leaving gaps does not result in accurate measures.
- Count the number of non-standard units required to measure the length of a given object using a single copy or multiple copies of a unit.
- Estimate and measure a given object using multiple copies of a non-standard unit and using a single copy of the same unit many times, and explain the results.
- Estimate and measure a given length, that is not a straight line, using non-standard units.

SS5

- Measure a given object, change the orientation, re-measure and explain the results.

SCO: **SS4: Measure length to the nearest non-standard unit by:**

- using multiple copies of a unit
- using a single copy of a unit (iteration process).

[C, ME, R, V]

SS5: Demonstrate that changing the orientation of an object does not alter the measurements of its attributes.

[C, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Ask students to predict the results prior to making their measurements. It is important for students to understand that the way a measurement unit is used is as important as the attribute being measured.
- Provide students with opportunities to measure an object and then re-measure the same object after it has been reoriented. They should compare results and discuss why or why not their measure is the same/different. Indicate that sometimes reorientation assists in the ease, not the value of the measurement.
- Use children's literature, such as *David's Father* by Robert Munsch to have students discuss non-standard measurement units and estimating lengths and distances.

Suggested Activities

- Provide students with frequent opportunities to measure the same object iterating a specific unit, and using multiple copies of that unit. They should compare results and discuss why or why not their measure is the same/different.
- Try the activity: "How Long is your Neighbour?" Students are to determine an effective measure of the height of a student in the class beginning with an estimate. Students are able to choose the unit of measure themselves. Explain that it might be easier to consider measuring their classmate when he/she is laying down. Discuss the similarity/difference in results from various groups of students

SCO: **SS4: Measure length to the nearest non-standard unit by:**

- using multiple copies of a unit
- using a single copy of a unit (iteration process).

[C, ME, R, V]

SS5: Demonstrate that changing the orientation of an object does not alter the measurements of its attributes.

[C, R, V]

Assessment Strategies

- Have students measure the length of a piece of paper with a tile. Record the result. Make a second measurement along the same side of the paper after reorienting the paper. Record the result. Discuss your findings.
- Ask students to solve problems, such as:
- “Susan has used a tile to measure the table in the classroom. Kyle has used the same tile to measure the same side of the table but has a different result. How is this possible? You may use a diagram to show your thinking.

SCO: **SS6: Sort 2-D shapes and 3-D objects using two attributes, and explain the sorting rule.**
[C, CN, R, V]

[C] Communication
[T] Technology

[PS] Problem Solving
[V] Visualization

[CN] Connections
[R] Reasoning

[ME] Mental Math
and Estimation

Scope and Sequence

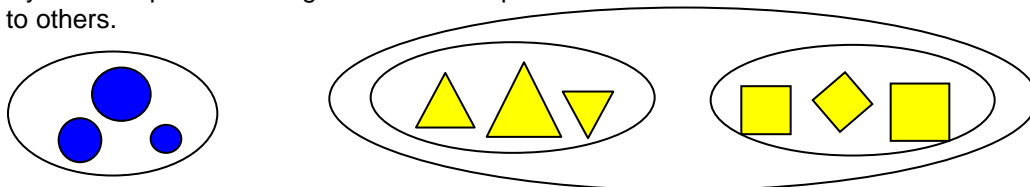
Grade One	Grade Two	Grade Three
SS2: Sort 3-D objects and 2-D shapes using one attribute, and explain the sorting rule.	SS6: Sort 2-D shapes and 3-D objects using two attributes, and explain the sorting rule.	SS7: Sort regular and irregular polygons, including: triangles; quadrilaterals; pentagons; hexagons; and octagons according to the number of sides.

Elaboration

The study of **two-dimensional shapes** and **three-dimensional objects** is essential as we strive to describe, analyze and understand the world we live in. In earlier grades, students will have had many opportunities to explore shapes through sorting, patterning, and building activities. At this level, activities that will further develop these skills should be provided. Children will begin at different levels of development so it will be necessary to provide activities with a range of complexity.

“Children need experiences with a rich variety of both two- and three-dimensional shapes. It is useful for students to be able to identify common shapes, notice likenesses and differences among shapes, become aware of the properties that different shapes have, and eventually use these properties to further define and understand their geometric world. The Van Hiele levels of geometric thought describe how we think and what types of geometric ideas we think about, rather than how much knowledge we have” (Van de Walle & Lovin, K-3, p. 188, 193). Sorting activities help to develop visual discrimination. It is important to encourage students to look for alternative ways of sorting; this necessitates the further investigation of objects. According to the Van Hiele model, most students in grade two are at the “visualization” level. They rely mostly on the appearance of the shape or object when doing sorting activities. There are many different **attributes** or characteristics of shapes and objects. Students are most familiar with attributes that describe the whole shape, such as colour, size, shape, or texture. Other attributes refer to parts of the shape or object, such as number of sides or faces, number of vertices (points), or lengths or edges. Students will often use non-geometric language to describe the attributes, such as “crooked”, “wavy”. Students may also sort according to the orientation of the shape, such as a square turned may be seen as a diamond.

In grade two, students need to build on their prior experiences to sort objects and shapes using **two attributes**. If students find it difficult to sort by two attributes at the same time, they may find it helpful to sort by one attribute and combine two of the sorted groups. For example, in the diagram below, students may have first sorted the shapes by number of sides and then combined the last two groups to make a single group that includes yellow shapes and straight sides. It is important that students are able to explain their sorting rule to others.



SCO: **SS6: Sort 2-D shapes and 3-D objects using two attributes, and explain the sorting rule.**
[C, CN, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Determine the differences between two given pre-sorted sets and explain the sorting rule.
- Identify and name two common attributes of items within a given sorted group.
- Sort a given set of 2-D shapes (regular and irregular) according to two attributes and explain the sorting rule.
- Sort a given set of 3-D objects according to two attributes and explain the sorting rule.

SCO: **SS6: Sort 2-D shapes and 3-D objects using two attributes, and explain the sorting rule.**
[C, CN, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Include a great variety of sorting and classifying activities.
- Focus on how shapes and objects are alike and different.
- Provide students with many opportunities to draw, build, make, and take apart shapes in both two- and three-dimensions. These activities should be designed around specific characteristics to develop an understanding of geometric properties. (Van de Walle, K-3, p. 192)
- Use collections of real objects, such as cans, boxes, or balls, for students to sort.
- Make a collection of 2-D shapes cut from posterboard. These should include not only shapes with which students are familiar, but also some less common shapes that would fit in potential sorting categories (e.g., a semi-circle or a cashew shape could be classified under “things with curved edges”). Because it is important for students to share their thinking with others, it is recommended that they work in small groups, talking about their ideas and strategies.
- Challenge students to test their ideas about shapes and objects. For example, can you find a triangle that has one square corner?

Suggested Activities

- Hold up a geometric solid, like a cone, and ask the students to find other objects that could be sorted into the same group as it. When a student adds an object to the group, have them explain what attribute it shares with the original object.
- Have students select two shapes or objects and tell how they are the same or how they are different.
- Create a set of objects or shapes (about 5) that have a secret sorting rule. Ask the students to add to your set (a drawing or a real item) and guess the rule.
- Play game of “One Way Different”. Select an object or shape as a starter piece. Students take turns placing objects on either side of the original shape. These objects must differ by one attribute from the one it is placed beside. Students need to explain why their piece works. For example, if the original object was a pyramid, the next object placed beside it could be a cone and the student could justify that the cone has curved faces.

SCO: **SS6: Sort 2-D shapes and 3-D objects using two attributes, and explain the sorting rule.**
[C, CN, R, V]

Assessment Strategies

- Provide a collection of about 8 shapes or objects. Have the students take turns sorting the shapes while the other group members try to guess the sorting rule being used. Each student should have a turn to sort the shapes. As a class, discuss all the ways that the shapes were sorted and try to determine if there are any ways that were missed.
- Provide several different 3-D objects. Ask the student to sort them using two attributes and to explain the sorting criteria. Ask him/her to sort them again, using different criteria.
- Provide several different 2-D shapes. Ask the student to sort them using two attributes and to explain the sorting criteria. Ask him/her to sort them again, using different criteria.
- Observe the language students use to describe their sorting rules. As students develop a higher level of thinking, they will begin to use more geometric language and focus more on the properties of the shapes.

<p>SCO: SS7: Describe, compare and construct 3-D objects, including:</p> <ul style="list-style-type: none"> • cubes • spheres • cones • cylinders • pyramids. <p>SS9: Identify 2-D shapes as parts of 3-D objects in the environment [C, CN, R, V]</p>
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[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Math and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Scope and Sequence

Grade One	Grade Two	Grade Three
<p>SS2: Sort 3-D objects and 2-D shapes using one attribute, and explain the sorting rule.</p> <p>SS3: Replicate composite 2-D shapes and 3-D objects.</p>	<p>SS7: Describe, compare and construct 3-D objects, including:</p> <ul style="list-style-type: none"> • cubes • spheres • cones • cylinders • pyramids. <p>SS9: Identify 2-D shapes as parts of 3-D objects in the environment</p>	<p>SS6: Describe 3-D objects according to the shape of the faces, and the number of edges and vertices.</p>

Elaboration

Students will continue to develop their depth of understanding of 3-D objects that was started in kindergarten. Grade two students need many varied opportunities to manipulate 3-D objects. Activities in which they describe, compare, and build 3-D objects, and discuss their observations help to develop essential geometric skills. It is through such activities that students will learn the names of 3-D shapes and begin to recognize their characteristics.

“As students develop mathematically, they are increasingly able to identify and name an object by examining its properties and using reasoning” (M. Small, 2008, p. 287).

Through exploration, students may notice that the faces of 3-D objects look like 2-D shapes. They should also discover that an object has certain attributes regardless of its orientation or size. Even though a pyramid may look different laying on one of its triangular faces, it is still a pyramid with the same attributes. They should also recognize that it is still pyramid whether it is tall or short.

It is very important to encourage students to use accurate language when describing objects. Students should be comfortable using such terms as **cube, sphere, cone, cylinder, pyramid**, and may extend their exploration to prisms.

Students should be given many opportunities to build 3-D objects using a variety of materials as it increases their visualization skills. It is also important to provide students with these experiences and focus less on the accuracy of the end product. Students should also be encouraged to make constructions using a combination of 3-D objects.

SCO: **SS7: Describe, compare and construct 3-D objects, including:**

- **cubes**
- **spheres**
- **cones**
- **cylinders**
- **pyramids.**

SS9: Identify 2-D shapes as parts of 3-D objects in the environment

[C, CN, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

SS7

- Sort a given set of 3-D objects and explain the sorting rule.
- Identify common attributes of cubes, spheres, cones, cylinders and pyramids from given sets of the same 3-D objects.
- Identify and describe given 3-D objects with different dimensions.
- Identify and describe given 3-D objects with different orientations.
- Create and describe a representation of a given 3-D object using materials such as modeling clay.
- Identify examples of cubes, spheres, cones, cylinders and pyramids found in the environment.

SS9

- Compare and match a given 2-D shape, such as a triangle, square, rectangle or circle, to the faces of 3-D objects in the environment.
- Name the 2-D faces of a given 3-D object.

SCO: **SS7: Describe, compare and construct 3-D objects, including:**

- **cubes**
- **spheres**
- **cones**
- **cylinders**
- **pyramids.**

SS9: Identify 2-D shapes as parts of 3-D objects in the environment

[C, CN, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Have students bring in 3-D objects that could be described as a cube, sphere, cone, cylinder, or pyramid. Students should explain their reasons for its classification focusing on particular properties. For example, a student may bring in a funnel and classify it as both a cone and a cylinder.
- Have the students explore how they can create a cone, sphere, or cylinder using materials in the classroom. Expect to see use of modeling clay, pipe cleaners, toothpicks, paper, and circles from the attribute blocks. Have the students describe their methods to a partner, and present to the class, or write about their methods in their journals.
- Provide opportunities for students to build with 3-D objects by following oral directions: for example, “Place the cube between the small cylinder and the cone and place the large cylinder behind the cube”. Invite students to make their own designs and then challenge their classmates to build the same structure from their oral directions.

Suggested Activities

- Give the student some toothpicks and clay and ask him/her to build a cube or a pyramid. Ask how many toothpicks were needed.
- Provide pairs of students with a small collection of 3-D objects. Have them build a creature or robot or something else of their choosing. Have them present their creation to the class and describe it using the names of the 3-D objects they used in their creation.
- Tell students that you have a 3-D object in a bag. One of its faces is round (a circle). Ask what the object could be. Follow up with having the students find an object in the class that has the same face.
- Have students trace the faces of a 3-D object and then search for another object in the classroom that a face that is also the same face.
- Challenge students to build your “secret object”. Give them clues such as, “my object uses 10 cubes and has a rectangle base and it looks like stair steps” or “my object uses 8 cubes and has a square base and it looks like the letter L”.
- Give the students 10 green triangular pattern blocks. Ask them to make as many different-size triangle as they can and to record their answers by making a drawing of each.
- Ask students to look around the room and see what 3-D shapes they can see and name. List the shape and the object name on chart paper.
- Put students in small groups and have each group create a book of 3-D shapes by cutting pictures from catalogues and magazines.

SCO: **SS7: Describe, compare and construct 3-D objects, including:**

- **cubes**
- **spheres**
- **cones**
- **cylinders**
- **pyramids.**

SS9: Identify 2-D shapes as parts of 3-D objects in the environment

[C, CN, R, V]

Assessment Strategies

- Ask students to describe how a cube and square pyramid are alike and how they differ. Repeat question using two other 3-D objects such as a cone and cylinder.
- Tell students that you traced around one of the faces of a 3-D object and the shape that you drew was a square. Ask what the object could be.
- Provide students with a blackline master that has drawings of a variety of 2-D shapes. Ask students to use a familiar 3-D object from their environment (toy or article from home) and match one of its faces to one or more of the shapes from the sheet.

<p>SCO: SS8: Describe, compare and construct 2-D shapes, including:</p> <ul style="list-style-type: none"> • triangles • squares • rectangles • circles. <p>SS9: Identify 2-D shapes as parts of 3-D objects in the environment [C, CN, R, V]</p>

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Math and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Scope and Sequence

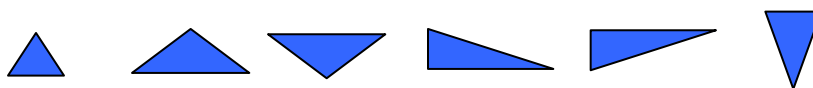
Grade One	Grade Two	Grade Three
<p>SS2: Sort 3-D objects and 2-D shapes using one attribute, and explain the sorting rule.</p> <p>SS3: Replicate composite 2-D shapes and 3-D objects.</p>	<p>SS8: Describe, compare and construct 2-D shapes, including:</p> <ul style="list-style-type: none"> • triangles • squares • rectangles • circles. <p>SS9: Identify 2-D shapes as parts of 3-D objects in the environment</p>	<p>SS7: Sort regular and irregular polygons, including: triangles; quadrilaterals; pentagons; hexagons; and octagons according to the number of sides.</p>

Elaboration

Many students will be familiar with many of the **2-D shapes** from their experiences, but they may not necessarily know the **attributes**. Students in the early grades need many varied opportunities to manipulate 2-D shapes. Activities in which they sort, make and recognize patterns, build shapes, and talk about what is happening help to develop essential geometric skills. It is through such activities that students will learn the names of 2-D shapes and begin to recognize their characteristics.

It is very important to encourage students to use accurate language when describing objects. Students should be comfortable using such terms as **triangle, square, rectangle, circle**, and may extend their exploration to other 2-D shapes.

Students will initially see figures based on visual characteristics and will use non-geometric language to describe the shapes, but need to be given opportunities to explore common attributes of these basic shapes. They need to be encouraged to focus on what are the relevant characteristics of all shapes within that class rather than a single shape. Students should be aware that size, orientation, or color does not determine the type of 2-D shape. For example, students should recognize all of the following as triangles.



Students should be encouraged to observe the attributes of many shapes. Describing shapes allows to focus on their basic characteristics. Use questioning to focus student thinking; for example:

- What other shapes are similar to this one? In what way are they alike.

What does this object look like?

SCO: **SS8: Describe, compare and construct 2-D shapes, including:**

- triangles
- squares
- rectangles
- circles.

SS9: Identify 2-D shapes as parts of 3-D objects in the environment

[C, CN, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

SS8

- Sort a given set of 2-D shapes and explain the sorting rule.
- Identify common attributes of triangles, squares, rectangles and circles from given sets of the same type of 2-D shapes.
- Identify given 2-D shapes with different dimensions.
- Identify given 2-D shapes with different orientations.
- Create a model to represent a given 2-D shape.
- Create a pictorial representation of a given 2-D shape.

SS9

- Compare and match a given 2-D shape, such as a triangle, square, rectangle or circle, to the faces of 3-D objects in the environment.
- Name the 2-D faces of a given 3-D object

SCO: **SS8: Describe, compare and construct 2-D shapes, including:**

- triangles
- squares
- rectangles
- circles.

SS9: Identify 2-D shapes as parts of 3-D objects in the environment

[C, CN, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Use a variety of models of the 2-D shapes to ensure students are provided with many different examples of triangles, squares, rectangles, or circles.
- Use 5 x 5 geoboards for students to explore how many different squares, rectangles, or triangles they can make.
- Encourage students to provide explanations beyond simple naming of shapes. For example, if the student says it is a triangle because it looks like a triangle, ask them to describe the characteristics of triangles.
- Include activities that focus on the entire class of shapes (e.g., all rectangles) and their likenesses and differences.

Suggested Activities

- Provide students with the large triangle, square, and rectangle of an attribute-block set. Ask them to work in pairs, comparing the three shapes, and listing all the ways in which they are the same and how they differ. They should be able to properly identify the shapes and to compare such things as number of sides or length of sides.
- Provide a drawing which incorporates many shapes. Ask the student to point out the triangle (among the shapes).
- Give students a combination of 2-D and 3-D shapes. Ask them to investigate which of the 2-D shapes appears most often as a face of a 3-D shape.
- Include activities in which students are required to find shapes in pictures of objects and their environment.
- Have children to hunt around the school to find various shapes (e.g., squares, triangles, circles, rectangles). Have them share their findings and discuss why certain shapes are more common than others.
- Put students in small groups and have each group create a book of 2-D shapes by cutting pictures from catalogues and magazines.
- Have students use shapes to spell the name of the shape or make the shape. For example, use a collection of triangles to form the letters that spell "triangle" or create the sides of a larger triangle.
- Have students physically build the shapes using their hands/fingers or in groups using their bodies.

SCO: **SS8: Describe, compare and construct 2-D shapes, including:**

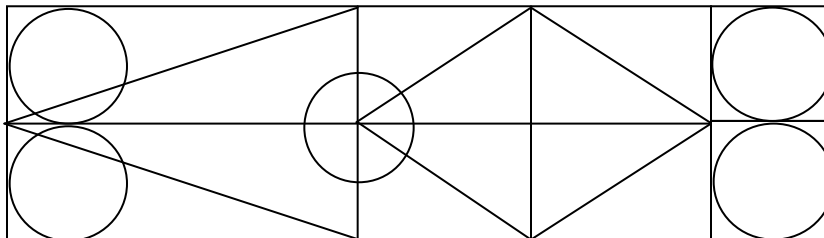
- triangles
- squares
- rectangles
- circles.

SS9: Identify 2-D shapes as parts of 3-D objects in the environment

[C, CN, R, V]

Assessment Strategies

- Listen to students' observations as they work on activities to determine whether they understand common characteristics of shapes.
- Ask the student to describe or draw different triangles (rectangles).
- Have students create a picture using only two shapes (i.e., circles and squares). Have them share their pictures with the class and discuss what they used.
- Make a rectangle (or triangle or square) on a geoboard and have students make 2 different rectangles on their geoboards.
- Provide students with a set of tangrams. Have students use 2 or more of the pieces to create a square or a triangle.
- Provide students with a design that includes a variety of 2-D shapes, such as the one below. Ask them to identify for you the different shapes they can find. They can use different colours to trace around each shape.



STATISTICS AND PROBABILITY

SCO: SP1: Gather and record data about self and others to answer questions. [C, CN, PS, V] SP2: Construct and interpret concrete graphs and pictographs to solve problems. [C, CN, PS, R, V]

[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Math and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Scope and Sequence

Grade One	Grade Two	Grade Three
	SP1 Gather and record data about self and others to answer questions. SP2 Construct and interpret concrete graphs and pictographs to solve problems.	SP1 Collect first-hand data and organize it using: tally marks; line plots; charts; lists to answer questions. SP2 Construct, label and interpret bar graphs to solve problems.

Elaboration

To make sense of their world, young students often find themselves **collecting and organizing data**, either verbally, pictorially, or in charts. Even though data management has not been formally taught in mathematics prior to grade two, it is reasonable to expect that students will have had experiences with collecting data and various types of graphs in other content areas. The focus of data management explorations should be that its purpose is to answer questions. The emphasis should always be on how to present and interpret data. The **context** should be realistic and of interest to the students. The data should be used to solve a problem and the graphs should clearly communicate the information that was collected.

Students should be encouraged to organize the data they collect in the course of classroom investigations. **Tally marks** are one of simplest ways to record organize data (it also promotes skip counting by 5's and 10's), but students may also use checkmarks or objects to record their data.

The most basic type of **concrete graph** is a “people” graph, in which the children themselves form the graph. It is important initially to allow children to form lines on their own so they learn the importance of all “lines” starting at the same level and of matching students in the “lines” in some form of one-to-one correspondence. Eventually, it is helpful to use some form of graphing mat on which each student stands in one box. If the children tape their name tags to their boxes before stepping out, they will be able to view the graph as a whole. Concrete graphs are bar graphs made of actual materials (such as sneakers, apples, caps, colour tiles, linking cubes, etc.). **Pictographs** are picture graphs that use a drawing of some type that represent what is being graphed. The pictographs should be based on a one-to-one correspondence (i.e., a picture represents one item, not a group of items). Since it is easier for young children to understand real and picture graphs, it is important for them to create these before moving on to symbolic bar graphs.

Students should create and **interpret graphs** that run horizontally and those that run vertically. They should also explore the common attributes of graphs, such as titles, labels, columns, and rows. Once a graph is completed, students need to discuss what the graph shows and what other questions it may answer.

SCO: **SP1: Gather and record data about self and others to answer questions.**

[C, CN, PS, V]

SP2: Construct and interpret concrete graphs and pictographs to solve problems.

[C, CN, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

SP1

- Formulate a question that can be answered by gathering information about self and others.
- Organize data as it is collected using concrete objects, tallies, checkmarks, charts or lists.
- Answer questions using collected data.

SP2

- Determine the common attributes of concrete graphs by comparing a given set of concrete graphs.
- Determine the common attributes of pictographs by comparing a given set of pictographs.
- Answer questions pertaining to a given concrete graph or pictograph.
- Create a concrete graph to display a given set of data and draw conclusions.
- Create a pictograph to represent a given set of data using one-to-one correspondence.
- Solve a given problem by constructing and interpreting a concrete graph or pictograph.

SCO: **SP1: Gather and record data about self and others to answer questions.**

[C, CN, PS, V]

SP2: Construct and interpret concrete graphs and pictographs to solve problems.

[C, CN, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Emphasis should be placed on answering real questions and ways to present data and how to interpret the data that is collected.
- Ensure that data management activities are relevant and of interest to the students in your class. Each class member should be able to participate and contribute data to the investigation.
- Encourage students to conduct small surveys to collect data.
- Use a piece of vinyl and tape to create a floor mat grid on which children can stand to form a graph. Have students place name cards on the grid before they step off it so everyone can see the whole graph that was created.
- Make use of opportunities to integrate graphing concepts into other subject areas, such as “You and Your World”.

Suggested Activities

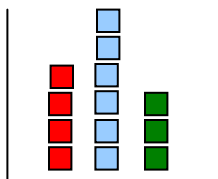
- Ask students to arrange themselves into a concrete people graph to compare the number of students who are wearing shoes with laces wearing shoes with Velcro.
- Ask the student to look at the way the information in the following graph is displayed.

Pizza: ☺ ☺ ☺ ☺ ☺ ☺

Burgers: ☺ ☺ ☺ ☺

Say, “It looks like the same number of children like pizza and burgers? Is that correct? What wrong with the way the graphs was made?”

- Ask the students to create a pictograph graph to show the number of children in the class who play various games (sports) or musical instruments. Have them write two questions that their graph will answer.
- Show students a graph without a title or labels and have them come up with different sets of data that the graph could represent.



SCO: **SP1: Gather and record data about self and others to answer questions.**

[C, CN, PS, V]

SP2: Construct and interpret concrete graphs and pictographs to solve problems.

[C, CN, PS, R, V]

Assessment Strategies

- Show students a concrete graph or pictograph on a topic of interest to students. Have them answer questions about the graph and have them make up their own question that the graph would answer.
- Have students create a pictograph to show how many students in the class know how to swim (or skate).
- Provide students with a collection of about 25 linking cubes in 3 or 4 different colours. Ask them to organize the cubes and record the data in a chart using tally marks or another method. Provide them with grid paper and have them create a graph to display the data.

LIST OF GRADE 2 SPECIFIC CURRICULUM OUTCOMES

NUMBER

- N1 Say the number sequence from 0 to 100 by 2s, 5s and 10s, forward and backward, using starting points that are multiples of 2, 5 and 10 respectively; 10s using starting points from 1 to 9; and 2s starting from 1.
- N2 Demonstrate if a number (up to 100) is even or odd.
- N3 Describe order or relative position using ordinal numbers (up to tenth).
- N4 Represent and describe numbers to 100, concretely, pictorially and symbolically.
- N5 Compare and order numbers up to 100.
- N6 Estimate quantities to 100 using referents.
- N7 Illustrate, concretely and pictorially, the meaning of place value for numerals to 100.
- N8 Demonstrate and explain the effect of adding zero to or subtracting zero from any number.
- N9 Demonstrate an understanding of addition (limited to 1 and 2-digit numerals) with answers to 100 and the corresponding subtraction by using personal strategies for adding and subtracting with and without the support of manipulatives; creating and solving problems that involve addition and subtraction; explaining that the order in which numbers are added does not affect the sum; and explaining that the order in which numbers are subtracted may affect the difference.
- N10 Apply mental mathematics strategies, such as: using doubles; making 10; one more, one less; two more, two less; building on a known double; and addition for subtraction to determine basic addition facts to 18 and related subtraction facts.

PATTERNS AND RELATIONS

(Patterns)

- PR1 Demonstrate an understanding of repeating patterns (three to five elements) by describing; extending; comparing; and creating patterns using manipulatives, diagrams, sounds and actions.
- PR2 Demonstrate an understanding of increasing patterns by describing; reproducing; extending; and creating patterns using manipulatives, diagrams, sounds and actions (numbers to 100).

(Variables and Equations)

- PR3 Demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100).
- PR4 Record equalities and inequalities symbolically using the equal symbol or the not equal symbol.

SHAPE AND SPACE

(Measurement)

- SS1 Relate the number of days to a week and the number of months to a year in a problem-solving context.
- SS2 Relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass (weight).
- SS3 Compare and order objects by length, height, distance around and mass (weight) using nonstandard units, and make statements of comparison.
- SS4 Measure length to the nearest non-standard unit by using multiple copies of a unit; and using a single copy of a unit (iteration process).
- SS5 Demonstrate that changing the orientation of an object does not alter the measurements of its attributes.

(3-D Objects and 2-D Shapes)

- SS6 Sort 2-D shapes and 3-D objects using two attributes, and explain the sorting rule.
- SS7 Describe, compare and construct 3-D objects, including: cubes; spheres; cones; cylinders; and pyramids.
- SS8 Describe, compare and construct 2-D shapes, including: triangles; squares; rectangles; and circles.
- SS9 Identify 2-D shapes as parts of 3-D objects in the environment.

STATISTICS AND PROBABILITY

(Data Analysis)

- SP1 Gather and record data about self and others to answer questions.
- SP2 Construct and interpret concrete graphs and pictographs to solve problems.

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