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Education and Early Childhood Development
English Programs

Prince Edward Island Mathematics Curriculum

Mathematics

Grade 4

CURRICULUM

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Prince Edward Island
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Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base. Developers have examined the curriculum proposed throughout Canada and secured the latest research in the teaching of mathematics, and the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for K-9 Mathematics* (2006) has been adopted as the basis for a revised mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories, and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP, and on the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➢ Essential Graduation Learnings

Essential graduation learnings (EGLs) are statements describing the knowledge, skills, and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills, and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work, and study today and in the future. Essential graduation learnings are cross curricular, and curriculum in all subject areas is focussed to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will demonstrate knowledge, skills, and attitudes expressed as essential graduation learnings, and will be expected to

- respond with critical awareness to various forms of the arts, and be able to express themselves through the arts;
- assess social, cultural, economic, and environmental interdependence in a local and global context;
- use the listening, viewing, speaking, and writing modes of language(s), and mathematical and scientific concepts and symbols, to think, learn, and communicate effectively;
- continue to learn and to pursue an active, healthy lifestyle;
- use the strategies and processes needed to solve a wide variety of problems, including those requiring language and mathematical and scientific concepts;
- use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

➢ Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate to all educational partners high expectations for students in mathematics education. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to

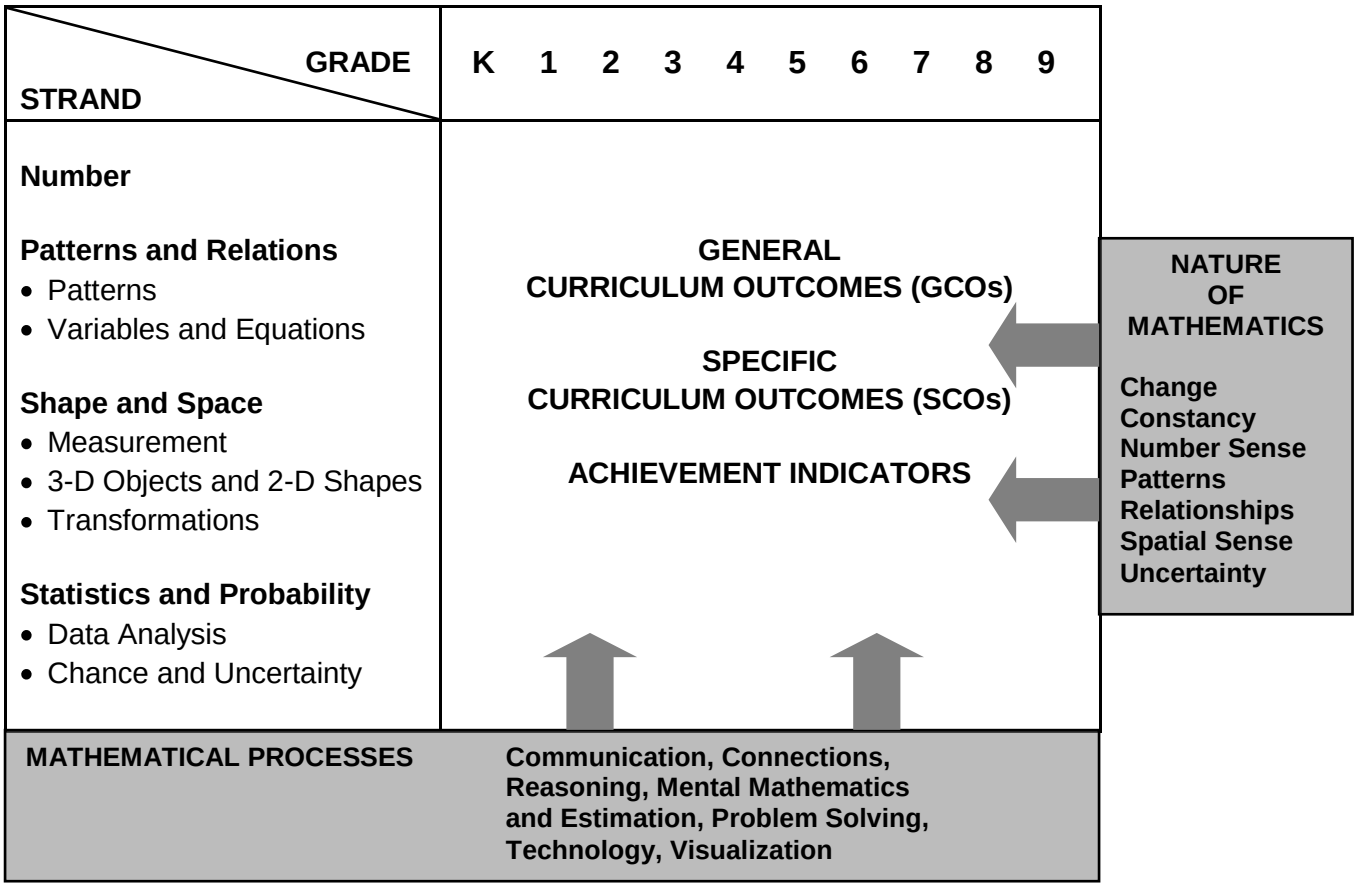
- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning;
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks;
- exhibit curiosity.

Conceptual Framework for K-9 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into four strands, namely Number, Patterns and Relations, Shape and Space, and Statistics and Probability. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connections among concepts both within and across strands. Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- Decreasing emphasis on rote calculation, drill, and practice, and the size of numbers used in paper and pencil calculations makes more time available for concept development.
- Problem solving, reasoning, and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➢ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]**
- develop visualization skills to assist in processing information, making connections, and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

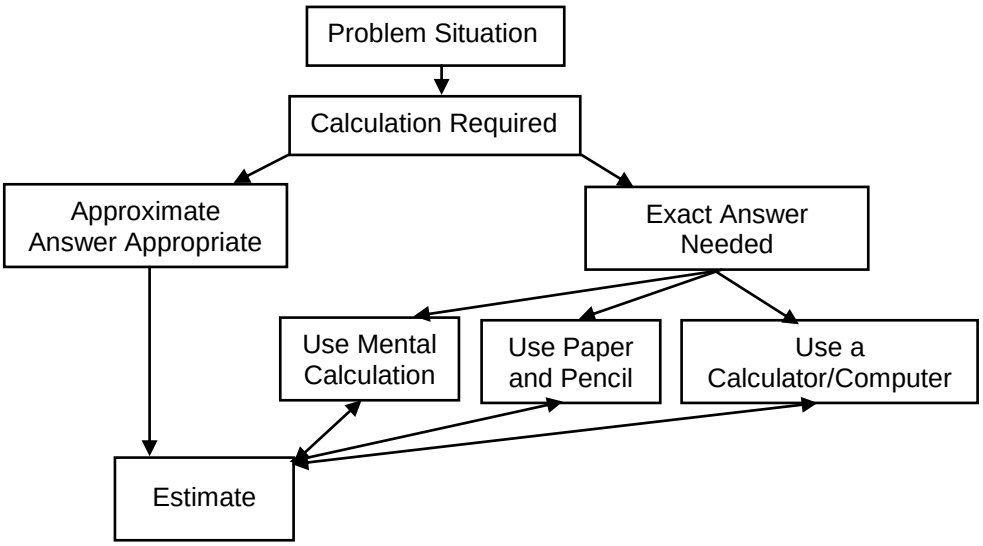
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves calculation without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need - more than ever before - with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001). Mental mathematics "provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers" (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know when to estimate, what strategy to use, and how to use it. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you. . . ?" or "How could you. . . ?" the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not

a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- using estimation
- guessing and checking
- looking for a pattern
- making an organized list or table
- using a model
- working backwards
- using a formula
- using a graph, diagram, or flow chart
- solving a simpler problem
- using algebra.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations;
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K-3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform, and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and knowledge of several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

➢ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics which are woven throughout this document. These components include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180^0.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (*The Primary Program*, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, and flexible and intuitive with numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collecting and analysing of data, and describing relationships visually, symbolically, orally, or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to interpret representations of 2-D shapes and 3-D objects, and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 2-D shapes and 3-D objects.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to use dimensions and make predictions about the results of changing dimensions.

- Knowing the dimensions of an object enables students to communicate about the object and create representations.
- The volume of a rectangular solid can be calculated from given dimensions.
- Doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of

probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Young children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Their mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Initial problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➢ Connections across the Curriculum

There are many possibilities for connecting Grade 4 mathematical learning with the learning occurring in other subject areas. Making connections between subject areas gives students experiences with transferring knowledge and provides rich contexts in which students are able to initiate, make sense of, and extend their learnings. When connections between subject areas are made, the possibilities for transdisciplinary inquiries and deeper understanding arise. When making such connections, however, teachers must be cautious not to lose the integrity of the learning in any of the subjects.

➢ Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding, and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should replace some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill-oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a teacher can give a parent clearer understanding of the mathematics curriculum and of the child's progress in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

➢ Diversity in Student Needs

Every class has students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters, but should be designed to help all students (whether strong, weak or average) to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson from which all students come away with a better understanding of what the solution to an equation really means.

➢ Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean not only that enrolments of students of both genders and various cultural backgrounds in public school mathematics courses should reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➢ Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English deficiencies and cultural differences must not be barriers to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and co-ordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication "as an essential part of mathematics and mathematics education (p.60)." The *Standards* elaborate that all students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading, and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers and will facilitate "communicating to learn mathematics and learning to communicate mathematically" (NCTM, p.60).

To this end,

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counsellors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not on other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated with appropriate language support to both students and parents;
- to verify that barriers have been removed, educators should monitor enrolment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➢ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development - such as poverty alleviation, human rights, health, environmental protection, and climate change - into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental, and economic perspective, and exploring how those factors are interrelated and interdependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database

Resources for Rethinking, found at **http://r4r.ca/en**. It provides teachers with access to materials that integrate ecological, social, and economic spheres through active, relevant, interdisciplinary learning.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, or teaching has been effective, or how best to address student learning needs. The quality of the assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others.

➢ Assessment

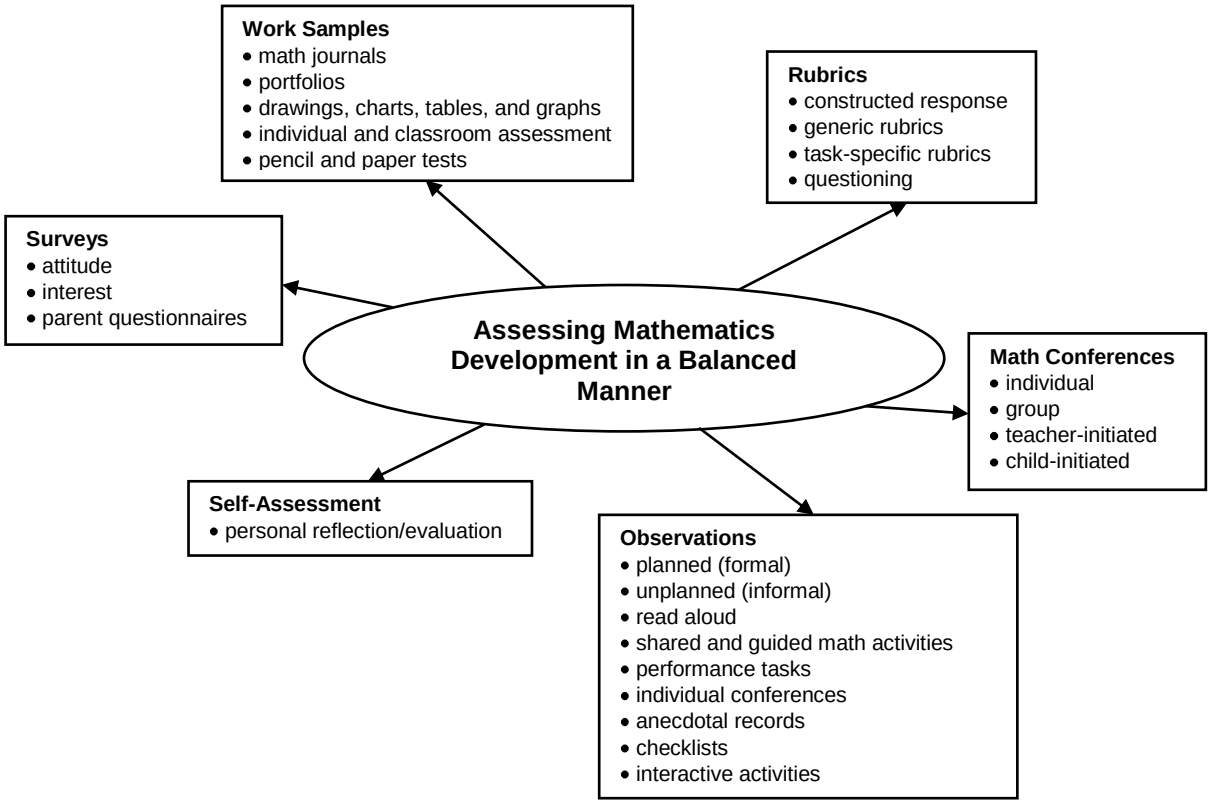
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children's learning;
- providing information to teachers on the effectiveness of their teaching, the program, and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources, including

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment.

This balanced approach for assessing mathematics development is illustrated in the diagram below.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning, and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used

a. to engage students in their own learning and self-assessment;
b. to help students understand what is important in the mathematical concepts and particular tasks they encounter;
c. to develop effective habits of metacognition and self-coaching;
d. to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used

e. to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
f. to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
g. to provide feedback to students about how they are doing and how they might improve;
h. to provide feedback to other professionals and to parents about how to support students' learning.

Assessment *of* learning is used

i. to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
j. to facilitate reporting;
k. to provide the basis for sound decision-making about next steps in a student's learning.

➢ Evaluation

Evaluation is the process of analysing, reflecting upon, and summarizing assessment information, and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires

- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information;
- using a high level of professional judgment in making decisions based upon that information.

➢ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes, and phone calls.

➢ Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration, and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles, as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes, or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.
- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.

- Assessment reports should be clear, accurate, and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that

- the best interests of the student are paramount;
- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes;
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he/she knows and can do.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island mathematics curriculum are organized into four strands across the grades K-9. They are Number, Patterns and Relations, Shape and Space, and Statistics and Probability. These strands are further subdivided into sub-strands, which are the general curriculum outcomes (GCOs). They are overarching statements about what students are expected to learn in each strand or sub-strand from grades K-9.

Strand	General Curriculum Outcome (GCO)
Number (N)	**Number:** Develop number sense.
Patterns and Relations (PR)	**Patterns**: Use patterns to describe the world and solve problems.
	Variables and Equations: Represent algebraic expressions in multiple ways.
Shape and Space (SS)	**Measurement**: Use direct and indirect measure to solve problems.
	3-D Objects and 2-D Shapes: Describe the characteristics of 3-D objects and 2-D shapes, and analyse the relationships among them.
	Transformations: Describe and analyse position and motion of objects and shapes.
Statistics and Probability (SP)	**Data Analysis**: Collect, display, and analyse data to solve problems.
	Chance and Uncertainty: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific outcome.

The first two pages for each outcome contain the following information:

- the corresponding **strand** and **general curriculum outcome**
- the **Specific Curriculum Outcome**(s) and the mathematical **processes** which link this content to instructional methodology
- the **scope and sequence** of concept development related to this outcome(s) from K - 2
- an **elaboration** of the outcome
- a list of **achievement indicators**

  Students who have achieved a particular outcome should be able to demonstrate their understanding in the manner specified by the achievement indicators. It is important to remember, however, that these indicators are not intended to be an exhaustive list for each outcome. Teachers may choose to use additional indicators as evidence that the desired learning has been achieved.

The last two pages for each outcome contain lists of **instructional strategies** and **strategies for assessment**.
The primary use of this section of the guide is as an ***assessment for learning*** (formative assessment) tool to assist teachers in planning instruction to improve learning. However, teachers may also find the ideas and suggestions useful in gathering ***assessment of learning*** (summative assessment) data to provide information on student achievement.

The second half of this curriculum guide contains a supplement which provides suggestions and recommendations for using *Math Makes Sense 4* as the primary resource for addressing curriculum outcomes.

NUMBER

SCO: **N1: Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.**
[C, CN, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
N2 Represent and describe numbers to 1000, concretely, pictorially and symbolically.	**N1** Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.	**N1** Represent and describe whole numbers to 1 000 000.

Elaboration

Students should recognize the value represented by each digit in a number, as well as what the number means as a whole. The “2” in 2300 means 2 thousands whereas the “2” in 3200 means 2 hundreds.

Students should have many opportunities to:

- **model** numbers containing zeroes. For example, 1003 means 1 thousand and 3 ones but is read as “one thousand, three.”
- **read** numbers several ways. For example, 9347 is read 9 thousand, three hundred forty-seven but might also be expressed as 93 hundred, 47 (other examples may include:9 thousands, 34 tens, 7 ones; 9 thousands, 33 tens, 17 ones);
- **record** numbers. For example, ask students to **write** twenty-eight hundred sixty; a number which is eighty less than ninety thousand; as well as write numbers in standard form and **expanded notation** (7453 = 7000 + 400 + 50 + 3).

Through these experiences, students will develop flexibility in identifying, modeling, and representing numbers up to 10 000. It is also important for students to gain an understanding of the relative size (magnitude) of numbers through real life contexts that are personally meaningful. Use numbers from student’s experiences, such as capacity for local arenas, or population of the school or community. Students can use these **personal referents** to think of other large numbers. They can also use **benchmarks** that students may find helpful are multiples of 100 and 1000, as well as 250, 500, 750, 2500, 5000, and 7500.

Include situations in which students use a variety of models, such as:

- base-ten blocks (e.g., to model 10 000 have the class make a long rod with 10 big cubes. It will be a 10 thousand rod. Students should recognize that this also models 10 000 unit cubes.)
- money (e.g., How many $100 bills are there in $9347?)
- place value charts.

Thousands			Ones		
H	T	O	H	T	O

The focus of instruction should be on ensuring students develop a strong sense of number.

SCO: **N1: Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.**
[C, CN, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Write a given numeral 0 – 10 000 in words.
- Represent a given numeral using a place value chart or diagrams.
- Describe the meaning of each digit in a given numeral.
- Express a given numeral in expanded notation, e.g., 4301 = 4000 + 300 + 1.
- Write the numeral represented by a given expanded notation. e.g. 2000 + 400 + 60 = 2460.
- Explain and show the meaning of each digit in a given 4-digit numeral with all digits the same, e.g., for the numeral 2222, the first digit represents two thousands, the second digit two hundreds, the third digit two tens and the fourth digit two ones.
- Write a given numeral using proper spacing without commas, e.g., 4567 or 4 567, 10 000.
- Read numerals up to four-digits without using the word “and,” e.g., 365 is read as “three hundred, sixty-five; 5321 is read as “five thousand, three hundred, twenty one.” The word “and” is reserved for reading decimal numbers, e.g., 3.8 is “three *and* eight tenths”.

SCO: **N1: Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.**
[C, CN, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Invite students to investigate the length of a line comprising 10 000 pennies. Encourage students to share the various strategies they used to investigate this problem. It is also important to have them share strategies that they considered, but rejected.
- Use base ten blocks or have students draw pictorial representations of the blocks. Have students use them to explore what numbers might be represented by using exactly 10 base ten blocks. (Note: It is important to use the correct vocabulary when referring to the blocks; “flat”, not “hundred flat” and “rod”, not “ten rod”, etc., so students are flexible in their thinking of the models when working with decimals.)

Suggested Activities

- Provide a stack of 4 sets of shuffled cards numbered 0 - 9. Ask the students to select 5 cards and arrange them to make the greatest possible number. Ask them to record and read the number and to rearrange the cards to make the least possible number. Have this number recorded under the larger number. As an extension, have the students estimate the difference between the two numbers. This activity is an ideal opportunity for students to practise front-end subtraction (left-to-right calculations).
- Have students, as a class, create a “ten thousands” chart. Provide each small group of students with hundred grids (or other pictorial representations such as arrays of dots) and have them create a model to represent 1000. Combine these models to create a class representation of 10 000.
- Ask students to create a four-digit number using 9, 2, 7, and 5. The digit in the hundreds place must be two more than the ones place. List all of the possible numbers.
- Ask questions about the reasonableness of numbers such as, “Would it be reasonable for an elementary school to have 9600 students?” or “Would it be reasonable for an elevator to hold 20 people?” “Would someone be able to drive 26 hundred kilometres in a day?” “Would it be reasonable to pay $5 000 for a boat/book/computer?” Investigate and discuss possible answers. Have students create their own “reasonable” questions about a variety of topics.
- Have students find large numbers from newspapers and magazines. Ask them to share and discuss the numbers within their group. Have students read, write, and model the numbers in different ways.

SCO: **N1: Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.**
[C, CN, V]

Assessment Strategies

- Ask the student to use base-ten materials to model 2016 in three different ways. Have him/her explain the models.
- Ask the students to record a series of numbers that have been read to them (such as eight thousand eighty-two, sixteen hundred five). Include examples such as the greatest 4-digit number or a number one hundred less than the greatest 4-digit number.
- Ask: How are 903 and 9003 different? Similar?
- Tell the student that a boat costs $6135. Ask: If one were to pay for it in $100 bills, how many would be needed? Extend by asking how many $10 bills would be needed.

SCO: **N2: Compare and order numbers to 10 000.**
[C, CN]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
N3 Compare and order numbers to 1000.	**N2** Compare and order numbers to 10 000.	

Elaboration

Comparing and **ordering** numbers is fundamental to understanding numbers. Students should investigate meaningful contexts to compare and order two or more numbers, both with and without models. For example, ask them to compare/order populations of communities or capacities of arenas. The capacity of the Charlottetown Civic Center arena is 3400 fixed seats. At the Bell Centre in Montreal, it's 21 273.

Students must realize that when comparing two numbers with the same number of digits, the digit with the greatest value needs to be addressed first. For example, when asked to explain why one number is greater or less than another, they might say that 2542 < 3653 because 2542 is less than 3 thousands while 3653 is more than 3 thousands. When comparing 6456 and 6546, students will begin comparing the thousands and move to the right.

Students must recognize that when comparing the size of a number, the 4 in 4289 has a greater value than the 9 and they should be able to provide an explanation.

SCO: **N2: Compare and order numbers to 10 000.**
[C, CN]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

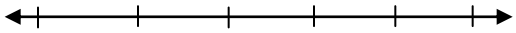
- Order a given set of numbers in ascending or descending order and explain the order by making references to place value.
- Create and order three different 4-digit numerals.
- Identify the missing numbers in an ordered sequence or on a number line.
- Identify incorrectly placed numbers in an ordered sequence or on a number line.

SCO: **N2: Compare and order numbers to 10 000.**
[C, CN]

Instructional Strategies

Consider the following strategies when planning lessons:

- Provide the students with opportunities to practise comparing numbers such as 12 098 and 12 210 and ask them to explain their reasoning.
- Have students discuss what would be the most appropriate benchmarks for various number lines, such as 0 to 50, 90 to 150, 200 to 1000, or 243 to 2448.
- Provide situations in which students name numbers which are greater than or less than a given number (Note: In some cases the amount greater or less could be specified, such as 29 more or 3000 less, etc) and name numbers which are between given numbers.
- Use a variety of number lines, including open number lines, in which students can place numbers and/or correct placed numbers.



Suggested Activities

- Display a 4-digit number on an overhead calculator (or on a card, or on the board). Have students enter on their calculators a number which differs by 1 digit. Have them read their numbers and ask others to determine if they are greater than or less than the number on the overhead. Collect five, or more, of their numbers and ask the students to order them on a number line. Explain.
- Assign pairs of students the task of making challenging number card for their classmates to order.
- Provide a list of populations of communities within your area ranging from a few hundred to about ten thousand. Ask students to order them from least to greatest. As an extension they could cluster the populations and graph them.
- Provide the following riddle: I am thinking of a number. It is between 8000 and 10 000. All the digits are even and the sum of the digits is 16. What are some possibilities? Use an open number line to display their numbers. Challenge students to write their own riddles.
- Tape numbers on students' backs and ask them to order themselves, without seeing their own number, from least to greatest without talking.
- When asked to find which of 9199 and 9210 is greater, Sadie said, "That is easy!" How did Sadie determine the greater one?
- Have the students find large numbers from newspapers and magazines. Ask them to create a collage that would illustrate the order of the numbers from least to greatest.
- Prepare cards for students to order from least to greatest. For example: 6183, 9104, 9080, 7102, 6604, 1999, 6540.
- Ask students to decide which is worth more: 4356 quarters, 8462 dimes or 9999 pennies. Have the students predict first, then use calculators to help solve the problem.
- Have students place numbers on the line using several benchmarks numbers to guide them.

SCO: **N2: Compare and order numbers to 10 000.**
[C, CN]

Assessment Strategies

- Give the students some number cards and ask them to order them from greatest to least.
- As a writing assignment, ask the student how he/she might advise a younger student to determine which of 2 numbers is greater .(interview or
- Ask the student to record two numbers to meet these requirements: the first has 3 in the thousands place, but is less than the second which has 3 in the hundreds place.
- Ask the students to write a number that has 980 tens.
- Ask the students to write a number that would fall about half way between 9490 and 10 000.
- Tell the students that you are thinking of a 5-digit number that has 2 thousands, a greater number of tens, and an even greater number of ones. Ask them to give three possibilities.
- Have the students create all of the possible numbers using the digits: 8, 9, 7, 6. Have students place their answers on a number line.
- Tell the student that Jodi's number had 9 hundreds, but Fran's had only 6 hundreds. Fran's number was greater. Ask: How was this possible?
- Ask: Which number below must be greater? Explain why.
  ☐4 ☐☐2 or 9 ☐☐3
- Ask the student how many whole numbers are greater than 8000, but less than 8750.

SCO: **N3: Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by:**
- **using personal strategies for adding and subtracting**
- **estimating sums and differences**
- **solving problems involving addition and subtraction.**

[C, CN, ME, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
N8 Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context. **N9** Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2 and 3-digit numerals) by: using personal strategies for adding and subtracting with and without the support of manipulatives; creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially and symbolically.	**N3** Demonstrate an understanding of addition of numbers with sums to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by: • *using personal strategies for adding and subtracting* • *estimating sums and differences* • *solving problems involving addition and subtraction*.	**N2** Use estimation strategies, including: • *front-end rounding* • *compensation* • *compatible numbers* *in problem-solving contexts*.

Elaboration

Personal strategies need to make sense to students and be accurate, as well as efficient. These alternatives are as valid as a traditional algorithm, which is the ultimate goal once their understanding of the operations has been developed (note: When introducing addition and subtraction with 3-and 4-digit numbers, use base-ten materials to model the operations).

Students need to recognize that **estimation** is a useful skill in their lives. To be efficient when estimating sums and differences mentally, students must be able to access a strategy quickly and they need a variety from which to choose. Some strategies to consider: using **benchmarks**, **rounding**, **front-end addition** and **subtraction (left-to-right calculations)**, and clustering of **compatible numbers**.

Students should have many opportunities to solve and create word problems for the purpose of answering real-life questions, preferably choosing topics of interest to them. These opportunities provide students with a chance to practise their computational skills and clarify their mathematical thinking.

Computational fluency is a balance between conceptual understanding (thinking about the structure of numbers and the relationship among numbers and the operations) and computational proficiency (includes both efficiency and accuracy). (NCTM, 2000, p.35)

SCO: **N3: Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by:**
- **using personal strategies for adding and subtracting**
- **estimating sums and differences**
- **solving problems involving addition and subtraction.**

[C, CN, ME, PS, R]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Explain how to keep track of digits that have the same place value when adding numbers, limited to 3- and 4-digit numerals.
- Explain how to keep track of digits that have the same place value when subtracting numbers, limited to 3- and 4-digit numerals.
- Represent concretely, pictorially, symbolically the addition and subtraction of whole numbers up 4-digit by 4-digit.
- Describe a situation in which an estimate rather than an exact answer is sufficient.
- Estimate sums and differences using different strategies, i.e., front-end estimation and compensation.
- Solve problems that involve addition and subtraction of whole numbers (one or more steps/where some numbers may be irrelevant). Explain solutions to problems.
- Create a problem given a number sentence for addition or subtraction.
- Solve problems that involve addition and subtraction in more than one way, limited to 3- and 4-digit numerals. For example, 385 + ☐ = 500 or 500 – 385 = ☐.

SCO: **N3: Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by:**
- **using personal strategies for adding and subtracting**
- **estimating sums and differences**
- **solving problems involving addition and subtraction.**

[C, CN, ME, PS, R]

Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Encourage students to estimate prior to calculating the answer.
- Use a variety of models, such as base ten blocks and number lines to assist in estimation.
- Help students to develop and apply a variety of estimation strategies, including:

  rounding: 439 + 52 is approximately 440 + 50.

  front-end for addition and subtraction: 138 + 24; 100 + 200 = 300. 476 – 348; 400 – 300 = 100

  ***adjusted* front-end for addition and subtraction:** 138 + 24; 100 + 200 = 300 and 30 + 40 is 70 for an estimate of 370. Depending on their *number sense*, some students may also consider the ones in their estimate and refine their answer to about 380. 476 – 348; 400 – 300 is 100, 70 – 40 is 30, 6 and 8 are about the same so I'll ignore them; my estimate is 130.

- Use computation strategies for solving problems, such as the open number line.
- Explore personal strategies such as those that can be found in *Teaching Student-Centered Mathematics, Grades 3-5,* Van de Walle and Lovin, p. 109 -111.
  Reinforce proper math vocabulary. "Regrouping" or "trading" is preferred to terms like "borrowing" or "carrying".

Suggested Activities

- Have students paraphrase various story problems to enhance understanding and to recognize which numbers in a problem refer to a part or to a whole. Share solutions.
- Provide the students with an addition number sentence, such as the following: 328 + 462 = 330 + 460. Have them decide if the number sentence is true or false and explain how they know. Encourage the students to think of the equal sign as "the same as" so that they are deciding whether the two sides balance each other.
- Ask the students to find two numbers with a difference of about 150 and a sum of about 500.
- Present the students with problems and have them decide which problems can be answered with an estimate only and which problems require calculation as well as an estimate. For example:

Will a container that holds 2000 mL be large enough to hold 1350 mL of water from another container as well as 1015 mL of water from a different container?

SCO: **N3: Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by:**
- **using personal strategies for adding and subtracting**
- **estimating sums and differences**
- **solving problems involving addition and subtraction.**

[C, CN, ME, PS, R]

Assessment Strategies

- Model the addition of 1273 and 485 using concrete and/or visual representations and record the process symbolically. Students should be able to explain their method.
- Subtract 248 from 5073 and model the subtraction using concrete or visual representations. Record the process symbolically. Students should be able to explain their method.
- Create an addition or subtraction story problem for the number sentence: 330 – 185 = ☐ or 185 + ☐ = 330.
- Determine the sum/difference of 3185 and 628 using a personal strategy and explain how the strategy works.
- Present the students with the following problem:
  You drink 250 mL of milk on the first day, 375 mL of milk the second day and 450 mL of milk on the third day. About how many millilitres of milk did you drink during these three days? Stimulate the students' thinking by asking whether 900 mL would be a good estimate for the answer.
- Tell the student that, to estimate 583 - 165, Jeff said, “575 subtract 175.” Ask him/her if the estimate will be high or low, and to explain why Jeff might have chosen to estimate in this way.

SCO: **N4: Explain the properties of 0 and 1 for multiplication and the property of 1 for division.** [C, CN, R]

N5: Describe and apply mental mathematics strategies, such as:
- **skip counting from a known fact**
- **using doubling or halving**
- **using doubling or halving and adding or subtracting one more group**
- **using patterns in the 9s facts**

to determine basic multiplication facts to 9 × 9 and related division facts. [C, CN, ME, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
N10 Apply mental mathematics strategies and number properties to determine answers for basic addition facts and related subtraction facts (to 18). **N11** Demonstrate an understanding of multiplication to 5 × 5	**N4** Explain the properties of 0 and 1 for multiplication and the property of 1 for division. **N5** Describe and apply mental mathematics strategies, such as: • skip counting from a known fact • using doubling or halving • using doubling or halving and adding or subtracting one more group • using patterns in the 9s facts • using repeated doubling to determine basic multiplication facts to 9 × 9 and related division facts.	**N3** Apply mental mathematics strategies and number properties, such as: • skip counting from a known fact • using doubling or halving • using patterns in the 9s facts • using repeated doubling or halving to determine answers for basic multiplication facts to 81 and related division facts.

Elaboration

It is important to address zero and one in multiplication. A number line can be used to illustrate both of these properties. To explore that the **product** is 0 when multiplying by 0, 3×0 can be shown by making 3 hops of 0 or making 0 hops of 3. The property of multiplying and dividing by 1 can similarly be explored on the number line. Provide opportunities for students to not only solve multiplication and division problems, but create their own problems requiring the use of these operations.

Developing basic **multiplication facts** to 9 × 9 and related division facts requires that the students have a strong foundation in patterns, number relationships, place value, and the meaning, relationships and properties of operations as described below:
- patterns are used in developing mental strategies, such as skip counting from a known fact and using the constant sum of the digits in products with the 9s facts;
- number relationships are evident when using the properties of operations or other strategies, such as repeated doubling; e.g., $4 \times 6 = (2 \times 6) \times 2 = 24$;
- place value is used extensively in various strategies, such as doubling and adding or subtracting one more group; e.g., $3 \times 7 = 2 \times 7 + 7 = 14 + 7 = 21$; $9 \times 9 = 10 \times 9 – 9 = 81$;
- the meaning of multiplication and division and the connection between the operations is crucial as the students develop understanding of multiplication and division facts. Students who have learned their multiplication facts have automatically learned their division facts.

SCO**: N4: Explain the properties of 0 and 1 for multiplication and the property of 1 for division.** [C, CN, R]

N5: Describe and apply mental mathematics strategies, such as:

- **skip counting from a known fact**
- **using doubling or halving**
- **using doubling or halving and adding or subtracting one more group**
- **using patterns in the 9s facts**

to determine basic multiplication facts to 9 × 9 and related division facts. [C, CN, ME, PS, R]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

N4

- Explain the property for determining the answer when multiplying numbers by one.
- Explain the property for determining the answer when multiplying numbers by zero.
- Explain the property for determining the answer when dividing numbers by one.

N5

- Provide examples for applying mental mathematics strategies:

  doubling (for 4×3, think $2 \times 3 = 6$, so $4 \times 3 = 6 + 6$)

  doubling and adding one more group (for 3×7, think $2 \times 7 = 14$, and $14 + 7 = 21$

  using known facts (for example, when multiplying 9×6, think $10 \times 6 = 60$, and $60 – 6 = 54$)

  halving (if 4×6 is equal to 24, then 2×6 is equal to 12)

  think division for multiplication facts (for $64 \div 8$, think $8 \times \square = 64$)

SCO**: N4: Explain the properties of 0 and 1 for multiplication and the property of 1 for division.** [C, CN, R]

N5: Describe and apply mental mathematics strategies, such as:

- **skip counting from a known fact**
- **using doubling or halving**
- **using doubling or halving and adding or subtracting one more group**
- **using patterns in the 9s facts**

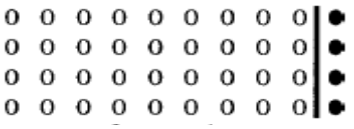
to determine basic multiplication facts to 9 × 9 and related division facts. [C, CN, ME, PS, R]

Instructional Strategies

Consider the following strategies when planning lessons:

- Use various concrete materials and pictorial representations to demonstrate the multiplication and division of zero. For example, use paper plates for the concept of multiplying by zero. Show six plates with zero counters on each. Ask: ″How many plates are there?″ (six) How many counters are there on each plate? ¨(zero)". Six groups of zero are how many? $6 \times 0 = 0$.
- Introduce a strategy with the use of materials, practise the strategy, and continue to introduce and practise new strategies. When students have two or more strategies, it is important to focus on strategy selection; choosing the strategy that will be most efficient to determine a particular fact.
- Encourage students to visualize the process for the strategy they are using.
  For example, using an array model.

$4 \times 9 = 4 \times 10$, or 40, subtract 4 is 36.



- Have students begin with what they know. For example, to figure out 6×8, one student might think, "I know $5 \times 8 = 40$ and one more 8 is 48." Another might think, "I know 3×8 is 24 and twice 24 is 48."
- Use the properties of multiplication in developing mental strategies: the associative property; e.g., $(2 \times 2) \times 6 = 2 \times (2 \times 6)$; the commutative property: 3×4 is read 3 sets, or groups of 4; the product however, is the same if the factors are reversed (4×3); distributive property: $4 \times 8 = (4 \times 5) + (4 \times 3) = 20 + 12 = 32$
- Address the misconception that multiplication always makes the product greater: any number multiplied or divided by 1 remains unchanged.

Suggested Activities

- Place students in pairs to practise the "double and double again" strategy for facts such as 4×7. (For example, 4×7 is double 2×7 which is 7 + 7. Since 2 sevens is 14, 4×7 is 28.) Students should take turns asking facts and providing answers by repeated doubling.
- Have students play the "Target Game."3 multiplied by what number is closest to the target number, without going over? Place individual items like the following on flashcards or on the overhead.
  $5 \times \square \longrightarrow$ 43 (Target) $\square$ are left over
- Tell the students that the "6" button on the calculator is not working. Have them suggest ways to solve "6×64" without using this button.

SCO**: N4: Explain the properties of 0 and 1 for multiplication and the property of 1 for division.**
[C, CN, R]

N5: Describe and apply mental mathematics strategies, such as:

- **skip counting from a known fact**
- **using doubling or halving**
- **using doubling or halving and adding or subtracting one more group**
- **using patterns in the 9s facts**

to determine basic multiplication facts to 9 × 9 and related division facts.
[C, CN, ME, PS, R]

Assessment Strategies

- Provide opportunities for students to demonstrate their understanding of these properties by posing questions and tasks such as:
  What general statement can you make about multiplying any number by one?
  What general statement can you make about dividing any number by one?
  Create a problem in which you are dividing a number by one.
  Create a problem in which you are multiplying a number by zero.
  What general statement can you make about multiplying any number by zero?
- To assess understanding of various thinking strategies for the multiplication facts, consider tasks and questions such as:
  Explain how would you find the answer to 30 ÷ 5 = by relating it to multiplication?
  What are two different strategies that we could use to figure out the answer to 6 × 7 = ?
  How does knowing 6 × 5 = 30 help you with 12 × 5 = ?
  How could 8 × 10 help you with 8 × 9 = ?
  Use counters or other materials to show how 5 × 8 is the same as (3 × 8) + (2 × 8).
  Jasmine said that she was supposed to divide 75 by 5, but found it easier to divide 150 by 10. Explain Jasmine's method. When is this strategy most useful?

SCO: **N6: Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by:**
- **using personal strategies for multiplication with and without concrete materials**
- **using arrays to represent multiplication**
- **connecting concrete representations to symbolic representations**
- **estimating products.**

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

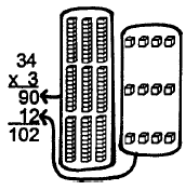
Scope and Sequence

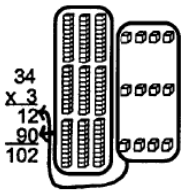
Grade Three	Grade Four	Grade Five
N11 Demonstrate an understanding of multiplication to 5 × 5.	**N6** Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by: • using personal strategies for multiplication with and without concrete materials • using arrays to represent multiplication • connecting concrete representations to symbolic representations • estimating products.	**N2** Use estimation strategies, including: • front-end rounding • compensation • compatible numbers in problem-solving contexts. **N5** Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems.

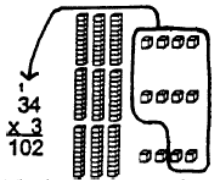
Elaboration

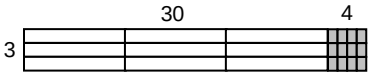
Encourage the students to **estimate** products and explore their own methods prior to learning the traditional **algorithm** or procedure for finding the **product**. These "personal algorithms" often serve as the procedure of choice, but students need to strive to use the most efficient strategy to solve a particular problem.

Students should use a variety of models to investigate multiplication problems to help develop an understanding of the connection between the model and the symbols. It is important to start with a word problem and then have students use materials to determine the product. Base-ten blocks serve as a tool for understanding the multiplication operation. It is important that the students use language as they manipulate the materials and record the corresponding symbols for the product. It is not expected that students would be explicitly taught all possible algorithms, but provide opportunities to discover which is most efficient for the numbers included in a given problem. Some examples of possible models for multiplication include:









Students should have many opportunities to solve and create word problems for the purpose of answering real-life questions, preferably choosing topics of interest to them. These opportunities provide students with a chance to practise their computational skills and clarify their mathematical thinking.

SCO: **N6: Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by:**
- **using personal strategies for multiplication with and without concrete materials**
- **using arrays to represent multiplication**
- **connecting concrete representations to symbolic representations**
- **estimating products.**

[C, CN, ME, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Model a given multiplication problem using the distributive property, e.g., 8 × 365 = (8 × 300) + (8 × 60) + (8 × 5).
- Use concrete materials, such as base ten blocks or their pictorial representations, to represent multiplication and record the process symbolically.
- Create and solve a multiplication problem that is limited to 2- or 3-digits by 1-digit.
- Estimate a product using a personal strategy, e.g., 2 × 243 is close to or a little more than 2 × 200, or close to or a little less than 2 × 250.
- Model and solve a given multiplication problem with and without an array and record the process.

SCO: **N6: Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by:**
- **using personal strategies for multiplication with and without concrete materials**
- **using arrays to represent multiplication**
- **connecting concrete representations to symbolic representations**
- **estimating products.**

[C, CN, ME, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Provide regular practice in estimation, accompanied by the sharing of strategies. When assessing estimation, the amount of time provided must be controlled in order to determine whether students are proficient in this skill. The goal is for students to automatically estimate in problem solving situations, not only when instructed to do so.
- Have the students estimate the product to the problem before calculating so that they are better able to determine the reasonableness of their answers.
- Provide a variety of problems representing the different multiplication situations with varying degrees of difficulty to differentiate instruction.
- Provide time for students to create their personal strategies to solve the problem and share these strategies with members of their group or with the entire class.
- Challenge the students to solve the problem another way, do a similar problem without models or clarify the explanation of their personal strategies.

Suggested Activities

- Ask students how they would use the front-end mental multiplication strategy for questions such as

  $3 \times 125 = 375$ $(3 \times 100 + 3 \times 20 + 3 \times 5)$ and encourage strategies such as $(3 \times 100 + 3 \times 25)$.
- Ask the students to fill in the blanks with 3, 4, and 5 in three different ways and find all the possible products. □ × =
- Provide students with problems to solve:

  You travel 375 km each day for 3 days. Will you reach the cabin that is 1200 km away by the end of the third day?

  You set up 6 rows of chairs with 28 chairs in each row in the gym. Are there enough chairs to seat 180 people? How many chairs did you set up?

  A toad jumps 135 cm on the first jump and twice as far on the second jump. About how far does it jump in all?

  You jog for 175 minutes each week. How many minutes do you jog in 28 days?
- Have students use a supermarket flyer. Ask them to select 6 of one item, 4 of another and 10 of a third item and give an estimate for the total.

SCO: **N6: Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by:**
- **using personal strategies for multiplication with and without concrete materials**
- **using arrays to represent multiplication**
- **connecting concrete representations to symbolic representations**
- **estimating products.**

[C, CN, ME, PS, R, V]

Assessment Strategies

- Tell the student that he/she has $60. Ask, “Do you have enough money to buy 3 CDs if each costs $17? How do you know?”.
- Ask the students if they can reach the cottage that is 1200 km away if they travel 375 km each day for 3 days. Explain your thinking.
- Write all the possible number sentences that are represented in the following array. Explain how each number sentence relates to the array.

  ********** ********** ********** * *
  ********** ********** ********** * *
  ********** ********** ********** * *

- Ask students to model 24×6. Have them explain the model.
- Tell students that for a school assembly, 9 rows of 38 chairs have been placed in the gym. Are there enough chairs for 370 students? Explain your thinking.
- Ask students to create and solve a realistic problem that includes the factors 6 and 329.
- Have students solve this problem. *You save 6 times as much money this year as you saved last year. If you saved $125 last year, how much money did you save this year?*

SCO: **N7: Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by:**
- **using personal strategies for dividing with and without concrete materials**
- **estimating quotients**
- **relating division to multiplication.**

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

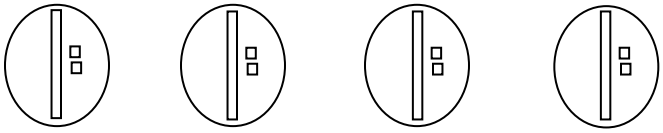
Scope and Sequence

Grade Three	Grade Four	Grade Five
N12 Demonstrate an understanding of division (limited to division related to multiplication facts up to 5 × 5)	**N7** Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by: • using personal strategies for dividing with and without concrete materials • estimating quotients • relating division to multiplication.	**N6** Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems.

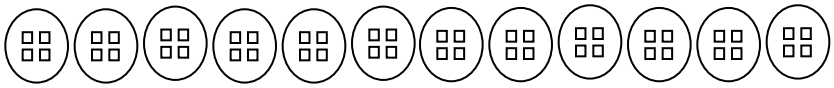
Elaboration

The concept of division needs to be taught in conjunction with multiplication. Students need to know the two meanings for division. These are as follows:

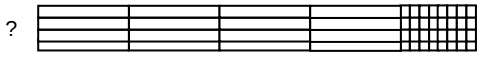
- identifying how many in each group (sharing): 48 pencils in 4 containers (base ten blocks as models).



- identifying how many groups (repeated subtraction): 48 pencils, 4 in each container.



Students can also use the area model with a missing dimension to show the relationship with multiplication.



Students should understand that the **remainder** (the number of units left over) must be less than the **divisor**. Models help to clarify this idea. In grade four, students are expected to express remainders as a digit and not as a fraction or decimal (e.g., a remainder of 7 is written as R7). Students also need to know that the answer for a division sentence is the **quotient** and the number to be divided is the **dividend**.

Students should have many opportunities to solve and create word problems for the purpose of answering real-life questions of personal interest. These opportunities provide students with a chance to practise their computational skills and clarify their mathematical thinking.

SCO: **N7: Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by:**
- **using personal strategies for dividing with and without concrete materials**
- **estimating quotients**
- **relating division to multiplication.**

[C, CN, ME, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Solve a given division problem without a remainder using arrays or base ten materials.
- Solve a given division problem with a remainder using arrays or base ten materials.
- Solve a given division problem using a personal strategy and record the process.
- Create and solve a word problem involving a 1- or 2-digit dividend.
- Estimate a quotient using a personal strategy, e.g., 86 ÷ 4 is close to 80 ÷ 4 or close to 80 ÷ 5.

(It is <u>not</u> intended that remainders be expressed as decimals or fractions.)

SCO: **N7: Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by:**
- **using personal strategies for dividing with and without concrete materials**
- **estimating quotients**
- **relating division to multiplication.**

[C, CN, ME, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Present division questions in context to identify either the sharing (how many in each group: partitioning) or the repeated subtraction (how many groups) meaning.
- Provide a variety of problem structures that include both of the meanings of division used in a real-life context.
- Provide regular practice in estimation, accompanied by the sharing of strategies.
- Have students create and share problems that include both of the meanings of division. It is helpful for many students to model this for them.

Suggested Activities

- Ask students to use a model to explain to a classmate how to share 86 marbles among five people. Discuss the different strategies used.
- Ask students to use a model to explain to a classmate how to find the number of groups of 6 in 28.
- Ask students to make up division problems about situations in the classroom and post them. Encourage them to give examples of both the sharing and the measurement meanings of division. Invite others to try to guess what the division situations are. For example, 25 ÷ 6 (classmates divided into groups of 6. How many groups?).
- Provide a list of division questions to pairs of students and ask them to estimate a quotient and explain their strategy to their partner and tell whether the estimate is too high or too low and why
- Present the students with a problem and have them choose which of the number sentences provided could be used to solve the problem and why they chose it. Example: Diego saved $96 this month by doing odd jobs for the neighbours. Last month, he saved $8. How many times as much money did he save this month as last month?

  $96 \times 8 = \square$ $\square = 8 \times 96$ $8 \times \square = 96$
  $96 \times \square = 8$ $96 \div 8 = \square$ $8 \div 96 = \square$
  $\square \div 8 = 96$ $96 \div \square = 8$ $8 \div \square = 96$
- Present students with a variety of problems to solve, such as:
- Tell students that there are 77 baseball cards to be shared between 2 students. Ask them how they know that there will be a remainder. What about sharing them among 5 students? 7 students?
- Tina rode her bicycle every day for 8 days. She cycled 68 km. About how far did she ride each day?
- Use base ten models to solve: If the area of a rectangular field is $182m^2$ and the length is 14m, how wide is the field?
- Show students on an overhead projector a number of counters. Have the students count them. Get them to close their eyes while you change the amount by one or two. Ask the students to open their eyes and tell you how the group of counters has changed.

SCO: **N7: Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by:**
- **using personal strategies for dividing with and without concrete materials**
- **estimating quotients**
- **relating division to multiplication.**

[C, CN, ME, PS, R, V]

Assessment Strategies

- Have the students use/draw models to show 83 ÷ 3.
- Ask students to explain why the answer to 69 ÷ 3 has to be 10 more than the answer to 39 ÷ 3.
- Ask: How many digits are there in the quotient of $4\overline{)57}$; explain how you know.
- Present the student with the following problem:

  You have 72 marbles to share equally among 4 friends. How many marbles will each friend receive? Explain how you know.
- Ask the student to explain the connection between multiplication and division by using counters or base ten materials. If necessary, coach the student to make an array and show how the array shows both multiplication and division.
- Have the student estimate 93 ÷ 5 and tell whether the estimate is probably too high or too low and why. Ask the student to suggest another division question for which the same estimate would be appropriate.
- Provide a set of base ten blocks. Ask the student to model 3 different division questions of his/her choice and to write the division sentence for each.

SCO: **N8: Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to:**

- **name and record fractions for the parts of a whole or a set**
- **compare and order fractions**
- **model and explain that for different wholes, two identical fractions may not represent the same quantity**
- **provide examples of where fractions are used.**

[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
N13 Demonstrate an understanding of fractions by: explaining that a fraction represents a part of a whole; describing situations in which fractions are used; comparing fractions of the same whole with like denominators.	**N8** Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to: • name and record fractions for the parts of a whole or a set • compare and order fractions • model and explain that for different wholes, two identical fractions may not represent the same quantity • provide examples of where fractions are used.	**N7** Demonstrate an understanding of fractions by using concrete and pictorial representations to: • create sets of equivalent fractions • compare fractions with like and unlike denominators.

Elaboration

In order for students to construct a firm foundation for fraction concepts, they need to experience and discuss activities that promote the following understandings:

- Fractional parts are equal shares or equal-sized portions of a whole or unit.
- A unit can be an object or a collection of things. More abstractly, the unit is counted as 1. On the number line, the distance from 0 to 1 is the unit.
- Fractional parts have special names that tell how many parts of that size are needed to make the whole. For example, thirds require three parts to make a whole.
- The more fractional parts used to make a whole, the smaller the parts. For example, eighths are smaller than fifths.
- The **denominator** of a fraction indicates by what number the whole has been divided in order to produce the type of part under consideration. Thus, the denominator is a divisor. In practical terms, the denominator names the kind of fractional part that is under consideration. The **numerator** of a fraction counts or tells how many of the fraction parts (or the type indicated by the denominator) are under consideration. Therefore, the numerator is a multiplier—it indicates a multiple of the given fractional part (Van de Walle and Lovin, vol. 1, 2006, p. 251).

Presenting fractions in context will make them much more meaningful to students. It is important that students develop visual images for fractions and be able to tell “about how much” a particular fraction represents. Therefore, students should model fractions using a variety of materials. To strengthen their fraction number sense, it is also recommended that the size of the whole be changed regularly. In grade four, the focus is on students initially developing a firm understanding of fractions less than one.

SCO: **N8: Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to:**

- **name and record fractions for the parts of a whole or a set**
- **compare and order fractions**
- **model and explain that for different wholes, two identical fractions may not represent the same quantity**
- **provide examples of where fractions are used.**

[C, CN, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Represent a given fraction using concrete materials.
- Identify a fraction from its given concrete representation.
- Name and record the shaded and non-shaded parts of a given set.
- Name and record the shaded and non-shaded parts of a given whole.
- Represent a given fraction pictorially by shading parts of a given set.
- Represent a given fraction pictorially by shading parts of a given whole.
- Explain how denominators can be used to compare two given unit fractions with numerator 1.
- Order a given set of fractions that have the same numerator and explain the ordering.
- Order a given set of fractions that have the same denominator and explain the ordering.
- Identify which of the benchmarks: 0, $\frac{1}{2}$, or 1 is closer to a given fraction.
- Name fractions between two given benchmarks on a number line.
- Order a given set of fractions by placing them on a number line with given benchmarks.
- Provide examples of when two identical fractions may not represent the same quantity, e.g., half of a large apple is not equivalent to half of a small apple; half of ten oranges is not equivalent to half of sixteen oranges.
- Provide an example of a fraction that represents part of a set and, a fraction that represents part of a whole from everyday contexts.

SCO: **N8: Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to:**

- **name and record fractions for the parts of a whole or a set**
- **compare and order fractions**
- **model and explain that for different wholes, two identical fractions may not represent the same quantity**
- **provide examples of where fractions are used.**

[C, CN, PS, R, V]

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Instructional Strategies

Consider the following strategies when planning lessons:

- Present **three models** for fractions: 1) part of a region, 2) part of a set, 3) part of a length or measures.
- Ensure students develop an understanding that a fraction is not meaningful without knowing what the "whole" is.
- Develop conceptual understanding of **fractions** by allowing students to use physical materials to model and compare fractions:
  in which the denominators are the same; e.g., five-eighths is greater than three-eighths
  in which the numerators of the fractions are the same; e.g., three-quarters is greater than three-fifths
  in which the numerator of a fraction is compared to the denominator in deciding its relation to a given benchmark; e.g., three-eighths is less than one-half because three is less than half of eight (Van de Walle and Lovin, vol. 1, 2006, p. 265)
- Remember to use a horizontal line when writing fractions, instead of a slash: e.g., $\frac{3}{4}$

Suggested Activities

- Have the students order a set of fractions. Use sticky notes and put a fraction on 4-8 students' foreheads. The students need to place themselves in order without speaking. Invite students to determine what fraction of the letters in their names are vowels.
- Have students explore fraction relationships among pattern blocks and other materials.
- Show examples and non-examples of specified fractional parts. Have students identify the wholes that are correctly divided into requested fractional parts and those that are not. For each response, have students explain their reasoning. The activity should be done with a variety of models, including length and set models.
- Tell the student that you have 8 coins. Half of them are pennies. More than 1/8 of them are quarters. The others are nickels. Have the student use coins to represent the situation. How much money might you have? Students are then asked to create coin problems using proper fraction notation.
- Provide students with different sizes and shapes of paper and have them estimate and then tear-off different fractional parts, such as one-fifth. Have them explain their thinking. Students can compare their "fifths" as the size of these will vary depending on the size of the whole.

SCO: **N8: Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to:**

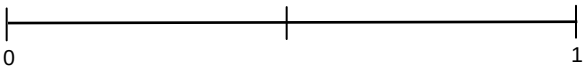
- **name and record fractions for the parts of a whole or a set**
- **compare and order fractions**
- **model and explain that for different wholes, two identical fractions may not represent the same quantity**
- **provide examples of where fractions are used.**

[C, CN, PS, R, V]

Assessment Strategies

- Have students place the following fractions on the number line below and verify their positions using models.

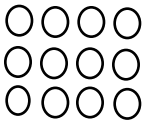
$\frac{5}{6}, \frac{1}{4}, \frac{7}{8}, \frac{5}{8}, \frac{1}{10}$



- Present the following problem to students: Kim ate $\frac{1}{4}$ of her pizza and David ate $\frac{3}{4}$ of his pizza. Kim said that she ate more pizza than David. Explain how Kim could be right by using diagrams and words.
- Place the following pairs of fractions before the student, one at a time. Tell the student to circle the larger fraction and explain orally how he or she knows that the fraction is larger. Then, have them select a manipulative and model the fractions to verify their choice.

$\frac{1}{5}$ $\frac{3}{5}$ $\frac{3}{8}$ $\frac{3}{5}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{4}{8}$ $\frac{3}{6}$ $\frac{3}{4}$ $\frac{9}{10}$

- Ask the student to tell why, whenever you see a representation of $\frac{1}{3}$, there is always a $\frac{2}{3}$ associated with it.
- Ask the student to colour $\frac{1}{4}$ of the circles.



SCO: **N9: Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically.**
[C, CN, R, V]
N10: Relate decimals to fractions (to hundredths).
[CN, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
	N9 Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically. **N10** Relate decimals to fractions (to hundredths).	**N8** Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically. **N9** Relate decimals to fractions and fractions to decimals (to thousandths). **N10** Compare and order decimals (to thousandths), by using: • benchmarks • place value • equivalent decimals.

Elaboration

Number sense with fractions and decimals requires that the students develop a conceptual understanding of fractions and decimals as numbers. To work effectively with fractions and decimals, the students should demonstrate the ability to:

- Represent numbers using words, models, diagrams and symbols and make connections among various representations.
- Give other names for numbers and justify the procedures used to generate the equivalent forms.
- Describe the relative magnitude of numbers by comparing them to common benchmarks, given simple estimates, ordering a set of number, and finding a number between two numbers.

Conceptual understanding of decimals requires that the students connect decimals to **whole numbers** and to fractions. Decimals are shown as an extension of the whole number system by introducing a new place value, the **tenth**'s place, to the right of the one's place. The tenth's place follows the pattern of the base ten number system by iterating one-tenth ten times to make one whole or a unit (Wheatley and Abshire 2002, p. 152). Similarly, the hundredth's place to the right of the tenth's place iterates one-**hundredth** ten times to make one-tenth.

The connection between decimals and fractions is developed conceptually when the students read decimals as fractions and represent them using the same visuals. For example, 0.8 is read as eight-tenths and can be represented using fraction strips or decimal strips (Wheatley and Abshire, 2002). Students should use a variety of materials to model and interpret decimal tenths and hundredths.

SCO: **N9: Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically.**
[C, CN, R, V]
N10: Relate decimals to fractions (to hundredths).
[CN, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

N9

- Write the decimal for a given concrete or pictorial representation of part of a set, part of a region or part of a unit of measure.
- Represent a given decimal using concrete materials or a pictorial representation.
- Explain the meaning of each digit in a given decimal with all digits the same.
- Represent a given decimal using money values (dimes and pennies).
- Record a given money value using decimals.
- Provide examples of everyday contexts in which tenths and hundredths are used.
- Model, using manipulatives or pictures, which a given tenth can be expressed as hundredths, e.g., 0.9 is equivalent to 0.90 or 9 dimes is equivalent to 90 pennies.

N10

- Read decimals as fractions, e.g., 0.5 is zero and five tenths.
- Express orally and in written form a given decimal in fractional form.
- Express orally and in written form a given fraction with a denominator of 10 or 100 as a decimal.
- Express a given pictorial or concrete representation as a fraction or decimal, e.g., 15 shaded squares on a hundred grid can be expressed as 0.15 or $\frac{15}{100}$.
- Express orally and in written form the decimal equivalent for a given fraction, e.g., $\frac{50}{100}$ can be expressed as 0.5

SCO: **N9: Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically.**
[C, CN, R, V]
N10: Relate decimals to fractions (to hundredths).
[CN, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Foster understanding of decimals by ensuring that they be read correctly. For example, 3.4 should be read as 3 and 4 tenths, not 3 point 4, or 3 decimal 4. It is also important that students understand the relationship between fractions and decimals. Example, 12.56 reads as 12 and 56 hundredths.
- Saying decimal numbers correctly will assist students in gaining an understanding of how decimals relate to fractions. By saying 12 and 56 hundredths, 56 is the numerator and 100 is the denominator. Plus, saying the number correctly reinforces that the digits to the right of the decimal are part of the whole number.
- Help students extend the place-value system to decimals by focusing on the basic pattern of ten. Remind students that 10 ones make 1 ten, 10 tens make 1 hundred, etc. Then, extend this pattern to help students understand that it takes 10 equal parts (tenths) to make 1 whole and 100 equal parts (hundredths) to make 1 whole. Explain the place value of the digits to the right of the one are tenths and hundredths.
- Investigate the relationship between 0.01, 0.1 and 1.0 by making analogies and using real-life objects which are sized proportionally.

Suggested Activities

- Use a variety of materials to model number with decimals to the hundredths. Ensure that some models show equivalent fractions/decimals. Shading in 2 tenths of a 100 grid represents the equivalent fraction/decimal of 20 hundredths. Provide students with ample opportunities to write the decimal and the fraction of what the model represents.
- Show students how to make a calculator “count” by ones by pressing +, 1, =, =, … Now have students press +, 0.1, =, =, … when the display reaches 0.9, stop and discuss what this means and what the display will look like with the next press. Many students will predict 0.10 (thinking that 10 comes after 9). When the tenth press produces a display of 1 (Note: calculators never display trailing zeroes), the discussion should revolve around regrouping 10 tenths for a whole. How many presses to get from one whole number to the next? Repeat counting by 0.01 (Van de Walle and Lovin, 2006)
- Ask students to show 2 tenths if a large cube represents one whole; if a flat represents one whole; if a rod represents one whole. Extend this to explore hundredths.

SCO: **N9: Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically.**
[C, CN, R, V]
N10: Relate decimals to fractions (to hundredths).
[CN, R, V]

Assessment Strategies

- Have students use a hundred grid to show a capital "T" that takes up more than 0.20 or of the grid and one that takes less than 0.20 of the grid. Express the shaded and un-shaded areas as fractions.
- Ask students where they would find decimal numbers in their daily lives.
- Ask the student to use a model of choice to explain why 0.40 and 0.4 are equivalent.
- Ask the student to give the number that is 0.01 more than, or less than, 3.24.
- Explain to the student that someone forgot to put the decimal in the number 1427. Ask where it would be if the number is less than 100.
- Ask students to read decimal numbers orally. Example: 2.5, 26.9, $127.60, 44.09, 0.02
- Have students write the numbers that you say to them. Example: 3.2, 87.06, 0.14, $5.40
- Plot common fraction and decimal equivalents on a number line. For example: one half, one fourth, one tenth, seventy-five hundredths.
- Have students count forward and backward from any number. For example, count on in tenths from 4.7 or count backwards in hundredths from 4.05.

SCO: **N11: Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by:**
- **using compatible numbers**
- **estimating sums and differences**
- **using mental math strategies to solve problems.**

[C, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

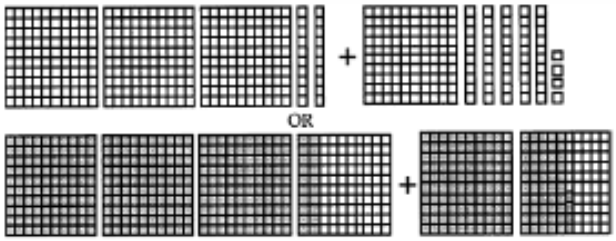
Scope and Sequence

Grade Three	Grade Four	Grade Five
N6 Describe and apply mental mathematics strategies for adding two 2-digit numerals. **N7** Describe and apply mental mathematics strategies for subtracting 2-digit numerals.	**N11** Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by: • using compatible numbers • estimating sums and differences • using mental math strategies to solve problems.	**N10** Relate decimals to fractions and fractions to decimals (to thousandths). **N11** Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).

Elaboration

It is essential that students recognize that all of the properties and techniques established for the addition and subtraction of whole numbers apply to decimals. Students should recognize that adding or subtracting **tenths** (e.g., 3 tenths and 4 tenths are 7 tenths) is similar to adding or subtracting quantities of other items (e.g., 3 apples and 4 apples are 7 apples). The same is true with **hundredths**. Rather than simply telling students to line up decimals vertically, or suggesting that they "add zeroes," they should be directed to think about what each **digit** represents and what parts go together. For example: 1.62 + 0.3, a student might think, 1 whole, 9 (6 + 3) tenths and 2 hundredths, or 1.92.

Base-ten blocks and hundredths grids continue to be useful models. If a flat represents one whole unit, then 3.2 + 1.54 would be modeled as:



Students need to recognize that **estimation** is a useful skill in their lives. To be efficient when estimating **sums** and **differences** mentally, students must be able to access a strategy quickly and they need a variety from which to choose. Situations must be provided regularly to ensure that students have sufficient practice with mental math strategies and that they use their skills as required. When a problem requires an exact answer, students should first determine if they are able to calculate it mentally; this should be an automatic response.

SCO: **N11: Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by:**
- **using compatible numbers**
- **estimating sums and differences**
- **using mental math strategies to solve problems.**

[C, ME, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Predict sums and differences of decimals using estimation strategies.
- Solve problems, including money and measurement, which involve addition and subtraction of decimals, limited to hundredths.
- Ask students to determine which problems do not require an exact solution.
- Determine the approximate solution of a given problem using compatible numbers.
- Determine an exact solution using mental computation strategies.
- Count back change for a given purchase.

SCO: **N11: Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by:**
- **using compatible numbers**
- **estimating sums and differences**
- **using mental math strategies to solve problems.**

[C, ME, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Encourage students to estimate prior to calculating answers.
- Use a variety of appropriate models, such as base ten blocks and number lines to assist students in their initial consideration of estimation.
- Use **e*stimation strategies*** including: Compatible numbers: e.g., 0.72 + 0.23 are close to 0.75 and 0.25 which are compatible numbers so the sum of the decimal numbers must be close to 1. Front-end addition: e.g., 32.3 + 24.5 + 14.1; a student might think "30 + 20 + 10 is 60 and the ones and tenths clustered together make about another 10 for a total of 70." Front-end subtraction: e.g., 4.76 – 3.48; a student might think "4 ones – 3 ones is 1 and 7 tenths – 4 tenths is 3 tenths for a difference of approximately 1 and 3 tenths." Rounding: e.g., 4.39 + 5.2 is approximately 4+ 5 for an estimate of 9.
- Use a think-aloud strategy to model a variety of mental computation strategies.

Suggested Activities

- Give students word problems that require the addition and/or subtraction of whole numbers and decimals. Particularly appropriate contexts are money and measurement (e.g., 3.45 m + 721.6 m; 12.4 kg - 7.25 kg).
- Ask students to determine how best to calculate various problems without a calculator. If they decide to use mental strategies, have them compute and share their strategies.
- Ask students which questions from a group of computations that they could solve mentally. Explain their thinking and identify the strategy they used.
- Ask the student to generate addition or subtraction number sentences using only decimal numbers that would result in an answer which is close to 50. Share their work.
- Ask the students to use a calculator, the digits 7, 5, 1, and 2, the symbols +, =, and the decimal point to produce 7.8 on the display.
- Have the student model a subtraction problem using base-ten, number line, pictorial representations or other suitable materials. Have them record their procedure using paper-pencil.

SCO: **N11: Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by:**
- **using compatible numbers**
- **estimating sums and differences**
- **using mental math strategies to solve problems.**

[C, ME, PS, R, V]

Assessment Strategies

- Ask the student to count back the change for $5.00, if the bill totaled $3.59
- Ask the student to make up a problem with multi-digit numbers for which the calculation could be done mentally. Have them solve it and explain their thinking.
- Ask: How can you know that 265 + 535 < 1000 without actually completing the addition? (Observe if they applied the compatible number strategy).
- Show the student $44.98 + $3.98 + $10.99. Ask him/her to predict the sum and then actually calculate the sum mentally.
- Ask the students to find the difference for 2.3 – 1.8 or other similar computations and explain how they got their answer.
- Tell the students that to solve 9.7 – 8.6, Syesha thought 86 + 11 are 97. Explain her thinking.

PATTERNS AND RELATIONS

SCO: **PR1: Identify and describe patterns found in tables and charts, including a multiplication chart.**
[C, CN, PS, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
PR1 Demonstrate an understanding of increasing patterns by: describing; extending; comparing; creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000). **PR2** Demonstrate an understanding of decreasing patterns by: describing; extending; comparing; creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).	**PR1** Identify and describe patterns found in tables and charts, including a multiplication chart.	**PR1** Determine the pattern rule to make predictions about subsequent elements.

Elaboration

Mathematics is often referred to as the science of patterns, as they are found in every mathematical concept and in everyday contexts. Patterns are found in physical and geometric situations as well as in numbers. The same pattern can be found in many different forms (Van de Walle and Lovin 2006, p. 290).

Students should be encouraged to identify and explain patterns that can be found in a variety of tables and charts, including addition and multiplication tables. These patterns can then be used to help students determine an unknown sum or difference. Students should be encouraged to find and explain patterns that occur in tables. It is important that students understand they can use these patterns to determine unknown products or quotients. Students should be familiar with tables that list either all the multiplication facts or some portion of them. For example, the three times table might be shown as:

×	0	1	2	3	4	5	6	7	8	9
3	0	3	6	9	?	15	18	21	24	27

Students should also explore the many patterns in the hundred chart. The hundred chart is a useful model to provide opportunities for students to find and describe a variety of patterns as well as identifying missing elements and errors. Students should use vocabulary, such as **vertical**, **horizontal**, **diagonal**, **row**, and **column** to help describe patterns.

SCO: **PR1: Identify and describe patterns found in tables and charts, including a multiplication chart.**
[C, CN, PS, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Identify and describe a variety of patterns in a multiplication chart.
- Determine the missing element(s) in a given table or chart.
- Identify error(s) in a given table or chart.
- Describe the pattern found in a given table or chart

SCO: **PR1: Identify and describe patterns found in tables and charts, including a multiplication chart.**
[C, CN, PS, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Explore patterns found on a **multiplication grid**, such as:
  numbers in each row and column increase by the same amount
  square numbers are found on the left-right diagonal
  numbers on the left-right diagonal increase by 1, 3, 5, 7,...
  row 4 is double row 2, row 6 is double row 3
  the grid is symmetrical (i.e., numbers are the same both above and under the left-right diagonal)
  when you add the corresponding products of rows 2 and 3, you get the product in row 5; for example, 2 × 4 (8) plus 3 × 4 (12) is the same as 5 × 4 (20)
  when you "cross multiply" any 4 numbers that form a square on the grid, the product is always the same; for example, 2 × 6 = 3 × 4 - also, when you "cross add" these numbers and subtract the sums, you get 1
- Explore patterns found on an **addition chart,** such as:
  even numbers are located on the main diagonal (upper left to lower right), so the sum of a number with itself is always even
  numbers increase by ones across a row, since one more is added for each step to the right
  there are three 2s, four 3s, five 4s, etc.
  the diagonals of any four numbers that form a square will have the same sum
- Explore the many patterns in the **hundred chart**. For example:
  Select four numbers that form a square. Add the two numbers on the diagonal, such as, 59 + 68 and 58 + 69. The sums are equal.

Suggested Activities

- Ask students to look for the even and the odd numbers on a chart and see if they can find the pattern.
- Provide the student with a multiplication grid, addition grid, hundred chart. Ask him/her to describe some of the patterns he/she observes.
- Have students extend several hundreds charts so they can see from 1 to 100, 101 to 200, up to 999. On these charts, use coloured counters to cover numbers forming a pattern and encourage the students to explore the place value representation of the covered numbers; for example, the pattern 13, 23, 33, 43, .., depicted as a vertical column of counters, represents increasing the number by 10 each time.
- Have students explore different versions of hundred charts by changing the order of the numbers, for example, the numbers can spiral or be placed in a different shaped chart, such as a triangle.
- Ask the students to show how one could use the multiplication grid to practice skip counting.

SCO: **PR1: Identify and describe patterns found in tables and charts, including a multiplication chart.**
[C, CN, PS, V]

Assessment Strategies

- Ask the students to explain why some column/rows on a multiplication grid have both even and odd numbers.
- Provide a chart or grid with missing numbers and ask students to fill in the missing numbers.
- Provide the students with a multiplication grid. Ask him/her to describe some of the patterns he/she observes.
- Provide a chart/grid/table that has not been used in the class as a model and ask students to identify and explain the patterns that can be found on the chart/grid/table.
- Provide a chart/grid containing errors and ask students to identify and correct them.

SCO: **PR2: Reproduce a pattern shown in a table or chart using concrete materials.**
[C, CN, V]
PR3: Represent and describe patterns and relationships using charts and tables to solve problems.
[C, CN, PS, R, V]

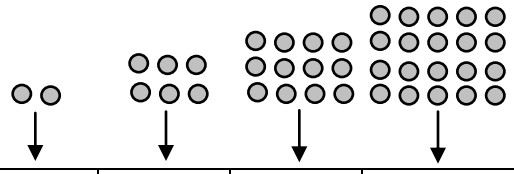
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
PR1 Demonstrate an understanding of increasing patterns by: describing; extending; comparing; creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000). **PR2** Demonstrate an understanding of decreasing patterns by: describing; extending; comparing; creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).	**PR2** Reproduce a pattern shown in a table or chart using concrete materials. **PR3** Represent and describe patterns and relationships using charts and tables to solve problems.	**PR1** Determine the pattern rule to make predictions about subsequent elements.

Elaboration

Once a table or chart is developed, students have two representations of a pattern: the one created with the drawing or materials and the numeric version that is in the table/ grid. When looking for relationships, some students focus on the table and others will focus on the physical pattern. It is important for students to see that whatever relationships they discover, they exist in both forms. When a relationship is found in a table, challenge students to see how that pattern plays out in a physical version (Van de Walle and Lovin 2006, p. 295).



Step	1	2	3	4	5	6	?	…	10
No. of dots	2	6	12	20	30	?	?	…	?

Growing patterns also have a numeric component, the number of objects in each step. A table or "T-chart" can be constructed. Once a table is used for the growing pattern, the materials may become unnecessary. This also leads to the next step which would be to predict what will happen at a particular step (Van de Walle and Lovin 2006, p. 293-294).

SCO: **PR2: Reproduce a pattern shown in a table or chart using concrete materials.**
[C, CN, V]
PR3: Represent and describe patterns and relationships using charts and tables to solve problems.
[C, CN, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

PR2

- Create a concrete representation of a given pattern displayed in a table or chart.
- Explain why the same relationship exists between the pattern in a table and its concrete representation.

PR3

- Extend patterns found in a table or chart to solve a given problem.
- Translate the information provided in a given problem into a table or chart.
- Identify and extend the patterns in a table or chart to solve a given problem

SCO: **PR2: Reproduce a pattern shown in a table or chart using concrete materials.**
[C, CN, V]
PR3: Represent and describe patterns and relationships using charts and tables to solve problems.
[C, CN, PS, R, V]

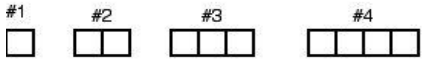
Instructional Strategies

Consider the following strategies when planning lessons:

- Have students not only practice extending patterns with materials and drawings but translate patterns from one medium to another. For example, red and blue pattern blocks become letters or triangles and squares translate to colored tiles. Have students explain how these patterns are mathematically alike.
- Engage students in constructing growing patterns with different materials (toothpicks, multi-link cubes, etc.) They may draw them on grid paper as well. Ask them to describe what is happening as the pattern grows, how is the new step related to the previous one.

Suggested Activities

- Present students with a geometric design series and have them extend the pattern and develop a "T- chart" to go with it. Ask students what the 10^{th} step would be, 12^{th}, 20^{th}, etc. For example:



Squares are the basic building blocks for this series.

Design #	Number of squares
1	1
2	2
3	3
4	4

- Provide a table or "T-chart" involving one arithmetic operation in the pattern, such as the one below. Describe what the data could be about and complete the table.

1	2	3	4	5	6	7	8	9
3	6	9	12	**?**	**?**	**?**	**?**	**?**

SCO: **PR2: Reproduce a pattern shown in a table or chart using concrete materials.**
[C, CN, V]
PR3: Represent and describe patterns and relationships using charts and tables to solve problems.
[C, CN, PS, R, V]

Assessment Strategies

- Provide a table or chart and have students create a concrete representation of the given pattern displayed in the table/chart.
- Provide several examples of tables and their concrete representations. Ask students to find the pairs.
- Ask students to fill in the missing parts of a table or graph. Drawings or materials may be used to complete these.

SCO: PR4: Identify and explain mathematical relationships using charts and diagrams to solve problems.
[CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
PR1 Demonstrate an understanding of increasing patterns by: describing; extending; comparing; creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000). **PR2** Demonstrate an understanding of decreasing patterns by: describing; extending; comparing; creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).	**PR4** Identify and explain mathematical relationships using charts and diagrams to solve problems.	**PR1** Determine the pattern rule to make predictions about subsequent elements.

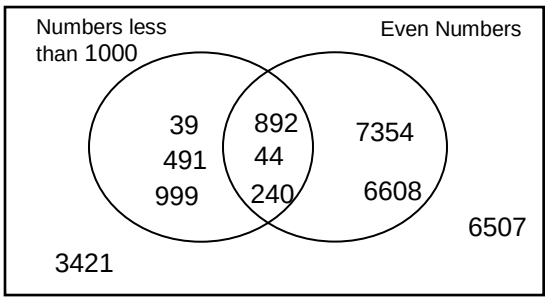
Elaboration

In everyday life we sort things by comparison relationships, for example by colour and size. Such relationships also apply to number, as numbers also have attributes or certain characteristics which make them similar or different from other numbers. Students need to explore this particular concept of number by being involved in experiences where they are expected to recognize, describe and identify relationships and number characteristics such that they have experienced with more everyday objects. Sorting and classifying objects will help students with organizing and categorizing data. Sorting is the action of grouping (or organizing) objects (or data); classification (or categorization) is the naming of the groups of objects (or data).

By grade 4, students are expected to use more sophisticated sorting tools such as a **Carroll** or **Venn diagram**. These organizational tools are particularly useful as a form of data display when the categories for the sorting situation overlap. Carroll diagrams are tables that work much like Venn diagrams. For Carroll diagrams, two attributes are being used for sorting, with one attribute of each characteristic being the focus (Small 2008, p. 521). A table is created with four cells to show the four possible combinations of these two attributes. Either the items themselves, or the count of how many items of each type, are put in the cells.

	Even	Odd
Numbers less than 1000	892, 44, 240	39, 491, 999
Numbers more than 1000	7354, 6608	3421, 6507

Carroll Diagram



Venn Diagram

SCO: PR4: Identify and explain mathematical relationships using charts and diagrams to solve problems.
[CN, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Complete a Carroll diagram by entering given data into correct squares to solve a given problem.
- Determine where new elements belong in a given Carroll diagram.
- Solve a given problem using a Carroll diagram.
- Identify a sorting rule for a given Venn diagram.
- Describe the relationship shown in a given Venn diagram when the circles intersect, when one circle is contained in the other and when the circles are separate.
- Determine where new elements belong in a given Venn diagram.
- Solve a given problem by using a chart or diagram to identify mathematical relationships

SCO: PR4: Identify and explain mathematical relationships using charts and diagrams to solve problems.
[CN, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Introduce the notion of how cross-classification is shown in Venn diagrams by setting out two hoops, side by side, and label each one with a sorting rule. Ensure the sorting rules and objects to be sorted lend themselves to cross-classification. Note that the circles of a Venn diagram do not have to overlap. They can be two separate circles if the attributes are exclusive. They could be separate if the items involved in the sort do not exhibit the same characteristics, even if they have the potential to do so (Small 2008, p. 521). If only one attribute is being used to sort the objects, then there would only be one circle in the Venn diagram.
- Reinforce the proper math vocabulary during sorting activities. The word “and” indicates that each item in the group would have all attributes of both categories where “or” makes the distinction between the two categories under consideration.
- Ensure that students include all of the data being considered from their sorting situation in their Venn or Carroll diagram. Whenever students create a Venn diagram, ensure that they draw a rectangle around the circle(s).
- Post a list of possible attributes (or characteristics) of numbers and encourage students to refer to the list as they examine Venn or Carroll diagrams involving numbers.

Suggested Activities

- Give students various 3-D objects or 2-D shapes. One student selects six objects or shapes, chooses 2 mystery attributes and then sorts them. The other(s) then attempt to guess the sorting rule.
- Have students create a set of ten 3- or 4-digit numbers and sort them using two attributes. Request that they write the sorting rule.
- Provide students with data to organize using a Venn and Carroll diagram. Have them reflect on their preferred organizational tool. Justify their choice.
- Have students sort a set of numbers in different ways and explain their sorting rule(s).

SCO: PR4: Identify and explain mathematical relationships using charts and diagrams to solve problems.
[CN, PS, R, V]

Assessment Strategies

- Prepare various pre-sorted 2-D shapes or 3-D objects in Venn diagrams. Hold up different additional objects and ask students where it should go on the Venn diagram.
- Provide an unlabelled Venn diagram, containing pre-sorted sets of numbers, and ask students to determine the sorting rule and add one more number to each subset.
- Give various numeral cards containing numbers up to 4-digits and have students create a labeled Venn diagram.
- Provide a completed Carroll diagram and present students with additional numbers which might have one attribute, both attributes, or neither attribute. Have them explain where they should be placed in the diagram.
- Have students compare a completed Venn diagram to a related Carroll diagram and ask them to determine if they show the same information. Have them explain their thinking.

SCO: **PR5: Express a given problem as an equation in which a symbol is used to represent an unknown number.**
[CN, PS, R]
PR6: Solve one-step equations involving a symbol to represent an unknown number.
[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
PR4 Solve one-step addition and subtraction equations involving a symbol to represent an unknown number.	**PR5** Express a given problem as an equation in which a symbol is used to represent an unknown number. **PR6** Solve one-step equations involving a symbol to represent an unknown number.	**PR2** Express a given problem as an equation in which a letter variable is used to represent an unknown number (limited to whole numbers). **PR3** Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.

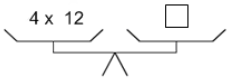
Elaboration

The various representations of patterns, including unknowns, provide valuable tools in making generalizations of mathematical relationships.

Equality is used to express **relationships**. The symbols used on either side of the equal sign represent a quantity. The equal sign is "a symbol of equivalence and balance" (NCTM 2000, p. 39).

Students should be comfortable using various symbols to represent the unknown, for example, a square, circle, or triangle.

Display a number of samples of balance scales, such as those below. Have students write an equation for each balance scale and then solve it. For example, for the first one: 8 + ☐ = 20, so ☐ = 12.



SCO: **PR5: Express a given problem as an equation in which a symbol is used to represent an unknown number.**
[CN, PS, R]
PR6: Solve one-step equations involving a symbol to represent an unknown number.
[C, CN, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

PR5

- Explain the purpose of the symbol, such as a triangle or circle, in a given addition, subtraction, multiplication or division equation with one unknown, e.g. 36 ÷ ☐ = 6
- Express a given pictorial or concrete representation of an equation in symbolic form.
- Identify the unknown in a story problem, represent the problem with an equation and solve the problem concretely, pictorially or symbolically.
- Create a problem in context for a given equation with one unknown.

PR6

- Solve a given one-step equation using manipulatives.
- Solve a given one-step equation using guess and test.
- Describe, orally, the meaning of a given one-step equation with one unknown.
- Solve a given equation when the unknown is on the left or right side of the equation.
- Represent and solve a given addition or subtraction problem involving a “part-part-whole” or comparison context using a symbol to represent the unknown.
- Represent and solve a given multiplication or division problem involving equal grouping or partitioning (equal sharing) using symbols to represent the unknown.

SCO: **PR5: Express a given problem as an equation in which a symbol is used to represent an unknown number.**
[CN, PS, R]
PR6: Solve one-step equations involving a symbol to represent an unknown number.
[C, CN, PS, R, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Build on the students' knowledge from the previous grade in using equations to write addition, subtraction, multiplication and division equations. Connect the concrete, pictorial and symbolic representations consistently as the students develop and demonstrate understanding of equations.
- Use everyday contexts for problems to which the students can relate so that they can translate the meaning of the problem into an appropriate equation using a symbol to represent the unknown number.
- Review the relationship between addition and subtraction number sentences as well as the relationship between multiplication and division number sentences.
- Have the students create problems for a variety of number sentences using the 4 operations.
- Encourage the students to write equations in various ways to represent the meaning of a given problem. 14 + Δ = 37 or Δ + 14 = 37; 5 × ☐ = 30 or ☐ × 5 = 30. Note that the order (commutative) property does not apply to subtraction and division.
- Explain that if the same variable, or unknown, is used repeatedly in the same equation, then there is only one possible solution for that variable or unknown; e.g. for ☐ + ☐ = 20; the unique solution is to place 10 in each of the squares. If, however, two different symbols are used, there may be a number of possible solutions, e.g., ☐ + △ = 16, some solutions include 0 + 16, 7 + 9, 12 + 4.

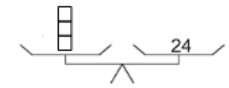
Suggested Activities

- Ask students to create problems to represent the following equations:

$15 + \square = 24$ $\quad \bigcirc + 15 = 24$ $\quad 24 = 15 + \hexagon$ $\quad 24 = \triangle + 15$

$24 - \pentagon = 15$ $\quad 24 - 15 = \diamond$ $\quad 15 = 24 - \square$ $\quad \triangleleft = 24 - 15$

- Show students a balance scale and ask them to work with a partner to find an equation that is represented for each of the examples below;



SCO: **PR5: Express a given problem as an equation in which a symbol is used to represent an unknown number.**
[CN, PS, R]
PR6: Solve one-step equations involving a symbol to represent an unknown number.
[C, CN, PS, R, V]

Assessment Strategies

- Tell what the box represents in the following equation: 15 – ☐ = 8.
- You have 24 marbles and your friend gives you some more marbles. Now you have 32 marbles in all. How many marbles did your friend give you?
  a. Write an equation to show what is happening in this problem.
  b. Solve the problem. Explain your thinking.
- Solve the following equation and use a diagram to explain the process.
  34 + 5 = ☐ + 12
- Solve the following equation and explain your thinking.
  Δ – 13 = 20
- Lori said that the box in the equation 6 + 8 = ☐ + 4 could represent more than one number. Is Lori correct? Why or why not?
- Ask the student to explain how to find the missing number in $4 \times \Delta = 100$.

SHAPE AND SPACE

SCO: **SS1: Read and record time using digital and analog clocks, including 24-hour clocks.** [C, CN, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
SS1 Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). **SS2** Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem solving context.	**SS1** Read and record time using digital and analog clocks, including 24-hour clocks.	

Elaboration

Although students have not had any explicit teaching related to reading and recording time using clocks by grade four, they have had opportunities in previous grades to explore the passage of time and have an understanding that there are 60 minutes in an hour. As well, students will have had many opportunities to use time through their own experiences with the real world.

By the end of grade four, students should be able to read and record time on 12-hour and 24-hour **analog** and **digital** clocks. Students should read times on clocks to provide information about relevant situations, such as - comparing start and finish times to determine how much time has passed - focusing on times when special events are going to happen.

Students may want to investigate the meaning for some terminology such as, A.M. and P.M. (A.M. is the abbreviation for *ante meridiem* meaning being before noon. P.M. is the abbreviation for *post meridiem* meaning being after noon.)

SCO: **SS1: Read and record time using digital and analog clocks, including 24-hour clocks.**
[C, CN, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- State the number of hours in a day.
- Express the time orally and numerically from a 12-hour analog clock.
- Express the time orally and numerically from a 24-hour analog clock.
- Express the time orally and numerically from a 12-hour digital clock.
- Describe time orally and numerically from a 24-hour digital clock.
- Describe time orally as “minutes to” or “minutes after” the hour.
- Explain the meaning of AM and PM, and provide an example of an activity that occurs during the AM and another that occurs during the PM.

SCO: **SS1: Read and record time using digital and analog clocks, including 24-hour clocks.** [C, CN, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Have students explore that the minute hand and hour hand on an analog clock are different lengths, and that the minute hand is at 6 for the :30 and at 12 or 24 for the :00. Students will be aware that the hour hand moves during the course of the hour, and that, at the :30, it is halfway between two numbers.
- Use an analog clock to introduce the terms “half past”, “quarter after”, and “quarter to”.
- Have students read time to the nearest five minutes. It is important that students are comfortable with skip counting by 5. This provides the opportunity for students to relate the numbers on a clock to the five times table.
- Use a clock that shows not only the numbers from 1 to 12, but also the minute amounts from 5 to 55 beside the numbers from 1 to 11. Furthermore, students will be aware that there are 5 minutes between the numbers on the clock. The long hand on the 3 represents 15 minutes, so two one-minute spaces past the 3 is 17 minutes, etc.

Suggested Activities

- Present students with an analog clock showing just the hour hand. Ask them to predict what the time might be. For example, if the hour hand is somewhere between the 4 and the 5, the time could be anything from *five **past** four* to *five **to** five* depending on the exact placement of the hour hand. Students couls also be asked to name an event/activity that often happens at about that time of day.
- Introduce the terms of analog, a.m. and p.m. and discuss the difference between the two and brainstorm activities for each.
- Ask the student to show, on an analog clock, the time (to the nearest half hour) at which they arrive at school, have lunch, go to bed, etc.
- Discuss when a 24-hour clock would be more appropriate to use than a 12-hour clock.
- Have students track events throughout a specific day by means of a time line divided into 15-minute segments. Students should record the time of the activity or event and note it at the appropriate spot on a time line.
- Have the student make a list of the times when the minute hand and the hour hand just about line up as well as other patterns, such as all of the times that include a 4 in a 24-hour period.
- Have students work in pairs to set up a schedule in which every student will get 10 minutes on the computer, starting at 8:30 a.m. Ask them if all students can have time on the computer before noon and, if not, how long it will take to finish after lunch. At what time will the last one finish? (Remind them to leave time for recess.)

SCO: **SS1: Read and record time using digital and analog clocks, including 24-hour clocks.** [C, CN, V]

Assessment Strategies

- Ask students what time might it be if the minute hand and hour hand are opposite one another.
- Ask the student to move the hands of an analog clock to match the time shown on a digital clock.
- Ask students to express the time orally and numerically that has been created on a 12-hour analog clock, 24-hour analog clock, and 12-hour digital clock.
- Ask students to name an activity they would typically do in the p.m.? a.m.?
- Ask students how many hours are in a day and a half.

SCO: **SS2: Read and record calendar dates in a variety of formats.**
[C, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
SS1 Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). **SS2** Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem solving context.	**SS2** Read and record calendar dates in a variety of formats.	

Elaboration

By grade four, students should already know the days of the week, the months of the year, and the four seasons. As well, students will have already developed a sense of the arrangement of our year in relation to the months and seasons; for example, January is the first month of a new year and is early in our winter season.

Using **calendars** throughout the school year strengthens the students' sense of time. Each month brings a new calendar to explore. Students would be familiar with calendars through their home and school experiences by grade 4. In previous grades, teachers may have explored calendars during explorations of units of time, such as days, weeks, months, and years. Calendars may also have been used to assist in developing number sense and for exploring patterns.

Students need to become aware of the variety of ways **dates** can be recorded. In grade four, students are expected to read, record, and interpret calendar dates in a variety of ways.

SCO: **SS2: Read and record calendar dates in a variety of formats.**
[C, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Write dates in a variety of formats, e.g., *yyyy/mm/dd*, *dd/mm/yyyy*, March 21, 2006, *dd/mm/yy.*
- Relate dates written in the format *yyyy/mm/dd* to dates on a calendar.
- Identify possible interpretations of a given date, e.g., 06/03/04.

SCO: **SS2: Read and record calendar dates in a variety of formats.**
[C, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Send students on a scavenger hunt and have them bring in different dates from magazines, posters, items printed from the Internet, cheques and newspapers. Share, discuss and display the variety of formats as a class.
- Have students predict how many days and/or weeks there are in a year. Verify using calendars.
- Have them explore what calendar dates can be confused with other dates when they are interpreted using various formats.
- Investigate a special holiday which has a date that fluctuates, such as Labour Day. Have students record the date(s) of this holiday over the past five years. Share their findings.

Suggested Activities

- Ask students to write about their favourite format for recording a calendar date and justify their choice.
- Have students interpret a particular date such as 06/04/03. Discuss that there is no standard or consistent format and why some dates may be misinterpreted unless you know the format.
- Provide students with a list of dates recorded in metric notation and have them order them from past to present.

SCO: **SS2: Read and record calendar dates in a variety of formats.**
[C, V]

Assessment Strategies

- Show the student a calendar for the year. Ask him/her to point out the day's date. Have them record it using the format month/day/year.
- Ask the student to identify two calendar dates which cannot be confused with other dates when they are interpreted regardless of the format.
- Have the student write their birth date using 3 different formats.
- Have the student identify their favourite day of year and write the date in metric notation. (year, month, day)

SCO: **SS3: Demonstrate an understanding of area of regular and irregular 2-D shapes by:**

- **recognizing that area is measured in square units**
- **selecting and justifying referents for the units cm^2 or m^2**
- **estimating area by using referents for cm_2 or m^2**
- **determining and recording area (cm^2 or m^2)**
- **constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.**

[C, CN, ME, PS, R, V]

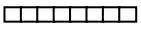
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
SS5 Demonstrate an understanding of perimeter of regular and irregular shapes by: estimating perimeter, using referents for cm or m; measuring and recording perimeter (cm, m); constructing different shapes for a given perimeter (cm, m); to demonstrate that many shapes are possible for a perimeter.	**SS3** Demonstrate an understanding of area of regular and irregular 2-D shapes by: • *recognizing that area is measured in square units* • *selecting and justifying referents for the units cm^2 or m^2* • *estimating area by using referents for cm^2 or m^2* • *determining and recording area* • *constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.*	**SS2** Design and construct different rectangles, given either perimeter or area, or both (whole numbers), and make generalizations.

Elaboration

Students should understand that the area of a shape can be expressed as the number of units required to cover a certain surface. Van de Walle and Lovin define area as "a measure of the space inside a region or how much it takes to cover a region" (2006, p. 234). The **square unit** is the most efficient unit to use for measuring area. It is helpful for students to use a **referent** for the single unit of measure and iterate this unit mentally to obtain the estimate; e.g., use the size of the fingernail on your smallest finger as a referent for 1 cm^2.

Once students have developed the meaning of measurement, it is time to move on to connect multiplication in an **array** format to determine the area of rectangles (Van de Walle and Lovin 2006, p.263). Students should relate the area of a rectangle to the product of the numbers describing its length and width. Conversely, any factor of the number representing the area of a rectangle can be one dimension of a rectangle with that area. For example, consider rectangles with an area of 8 square units.  OR 

It is important for students to explore not only the areas of rectangles, but areas of other shapes as well. Through these investigations students should recognize that objects of different shapes can have the same area. Encourage students to find shapes using partial squares.

Opportunities should be provided for students to estimate and calculate the area of various surfaces. Laying an acetate centimetre grid over objects is helpful when determining surface area. Students might investigate the area of shapes drawn on centimetre dot paper. Strategies for doing this include adding squares and half squares within the figure; placing a rectangle around the shape, determining its area, and subtracting the area of the "extra" pieces.

SCO: **SS3: Demonstrate an understanding of area of regular and irregular 2-D shapes by:**

- **recognizing that area is measured in square units**
- **selecting and justifying referents for the units cm^2 or m^2**
- **estimating area by using referents for cm_2 or m^2**
- **determining and recording area (cm^2 or m^2)**
- **constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.**

[C, CN, ME, PS, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Describe area as the measure of surface recorded in square units.
- Identify and explain why the square is the most efficient unit for measuring area.
- Provide a referent for a square centimetre and explain the choice.
- Provide a referent for a square metre and explain the choice.
- Determine which standard square unit is represented by a given referent.
- Estimate the area of a given 2-D shape using personal referents.
- Determine the area of a regular 2-D shape and explain the strategy.
- Determine the area of an irregular 2-D shape and explain the strategy.
- Construct a rectangle for a given area.
- Demonstrate that many rectangles are possible for a given area by drawing at least two different rectangles for the same given area.

SCO: **SS3: Demonstrate an understanding of area of regular and irregular 2-D shapes by:**

- **recognizing that area is measured in square units**
- **selecting and justifying referents for the units cm^2 or m^2**
- **estimating area by using referents for cm_2 or m^2**
- **determining and recording area (cm^2 or m^2)**
- **constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.**

[C, CN, ME, PS, R, V]

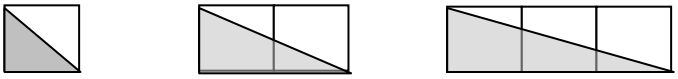
Instructional Strategies

Consider the following strategies when planning lessons:

- Use referents for area and estimating area. Review that referents are familiar objects which students can use to which students can refer for estimating (e.g., the width of the pinky finger is about 1 cm). Ask the students to suggest a suitable referent for 1 cm^2 and explain why they think it would work. Have them use this referent to estimate the area of a book cover. Ask them to check their estimate by finding the area of the book cover. Discuss possible referents for 1 m^2. Have the students use their referents and estimate the area of a large tabletop or a section of the classroom floor and check their estimates.
- Have students use colour tiles or grid paper to investigate the numbers from 1 to 30 to see how many different rectangles can be made for each. Students should record their results and look for patterns.
- Use a transparency of a centimetre grid to confirm the estimate of an area of an irregular shape.

Suggested Activities

- Have students explore how a diagonal of rectangle(s) divides the shape in half.



- Provide the students with rectangular papers that each measure 10 cm by 13 cm. Have them estimate how many copies of each shape of pattern block it would take to cover the rectangle. Then have the students measure the area using each of the shapes in turn.

____ ____ ____ 20 squares ____

- Make the design below on an overhead geoboard and ask a student to explain to the class how to find the area. Have the students alter the shape on their geoboards to increase the area by 1 cm^2.

- Provide the students with tiles and centimetre grid paper. Give them the following instructions: For each of the areas from 1 cm^2 to 20 cm^2, find all the possible rectangular arrays using whole numbers. For example, the possible arrays for an area of 6 cm^2 would be as follows:

SCO: **SS3: Demonstrate an understanding of area of regular and irregular 2-D shapes by:**

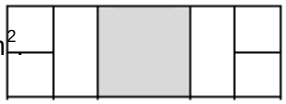
- **recognizing that area is measured in square units**
- **selecting and justifying referents for the units cm^2 or m^2**
- **estimating area by using referents for cm_2 or m^2**
- **determining and recording area (cm^2 or m^2)**
- **constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.**

[C, CN, ME, PS, R, V]

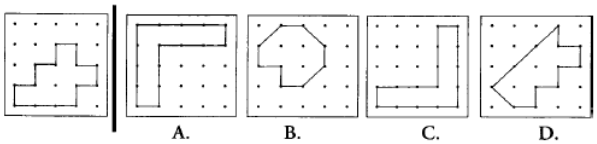
Assessment Strategies

- Ask students to predict how many different arrays can be made to represent 36 cm^2. Draw the arrays to check your prediction.
- Have students estimate the area for each of the following pairs of congruent shapes. Decide if the shaded part has the same area in each pair of shapes. Explain your thinking.

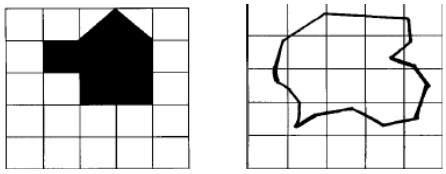
- Ask the student to estimate the area of a rectangle and explain what referent he or she used.
- Explain why area is measured in square units.
- Find the area of the shaded part.
  The area of the entire design below is 12 m^2.
  Explain your thinking.



- Ask students to circle the letters of the shapes that have the same area as the one on the left.



- Ask why it is easier to find the area of the shape on the left than the one on the right.
  Give an estimate for the area of the shape on the right.



SCO: **SS4: Describe and construct rectangular and triangular prisms.**
[C, CN, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
SS6 Describe 3-D objects according to the shape of the faces, and the number of edges and vertices.	**SS4** Describe and construct rectangular and triangular prisms.	**SS5** Describe and provide examples of edges and faces of 3-D objects and sides of 2-D shapes that are: parallel; intersecting; perpendicular; vertical; horizontal.

Elaboration

Prisms have first names based on the shape of their **bases** and second name, which is **prism**. Geometric models provide a perspective from which students can analyse and solve problems, and geometric interpretations can help make an abstract (symbolic) representation more easily understood (Curriculum and Evaluation Standards for School Mathematics, p.112). A good way to explore shapes is to use smaller shapes or tiles to create larger shapes. Different criteria or directions can provide the intended focus to the activity. Pattern blocks are very good for this, but many teacher-made materials can be used. While the pattern block pieces are prisms, they have been treated as 2-D shapes; however, stacking a number of triangles or squares would provide examples of different prisms. This stacking would help students conceptualize the uniform nature of prisms. Also, students can make skeletal models for prisms, using rolled newspapers and tape, straws and string, or toothpicks and miniature marshmallows. Commercial sets of 3-D objects usually have a variety of prisms.

Students should be given copies of **nets** of **rectangular** and **triangular** prisms to cut out and fold up. They should be encouraged to unfold them and examine the 2-D shapes that are connected to make each net. Have them visualize the folding up and unfolding. In addition to cutting out and assembling prepared nets, it is now expected that students will draw their own nets for rectangular and triangular prisms. They will also consider the various possibilities for these nets. Have the students trace on paper the various **faces** of the different prism to make its net. Have the students cut out the net and fold it up around the shape to see if it works. Ask them to record this net on grid paper. Have them cut one of the faces off and investigate the possible places it could be reattached to make a new net. Have them record each one on grid paper.

SCO: **SS4: Describe and construct rectangular and triangular prisms.**
[C, CN, R, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Identify and name common attributes of rectangular prisms from given sets of rectangular prisms.
- Identify and name common attributes of triangular prisms from given sets of triangular prisms.
- Sort a given set of rectangular and triangular prisms using the shape of the base.
- Construct and describe a model of rectangular and triangular prisms using materials, such as pattern blocks or modeling clay.
- Construct rectangular prisms from their nets.
- Construct triangular prisms from their nets.
- Identify examples of rectangular and triangular prisms found in the environment.

SCO: **SS4: Describe and construct rectangular and triangular prisms.**
[C, CN, R, V]

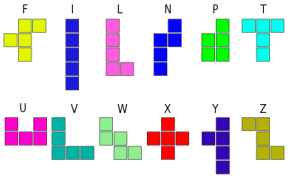
Instructional Strategies

Consider the following strategies when planning lessons:

- Encourage students to use the attributes of any prism (number of faces, number of edges, number of vertices, or shapes of the faces) to describe prisms.
- Determine if students recognize the same prism can be built by piling vertically or horizontally when building objects from the base, (i.e., orientation).
- Work with nets to investigate attributes, including alignment of faces, to determine if the net could successfully construct the 3-D object.

Suggested Activities

- Stack pattern blocks to make rectangular prisms and triangular prisms. Describe how they are alike and how they are different.
- Ask students to build skeletal models of two different triangular pyramids. Ask them how they are the same/different?
- Provide students with various nets of prisms for them to construct. Have them label each face of their model using the words “face” and “base” as well as identify their 3-D object.
- Have the students trace on paper the various faces of the different prisms to make its net. Have the students cut out the net and fold it up around the shape to see if it works. Ask them to record this net on grid paper. Have them cut off one of the faces and investigate the possible places it could be reattached to make a new net. Have them record each one on grid paper.
- Provide students with a square or rectangular prism and an 11-pin x 11-pin geoboard. Ask them to use elastics to construct a net for the prism. Ask them to discuss how they might move one of the faces to make a new net for the same prism. Have them check by recording the new net on square dot paper and cutting it out.
- Provide the students with a *Pentomino Puzzle* piece, a 2-D shape made by joining 5 squares along full sides that would fold to make a box with no top. (you can also draw these shapes onto larger grid paper for students to fold into prisms). Ask them to add an additional square to the shape so that the prism will have a cover. In how many places can this square be added?



SCO: **SS4: Describe and construct rectangular and triangular prisms.**
[C, CN, R, V]

Assessment Strategies

- Have students name the prism that best represents various real-life examples of 3-D objects.
- Ask students to build skeletal models of two different triangular pyramids. Ask them how they are the same/different?
- Have student work together to sort their collection of 3-D objects into two groups: rectangular and triangular prisms. Ask: what are the attributes of the shapes that made them alike? How are they different? What makes a rectangular prism a cube? What kind of prism would you have if you built from a rectangular base?
- Listen as students construct nets on geoboards and using grid/isometric paper; are they discussing the attributes of the object, are they using the correct vocabulary (faces, edges, vertices, congruency)?
- Ask the students to add additional faces to complete partial nets.
- Give small groups of students a set of 4 or 5 nets of rectangular or triangular prisms. Each set should consist of one net that can be made into the 3-D object, and 3 or 4 others which cannot be made into the 3-D object. Have the student analyse the nets, without manipulating them, to determine which one of the nets in the group could be used to create the 3-D object. Have them justify, and then test their predictions.

SCO: **SS5: Demonstrate an understanding of line symmetry by:**
- **identifying symmetrical 2-D shapes**
- **creating symmetrical 2-D shapes**
- **drawing one or more lines of symmetry in a 2-D shape.**

[C, CN, V]

SS6: Demonstrate an understanding of congruency, concretely and pictorially.
[C, CN, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

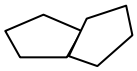
Scope and Sequence

Grade Three	Grade Four	Grade Five
	SS5 Demonstrate an understanding of line symmetry by: • identifying symmetrical 2-D shapes • creating symmetrical 2-D shapes • drawing one or more lines of symmetry in a 2-D shape. **SS6** Demonstrate an understanding of congruency, concretely and pictorially.	**SS8** Identify a single transformation including a translation, a rotation and a reflection of 2-D shapes.

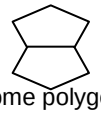
Elaboration

Congruency and **symmetry** are **geometric properties**. These properties can be used to determine what makes some shapes alike and different. Congruent 2-D shapes are "geometric figures that have the same size and shape" (Alberta Education 1990, p. 198). Symmetrical 2-D shapes are geometric figures "that can be folded in half so that the two parts are congruent" (Alberta Education 1990, p. 205).
Symmetrical and congruent shapes are closely connected. Any symmetrical shape can be divided into two congruent parts along the line of symmetry; however, not every **composite shape** made up of congruent figures is symmetrical. For example, this regular hexagon is symmetrical. The line of symmetry shown in the diagram divides the hexagon into two congruent shapes, each shape is a pentagon.

This composite shape is made up of two congruent pentagons. It is **not** symmetrical.

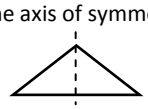


This composite shape is made up of two congruent pentagons. It **is** symmetrical.

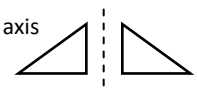


Students should begin to appreciate that reflective symmetry is a characteristic of some polygons and not others. These polygons can be described by stating how many lines of reflective symmetry they have.
A shape remains the same size and shape when transformed using translations, reflections or rotations; i.e., the object and the image in these transformations are congruent. Symmetrical shapes form a subset of reflections. A reflection results in a symmetrical composite shape when the mirror line used to reflect a shape aligns with one side of the shape. For example:

This reflection results in a composite shape that is symmetrical.
The mirror line and the axis of symmetry coincide.



This reflection does not result in a composite shape, but rather two separate shapes that are congruent.
The mirror line is shown, but it is not the axis of symmetry.



SCO: **SS5: Demonstrate an understanding of line symmetry by:**
- **identifying symmetrical 2-D shapes**
- **creating symmetrical 2-D shapes**
- **drawing one or more lines of symmetry in a 2-D shape.**

[C, CN, V]

SS6: Demonstrate an understanding of congruency, concretely and pictorially.
[C, CN, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

SS5

- Identify the characteristics of given symmetrical and nonsymmetrical 2-D shapes.
- Sort a given set of 2-D shapes as symmetrical and nonsymmetrical and explain the process.
- Complete a symmetrical 2-D shape given half the shape and its line of symmetry and explain the process.
- Identify lines of symmetry of a given set of 2-D shapes and explain why each shape is symmetrical.
- Determine whether or not a given 2-D shape is symmetrical by using a Mira or by folding and superimposing.
- Create a symmetrical shape with and without manipulatives and explain the process.
- Provide examples of symmetrical shapes found in the environment and identify the line(s) of symmetry.
- Sort a given set of 2-D shapes as those that have no lines of symmetry, one line of symmetry or more than one line of symmetry.

SS6

- Determine if two given 2-D shapes are congruent and explain the strategy used.
- Create a shape that is congruent to a given 2-D shape and explain why the two shapes are congruent.
- Identify congruent 2-D shapes from a given set of shapes shown in different orientations.
- Identify corresponding vertices and sides of two given congruent shapes.
- Explain the connections between congruence and symmetry using 2-D shapes.

SCO: **SS5: Demonstrate an understanding of line symmetry by:**
- **identifying symmetrical 2-D shapes**
- **creating symmetrical 2-D shapes**
- **drawing one or more lines of symmetry in a 2-D shape.**

[C, CN, V]

SS6: Demonstrate an understanding of congruency, concretely and pictorially.
[C, CN, V]

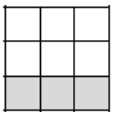
Instructional Strategies

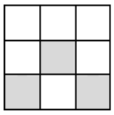
Consider the following strategies when planning lessons:

- Use everyday contexts to introduce congruence and symmetry, drawing on the students' prior experiences in the real world and knowledge on 2-D shapes.
- Include many hands-on activities to establish the concept of congruence prior to symmetry.
- Have the students share their ideas about sorting various sets of 2-D shapes and encourage students to sort shapes in different ways. Provide follow-up activities to address any misconceptions.
- Have the students justify their thinking by explaining why they use a particular strategy to create congruent and/or symmetrical 2-D shapes.
- Explore that the line of symmetry can be vertical, horizontal, or slanted.
- Explore that reflective symmetry is a characteristic of some polygons and not others. These polygons can be described by stating how many lines of reflective symmetry they have. For example, after investigations, students should find that a square has 4 lines of reflective symmetry.
- Provide students with experiences to help them understand that a symmetry line is where a polygon can be folded onto itself so that each half matches exactly, or where a mirror can be placed so that the reflection on one side matches the shape on the other.

Suggested Activities

- Provide a variety of shapes and ask the students to sort them, grouping those with reflective symmetry and those without reflective symmetry.
- Ask each student to draw a picture of a shape or create a design that exhibits symmetry
- Ask students to draw examples of triangles with symmetry and triangles without symmetry.
- Have students draw on squared dot paper examples of the different quadrilaterals. Cut them out and fold them to find the lines of symmetry. Use pictures of shapes with Miras also. Share and discuss the lines of symmetry.
- Have the students create congruent designs on geoboards and draw the designs on square dot paper or geopaper. They could cut out one design from the dot paper and superimpose it on the other design to check for congruency.
- Provide examples of 2-D shapes with one line of symmetry, two lines of symmetry and no lines of symmetry. Have the students draw the lines of symmetry and sort the shapes.
- Provide examples of 3 by 3 squares on grid paper. Shade 3 small squares so that the figure has one line of symmetry. Challenge the students to make as many different patterns with one line of symmetry by shading in three small squares or make figures with more than two lines of symmetry.





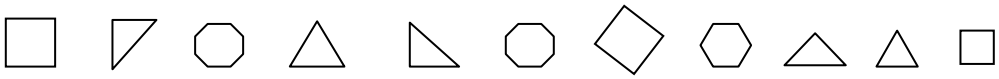
SCO: **SS5: Demonstrate an understanding of line symmetry by:**
- **identifying symmetrical 2-D shapes**
- **creating symmetrical 2-D shapes**
- **drawing one or more lines of symmetry in a 2-D shape.**

[C, CN, V]

SS6: Demonstrate an understanding of congruency, concretely and pictorially.
[C, CN, V]

Assessment Strategies

- Ask the student to name different 2D shapes, to state how many lines of symmetry each has, and to show where the lines of symmetry are.
- Provide the diagrams of 2-D shapes some of which are congruent, such as the following:



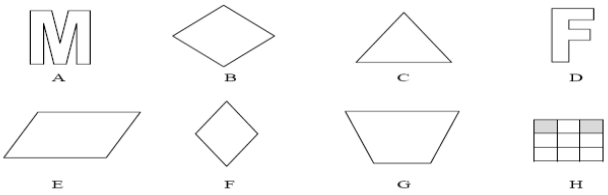
Ask the students to put a check mark on shapes that are congruent to 

put an X on shapes that are congruent to 

shade in the shapes that are congruent to 

Suggest that they trace and cut out the three shapes and then superimpose them on the given shapes to prove congruency.

- Give three examples of symmetrical shapes in your everyday world.
- Place the following labeled 2-D shapes before the student.



Have students circle all the symmetrical shapes. Then ask them student to draw all the lines of reflective or line symmetry on the symmetrical shapes. Finally, have the student sort the shapes by the number of lines of symmetry in each shape: no lines of symmetry; one line of symmetry; more than 1 line of symmetry.

STATISTICS AND PROBABILITY

SCO: **SP1: Demonstrate an understanding of many-to-one correspondence.**
[C, R, T, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
SP1 Collect first-hand data and organize it using: tally marks; line plots; charts; lists to answer questions. **SP2** Construct, label and interpret bar graphs to solve problems.	**SP1** Demonstrate an understanding of many-to-one correspondence.	**SP2** Construct and interpret double bar graphs to draw conclusions.

Elaboration

Prior to grade four students have had opportunities to collect and display data in **pictographs** and **bar graphs**. As they investigate a wider range of topics, they may discover that the data they collect is too large to display in a graph using a one-to-one correspondence (i.e. having each symbol or number on the bar graph represent one piece of data). Students need to be introduced to the concept of using a **many-to-one correspondence** or **scale** when they are creating graphs to display large amounts of data. In grade four, students should begin to make decisions about what symbol to use and what that symbol should represent. These decisions are based on the data being used.

Students need to be given many opportunities to explore what scale is most appropriate for their set of data. For example, if they want to display a graph to show their marble collection and they have 36 blue, 28 red, and 42 clear, students may decide to draw symbols to each represent 5 marbles or create a scale in a bar graph that increases by 2. If the numbers were all less than 20, it is usually more appropriate to use a one-to-one correspondence. However, if the numbers were much larger, students may find it better to use 10 as the scale. Students should discuss their data displays and be able to explain why they chose their scale. It is important for students to ensure that the scale in their data displays is consistent. For example, if they are creating a bar graph that has a scale of two, all of the numbers need to increase by 2 (2, 4, 6, 8, 10, 12 … and not 2, 4, 6, 7, 8, 9, 10, 12…). Depending on the data and the scale that is selected, it may become necessary to create partial symbols and bars that fall between numbers.

As students compare their own graphs and those from other sources, they should examine how the graphs are similar and different. Students should discuss why they think the interval or correspondence was chosen and what other scales may have also been used.

Deciding on what scale to use requires students to apply their knowledge of multiplication and therefore, it is very helpful for students to have a good knowledge of these facts.

SCO: **SP1: Demonstrate an understanding of many-to-one correspondence.**
[C, R, T, V]

Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Compare graphs in which different intervals or correspondences are used and explain why the interval or correspondence was used.
- Compare graphs in which the same data has been displayed using one-to-one and many-to-one correspondences, and explain how they are the same and different.
- Explain why many-to-one correspondence is sometimes used rather than one-to-one correspondence.
- Find examples of graphs in which many-to-one correspondence is used in print and electronic media, such as newspapers, magazines and the Internet, and describe the correspondence used.

SCO: **SP1: Demonstrate an understanding of many-to-one correspondence.**
[C, R, T, V]

Instructional Strategies

Consider the following strategies when planning lessons:

- Allow students to decide on the scale for their data displays, but ensure that they can justify their choice.
- Have students work with a variety of sets of data, so they will have experiences creating different scales.
- Use data display software or websites for students to quickly compare graphs that have different scales.

Suggested Activities

- Have the student redraw a pictograph so that each symbol represents 4, instead of 2. Ask the student which graph he/she prefers and to give reasons for the choice. Ask if there is another way to display the data which might be clearer.
- Provide data for a bar graph: such as Favourite Sports (hockey 36, baseball 22, basketball 30, volleyball 16). Have the student select a scale.
- Ask the students to determine the scale for a bar graph to display the number of students traveling on each different school bus in the morning. Each step along a bar is to represent more than one student.
- Have students explore other applications of many-to-one correspondence, such as the use of scale in mapping.

SCO: **SP1: Demonstrate an understanding of many-to-one correspondence.**
[C, R, T, V]

Assessment Strategies

- Ask why a symbol in a pictograph usually represents more than 1.
- Provide students with two graphs: one that displays one-to-one correspondence and the other displays many-to-one correspondence. Explain the similarities and differences.
- Provide students with a set of data and have them create a scale for it. Ask the student to support their choice.
- Ask the student for an example of when it would be appropriate to use a one-to-one correspondence using a real-life context.
- Ask the student for an example of when it would be more appropriate to use a many-to-one correspondence using a real-life context.

SCO: **SP2: Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions. [C, PS, R, V]**

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence

Grade Three	Grade Four	Grade Five
SP1 Collect first-hand data and organize it using: tally marks; line plots; charts; lists to answer questions. **SP2** Construct, label and interpret bar graphs to solve problems.	**SP2** Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.	**SP2** Construct and interpret double bar graphs to draw conclusions.

Elaboration

The emphasis should be to help students see that graphs tell us about information and that different types of representations tell different things about the data. The value of having students actually construct their own graphs is not so much that they learn the techniques but that they are personally invested in the data and that they learn how a graph conveys information (Van de Walle and Lovin 2006, p. 329).

Students extend their understanding of constructing graphs and interpreting data from previous grades by exploring **vertical** and **horizontal** displays that require a many-to-one correspondence. When creating **pictographs** and **bar graphs**, it is important for their displays to include a **title**, **labels**, and a **legend** or **key** (when applicable).

Once students have constructed a graph, it is important for students to have an opportunity to make observations and interpret the data. They should also have experiences discussing other graphs that they can find, such as in newspapers and magazines, and on television and the Internet.

SCO: **SP2: Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.**
[C, PS, R, V]

Achievement Indicators

- *Students who have achieved this outcome(s) should be able to:*

  - Identify an interval and correspondence for displaying a given set of data in a graph and justify the choice.
  - Create and label (with categories, title and legend) a pictograph to display a given set of data using many-to-one correspondence, and justify the choice of correspondence used.
  - Create and label (with axes and title) a bar graph to display a given set of data using many-to-one correspondence, and justify the choice of interval used.
  - Answer a given question using a given graph in which data is displayed using many-to-one correspondence.

SCO: **SP2: Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions. [C, PS, R, V]**

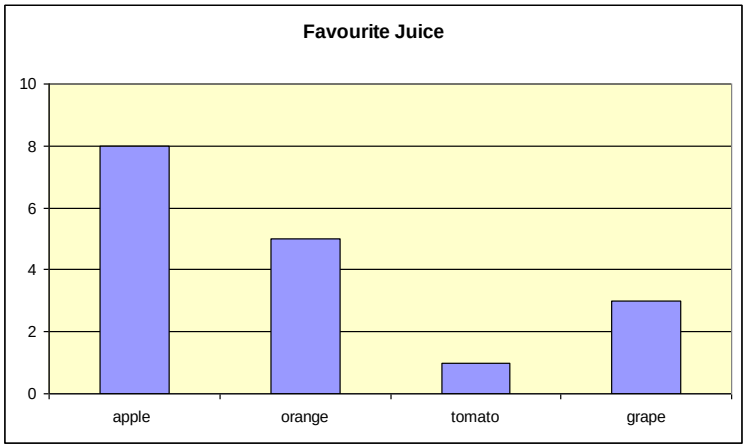
Instructional Strategies

Consider the following strategies when planning lessons:

- Provide experiences to ensure that when creating bar graphs and pictographs, student have an understanding of the importance of a common base line and many-to-one matching of objects in the various categories.
- Use pictographs based on a many-to-one correspondence (i.e., a picture represents a group of items).
- Have students interpret and create various bar graphs and pictographs that run horizontally and vertically.
- Create graphs primarily in the context of other investigations, rather than as an isolated activity to achieve the curriculum outcome.

Suggested Activities

- Suggest that students create a graph that shows the most popular authors, movies, types of food, etc. of class members. Have some students create a bar graph that shows the results of data in scale of 2 and other groups can use a scale of 3, 4, and 5. Have students explain which graph displays the most appropriate use of the data.
- Show a graph like the one below. Explain that each tile represents 3 people and ask questions, such as: “How many like apple juice? How many more like apple juice than tomato juice? How many students answered the questions about their favourite juice?” Order the juices from most popular to least popular.

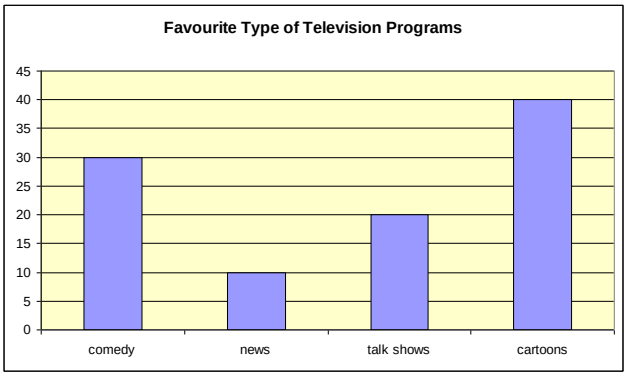


- Have students discuss what kinds of information they can get from reading pre-made bar graphs and pictographs that display the use of many-to-one correspondence.

SCO: **SP2: Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.**
[C, PS, R, V]

Assessment Strategies

- What questions might be answered by interpreting this graph?



- Create and label (with categories, title and legend) a pictograph and bar graph using the table below about Favourite Movies using many-to-one correspondence, and justify the choice of correspondence used.

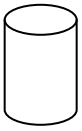
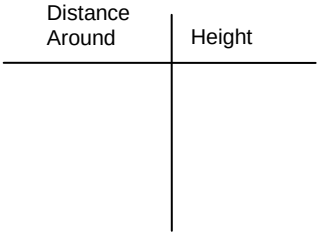
Adventure	9
Comedy	8
Drama	5
Science Fiction	10

Curriculum Guide Supplement
Math Makes Sense 4
Unit Plans

September	October	November	December	January
Mental Math: Fact Learning, Mental Computation & Estimation Should Be A Daily Routine				
UNIT 1: Patterns and Equations 16 Classes September 9 - 30	UNIT 2: Whole Numbers 32 Classes October 1 – November 20		UNIT 4: Measurement 22 Classes November 21-December 22	UNIT 3: Multiplication and Division Facts 19 Classes January 5 - 29
Mental Math: Fact Learning, Mental Computation & Estimation Should Be A Daily Routine				
February	**March**	**April**	**May**	**June**
UNIT 5: Fractions and Decimals 35 Classes February 1 – March 26		UNIT 6: Geometry 15 Classes March 29 – April 16	UNIT 8: Multiplying and Dividing Larger Numbers 30 Classes April 19 – May 28	UNIT 7: Data Analysis 10 Classes June 1 - 18

Note: All dates, as well as the number of classes suggested, are estimates only.

Unit 1: Patterns and Equations

Preparing to Teach This Unit	Review Specific Curriculum Outcomes **PR1, 2, 3, 5 & 6** in your Curriculum Guide as these are the SCOs the unit is attempting to address. Think about the two guiding questions***, "What do I want my students to learn? What do I want my students to understand and be able to do?"*** The ***Achievement Indicators*** and the ***Assessment Strategies*** for these outcomes will help you answer these questions and give you a better understanding of what it is you need to emphasize in you teaching. You should also refer to the ***Unit Rubrics*** on p.31 and p.33 in the Teacher's Guide, the ***Ongoing Observations*** sheet on p.32 and the ***Assessment for Learning*** sections which are components of each individual lesson.
Investigation*: It's All in the Can*	This is a great lesson to engage students in a hands-on activity to investigate the relationship between the circumference of a cylinder and its height. Be sure to include at least one "Pringles" can so that students have a *counterintuitive* experience when the dimensions of this type of cylinder are estimated and then measured. Use the Observation Checklist (TG v) to help you assess conceptual understanding, procedural knowledge, and problem solving skills. Tally chart Which is Greater?  

Lesson 1: *Patterns in Charts* **SCO N1**	This lesson introduces the term "pattern rule". The key idea for students to understand is that pattern rules are different ways to describe what's happening to make the pattern. Different people will describe the pattern rule in different ways. **Q2: Do Together** Make a chart or overhead transparency of the calendar page for April. Use two different color markers, one to circle Anthony's guitar lesson days, and one for his sister's piano lessons. **Q3: Do Together** If you have a large hundred chart in your classroom, you can remove some of the numbers and have students identify what they are and how they know. You can also make a transparency of a hundred chart and cover several numbers with opaque counters. It would also be a good ideas to extend this type of pattern activity to larger charts such as a 300 chart. **Q4: *(Numbers, Pictures, Words)*** It might be difficult for most students to identify five different patterns. Two or three patterns should be sufficient. **Q6: Do Together** Use a transparency of PM 16 to show how an addition chart works and notice the patterns in the sums. Using PM 18, you can make a transparency of the addition chart that appears in the student text and then help students search for any patterns. When a pattern is interrupted, it is because of an addition error. Identify what these errors are and help students use mental math to add the two numbers correctly. ➢ 30 + 11 *"Thirty plus ten is forty, and 40 plus 1 is 41"* ➢ 17 + 22 *"Twenty and ten is thirty, and seven and two are nine, so thirty and nine make 39".* ➢ 26 + 15 *"Twenty plus ten is thirty; six plus five is eleven; Thirty plus eleven is like thirty plus ten and one more, so that's 41."*

<table>
<tr>
<td>Lesson 2: Extending Number Patterns
SCO PR3</td>
<td>In the Lesson Organizer, the list of student materials suggests photocopying a variety of charts from the Program Masters to be used as tables. Save on paper by getting students to draw these charts in their notebooks using a pencil and ruler.

(TG 8) Before. Use an overhead transparency of the dot paper to show students how to get started with drawing the squares and recording the number of dots. Create a t-chart with at least 10 rows and fill in the first two rows together. Help students accurately count the dots in the squares. Identify the number pattern and use it to determine how many dots will be in the 10th square.

(TG 9) Explore. This task is more complex than the previous one because it describes width and length as units. Students are often confused by the meaning of unit, and will count the pegs rather than the spaces between them. It is recommended that teachers create a chart similar to the one in the student text and then make each rectangle one at a time, recording the information in the chart as you go.

<table>
<tr><th>Rectangle</th><th>Length</th><th>Width</th><th># Pegs Touching</th></tr>
<tr><td>1</td><td>2</td><td>1</td><td>6</td></tr>
<tr><td>2</td><td>3</td><td>2</td><td>10</td></tr>
<tr><td>3</td><td>4</td><td>3</td><td>16</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
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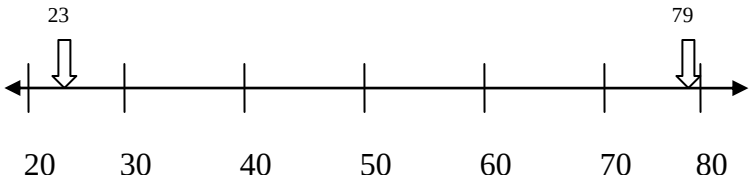
</table>

Q1 – 3: Do Together
These activities reinforce the concept that numerical relationships and patterns can be recorded in charts and that we can use the patterns to make predictions. Allow students to draw the next 2 figures in Q1 and then help them enter the information for the first 6 figures. Discuss the patterns and predict for the 7th, 9th and 10th figures. Have the same type of discussion for Q2 and Q3. For the pizza question, show students how to extend the table to accommodate a pizza with 48 slices.</td>
</tr>
<tr>
<td>Lesson 3: Representing Patterns
SCO PR2</td>
<td>The essential understanding in this lesson is that you can have two representations of a pattern; one shown in a table or grid and the physical version made with materials or in a drawing. When looking for relationships, some students focus on the table and others will focus on the physical pattern. It is important for students to see that whatever relationships they discover, they exist in both forms.
The examples shown in the Explore and Connect (TG 12-13) of Lesson 3 imply that the patterns should look like the examples in the book. This is not necessary. There are many different ways to represent the pattern in a table with materials. Do not expect too much from students in terms of what their patterns look like.</td>
</tr>
</table>

	Instead, check to see if each figure is consistently different from the one before it based on the number pattern in the table or chart. In the ***Before*** section of this lesson, you can use connecting cubes rather than pattern blocks to represent the number of scoops on each cone. **Q1:** The pattern does not need to look like the one in the guide. **Q2:** Note that if using pattern blocks, each student needs to work with congruent pieces (all squares, all trapezoids, all triangles etc.) It might be better to use another manipulative such as counters, connecting cubes, or color tiles which are more readily available. **Q4:** The pattern rule in this activity is quite simple, but the question is poorly worded. Teachers are advised to have students use connecting cubes (multi-links) rather that colored tape, to construct the various lengths, identify the rule, and predict the length of various figures from the pattern. **Q5: *(Numbers, Pictures, Words)*** Either do this together or omit it.
Lesson 4: *Equations Involving Addition and Subtraction* **SCO PR5, PR6**	**Explore:** The teacher should lead this activity using the overhead. It may be the first time that many students have seen symbols being used for the unknown part of an equation. Help them understand what the symbols represent with an emphasis on the equals (=) sign as a symbol of *balance.* Read each story one at a time and see if students can identify the equation that matches it. Besides base ten blocks, you could use counters or a number line. ***Connect***: It is important that students are given the opportunity to think and reason with different types of story structures. Before working through the example in the book which is a Join, Change Unknown problem involving the numbers 12 and 32, make up some similar problems with the same structure using smaller numbers and have students work with counters to model and solve each problem. For example: ➢ *Andrea and her brother Mark read 13 books between them. Andrea read 7 books. How many did Mark read?* ➢ *There were 16 colored Christmas balls in the box. Nine of the balls were red and the rest were green. How many were green?* Ask students to explain how they used counters to model each problem (there may be more than one way) and use numbers and symbols to record the process. **Q4-5:** These are two good story problems for students to solve. However, do not expect them to be able to record the method they used as an equation. They might use counters, or draw a picture or use some other problem solving method to make sense of each situation and arrive at the answer. However, the teacher should be the one who introduces the equation that represents each different method

	Q6: Do this one together after students have had a chance to think about what needs to be done. It seems more obvious that this is a subtraction situation rather than one that you would use "Guess and Test" for. You could set it up either as a subtraction sentence (vertical or horizontal) or you could use a number line to reinforce the concept that subtraction is asking us to find the *difference* between two numbers. Start with the smaller number and count on to 30 and then add this distance to the rest of the way to 79.  **Q7:** This is a good question to do, but it should be read to the students so that they understand the situation. They then need to pick the equation which best represents the story and explain why. The class should also discuss why the other equations are not the correct ones. **Q9:** Teacher should model this **Q10:** ***(Numbers, Pictures, Words*** Good assessment checkpoint.
Lesson 5: *Equations Involving Multiplication and Division* **SCO PR5, PR6**	***Connect:*** This example of the apples being shared with the 5 friends is a bit misleading and confusing. The 25 apples are shared equally among the five friends and we want to know how many each friend got. That would be represented by 25 ÷ 5 = ◊. To get the equation like the one in the book 25 ÷ ◊ = 5, you would need a story like, "Sarah had 25 apples. How should she divide the apples among her five friends so that each of them gets an equal share? The important thing here is that students recognize that division has two meanings. 25 ÷ 5 = ◊ means, *"25 is divided equally into 5 groups. How many are in each group?"* OR *"25 is divided into groups of 5. How many groups are there"?* The rest of the **Connect** looks at three different ways to think about and solve a division problem. **Q5:** Students should be able to do this on their own. However, 2 x 4 = ◊ and 4 x 2 = ◊ are both acceptable equations.
Lesson 6: *Strategies Toolkit*	***Explore:*** Do this activity with the students to help them see how you would use *guess and test* to figure out the value of each shape. Draw each of the equations on the chalkboard. Start with the last one and try values of 3 for the squares and 2 for the triangle. Plug these values into the first equation. If it doesn't work there, go back to where you started and try different values until they work for all three equations. Repeat this method with the first problem in **Connect**. Q1-2: Students should be able to try these on their own. Q3: Use a Think-Pair-Share strategy for this question. Students

	look at the problem individually, then discuss their thinking with a partner, and then with the whole group. Students who are having difficulty with this problem, could assign values to the different shapes. For example, if a pyramid is worth 2, then the sphere must be 6 and the cube must be 1. The sphere has the greatest mass.
Unit 1 Test: *Show What You Know*	Do not expect your students to be able to complete this work independently at the beginning of the year. Use the items as teaching activities to ***review and reinforce*** the kind of pattern work students have been doing in this unit.
Unit Problem: *Calendar Patterns*	Using calendars to search for patterns is an effective strategy because fewer numbers are involved than in a hundred chart and the 1 is not always in the top left corner. This problem is in 3 parts. The first 2 should be introduced and discussed by the teacher. Part 3 may be a bit more challenging, but if you check the ***Assessment for Learning*** tab on page 29 of the TG you will find an excellent suggestion for scaffolding this activity for all students.
Assessment	Review the Achievement Indicators and the Assessment Strategies in the Grade 4 Mathematics Curriculum Guide for SCOs PR1, PR2, PR3, PR5, and PR6. You may also use the ***Unit Rubrics*** on P.31 and 33 in the Teachers Guide in conjunction with the ***Ongoing Observations*** (p.32) and ***Assessment for Learning*** sections (for each lesson) as tools to help you gather relevant assessment information. Consider these three guiding questions: *"What conclusions can be made from assessment information? How effective have my instructional approaches been? What are the next steps in instruction?"*

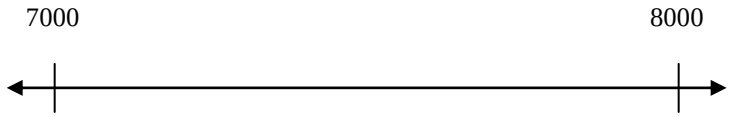
Unit 2: Whole Numbers

<table>
<tr><td>Preparing to Teach This Unit</td><td>Review Specific Curriculum Outcomes N1, 2, 3, and PR4 in your Curriculum Guide as these are the SCOs the unit is attempting to address. Think about the two guiding questions, “What do I want my students to learn? What do I want my students to understand and be able to do?” The Achievement Indicators and the Assessment Strategies for these outcomes will help you answer these questions and give you a better understanding of what it is you need to emphasize in your teaching. You should also refer to the Unit Rubrics on p.51 and p.53 in the Teacher’s Guide, the Ongoing Observations sheet on p.52 and the Assessment for Learning sections which are components of each individual lesson.</td></tr>
<tr><td>Launch: Those Amazing Elephants</td><td>This activity provides an opportunity to informally introduce a Carroll Diagram as a way to organize some information about Asian and African elephants. Begin by discussing the meaning of typical and then draw a Carroll Diagram on a chart or on the chalkboard. Discuss the differences and guide students to make reasonable estimates.

<table>
<tr><td></td><td>African</td><td>Asian</td></tr>
<tr><td>F</td><td>3600 kg</td><td>2720 kg</td></tr>
<tr><td>M</td><td>5500 kg</td><td>4990 kg</td></tr>
</table>

There are 3 questions in the student text p.33 that should be discussed as a group.
➢ The first one compares the mass of female elephants. Some students may recognize that a subtraction equation could be used to find out how much greater the mass of an African elephant is. The teacher should also use a number line to reinforce the concept that subtraction is asking us to find the difference between two numbers. Use a scale showing 2000kg, 2500kg, 3000kg, 3500kg, and 4000 kg. Draw an arrow or make a point on the number line to show where the Asian female’s mass is (2720 kg) and the African female’s mass (3600 kg). Starting with the smaller mass, it’s 80 kg to get to 2800, and then 200 more kg to get to 3000, and finally 600 kg to get to the mass of the African elephant. That’s a difference of 80+200+600 = 880 kg.
➢ To help students estimate the difference between the baby elephant’s mass (145 kg) and a fully grown adult male (4990 kg) you could use a line in a slightly different way. Show 145 kg at one end and 4990 kg at the other end. Ask, “If the baby elephant weighed 100 kg, what would you estimate the difference to be? (4800 kg, or 4900 kg). Since we know that a baby elephant weighs more that 100 kg, what would a better estimate of the difference be? (4750 kg or 4850 kg) All of these estimates are reasonable. The goal is to help students</td></tr>
</table>

	see that you can make small adjustments to get closer and closer to the actual answer. ➢ For the last question, students are estimating the difference between 5500 kg and 10 000 kg. You might help them use the following line of reasoning: *"5500 is about half of 10 000; it's actually more, so an estimate of 5000 kg would be about 500 kg more. An estimate of 4000 would be about as close only that would be 500 kg less."* Some students may recognize that the exact difference can be found by adding 5500 to 4500 to get 10 000kg
Lesson 1: *Whole Numbers to 10 000* **SCO N1**	***Before*** Provide rods and unit cubes from the base ten blocks for students model some two digit numbers. Depending on the availability of flats, provide some of these so that 3-digit numbers can be represented. If there are not enough of these to go around, provide them at the front of the class and have individual students come up and model numbers you write down. Note: Do not use the word ***and*** when reading whole numbers; ***"one hundred forty three"*** not ***"one hundred and forty three."*** While working with numbers in the hundreds, model how a given number can be represented in more than one way and encourage students to do the same. For example, 343 can be modeled with 3 flats, 4 rods and 3 unit cubes or with 2 flats, 14 rods and 3 unit cubes, or with 3 flats, 2 rods, and 23 unit cubes etc. Be sure to include numbers such as 207, 509, 420. Extend this to 4-digit numbers, again having students come up to the front to model with the base ten materials. Move to the pictorial stage by showing students how they draw a picture of the base ten model using dots, sticks, squares and cubes. ***After*** Draw a place value chart on the board and have students record the base ten models in the chart. Just do a few of these to introduce how the place value chart works and how it connects to ***expanded form***. **Q2:** Do this together using chart, chalkboard, or overhead projector **Q4(b):** Use a place value chart and do this together **Q11(a,b,c):** This is a good thinking activity involving a calculator. Depending on how much experience they have had with calculators, students may need help clearing error messages and getting a new screen to work with. **Q12 *(Numbers, Pictures, Words)*** would be a good assessment activity to include as a journal entry. Q13: Differentiate how many different ways you require of students. Struggling students may need to continue to work with 3-digit numbers. In the ***Assessment for Learning*** section p.7, the suggestion that you provide place value charts for students who don't recognize

	zero as a place holder is an excellent idea and one that probably should be extended to most of your students especially at the beginning.
Lesson 2: *Comparing and Ordering Numbers* **SCO N2**	Math Note: You can make and provide laminated place value mats for students to work with or they could use their dry erase boards to draw their own mats. ***Before*** Be sure you write the numbers you are asking students to compare (greater/less than) on the chart, chalkboard or overhead. ***Explore*** Ask students to make up five or six different numbers using the given digits and put them in order. (Early finishers should be challenged to find all 24, and will probably see the need to make some king of organized list to keep track of their work) ***Connect*** If you use base ten blocks, do it as a class and have students come up to work with your set of materials. They can also draw dots, sticks, squares and cubes to represent the numbers. For the last item in the Connect section, draw the number line on the board and have students come up and show where they think 1423, 2143, and 2413 belong. Use additional groups of numbers so that more students can be involved. **Q9: *(Numbers, Pictures Words)*** This is a good assessment checkpoint. To help students understand the range of numbers that is allowed, draw a number line with 7000 on one end and 8000 on the other. Later, when students have created and ordered their 4-digit numbers, have several come up and position some of these numbers on the number line.  **Q10:** Spend some time helping students read the numbers correctly and look for the number pattern in each sequence. Do the first two together. **Q11:** After students have ordered the numbers correctly, ask several to read the sequence aloud. **Q12:** Same notes as for Q10.

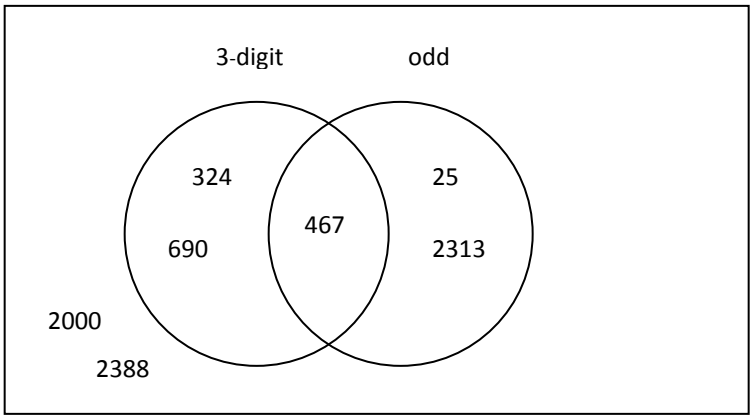
Lesson 3: *Sorting Numbers*
SCO N1, PR4

Math Note: "Loops of string" is not the easiest model to use when learning about Venn Diagrams. Try Hula-Hoops or other rings that may be available in your school

Explore

This section will need considerable direction from the teacher. It would be a good idea to first spend some time working with a collection of objects to identify some of the ***attributes***. For example, in a collection of buttons, you might have some with 2, 3, or 4 holes, round, square, or triangle buttons, and different colors of buttons. A Venn Diagram could include a circle for buttons with 2 holes and another circle for round buttons. Since there would be buttons that are both round and have 2 holes, the circles would need to overlap to create a ***shared region***
You could also do similar things with pattern blocks. Some common attributes would be, *3-sides, 4-sides, more than 4 sides, fewer than 4 sides, color, "big", "small".*

Create a list of attributes that you and your students identify in the numbers at the top of p.42 in the student book. For example, 2-, 3-, 4-digits, more than 3 digits, fewer than 3-digits, >1000, <1000, even, odd, etc.



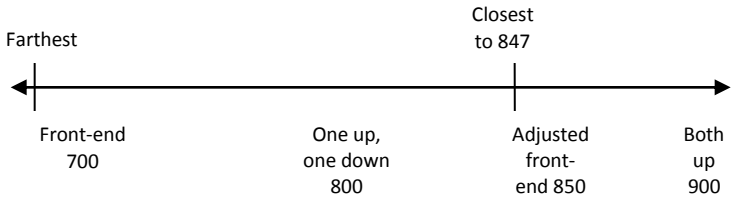
Discuss the 3 Venn Diagrams in **Connect** in the student book. What information does each show? The second example is not one that is often seen, but it is still a type of Venn Diagram.

For the Carroll Diagram, refer back to the Diagram you created in the **Launch:** *Those Amazing Elephants*. The diagram in the student book uses the categories *odd* and *not odd*. If a number is not odd, then it's ***even***. Use this term with students.

Q1: Begin by having students find several numbers that are greater than 500. Then see if they can identify some even

	numbers. Ask*, "Are there some numbers that are both even and greater than 500?" Yes, some numbers have both attributes, so we need a Venn Diagram that has two circles that overlap."* **Q2:** Rather than have students copy the Venn Diagram in their book, the teachers should put it on the chalkboard or overhead and do the activity with the group. For (c) write the numbers on sticky notes and have students come up to the chart and put them in the right region. **Q3:** Same as Q2. **Q4**: Do together to help students make sense of the information displayed. **Q8**: You should use a calendar for the Month of April to identify the dates that Joe and Cher work. Students should then be able to take these numbers and create a Venn Diagram with the labels ***Dates Joe Works*** and ***Dates Cher Works***. Since they also work on some of the same days, the circles should overlap. **Q6**: Since students have not worked with Carroll Diagrams very much in this unit, do this question together. Review the meaning of ***multiples.*** Counting by 5s is a good place to start to remind students that multiples are related to skip counting and the multiplication tables. **Q7*: (Numbers, Pictures, Words)*** The essential learning for SCO PR4 as it relates to this question is that students can use a Carroll Diagram to display and interpret information. They may not know what *"is divisible by 5"* means. Explain that if they say the number when they count by 5s, then the number is divisible by 5 (it's also a *multiple* of 5!). Students should be able to do most of this question independently, and it is a good assessment checkpoint. In the ***Assessment for Learning*** section (TG p15), notice that under *Procedural Knowledge*, the emphasis is on students **using** Venn Diagrams to sort numbers. They may not be proficient at creating and labeling. It also highlights the importance of identifying and using **patterns** so that students can determine why numbers are sorted in a particular way and can apply their understanding by adding other numbers to the correct region of the diagram.
Lessons 4 through 9	Much of what is presented in these 6 lessons can be achieved by using the resource ***Mental Math: Fact Learning, Mental Computation, Estimation Grade 4 Teachers Guide***. In this guide, strategies for estimation and mental computation are developed through a gradual progression of increasingly complex numbers to ensure that students have early success in applying a particular strategy. Strategies are introduced one at a time and practiced and reinforced over several days. The introduction to a strategy takes the most time, so this could be during regular math class. The reinforcement and practice (5-10 min. each day) would

	be part of your mental math time. There are some good "story" problems in Math Makes Sense that give students a chance to apply their estimation and mental computation skills in context. The difficulty pilot teachers had with many of the exercises in Math Makes Sense is that the numbers are often too large and more than one strategy is introduced in a lesson and this caused some confusion for students. In ***Mental Math: Fact Learning, Mental Computation, Estimation Grade 4 Teachers Guide***, the following estimation strategies are introduced (p.49-54) ➢ Rounding to nearest 100 or 1000 in addition and subtraction ➢ What to do when 5 or 50 or 500 is involved (different for subtraction) ➢ *Adjusted* Front End estimation for addition and subtraction (*adjusted* simply means that you start to look at some of the other place values instead of just the largest place value so that you'll get a better estimate) All of these strategies are in Lessons 4 and 8 of Math Makes Sense, but there are too many all at once. It also introduces a new rounding strategy for subtraction; ***rounding the subtrahend only*** to the nearest 100 or 1000. This is an effective and useful strategy for students to learn. Mental computation strategies in the grade 4 Mental Math Teachers Guide include (p.31-38) ➢ Front-End Addition (2 different but similar ways) ➢ Compensation (for 371 + 18, think 371 + 20 and then subtract 2 from 391) referred to as "make friendly numbers" in Math Makes Sense. ➢ Make 10, 100, 1000 (similar to compensation, except you don't have to make any adjustment at the end) For example, for 450 + 62, think 450 + 50 (from the 62) = 500 + 12 (the remainder of 62) is 512. If teachers choose to closely Follow Math Makes Sense Lessons 3-8, the following suggestions may help.
Lesson 4: *Estimating Sums* **SCO N3**	***Explore*** Do this together, since students are notoriously resistant to the idea that estimation is an important skill in math. When working with estimation, use words and phrases such as *about, almost, between, approximately, a little more than, a little less than, close to,* and *near*. (see Mental Math: Fact Learning, Mental Computation, Estimation Grade 4 Teachers Guide p.49) To estimate the cost of the DVD player ($204) and TV ($589), you could look just at the first number in each and add them to get an estimate of about $700. Some students may notice that this estimate is pretty low, because the TV is almost $600 and if you added $600 and $200 you would have an estimate of $800 which is closer to the real answer. If you were really going to the store with money in your pocket you'd want to have an even better estimate (if not the actual cost). One way to do that is to round the prices to the nearest $10, so for the TV it would be $590 and for the DVD player it would be $200 for an estimate of $790. Notice

	here that if you only went to the store with $790, you wouldn't have enough money! Sometimes it's better to estimate a bit higher. ***Connect*** Do this section together with students. For the first bullet p.47 in the text demonstrate what happens when you: ➢ Round both number up ➢ Round one up and one down ➢ Use front-end addition ➢ Use adjusted front-end addition Draw a number line with the labels Farthest on the left and closest on the right. The actual answer of 847 should be written on the right. The results of each type of estimate can be positioned on the line. Students need to see that all of these estimates are reasonable but that some are closer to the actual than others. Depending on the situation, you may just need a "ball park" estimate. At other times, it's better to be closer to the actual answer.  **Q1:** Ask for estimates from students. Accept all reasonable answers. **Q2 and 3.** Do together. Copy onto chart or chalkboard. Doing these as a group helps students see that different estimates are possible and that some are closer on others; depends on your purpose and needs. **Q4:** This is a reasonable problem for students to solve. Read it over with them. **Q5:** Omit **Q6**: Discuss these two estimates which involve rounding to the nearest thousand. Leave out the calculator part. Discuss ways students could make an even closer estimate. (*adjusted* front-end strategy) **Q7:** Discuss the answer to this together. It should have already come up in your teaching. **Q8: *(Numbers, Pictures, Words)*** Good assessment checkpoint.
Lesson 5 *Using Mental Math to Add* **SCO N3**	***Before*** If you ask students to mentally add the numbers in the teachers guide, be sure to write them down so they can see them as well

	as hear them. ***Explore*** The first example in the student book (227 + 134) is very challenging because it involves regrouping. Use smaller numbers such as 24 + 32 or 243 + 3 52. Always scaffold the activities in this unit so that students have reasonable entry points. ***Connect*** "Trang's" example (170 + 348) is difficult because it crosses over into the next hundred by counting by 10s. Use an easier one. The two strategies introduced by the children here are the same as "Make 10, 100, 1000" and the front-end strategy "Break Up and Bridge" that's in the Mental Math guide. ***Scaffold*** the last item in this section (top p.50). Start with numbers that don't require regrouping. [353 + 240, 353 + 210, **Q1**: Items a, b, d, e, and g are too difficult for most kids at this stage. Use easier examples. **Q2**: Items b, c e, g and h are too difficult. Use easier examples. **Q3:** Change the numbers in this problem to 160 and 230. **Q4:** (Numbers, Pictures, Words) This is a good assessment checkpoint because it will allow students to write a problem that is reasonable for them to do. ***Reflect*** Modify this task by presenting an addition sentence such as 227 + 312 and ask students to explain using numbers, pictures and words how they would add these two numbers mentally. For struggling students, consider 2-digit addends such as 26 + 32, or 2-digit plus a single digit with no regrouping; 44 + 5 = In the Assessment for Learning section under Check Further, the example 179 + 234 is very difficult. Provide an easier computation for students to apply their mental strategies. Continue to provide practice for mental addition of 2-digit and 3-digit numbers using the strategies in the Grade 4 Mental Math Teachers Guide.

Lesson 6: *Adding 3-digit Numbers* Lesson 7: *Adding 4-digit numbers* **SCO N3**	Both lessons in Math Makes Sense introduce ***alternative procedures*** for addition (and for subtraction in lessons 10 and 12). In the grade 4 curriculum **SCO N3** states that one way students demonstrate their understanding of addition and subtraction is by using **"personal strategies"**. These strategies are as valid as a traditional algorithm which is the ultimate goal once their understanding of the operations has been developed. In order for students to develop personal strategies, they need to have a good understanding of **place value**, which is an ***essential understanding*** in elementary math. Alternative procedures include ***Mental Math***. Lesson 6: ***Before*** This is a very difficult example to start off with. It involves regrouping ones to tens, and then tens to hundreds. An easier example would be 361 + 237. Discuss and record some different ways that these two numbers could be added. ***Explore*** Do together. This is a difficult pair of numbers to work with and the suggestions in the teachers guide require a higher level of number sense than most Grade 4 students are ready for. Use a different set of numbers that don't require so much regrouping so that strategies can develop and students experience success. ***Connect*** Use "think aloud" to model what Abigail did to add 357 and 275 mentally. In the second method, base ten blocks are introduced as a way to show what happens when we add from right to left using pencil and paper. It represents a physical model of the traditional algorithm. ***Practice*** Review some of the estimation strategies you have been working on in class. **Q1 & 2:** Use easier numbers that require less regrouping. See MM Teachers Guide. **Q3:** Help students set this problem up so that they can arrive at a solution. Consider changing the numbers to 130 and 254. **Q4:** The question should ask, *"What 3-digit numbers might they be?"* **Q5: *(Numbers, Pictures, Words***) This is a good question to ask to see who has some understanding of regrouping. **Q6:** Do together. It's a good example for using *compensation.* 389 + 209 = 389 + 210 = 599-1=598 **Q7:** Use *front-end addition* and a *place value chart* (expanded form) and model how these two numbers might be added.

Q8: Have students record the numbers. Ask them to use *compensation* to add them.

Q9: Do together. Use a place value chart.

Q10(b) Ask students to show how they would add the numbers in a way different from the right to left method.

Q11: This has many possible answers. Good activity for early finishers and for those who need a challenge.

Reflect: Excellent prompt!

Lesson 7: ***Before***
Most students probably won't be able to name all the different ways they have been learning to add 2- and 3-digit numbers (the teachers guide identifies 6). It may be a better strategy to review the different ways as a group, with examples, so that students can see that the same strategies can be used with 4-digit numbers. Note: If you are using Mental Math: Fact Learning, Mental Computation, Estimation Grade 4 as your primary resources for introducing and developing mental computation strategies, the practice items start with 2-digit numbers and move up to numbers involving 4-digits. The strategies include Front-End (two variations), Compensation, and Make 10s, 100s, 1000s.

Explore
(Consider an easier example which doesn't involve regrouping in two places such as 1272 + 2163). For the example in the book, here are two possible ways to add 1275 and 2168.
Front-End (adding from the left)
1000 + 2000 = 3000
200 + 100 = 300
7 tens + 6 tens = 13 tens (130)
5 + 8 = 13
so 3000 + 300 + 130 + 13 = 3000 + 430 + 13 = 3443

Using a Place Value Chart

Thousands	Hundreds	Tens	Ones
3	3	13	13
3	4	4	3

Connect:
Another difficult computation, this example involves regrouping in three different places! Consider a few easier ones (2-digit and 3-digit numbers) to help scaffold the strategies.

In the last item in the Connect section, Zena uses ***Column Addition*** starting on the left. ***Expanded Form*** might be better

understood by students and works just as well.

1976 + 2868 =
3000 + 1700 + 130 + 14 =
4700 + 144 = 4844

Q1 & 2: Do these together with students. Use the overhead or chalkboard to estimate first and then model a strategy for adding the numbers.

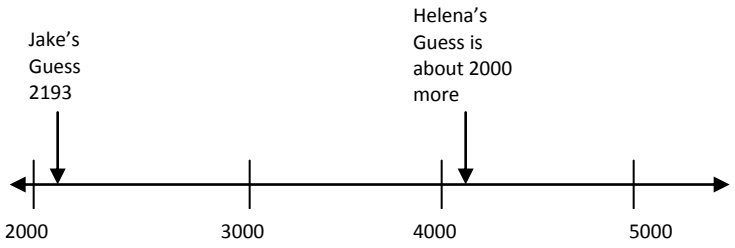
Q4: Read this problem aloud with students. Discuss how these numbers should be written.

Q5: Scaffold this by writing 880 on the board. What are two 3-digit numbers that add to make this sum? Write down any reasonable 3-digit number less than 880 that allows another 3-digit number to be the other addend. For example, 250. What needs to be added to 250 to give a total of 880? It could look like this:

$$\begin{array}{r} 250 \\ +\quad\quad \\ \hline 880 \end{array}$$

Do several more examples, including 4-digits, and then allow students to work on Q5.

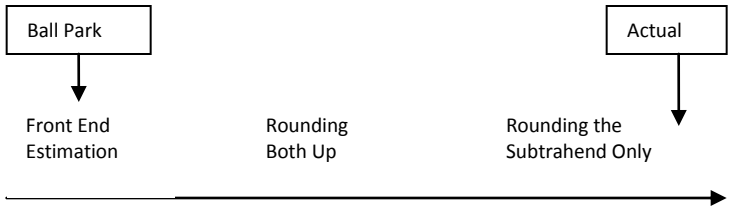
Q6: Read this question over with students to make sure they understand what is meant by *greater than*. A number line may help students picture what the question looks like.

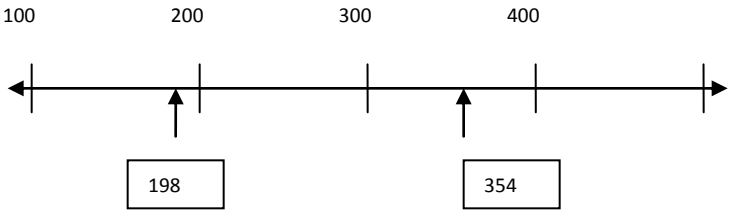


Q8-9: Read all problems together and work through them one at a time using various strategies.

Reflect:
Students will probably have difficulty understanding what this question is asking for. A better question might be, *How would you use a place value chart to add 2456 and 3237?*

In the ***Assessment for Learning*** section (TG p.28) it suggests "starting with 2- and 3-digit numbers that involve no regrouping" to help build confidence in struggling students. It makes good sense to apply this scaffolding strategy with the majority of students in

	your class.
Lesson 8: *Estimating Differences* **SCO N3**	Rounding in subtraction is different from rounding in addition. In subtraction, both numbers **should be rounded in the same direction**. If you round one up and one down (which often happens in addition) the difference between the two numbers will be even greater than the actual answer. Use a number line and reinforce the idea that subtraction is asking us to find the *difference* between two numbers. Use another line to show how different strategies produce estimates further from or closer to the actual answer. Be sure to help students understand that in many estimating situations, a broad "ball park" estimate is fine, and in other cases you want a closer estimate.  **Q3:** Do together to help students identify "the closest hundred". Use a variety of number lines. **Q5:** Do together. Rather than "solve the problem", say, "Estimate the answer to your problem". **Q6*: (Numbers , Pictures, Words)*** Connect this question to the one you modeled in the previous lesson. That one wanted exact numbers; this one is looking for estimates. **Q8:** Do together. **Q9-12:** Do together to ensure students understand what is being asked and have a strategy for coming up with *approximate* answers. In the ***Assessment for Learning*** section under *Procedural Knowledge*, students will probably have difficulty explaining why they used a particular strategy, other than to say that they just like it or know how to use it.

Lesson 9: *Using Mental Math to Subtract* **SCO N3**	***Explore:*** See if anyone can suggest a way to find the answer to this question and do together. A number line will help students visualize the "**Up Through 10, 100"** strategy which is part of the *think addition* process.  *What is <u>the difference</u> between these two numbers?* *"Starting at 198, it's 2 to get to 200, then 100 more to get to 300 (that's 102), then 54 more for a difference of 156."* Another strategy students could use is to change 198 to 200 and 354 to 356 (2 was added to each number. This keeps the difference between the two numbers the same, but the subtraction is now easier to do). 356 – 200 = 156 **Q1 & 2**: Have students use their dry erase boards and share the strategies they use. Write each of the problems on the overhead, chalkboard, or on a piece of paper. Read the ***Assessment for Learning*** section in this lesson. The Mental Math Teachers Guide has lots of practice items to develop confidence in applying strategies.
Lesson 10: Subtracting 3-Digit Numbers **SCO N3**	***Connect:*** To make this a bit easier for students, change the numbers to 565 – 310 = ; *"565 – 300 is 265, but I didn't subtract enough; I still need to subtract 10 more, so my final answer is 255."* In the ***Reaching All Learners*** section, under **Common Misconceptions: How to Help**, do not use the first method to model the subtraction. It may be too confusing for students. Instead, follow the <u>second</u> method which involves modeling only the number that students are subtracting from – the minuend. In the last item in the **Connect** section, Jan uses "counting on" (also known as Up Through 10, 100 in the Mental Math guide) to subtract 397 from 622. Model how you would jot down some of the sub-steps involved in this computation and encourage students to do the same to "keep track of their thinking." For example: 622 – 397 = Starting at 397, it's 3 more to get to 400 [3], then 200 to get to 600 [200], then 22 more to get to 622 [22], so that's 3 + 200 + 22 =225 **Q2**: Do this together to identify which ones students need to complete.

	Q3: Do together one at a time by having students come up wuth an estimate before they actually do each subtraction. **Q4 & 5:** Discuss these two questions together to make sure students understand what is being asked and can suggest a way to solve each problem. **Q6:** ***(Numbers, Pictures, Words)*** Read this assessment checkpoint question over with students. Encourage them to use whatever method makes the most sense to them to solve the problem. In the ***Assessment for Learning*** section, under **Conceptual Understanding**, reference is made to aligning the digits correctly in a vertical format. This is more Procedural Knowledge than Conceptual Understanding. A higher level of understanding would be evident if a student wrote the numbers horizontally and performed the subtraction eg. 326 – 47 =
Lesson 11: *Strategies Toolkit*	***Explore & Connect*** This lesson introduces the problem solving strategy**, Make an Organized List**. Work through each problem in these two sections with the students so that they see how you model the strategy. **Q1:** The first Practice item is quite challenging because of the numbers involved. Provide easier numbers and model how you would start a list. For example: ➢ ***The Huda family picked 550 cucumbers in two days. They picked 100 more on the second day than they did on the first. How many cucumbers did the family pick each day?*** Day 1 | Day 2 | Total 100 | 200 | 300 200 | 300 | 500 250 | 350 | 600 225 | 325 | 550 550 Cucumbers 100 more on Day 2 **Q2:** Draw a table as above. Use headings of **10¢**, **5¢**and **Total.** Do the first couple of rows together so that students see how you have to add the total for the dimes to the total for the nickels. Begin with 2 dimes and 2 nickels (30¢) and then 3 dimes and 3 nickels (45¢). Students should be able to continue adding to the list until they find the correct combination that adds to 90¢.
Lesson 12: *Subtracting 4-Digit Numbers* **SCO N3**	***Connect*** The two methods for subtraction shown on p.71 of the student text involve difficult pairs of numbers. Kate's strategy, which is to

	change each number by the same amount in order to make the subtraction easier, requires her to subtract 31 from 856 which is not easy to do. A better practice item for this strategy would be 2030 – 850 which could be altered to produce 2000 – 820 = If students are unsure why this strategy works, go back to the number line model and show how the difference between the 2 numbers is maintained, but the numbers are easier to work with; they are more "friendly". Maksim uses a ***compensation*** strategy. He changes the subtrahend to make the calculation a bit easier but this means he has subtracted 44 too much, so he has to add it back on to his final answer. An easier pair of numbers for students to work with would be 1953 – 498 = "1953 subtract 500 is 1453, but I subtracted 2 too many so I'll add it back on for a final answer of 1455. If these numbers are still too challenging for some, use 3-digits numbers only or even 2-digit numbers. **Q3:** All of these subtractions are difficult and involve regrouping in more than one place. Provide much easier examples so that students are successful in applying a strategy. **Q4:** Have students draw a place value chart on their dry erase boards (or draw one on the chalk board). Have students fill in the numbers for "Seven Thousand One", "Eight Thousand Twelve" etc. Model (with the help of students) how each subtraction might be solved. For (a) 7001 – 356 = , one method would be "Up through 10, 100". "Starting with 356, it's 4 to get to 360 [4], then 40 more to get to 400 [40]; 600 more gets me to 1000 [600], 6000 more puts me at 7000 [6000] and then 1 more to get to 7001 [1]; so, 4 + 40 + 600 + 6000 + 1 = 6645. The standard right to left algorithm should also be shown, but because of the zeros involved in the minuend, many students will struggle with the procedure. Scaffold this type of problem with examples such as 305 – 226 = , 4054 – 1063 = etc. **Q5:** ***(Numbers, Pictures, Words)*** Discuss each one of these questions one at a time and have students record answers independently. **Q6**: Draw a ***Time Line*** on the board and show how it's a kind of number line. Help students use one or more strategies to find the difference between 1215 and 2008. **Q7:** This question allows students to use a calculator as a tool for problem solving. Model how you would use ***Guess and Test*** and ***Make an Organized List*** to keep track of the numbers you try. The task can and should also be modified to suit individual learners' needs. Students who are struggling might start with just 2 boxes in each row and gradually move up to 3 and 4 boxes. More capable students might be asked to list the *five* greatest or least differences they can make. **Q8:** If you choose to do this question it will be a difficult problem solving activity for most students. Challenge only the most

	capable ones in your class to work on it independently. You could have a discussion about what the numbers could be. For example, S can be any number except zero, because numbers don't start with zero. Z can't be zero either, because you don't write 000 when the answer is zero. H can be any number from 0-9. Perhaps students could try plugging in some values. If they start with S = 1 and H = 0, they'll solve the problem!
Lesson 13: *Solving Addition and Subtraction Problems* **SCO N3**	Throughout this unit, it has been necessary to look carefully at the numbers involved in the exercises and provide easier examples for students to work with. Lesson 13 is no exception. Make changes to the numbers as you see fir. For each question in this lesson, including the ***Explore*** and ***Connect***, discuss the problem situation with students. Some of these problems are multi-step and students will need help making sense of them. Have them suggest the most appropriate operation in each case and record the operation on the chalk board or overhead for students to work on. Use 2- or 3- digit numbers instead of those in the book if necessary. See ***Assessment for Learning*** under "What to Do if You Don't See It".
Unit Problem: *Those Amazing Elephants*	Questions 1 and 3 should be done together. Questions 2 and 4 should be read and discussed as a group, but students can complete the work independently in small groups or individually.
Assessment	➢ **The Unit Quiz, *Show What You Know* p.76-77 in the student text should not be the only assessment tool you use to determine the degree to which students have achieved the curriculum outcomes.** ➢ Review the Achievement Indicators and the Assessment Strategies in the Grade 4 Mathematics Curriculum Guide for SCOs N1, N2, N3 and PR4. You may also use the ***Unit Rubrics*** on P.51 and 53 in the Teachers Guide in conjunction with the ***Ongoing Observations*** (p.52) and ***Assessment for Learning*** sections (for each lesson) as tools to help you gather relevant assessment information. ➢ Consider these three guiding questions: ***"What conclusions can be made from assessment information? How effective have my instructional approaches been? What are the next steps in instruction?"***

Unit 3: Multiplication and Division Facts

Many of the lessons in this unit should be treated as an extension to the work you are doing in the *Grade 4 Mental Math Teacher's Guide* and/or *Thinking Strategies: Multiplication*	
Preparing to Teach This Unit	Review Specific Curriculum Outcomes **N4, N5,** and **PR1, PR5 and PR6** in your Curriculum Guide as these are the SCOs the unit is attempting to address. Think about the two guiding questions***, "What do I want my students to learn? What do I want my students to understand and be able to do?"*** The ***Achievement Indicators*** and the ***Assessment Strategies*** for these outcomes will help you answer these questions and give you a better understanding of what it is you need to emphasize in you teaching. You should also refer to the ***Unit Rubrics*** on p.45 and p.47 in the Teacher's Guide, the ***Ongoing Observations*** sheet on p.46 and the ***Assessment for Learning*** sections which are components of each individual lesson. In addition to Math Makes Sense, the resource ***Mental Math: Fact Learning, Mental Computation, Estimation*** should play an important role in your daily planning and teaching. The grade 4 teacher's guide provides a variety of thinking strategies to help students master the multiplication facts. ***Thinking Strategies: Multiplication*** by Celia Baron is another resource that teachers have at their disposal to assist them in helping their students achieve mastery of the multiplication facts by the end of grade 4. Multiplication and division facts are relatively new ideas for grade 4 students, and will be a major focus this year. However, that does not mean that practice and reinforcement of addition and subtraction skills is no longer important. Mental Math time in your class should include daily, brief practice and reinforcement of basic addition and subtraction facts. As well, practice in applying mental computation strategies such as *Front-End Addition, Compensation, Compatibles,* and *Make 10, 100*, should be ongoing so that both components support one another.
Launch*: Here Comes the Band*	Consider rephrasing the last two questions on p.81 of the student book to read*, "Can you count the people in the band in a different way?" "How many different ways can you count the people in the band?"* In the ***Assessment for Learning*** section, there are two excellent suggestions for helping students who do not have well developed "skip counting" skills.
Lesson 1: *Using Doubles to Multiply* **SCO N5**	An array model is used throughout this unit to illustrate multiplication facts. The first factor represents the number of rows and the second factor, the number of counters in a row. It is not important that students remember this format. It is more important that they realize the order of the factors does not change the product. For example, by turning an array such as 4 x 2 around, you get 2 x 4, but the product is still 8. Help students understand this *"turnaround"* or ***commutative property*** for multiplication from the very start. It will reduce the 100 multiplication facts to about 50! You can use round counters or square counters to build arrays, though using square counters leads to a later

understanding of *area* as being *square units.*

In this lesson, one strategy for helping students think about the four-times table is to think “double two-times”. Using the array model, students will be able to clearly see that 4 x 6 and 6 x 4 is double the size of 2 x 6, 6 x 2. Double 6 is 12 and double 12 is 24, so 4 x 6 and 6 x 4 is 24! Students will need to practice doubling numbers mentally as a warm up to this lesson. For example, they could use their dry erase boards to double 4, double 6, double 7 etc. Doubling 2-digit products in the 2-times table (12, 14, 16, 18) is slightly more problematic, but something students in grade 4 are capable of doing. Over time, students should be asked to double many 2-digit numbers mentally>

Easier	Harder
1-15	16 - 19
20 - 25	26 - 29
30 - 35	36 - 39

Connect
it may be wise to leave *doubling to multiply by 8* until a later time and focus more on the relationship between the 2-times and the 4-times tables. There are other strategies that may help students with the eight-times table. The last bullet in this section (top p.84 in student text) is a good activity for students to explore. Have them make an array (stick with facts in the 2-times table) and then double one of the factors to make an array double the size. ***Doubling one of the factors doubles the product*** is an important concept for students to internalize.

Q3, 4, 5: Do these together. Some students may “just know” the answer to a multiplication fact. They should not be asked to explain the “strategy” they used. However, they might be asked, *“How does 2 x 4, 4 x 2 help you with 4 x 4?”*

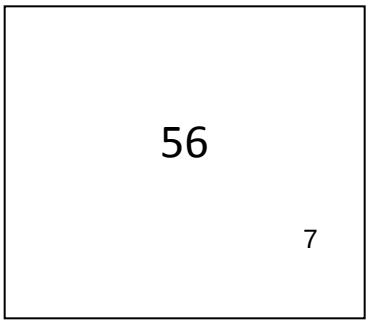
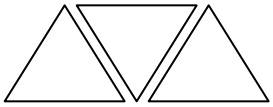
Q6: ***(Numbers, Pictures, Words)***
This should be a good formative assessment checkpoint to determine whether or not students have any idea of the relationship between doubling a factor and the product.

Q8: This problem needs to be worked through as a group. Make a table on the overhead or chalk board and show how you would start with “guess and test”. Patterns will begin to emerge.

	28 Wheels Number of Wagons | Number of Bicycles | Total Wheels 3 | 2 | 16 4 | 3 | 22 **Q10**: Review what the square, triangle, and circle represent. ***Reflect*** Suggest that students draw arrays in their explanation. In the ***Assessment for Learning*** section under *Adjust Instruction*, there is an excellent strategy for helping students develop an understanding of doubling. This would be effective for most students in your class.
Lesson 2: *Multiplying by 1, by 0, and by 10* **SCO N4**	***Explore*** Provide students with counters and "plates" (small squares of paper, etc.) and have them model each situation described. In the first one, Mark has *5 plates with 1 waffle on each plate* (write 5 x ("groups of") 1. It can also be described as *I waffle, 5 times* (write 1 x 5) The second example involves the fact 3 x 0, 0 x 3 and the last is multiplying by 10. Discuss the questions under ***Show and Share*** together. **Q6:** Do this problem together. Help students draw a picture of the waffle as different fruits are added and show them how each stage can be recorded as a multiplication. **Q8:** Introduce this to the whole group. Students can draw pictures of dimes and nickels, use repeated addition (10 + 10 + 10 + 10 + 10 + 10 etc.) or multiplication 6 x 10, 10 x 6, 9 x 5, 5 x 9. **Q10**: This would be a good question for a math journal. Encourage numbers, pictures and words in their answer.
Lesson 3: *Using Skip Counting to Multiply* **SCO N5**	Besides ***factors*** and ***products*****,** students are now introduced to the term ***multiples***. Help them to understand that multiples of a number are the products you get when that number is multiplied by another number. For example, the multiples of 3 are all the products in the 3-times table *(the products are also multiples of the other number!)* Use overhead transparencies of the hundred chart, multiplication table, and blank number lines that you can change the scale on (PM 32) to work through ***Explore*** and ***Connect*** with the students. Try using "jumps of 6" rather than "steps of 6". Students should see that *skip counting* and *jumping* are really the same thing. *Counting on from a known fact* is a challenging (but important) idea for students to think about. As you're working through the

	example in the student book, talk about *9 jumps of 5* and landing on 45. Nine jumps of 6 is the same number of jumps, but the jumps are a bit bigger. ➢ *How much bigger is each jump in 9 x 6 as compared to 9 x 5? (1).* ➢ *So if 9 x 5 is 45, then 9 x 6 is 45 + 9 which is like 45 plus 10 (55) and subtract 1.* ***(compensation strategy***) *9 x 6 = 54* **Q2 and 3:** These two questions are trying to help students see that if they can skip count by 3 and 4, they can figure out all the multiplication facts for those two tables. Most students will not be able to identify all the patterns as suggested in the ***Sample Solutions***. You can point out any that students do not see. **Q4:** This involves using a helping fact. Students have had limited exposure to this strategy, so you should work through a, b, and c as a group. Refer back to the number line model. ➢ 6 x 5 is six jumps of 5 (30); 7 x 5 is another jump of 5 so that's 30 + 5 = 35. ➢ 3 x 6 is three jumps of six (18), so 4 jumps of 6 is six more; 18 + 6 is the same as 20 + 4 ***(Make 10 strategy***) **Q5 and 6:** There are many ways that students may show their work. Accept words, pictures and equations such as 8 x 2 = 16 or 2 x 8 = 16. **Q8.** You can also use a standard 1-6 die for this game. Have students select target numbers between 12 and 50. **Q9:** ***(Numbers, Pictures, Words)*** Using a multiplication chart is not really part of the outcome. so this question has limited value as a formative assessment checkpoint.
Lesson 4: *Other Strategies for Multiplying* **SCO N5**	***Explore and Connect*** All of the multiplication strategies in these two sections are in the Fact Learning section of ***Mental Math: Fact Learning, Mental Computation, Estimation*** The *halve-double strategy* (p.96 in student book) for any fact that has at least I even factor is not introduced until Grade 6 in the Mental Math Guide, but it may be appropriate for many Grade 4 students as well. The difficulty with this lesson is that it introduces the 2-times, 5-times, and 10-times tables as ***helping facts*** all at the same time and then the ***halve-double*** strategy for facts with an even factor. Students could become easily overwhelmed and confused with so many different (though related) strategies to think about. In the *Mental Math* program, and in *Thinking Strategies: Multiplication*, strategies are introduced in isolation from one another and practiced and reinforced daily. If you are following the suggestions in these resources, you can leave out much of what's in lesson 4 although there are some good *application* problems (Q5 – Q11)

	that students could solve once they have dealt with the facts that are involved.
Lesson 5: *Using Patterns in a Multiplication Chart* **SCO N5**	***Explore and Connect*** Students should learn how a multiplication chart works and how it reinforces the commutative (order) property of multiplication. Students can also use it as a tool to keep track of the facts they have been introduced to or have perhaps mastered. A similar use for a multiplication grid can be found in Thinking Strategies: Multiplication p.3 along with other ideas to help students monitor their progress. Be sure to spend some time looking for all the different patterns that exist in a multiplication chart. This reinforces much of the pattern work from Unit 1. The pattern in the 9-times table is introduced here, but ***Mental Math: Fact Learning, Mental Computation, Estimation*** and ***Thinking Strategies: Multiplication*** do a much better job of teaching students how to use the patterns for fact learning. **Q1-6**: Do these together one at a time with students. They help reinforce the strategies that have been introduced. **Q7:** Independent work **Q9**: ***(Numbers, Pictures, Words)*** Good formative assessment checkpoint, but you should read the problem over with students to make sure they understand that they're trying to find out how far Yana walks in 5 weeks and that there are 7 days in a week. ***Reflect*** This is an assessment of strategy selection and is a reasonable one to ask students if you have been working with the strategies as introduced and developed in the *Grade 4 Mental Math Teachers Guide.*
Game: Array, Array!	This is an effective game to reinforce and develop mastery of the multiplication facts by using the array model. Rather than waste time having students draw and cut out arrays, make a master set of those listed on TG p.23 and then run off several sets for students to work with at centers. On the product side of each array, use a pencil to lightly write down one of the factors. This can be erased later as students become more proficient with their facts.

	
Lesson 6: Strategies Toolkit **SCO N3**	The problem solving strategy in this lesson is Use a Pattern. Begin by reading over the problem with students. Use the overhead and some equilateral triangles (pattern blocks or cut out your own) Start with 1 triangle. One person can sit on each side, so that would be 3 people. When you put 2 tables together, you don't make room for 6 people because one side of one table is now touching one side of the other table. Two tables will allow you to sit 4 people. Begin a chart for students to copy and extend.  Number of Tables / Spaces for People to Sit 1 — 3 2 — 4 3 — 5 4 5 Students should work with squares for ***Connect*** and hexagons for the first question in ***Practice*** and create chart/tables for both. # 2 in Practice can be solved by making a chart with **Blue** as the heading for the first column and **Red** for the second.
Lesson 7: *Using Arrays to Divide* **SCO N5**	This best use of this lesson is to help students see how division and multiplication are related operations. Using the array model, students have an understanding of what 3 x 4 looks like. Now they need to "work backwards" and see the 12 as divided into rows of 4 and know that there are 3 rows (12 ÷ 4 = 3) and that 12 divided into 3 rows will mean 4 counters in each row (12 ÷ 3 = 4). In ***Explore***, students are asked to make different arrays using 24 counters. Once everybody has made an array, record the different ones that students have come up with as multiplication facts. (1 x 24, 2 x 12, 3 x 8, and their turnarounds). If there are any that are not on the list, challenge students to search for another array. Describe each multiplication fact in terms of the story so that a division sentence can be written, For example, for 2 x 12, say, *"The 24 children were divided up into 2 equal rows. How many children were in each row?"* (12) 24 ÷ 2 = 12. For 12 x 2 say,

	"There were 24 children. !2 children were put in every row. How many rows would there be?" 24 ÷ 12 = 2 **Q2**. Just ask students to model these arrays with counters. Connect 5 ÷ 1 to *"How many ones are there in 5?"* and 5 ÷ 5 as *"How many fives are there in 5?"* **Q4**: All division facts can be solved using a "think multiplication" strategy. Model all of these for students and have them provide the answers. For 30 ÷ 6 =, ask *"Six times what is 30?" etc.* **Q5**: Do together and ***think aloud*** for students. For a) ask*, "Five times what is 35?"* For b) ask*, "Seven times what is 35?"* **Q6: *(Numbers, Pictures, Words)*** Read this over with students and clarify how many musicians are in the band (27) **Q7:** Provide a few examples of what *"a number divided by itself"* looks like. 7 ÷ 7 =, 6 ÷ 6 =, etc.
Lesson 8: *Relating Multiplication and Division* **SCO N5**	This lesson continues to develop to relationship between multiplication and division. Using "think multiplication" for the division facts should be standard practice. Because this understanding takes to develop in students, teachers should lead the group through each exercise one at a time. **Q9*: (Numbers, Pictures, Words)*** Good formative assessment checkpoint. Be sure to read the question over with students. Most of them should work with counters of some kind to model the situation described.
Lesson 9: *Dividing by Numbers From 1 to 9* **SCO N5**	***Before*** By this time, most students should be able to write the 2-times table without using a multiplication chart. Reinforce the term ***multiples*** and discuss how they are similar to ***products***. ➢ 3 x 2 = 6 *"Six is the product of 3 x 2 and 2 x 3. It is also a multiple of 3 and a multiple of 2*" Model all of the **Before** section using the multiplication chart transparency. As in lesson 8, this lesson continues to develop to relationship between multiplication and division. Using "think multiplication" for the division facts should be standard practice. Because this understanding takes to develop in students, teachers should lead the group through each exercise one at a time.

Lesson 10: *Pose and Solve Problems* **SCO N5**	***Explore*** When introducing the chart on p.113 in the student book, be sure that they understand that the numbers in the column are the regular number of players on a particular sports team. Some students may think the numbers represent how many children signed up to play. Work through the baseball and basketball teams with the kids. ➢ *There are 30 children.* ➢ *How many baseball teams can be made with that many? 30 ÷ 9 = 3 teams, but there will be 3 kids left out, so that won't work.* ➢ *For the basketball team, it's 30 ÷ 5 = 6 teams with nobody left out, so that's one sport. Etc.* ***Connect:*** Work through this problem with students. The question restricts the possibilities to 7 games. Consider ignoring this for the time being and show how a chart can be used to show all the combinations. For example. 22 Players Doubles Games | Players | Singles Games | Players | Total 1 | 4 | 9 | 18 | 22 2 | 8 | 7 | 14 | 22 3 | 12 | 5 | | 4 | | | | **Q3:** Do together **Q4**: Omit **Q6:** Omit
Assessment	➢ **The Unit Quiz, *Show What You Know* p.116-117 in the student text should not be the only assessment tool you use to determine the degree to which students have achieved the curriculum outcomes.** ➢ Review the Achievement Indicators and the Assessment Strategies in the Grade 4 Mathematics Curriculum Guide for SCOs N4, N5, PR1, PR5 and PR6. You may also use the ***Unit Rubric*** on P.45 in the Teachers Guide in conjunction with the ***Ongoing Observations*** (p.52) and ***Assessment for Learning*** sections (for each lesson) as tools to help you gather relevant assessment information. ➢ Consider these three guiding questions: ***"What conclusions can be made from assessment information? How effective have my instructional approaches been? What are the next steps in instruction?"***

Unit Problem: *Here Comes the Band*	Most Students should be able to work on the questions in the unit problem independently once they have been read and discussed as a group.
Unit 1-3 Cumulative Review	The following questions are recommended as review/reinforcement/ re-teaching questions: Q1, 3, 5, 6, 9, 12 Q7, 8, 10, 11 should be done as a group.

Unit 4: Measurement

Preparing to Teach This Unit	This unit is divided into two clusters, each addressing different aspects of Measurement. The first six lessons explore time as it is represented on a calendar and on analog, digital, and 24-hour clocks. Review Specific Curriculum Outcomes **SS1** and **SS2** in your curriculum guide for a more complete elaboration of the depth and breadth expected. For lessons 7-13 review **SCO SS3** which addresses concepts related to *area*. The ***Achievement Indicators*** and the ***Assessment Strategies*** for these outcomes will give you a better understanding of what it is you need to emphasize in your teaching when working through the unit. The ***Unit Rubric*** on p.55 in the Teacher's Guide includes both Time and Area on the same form, but you may wish to assess them separately using categories that work best for you. The ***Assessment for Learning*** sections which are found on the last page of each lesson in the Teachers Guide are very helpful in providing examples of what to look for when assessing learning and what to do when you don't see it.
Launch*: Design a Playground*	**This launch is better left until just before Lesson 7 since that is where the work on area begins in this unit**
Lesson 1: *Calendar Time* **SCO SS2**	This lesson will introduce students to the international standard (metric standard) for representing or recording calendar time. Many adults will not be familiar with this notation, but it is easy to learn and understand and avoids some of the confusion that is sometimes caused when dates are recorded in alternative ways. For example, is 8/10/51 August 10, 1951 or October 8, 1951? After students have explored some of the many ways that dates are often written and identified instances that are open for interpretation, the need for a *standard format* can be discussed and introduced. ***Explore*** The two calendar pages on p. 124 in the student text show the month of February in a leap year and in a normal year. You may find some of the following background information helpful in your teaching. ❖ A common year has 365 days and a leap year 366. ❖ A leap year happens every four years to synchronize with Earth's orbit around the sun which takes 365¼ days. The "Solar Year" is actually about 11 minutes less than 365¼ days, so to keep synchronized, the leap year is omitted 3 times every 400 years! ❖ Most years which are divisible by 4 are leap years, so 2008 was a leap year, but not 2009. ❖ Students may also be curious about when "leap year babies" celebrate their birthday, or how old they really are if their actual birth date is only once every 4 years. ***Connect*** Metric notation is introduced right away in this section. There are also some examples of alternative ways that dates are often recorded. The main purpose of these examples is to illustrate how they are open to interpretation.

	Q1 through 7: All of these tasks are very similar. Pick and choose a representative sample for students to complete rather than all of them. **Note: In Q2 students probably won't know the 5th month or the 9th month without rhyming off the sequence and keeping the count on their fingers (many adults use a similar strategy for some months).** **Q8**: This question can be omitted. It is the first time that a non-standard representation is used on these two pages and may cause unnecessary confusion. Help students become comfortable with the standard, metric notation. **Q9**: ***Numbers, Pictures, Words*** This problem is one that students should be able to do, for the most part, on their own especially if they are directed to the three calendar pages in question 7. If they don't use the calendar pages, they need to know how many days there are in the 7th and the 8th months and use the information in the problem to figure out the answer of 11 or 12 days depending on their interpretation of the word "away". Be open to both answers as long as students can explain their reasoning. ***Reflect*** This could be a good homework activity to involve parents and introduce metric notation.
Lesson 2: *Exploring Time* **SCO SS1**	**Time and Clock Reading** ▪ Telling time has very little to do with measurement of time conceptually. ▪ The skills of clock reading are related to the skills of reading any meter that uses pointers or hands on a numbered scale. ▪ Clock reading is a difficult skill to teach in the first and second grades, and yet nearly everyone learns to tell time by middle school. ***Explore*** The passage of time can be recorded in many different ways. Students may be familiar with a sand timer such as an hour glass, or the kind often included in board games. They may also be familiar with sundials even a metronome if they take piano lessons. The important thing for students to realize is that there are many ways to measure the passage of time. Clocks (digital, analog, and 24-hour) are tools that have evolved over hundreds of years and each present the same information in a different way.

Analog Clocks

Work with the analog clock will help reinforce the five times table in multiplication if the numerals are identified as minutes after the hour eg. 9 is 45 minutes after; 6 is 30 minutes after etc., so 9 x 5 and 5 x 9 is 45 and 6 x 5 and 5 x 6 is 30. (See Lesson 3)

- On an analog clock, the little hand indicates broad, approximate times - the nearest hour.
- When we look at the hour hand we focus on where it is pointing.

- The long hand indicates time *after* and *until* an hour in minutes.
- With the minute hand, the focus is on the distance it has gone around the clock or the distance yet to go for the hand to get back to the top.

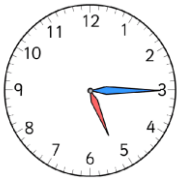
Begin with a one-handed clock (see Teaching Tip, Unit 4 Pro Guide p.9). A clock with only an hour hand can be read with reasonable accuracy. Use lots of *approximate* language

- *It's about seven o'clock.*
- *It's a little past 9 o'clock.*
- *It's halfway between 2 o'clock and 3 o'clock.*

Discuss what happens to the big hand as the little hand goes from one hour to the next. When the big hand is at 12 the hour hand is pointing exactly to a number.

- *If the hour hand is about halfway between numbers, about where would the minute hand be?*
  If the hour hand is a little past or before an hour, where could the little hand be?





As skills develop, suggest that students always look first at the little or hour hand to learn *about* what time it is, and then focus on the minute hand for precision.

Have students show the reading on a digital clock when shown an analog clock and vice versa. Stress *time after the hour* when comparing analog and digital clocks. Time *before* the hour or *till* the hour can come later.

Other Ways to "Read" Time

When we tell time we often use other words that mean the same thing as the minutes after. 45 minutes after is sometimes called "a quarter to…"; 30 minutes after can be called "half past". See if students can explain why some of these descriptions make sense. A chart or table would also be helpful, especially if you include the

<table>
<tr><td></td><td>clock face next to each one, and a discussion of the meaning of fourths.

<table>
<tr><th>Clock Numeral</th><th>Minutes After the Hour</th><th>Other Ways to Say This Time</th></tr>
<tr><td>6
3
9</td><td>30
15
45</td><td>Half past
Quarter after or ___ fifteen</td></tr>
</table>

Digital Clocks
Digital clocks permit students to read times easily, but do not relate times very well. To know that a digital reading of 7:58 is nearly 8 o’clock, the child must know
▸ that there are 60 minutes in an hour
▸ that 58 is close to 60
▸ and that 2 minutes is not a very long time

It is important to impress upon students that analog clocks/watches have been around for much longer than digital clocks and that they are still very much in use. Students need to learn how to read these clocks and relate them to their digital representation.

Q1 and 2: Help students read the times on the analog clocks. If you have been getting students to read time as “quarter after” or “half past”, the digital representation may not be as obvious to them.

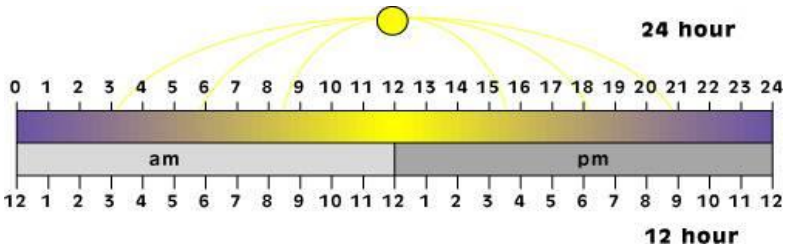
Q3-8: These are standard “learn to tell time” activities. Pick and choose which ones you want to include.

Numbers, Pictures, Words
This assessment question is open-ended in that students can have different starting and ending times for recess, although the duration of the break is 15 minutes. Drawing the analog clock (b) will be a challenge for most students at this point, Consider providing them with a blank analog clock face on which to draw the hands.

In Assessment For Learning (p.11) there are some excellent suggestions for developing the concept of quarters on a clock face.</td></tr>
<tr><td>Lesson 3: Telling Time
SCO SS1</td><td>Q4 - 8: Students should do many of these using play clocks, with the teacher providing the directions.

Note: Q7: Be sure students understand that 60 minutes is one hour.

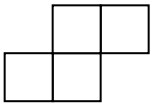
Reflect:
This is a good assessment question that students can use as a writing prompt and the At Home suggestion makes an excellent homework activity that parents could get involved with.</td></tr>
<tr><td>Lesson 4: Elapsed Time
SCO SS1</td><td>Be sure that students have play clocks to work with to help them work out elapsed time. Some students will be ready to work on</td></tr>
</table>

	these problems without having to refer to a clock.
Lesson 5: *Telling Time to the Minute* **SCO SS1**	This lesson is an extension of the work you have been doing to this point. Telling time to the five minutes leads to the more exact "time to the minute". The notation A.M and P.M. are introduced in this lesson. Understanding this notation will help with understanding of the 24-hour clock in Lesson 6.
Lesson 6: *The 24-hour clock* **SCO SS1**	 The 24 hour clock has also been called ***railway time*** and ***military*** or ***railroad time***. Modern 24 hour analog clocks and watches, and many digital clocks and watches, use the 24 hour time system, in which the 24 hours of the day are numbered from 1 to 24. The first 12 hours of the day are numbered from 1 to 12, but 1 in the afternoon is numbered 13, 2 is numbered 14, and so on, until 11 at night, which is numbered 23.The 24 hour time system is also widely available as an option for digital clocks or watches. If they can't manage the 24 hour time system, they'll need some kind of AM/PM indication. To convert between the 24 and 12 hour time systems, use this diagram: 
Launch: *Design a Playground*	It is appropriate to launch the next 7 lessons which explore the concept of area by having students look back to the Launch for this unit on p. 122-123 of their text. The last 2 questions on p. 123 should spark some disagreement and lively discussion about the amount of space that different objects seem to cover. It may be difficult to know for sure from looking at the picture, but the important concept for students to understand is that the term ***area*** refers to the *amount of space* an object takes up or *covers*. This will be a very "hands on" concept for students to explore as they ***construct*** their ideas and understanding of what area is.
Lesson 7: *Covering Shapes* **SCO: SS3**	By the time students get to grade 4, they will have worked with pattern blocks in a variety of ways, including activities similar to the **Explore** and **Connect** on p. 146-147. In the past, students may have covered a shape with different pieces of pattern blocks (hexagons, triangles, trapezoids etc.). Although they were able to cover the shape without leaving any gaps, there was little relationship to the *area* of the shape, because of the different pieces that were used. The words *Congruent* and *Area* are introduced in this lesson and should be added to your *Word Wall.*

	Refer to the *Assessment for Learning* on p. 29 which outlines the learning goals for the lesson. **Q1 & 2:** You will need to have quite a few sets of pattern blocks and lots of green triangles for students to use as the area unit. An alternative idea is to use the same shapes on p.146 and have students find the area in ***green triangles*** (the area of each shape will be twice what it was when the blue rhombus was used). **Q3 – 7:** Pick and choose from these questions. If students understand that area is the amount of space covered by congruent shapes and they can cover various shapes with pattern blocks and "count the area", it's time to move on to the next lesson which introduces the square as the standard unit for measuring area. In fact, lessons 7 and 8 could be combined into a tighter sequence of exploration.
Lesson 8: *Exploring Area* **SCO SS3**	Begin this lesson with the **Connect** on p. 151. Color tiles or square 2-sided counters work well. The concept of a rectangular array which students explored in the last unit, should support this related concept that area is expressed as the number of square units and that you can use multiplication to figure out the total number of squares instead of counting each one – though some students may still rely on some kind of counting strategy to work out the total. **Q1-5:** Students should be able to complete all of these questions though some may have difficulty expressing the total as a multiplication sentence. Help them see each rectangle as the number of rows and columns, but remind them that reversing the factors might change the way we "see" or position the rectangle, but it does not change the total number of squares. *"Four rows of six squares (4 x 6) and six rows of four squares (6 x 4) both have a total of 24 squares."* **Q6**: This would be a good question to do together because students will have different spatial abilities for estimating, and it will be useful to discuss various ways that they reason about the area of each shape. In the last two parts of this question, students might use some type of counting or multiplication to determine each area (including *partial products* and *subtraction*). **Q7:** This question will help develop elements of problem solving, and illustrate how different shape can have the same area. KT students could work with the square counters instead of drawing the rectangles. **Q8: *Numbers, Picture, Words*** Good formative assessment question. The shape doesn't have to be a rectangle, and it will be important for students to understand this. Provide grid paper and square counters or tiles for students to use.
Lesson 9: *Measuring Area in Square Centimetres* **SCO SS3**	***Before*** This is a very good idea for introducing the square centimetre as a common unit for recording the area of relatively small shapes.

	(You will eventually discuss why a square centimetre is <u>not</u> such a good unit for recording larger areas, specifically in Lesson 11). You will need to make an overhead transparency for the cm grid paper and the 4cm x 6cm rectangle, because a cardboard rectangle will appear black on the projector and you need to be able to overlay the grid paper on the rectangle to determine its area. Remind students of the work they did in the last lesson with the square counters or tiles and help them see how the squares on the grid paper are much smaller, in fact each one is 1cm square. So if we find that a shape can be covered by exactly 20 of these small squares, then we say that the shape has an area of 20 ***square centimetres***. We can write out these two words or we can use the ***shortcut*** method and write 20^2 cm. For a physical model of a square centimetre you could use the unit cube from the Base Ten materials and help students see that each face is one $cm^{2.}$ ***Explore*** If you don't want to make grid paper transparencies for every student, use the one you made earlier. Make a transparency of Master 4.29, and have individual students come up to the projector and overlay the grid on each shape to find the area. Again, some discussion about ways to find the total will reinforce the work from earlier lessons. ***Practice*** The concept of *benchmark* or *referent* is introduced superficially on p. 154. Students need some strategy for estimating the area of a shape in sq.cm and the width of a finger is suggested as something they could use. There might be other things that they could identify to use as a referent or benchmark as well. Walk students through the first 2 questions in this section to help them understand what you mean when you us the term *benchmark* and emphasize that this is just a strategy to help with estimation. **Q3, 4, 5:** Students should have no difficulty with these questions. **Q6: *Numbers, Pictures, Words*** This question has limited assessment value. Part C is probably the best of the three tasks. **Q7: Game** Worthwhile game to play, but see if students can tell; you the area of the 10cm x 10cm grid before they begin. **Reflect:** Omit.
Lesson 10: *Estimating and Measuring Area* **SCO SS3**	***Explore*** There's quite a bit of preparation needed to do this the way it is outlined. You may be just as far ahead by going to the ***Connect*** and introducing the idea of half squares and parts of squares when irregular shapes are involved. Do p. 157 as a group to develop spatial reasoning skills.

	Q1: Students should be able to do these three tasks independently. **Q2:** No need to do this one. **Q3, 4, & 5:** Choose **one** of these for students to work on, or let them choose one to do! **Q6:** Having students explain their thinking in this question will be very useful. However, there is limited value in asking them to replicate the shape on grid paper and many students will not be able to draw it accurately. The important thing is for them to explain how they would estimate its area. ***Reflect:*** This would make a good writing prompt.
Lesson 11: *Finding Area in Square Metres* **SCO SS3**	Begin this lesson by reviewing the use of the square centimetre to determine the area of some smaller shapes. Discuss the dimensions of the school playground or some other large area and what the dimensions of those shapes might be in centimetres. Help students realize that for larger areas, we use larger units because they are easier to work with and because it takes less time and effort to write them down. For example, we might measure the area of a large square of paper using square centimetres and find that it has an area of 10 000 cm^2. If we used a larger unit like a square metre, we would be able to say that the same shape has an area of one square metre or 1m^2 . That's a lot easier to write down and it means the same thing. ***Explore/Connect*** This would be an excellent activity to do with your students. Materials are easy to come by and it engages kids in some real hands-on, minds-on learning. **Q1 & 2:** The concept of referent (benchmark) is again raised. This time, students are challenged to come up with something that is about the size of a square metre. Most students will not be able to think about this on their own, so the teachers will need to guide them and help them with spatial reasoning and estimation. **Q3:** Depending on how many square metres cut from newspaper you have, you may want to use smaller number for the area. You will also need quite a bit of space, so you will probably want to move to somewhere in the school with enough floor space to accommodate the various shapes. The teacher should direct this activity to minimize confusion. **Q4, 5 and 6:** This is the same type of work done in earlier lessons, except that a square metre is the unit rather than a square centimetre. Students should have few difficulties with this work. **Q7 & 8:** (Benchmarks/Referents) Do this together one at a time. For example, ask, *("Is the classroom wall a fairly large thing or a small thing? What would be a good unit to measure its area with? What could we use as a benchmar to estimate its area?*"

	Q9: *Numbers, Pictures, Words* This would be a fair assessment task for students, although it's not clear how a student might *show his/her work.* If they make a table, the answer to the question will be obvious from the information presented and there will be no "work" to show! ***Reflect:*** Good question to have students write about in their journals.
Lesson 12: *Strategies Toolkit*	***Explore*** Use the overhead projector and square tiles. Have students come up one at a time and make a shape using 4 squares as described in the text. There will be 4 possible shapes and not 3 as shown in the teachers guide. Here is the missing one.  Some students may make a shape that has already been done but is in a different orientation. Help students to understand that if a shape can be picked up or turned around and it fits over another shape, then the two shapes are the same. ***Connect*** This challenge is reasonable for students to work at on their own or with a partner. You should remind them that the squares they are using are not square metres, but are used to *represent* square metres. We call this a *scale*. Each group should have at least 12 squares (about 40 would be better since there are three possible rectangles that could be made each with an area of 12 square metres) and grid paper to record the rectangles they make. ***Practice*** **Q2, 2, & 3:** Choose one of these for students to complete or have them choose the one they would like to do. ***Reflect:*** This is a very good prompt to get students thinking about how "**Draw a Picture**" can be used effectively to make sense of some kinds of problems.

Lesson 13: *Exploring Rectangles With Equal Areas* **SCO SS3**	All of the activities in this lesson deal with the same concept, shapes with the same area can look different. **There is no need to do all of these questions.** Q2 in particular (p.165) should be modified so that students don't have to waste time drawing the rectangles. The area can be determined from the figures in the book. It might be helpful to make the connection between the dimensions of the rectangles and their area in terms of factors and products which were introduced in Unit 3. For example, if we know that the area of a rectangle is 24 cm^2, (the product) we can list all the pairs of numbers that, when multiplied, equal 24 (factors). 1 x 24 = 24, 2 x 12 = 24, 3 x 8 = 24, 4 x 6 = 24. There aren't any more, so there are 4 possible rectangles that we could make having an area of 24 cm^2. Some understanding of this relationship will help students *construct shortcuts* for the problem. ***Reflect*** This will help students clarify their understanding and should be assigned as a writing activity.
Show What You Know: Unit 4	This can be a good review of what was learned in this unit. For Q 1-9, do one example from each question together as a class and then assign the rest for students to complete on their own. **Q10:** Omit. **Q11-14:** Students should have access to square counters for each of these. **Q15**: Omit. **Q16**: Remind students about factors and multiples and how this might help with these three tasks.
Unit Problem: *Design a Playground*	This could be a homework assignment or one to be completed with a partner. Students could name, illustrate and colour their "playgrounds". Consider giving out more than one sheet of grid paper so that students can tape them together to make a larger grid to work with.
Investigation: *The Icing on the Cake*	Introduce this investigation to the class by modeling a 2 x 2 square one-layer cake with snap cubes (cube-a-links™). Draw a table on chart paper or on the chalkboard and record the number of sides that have icing on them. Repeat for the next largest one-layer cake that's a square (3 x 3). Have students work on their own or with a partner to make the next two largest square cakes (4 x 4 and 5 x 5). Discuss the patterns that are evident from the data in the chart so that they can *predict* for the next size cake and the next and the next etc. Students don't actually build the cakes, but **work with the patterns in the table.** ***Display Your Work*** and ***Take it Further*** can be omitted.

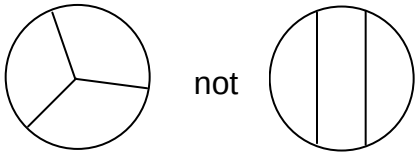
Unit 5: Fractions and Decimals

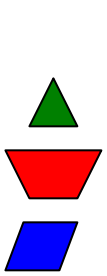
Preparing to Teach This Unit	This unit is divided into three clusters. The first 8 lessons develop understanding of basic fractions less than or equal to one and introduces the terms numerator and denominator. Review Specific Curriculum Outcome **N8** in your curriculum guide for a more complete elaboration of the depth and breadth expected. Lessons 9-11 will use a variety of models including base ten materials and money to show how *tenths* and *hundredths* can be written as *decimal* numbers. **N9 and N10** in your curriculum guide lists achievement indicators that describe what students should be able to do if they have achieved these outcomes. For lessons12-14 review **SCO N11** which addresses addition and subtraction of decimals (hundredths). Mental computation and estimation strategies for these operations will be developed. The ***Unit Rubric*** on p.51 and ***Ongoing Observations: Fractions and Decimals*** on p. 52 in the Teacher's Guide assesses understanding of ***concepts, procedures, problem solving skills*** and ***communication*** as they relate to fractions and decimals. These tools will help you gather the information you need to enhance student achievement of **SCOs N8, N9, N10 and N11.** In addition, The ***Assessment for Learning*** sections which are found on the last page of each lesson in the Teachers Guide are very helpful in providing examples of what to look for when assessing learning and what to do when you don't see it.
Launch*: Spring Activities Day*	For the first question, "What does $\frac{4}{12}$ mean?" the suggested response given in the guide will not be a typical one from most students. It is enough if they can say that Penny managed to put *4 of the 12 eggs* into the carton. You can write this as a fraction on the chalkboard. Ask them to look at the other egg cartons on the table and describe how many are in each carton using this kind of fraction language. Write each fraction on the board as you go, e.g., *"Five out of 12 eggs."* $\frac{5}{12}$. Ask what it would be if there was just one egg in the carton, and what it would be if there were twelve eggs. Write these as fractions as well. You can then compare the information presented in the table and emphasize that, because the *denominators* are the same, it's easy to say who came first, second and third just by looking at the *numerator* which shows how many of the 12 eggs each person was able to put into the carton.
Lesson 1: *Fractions of a Whole* **SCO N8**	***Before*** Rather than using the overhead, give each students 12 colour tiles or squares and ask them to make a rectangle of any size using two different colours. Call on a student to describe his or her rectangle in terms of the fraction represented by each colour. For example, ask*, "How many tiles did you use to build your rectangle? (9) How many of them are red? (4) So four of the nine squares are red (*$\frac{4}{9}$*). How many of them are blue? How would you write that as a fraction?*

	Do not spend too much time on the term "proper fraction". This is developed later. At this point, students in grade 4 will be learning about fractions which are less than 1. Emphasize what the ***numerator*** and ***denominator*** in a fraction *means* throughout this unit. **Q1-8**: Help develop the concept by doing each of these questions together as a group. In **Q4(c)** some students might "see" the shape as a cube and argue that the picture represents sixths. This would be correct for those students. Others will see the shape as a hexagon made with 3 rhombi as in a set of pattern blocks. These students will identify thirds. In **Q6**, after the fractions have been written for each shape, point to the denominator and ask, *"What does this number tell us? Do you remember what we call this part of a fraction?"* Repeat for the numerator. **Q9:** Assign this as a checkpoint for what you have just taught. Use Master 5.24. **Q10:** Optional **Q11:** Ask students to write fractions that describe each colour in their designs as a fraction of the total. **Q12: *Numbers, Pictures, Words*** This is an interesting assessment question. It will provide information about who understands the concept of half in this context and who can apply spatial reasoning when dealing with diagonals. ***Reflect:*** Ask students to think about what *five eighths* means and have them draw and write about it.
Lesson 2: *Fraction Benchmarks* **SCO N8**	***Explore*** Make a transparency of Master 5.25. Show one of the strips on the projector and ask, *"How many equal parts are there? (10) So this strip is divided into tenths. Is the number of equal parts that this is divided into called the **numerator** or the **denominator**?"* Mark **zero** and **one** at either ends of the strip and count while pointing, "*one tenth, two tenths, three tenths, four tenths, five tenths.* Notice that five tenths is half way. Write the fraction for **one half** on the strip. Write **five tenths** below it and discuss why both fractions mean the same amount. Look at each fraction in the box and ask students where each one would be on the paper strip. Ask if they are closer to 0, one-half, or to one. ***Connect*** Prepare three number lines (5ths, 10ths, 20ths) for the overhead. For each number line, ask students to identify what each is divided into (fifths, tenths, twentieths) and ask if various fractions are closer to 0, one-half, or 1 on each line (two fifths?, four fifths?, 0ne fifths? etc.). **Q1 & 2**. Save time by doing this one with students. Prepare a

	number line on the chalkboard for Q2 so students can come up and place the fractions. **Q3: *Numbers, Pictures, Words*** Students will probably not be ready for this kind of question because they have had so little experience in thinking about the relative sizes of fractions. The teacher can direct the learning by drawing various number lines each representing different fractions (thirds, eighths, tenths etc,). **Q4:** This is similar to the previous one and should be done together. **Q5**: Students can do this one on their own. Make sure they understand that they can divide the number line into whatever fraction they wish as long as the parts are equal. ***Reflect*** This will be a good assessment checkpoint. Make sure they have strips of paper to fold.
Lesson 3: Exploring *Fractions of a Set* **SCO N8**	**Explore** Use 20 two-sided counters if you have them. For each time, have students draw and colour the counters and record the fraction for each colour. This is a good opportunity for the teacher to walk around the room and see who understands the concepts of numerator/denominator. **Q1:** Do this together. The teacher should record each fraction one at a time, point to the denominator and ask, "What does this tell us?" Repeat for the numerator. Can students remember the names of these parts of a fraction? **Q2, 3, 4:** Students should complete these three questions in about 15 minutes. Check answers together. **Q5:** Orally. Teacher records the fraction. **Q6 & 7**: Together. Teacher draws circles or some basic shapes to represent the rabbits and the pairs of pants. Ask questions as before about the numerator and the denominator. **Q8:** Students do independently. **Q9*: Numbers, Pictures, Words* and *Reflect*** Both of these are valuable assessment questions.
Lesson 4: *Finding a Fraction of a Set* **SCO N8**	**Explore** This activity will too difficult for most students at this point. Come back to it once they have had more experience with finding the fraction of a set. For a statement such as $\frac{3}{8}$ of 16, it is important that students can tell you that: • There are 16 objects or counters in the whole set. • The denominator tell you the 16 are divided into 8 equal groups. • The numerator is the number that identifies how many of

	the equal groups are being counted. • 3 of the equal groups has a total of 6 counters. **Connect** Use counters on the overhead to show how sets can be sorted or divided into equal groups. The two examples in the book can also be sorted into halves as some students may point out. **Practice** **Q1, 2, 3:** Give each student 20 counters of some kind. Work through each question one at a time with the students. For example, for $\frac{1}{4}$ of 8, ask, *"How many counters do we need to sort? How many equal groups will there be? Go ahead and sort the 8 counters into 4 equal groups (fourths). How many counters are there in one fourth? Two fourths? Three fourths?"* Repeat for questions 2 and 3. **Q4 & 5**: Do the first one in each question together and let students do the others. **Q6**: Omit. **Q7:** Omit. Structure of this question may be misleading to many students. **Q8, 9, and 10**: These are good questions for modeling a **think-aloud** strategy. For example, in Q8: *"We know that one fourth of a set is 5 and we need to figure out how large the whole set is. If it's divided into fourths, how many fourths would there be in all? (4). So that would be 5 and 5 and 5 and 5…20. The whole set has 20."* ***Q*11**: Good question for students to grapple with. ***Reflect:*** Optional
Lesson 5: *Relating Fractional Parts of Different Wholes and Sets* **SCO N8**	***Explore*** Omit this section altogether. ***Connect*** Begin with the second part of this section (p187 in the text) since it follows directly from the previous lesson. Students will each need 28 counters to make the two different sets so they can compare them for size. Follow up this activity with the first idea under Connect (p.186). Cut some 8" x 11" paper into strips so that each student has 3 strips of 8" paper and 3 strips of 4" paper. Ask them to fold one long strip in half. *"How many equal parts has the strip of paper been folded into? (2) Shade in one half and write the fraction. Do the same thing with one of the short pieces of paper. Both shaded parts on your strips are one half. What do you notice about the size of each fraction? Can you explain why?"* Repeat for other fractions. ***Practice*** **Q1 & 2:** It is difficult for students to fold strips of paper into 3rds

	and 6ths. Try 4ths and 8ths. Q3: Students can do this independently Q4: ***Numbers, Pictures, Words*** This is a good assessment checkpoint. Discuss how a circle (pie) can be divided into 3rds  not ***Reflect:*** Optional
Lesson 6: *Strategies Toolkit* **SCO N8**	***Explore*** Introduce this lesson by talking through the first problem to make sure students understand the question. Then allow them to work on it independently. Make counters available to all students or encourage them to draw pictures if it will help them think through the situation described. Allow about 5 minutes for this activity and then assign the problem under ***Connect***. Note: The textbook tries to help students through the problem solving process by posing questions to help them ***Understand, Plan, Solve***, and ***Look Back***. (p.188-189) By allowing students to do the ***Explore*** in the way suggested above, this is probably not necessary. See if they can apply what they did in the first problem to this new, but similar situation. ***Practice*** Allow students to **choose one** of these problems to do. ***Reflect*** Omit
Lesson 7: *Comparing and Ordering Fractions* **SCO N8**	***Explore*** Before asking students to compare the fractions, have them work with pattern blocks to establish the fractional relationship of each piece to the yellow hexagon. Prepare a chart such as onthe following page: Why can't the square and the tan rhombus be used? (these shapes don't cover the yellow hexagon completely).



is _____ of the yellow hexagon.

is_____ of the yellow hexagon.

is _____ of the yellow hexagon

The first pair of fractions includes $\frac{1}{1}$. This is the first time students have seen this type. Spend some time discussing what it means. Start with a fraction such as $\frac{3}{3}$ which is easier to conceptualize.

Connect

The term ***unit fraction*** is introduced here, but it is not important for students to remember this vocabulary. There are 3 fractions which students are asked to order from least to greatest. Give them each three strips of paper the same length. And have them fold one into 4ths, one into 8ths, and one into 3rds. Have them shade one part of each and compare for size.

Practice

Q1: Review the symbols < and >. Some teachers use the "fish" analogy; *"The bigger fish eats the smaller fish."*
Do **a), b),** and **c)** together. Discuss what each shaded region is as a fraction.

Q2: Do **a)** and **b)** together. Model how you would draw two figures the same size, divide then up into equal parts, and then compare for size. Assign the rest of this question for students to do.

Q3: Students should be able to do this independently.

Q4: ***Numbers, Pictures, Words***
This is similar to what students did with paper folding in the ***Connect***.

Q5. Provide paper strips or encourage students to draw figures that can be partitioned. Do the first one orally and discuss. Assign the remaining three for students to complete. Check as a group.

Q6: Discuss and do together.

Reflect

Omit.

At Home Connection

This could be a good activity to actively engage parents and their children in the search for fractions.

Lesson 8: *Comparing/Ordering Fractions With Same Numerator or Denominator* **SCO N8**	***Explore*** Instead of measuring and cutting 20cm strips of paper, just cut some regular plain paper into strips lengthwise. Review how students made 4ths by folding a strip of paper in half and then half again and how folding again will result in 8ths. 3rds is a bit trickier to get right the first time, but let students struggle with the task and then 6ths will be the result when thirds are folded in half. Compare the fractions presented in the book. For the second part, discuss the reasoning that allows you to compare these two fractions. For example, "4ths are smaller than 3rds, because there are more pieces in fourths, so each piece has to be smaller than they would be if there was only 3 of them (thirds). ***Connect*** (p.195) This is showing 3 different ways to compare fractions, Folding paper strips, number lines, and using benchmarks. A good understanding of the first two is necessary before students will be able to use benchmarks effectively. ***Practice*** Q1 and 2: Students should do these questions on their own. Review <,>. **Q3:** Rather than a 12cm line, tell students that the line they draw can be any length. This will allow you see if they are able to visually partition the line into 4ths, 8ths, and 6ths. **Q5**: You do not need the number lines to answer this question because the numerators are all the same. Students should be able to apply some reasoning to these fractions. **Q6**: ***Numbers, Pictures, Words*** Good assessment item. **Q7**: Involves *fraction of a set* and is the first one students have encountered since lesson 4. However, they should be able to tell you who sharpened more pencils because the numerators are the same. As a challenge, see if they can tell you exactly how many each person sharpened. ***Reflect:*** The intent of this question is good. It might be better to ask, "What do you do when you want to compare the size of two fractions?"

Lesson 9: *Exploring Tenths* **SCO N9, N10**	***Explore*** Each student should have 1 flat and up to 10 rods. When you ask them what a rod represents if the flat is one whole, be sure to get them to explain their reasoning. The directions tell students to record their work on grid paper. This can be time consuming and not a worthwhile use of their time. ***Connect*** The arrange of rods in the picture does not matter ***Practice*** **Q1:** Because this is new notation, do the question together and have the students tell you each fraction. Record this on chart paper and include the decimal equivalent. **Q2:** Students should do this on their own. **Q3 and 4**: Do the first one in each question with the students and have them do the others. Be sure to draw a picture, divide it into the appropriate number of parts, shade the part being counted, record it as a fraction and then as a decimal. **Q5:** Students can do this on their own **Q6*: Numbers, Pictures, Words*** Read this with the students. Be sure to say, "seven tenths" for 0.7. Count the number of wedged in the circle to discover that it is showing 8ths and not 10ths. **Q7:** Start by asking students to look at the number line and tell you how many equal parts are shown (tenths). Point to 0.1 and say, "one tenth" and write the fraction $\frac{1}{10}$ beneath the decimal. Do the same thing for A, then assign the rest for students to complete. **Q8:** Read through this together. We want students reading these decimal numbers by saying "tenths". Decide on true or false together. ***Reflect.*** This is a worthwhile question to discuss as a group. You might want to ask it again later in the unit once students have a better understanding of decimal concepts.
Lesson 10: *Exploring Hundredths* **SCO N9, N10**	Writing tenths as a decimal is now extended to recording hundredths as a decimal. A similar process can be followed using base ten blocks as a model. The flat equals one whole, and the unit cube represents one hundredth. Money is also an important model to use in this context. ***Practice*** **Q1**: Do **a)** together and let students complete the rest. **Q2**: Be sure that students are able to read **c)** and **d)** as *"five*

	hundredths" and *"eighteen hundredths"* before they begin to shade their grids. **Q3:** If students have base ten blocks with which to model each decimal, it may not be necessary to have them shade in grids as a record. This may save some time. In any event, do a) with the group and assign the rest. **Q4 & 5**: Do the first one in each question with the group and assign the remainder for students to complete on their own. Check answers as you go. **Q6:** You will need to call on students to 'Read" each decimal, e.g. 0.03 = *"three hundredths"* **Q7:** Numbers, Pictures, Words This is a good question to do as a group. Dropping the trailing zero in 0.20 (20 hundredths) will be a new idea for students. Discuss why two tenths and twenty hundredths is the same. **Q8 & 9**: Together **Q10**: Orally **Q11:** Go slowly with this question. Do it with the students and model with Base ten blocks or money. For example, 0.77, can be read as *"seventy seven hundredths"* or as *"seven tenths plus 7 hundredths"*. You can use 77 pennies if you have them, or 7 dimes and 7 pennies. The same for the unit cubes and rods in a base ten set. **Q12**: Omit ***Reflect:*** Optional
Lesson 11: *Equivalent Decimals* **SCO N9**	***Practice*** Q11 in the previous lesson will help with the questions in this section. The word equivalent may not mean anything to some students, so be sure they understand that the task is to write tenths as hundredths and hundredths as tenths. **Q1, 2, & 3** Do **a)** together in each question and let students do the rest. **Q5**: Discuss this problem with the group. Do they notice the prices and the different weights on the bags? Can they tell you if the bags have different weights or are they the same? **Q6**: ***Numbers, Pictures, Words*** This is a good assessment checkpoint for students to work on. Be sure to read the two decimal numbers as "forty hundredths" and "four tenths" **Q7**: This is not a very challenging question for most students. ***Reflect:*** Optional

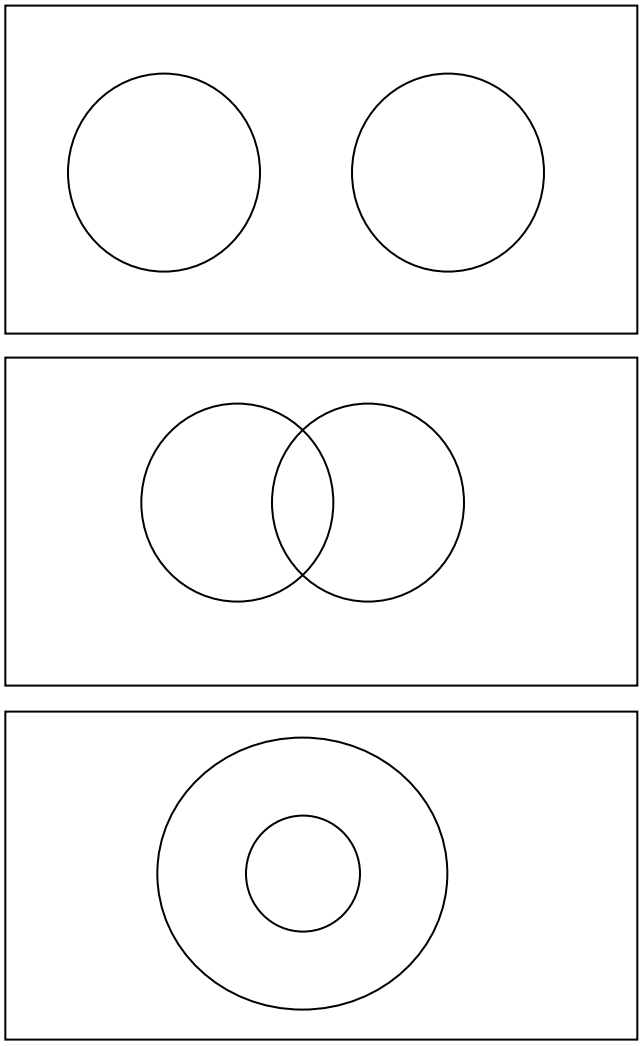
Lesson 12: *Adding Decimals to Tenths* **SCO N11**	***Before*** This is the first lesson in which whole numbers are included in the fractional amounts. Spend some time discussing the meaning of the whole number parts of the decimals. ***Explore*** Discuss the distances represented in the table on p.205 of the student text. 3.9 means three kilometers and part of another km. Ask students how much more (nine tenths) Is that close to another km? Since students will be asked to add decimals, start with easier problems that do not require any regrouping such as 3.5 + 4.2. When you eventually get to those that do require regrouping, use a think-aloud strategy such as: 3.9 + 4.4 = *"three km and four km makes 7 km. Nine tenths plus four tenths is thirteen tenths. (ten tenths makes another whole km, so now I have 8 km) and 3 tenths of a km more. 8.3 km"* ***Connect*** (Second arrow bullet p.206) Introduce with an easier computation (2.4 + 1.3) ***Practice*** **Q1, 2, & 3:** You can make up a list of addition exercises that progress from easier to more difficult (regrouping) and do these as a mental math activity. The use of base ten blocks in Q2 may not be necessary if you have introduced the addition process with this model earlier in the lesson. **Q4:** Do this question together. Check to see if students can correctly identify the distance to add by looking at the map. Record the distances to be added on a chart and turn it into a mental computation activity. **Q5: *Numbers, Pictures, Words*** This is a good activity for students to do because it encourages estimation. **Q6:** Optional. May be too time consuming. **Q7:** Students should be able to do this question on their own. **Q8 & 9:** Involves regrouping. Consider changing the numbers for some students. **Q10:** Read this question over to make sure students understand what it is asking, then assign it for them to complete. ***Reflect*** This is a good assessment checkpoint.
Lesson 13: *Subtracting Decimals to Tenths* **SCO N11**	***Explore and Connect*** Same procedure as previous lesson. Provide a gradual progression of computations that begin with those that do not

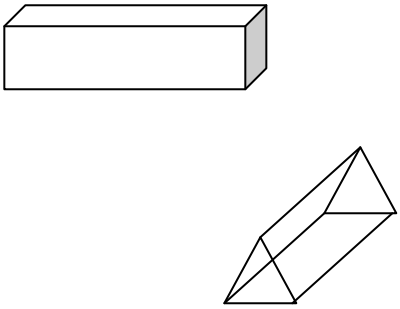
	require regrouping. “Think addition” for subtraction will be an important strategy to support throughout this lesson. ***Practice*** **Q3 – 7** Discuss each story problem one at a time. Ask questions that will help students make sense of the information. • Q3: Together • Q4: Independently • Q5: Together • Q6: Independently • Q7: Independently **Q8:** Optional ***Reflect:*** Optional
Lesson 14: Adding and Subtracting *Decimals to Hundredths* **SCO N11**	***Explore & Connect*** Extension of previous lesson Practice **Q1:** Discuss estimates first. Do **a)** together and assign the rest. **Q2**: Same as for Q1 **Q3:** Remind students that these are subtraction problems. **Q4**: Discuss this question with students and model how the first one might be done mentally using think addition, because the number 10 is involved. For example $5.42. *“8 cents brings me to $5.50* (write down 8), *then $.50 gets me to $6.00* (write down $.50) *and the $4.00 more gets me to $10.00* (write down $10.00). *So that’s 8 cents plus 50 cents ($.58) and 4 dollars is $4.58.* You can also model the standard subtraction algorithm, but with 3 zeros in the minuend, it is the hardest type of subtraction to complete with paper and pencil. **Q5:** Students should be able to do this on their own. **Q6**: Optional **Q7, 8, & 9:** Good problem solving activity. Read it over with your students. **Q10**: ***Numbers, Pictures, Words*** This is a challenging problem that some of your stronger students might enjoy doing. ***Reflect:*** Optional

Show What You Know (Unit 5)	**Q1& 2:** Review ***Numerator*** and ***Denominator.*** What does each part in a fraction tell us? Do a) together and assign the others. **Q3:** Remind students that they should either draw a number line or fold a strip of paper to help them think about fractions. **Q4 & 5:** Do **a)** together and assign the rest in each question. **Q6 & 7:** Review this concept briefly, and see if students are able to draw pictures that address the task. **Q8 & 9**: Independently **Q10 & 11** Review decimal tenths and hundredths (read correctly), then assign for students to complete. **Q12**: Challenge students to do this question mentally **Q13:** Line up numbers same as for adding or subtracting whole numbers or do mentally. Encourage students to do some of both. **Q14:** Independently
Unit Problem: Spring Activities Day	Optional lesson.

Unit 6: Geometry

Preparing to Teach This Unit	This unit is divided into two clusters. The first 4 lessons introduce students to prisms (a class of cylinders) and, in particular, rectangular and triangular prisms. Spatial skills are developed as students use "nets" to identify and construct various prisms. develop understanding of basic fractions less than or equal to one and introduces the terms numerator and denominator. Review Specific Curriculum Outcome **SS4** in your curriculum guide for a more complete elaboration of the depth and breadth expected. Lessons 5-7 introduce, for the first time in the mathematics curriculum, the concept of *symmetry in design*. A particular type of symmetry, line symmetry or fold symmetry, is introduced and explored. **SS5** in your curriculum guide lists achievement indicators that describe what students should be able to do if they have achieved this outcome. The ***Unit Rubric*** on p.35 and ***Ongoing Observations: Geometry*** on p. 36 in the Teacher's Guide assess understanding of ***concepts, procedures, problem solving skills*** and ***communication*** as they relate to these two outcomes. In addition, The ***Assessment for Learning*** sections which are found on the last page of each lesson in the Teachers Guide are very helpful in providing examples of what to look for when assessing learning and what to do when you don't see it. In the Mathematics Background section (see What Are the Big Ideas? P.ii in the Teachers Guide), the second bullet should read, "***Most objects in our world are three-dimensional objects***." Students need to understand that the geometric shapes that they may work with in school can be found in many "real-world" examples. The 6th bullet in this section uses the term "maps" to help explain fold or line symmetry. When discussing this idea with students, help them understand that this means the two halves of a symmetrical design will fit exactly on top of one another when it's folded along a line of symmetry. If there is no way to fold the design so that this happens, then the design does not have line symmetry.
Launch*: Building Castles*	The Assessment for Learning section on p.3 makes a good suggestion for helping students who struggle to describe 3-D objects in terms of their attributes. They may not be able to focus on the different attributes individually or they may not be sure what it is they are being asked to describe. The teacher can support students by asking specific questions that direct their attention to certain attributes one at a time. *"What do you notice about the sides of this shape? How many edges does this prism have? How many faces are on this cube?*

Lesson 1: *Objects in Our World* **SCO SS4**	***Before*** It will be important to discuss the Math Note regarding modified solids (p.4). Many students will question will question the shape of real world objects because they aren't exactly the same as those they learned about in school. For example, a coat hanger is a triangle that has been modified to make it more useful. ***Explore*** Students used Venn diagrams in Unit 2 to sort various numbers. Provide examples of these diagrams and review how they are used to display information. (textbook p.44)  The **Reaching All Learners** section (p.5), highlights a common misconception among students (and many adults as well) who believe that the ***base*** of a prism is the face upon which it sits. Because prisms are names for their bases (*rectangular prism, triangular prism*), it is important to consider how bases are identified. If a prism has two opposite faces that are congruent and parallel to one another, then the shape of this face gives the prism its name.



The first two prisms are ***rectangular prisms*** because opposite faces are congruent and parallel rectangles, so the base of each shape is a rectangle. The shape on the right, of course, is commonly called a ***cube*** made up of 6 congruent squares. In this context, however, it's important to remember that a square is a special kind of rectangle, so technically it is a rectangular prism.

The ***triangular prism*** is so named because the two triangles are opposite each other, congruent, and parallel. While this prism does have three congruent (and possibly opposite) rectangular faces, they are not parallel to one another and therefore cannot be considered the base of the prism.

Practice

Q1-3: These questions use the geometric language of *attributes* of triangular and rectangular prisms. Since this term may be new to many students, it will be important for the teacher to direct their attention to the *shape* and number of *faces*, the number of *edges* and *vertices*, and concepts like *congruent, opposite*, and *parallel*. However, do not expect students to be able to commit all of these attributes to memory or be able to list all of them on paper. At this point, the goal is to help students notice how these prisms are the same and different from one another.

Q4: It will be hard for students to sort these objects just by looking at pictures of them in the book. They should have 3-D objects to hold, examine, and sort according to attributes you want them to look for. Concepts such as *faces, congruent, opposite, edges* and ***vertices*** can all be incorporated into this sorting activity.

Q5: Do one or two examples with students to get them started. For example, two attributes might be "Has square faces" or "Has triangular faces". You could demonstrate the use of a Venn diagram to sort the shapes according to these two attributes. Encourage students to identify two other attributes by which the shapes could be sorted.

Q6: *Numbers, Pictures, Words*

The structure of this task (not the concept) may be difficult to for some students to "get their head around". Consider introducing the activity by reading each statement one-at-a-time and asking if it's ***true*** or ***false***. Since all statements are true, ask if each one is

	true for ***all*** prisms or just for ***some*** and ask for examples. From this discussion, students should be able to complete the statement using the words *all* or *some* at the beginning ***Reflect:*** Good assessment checkpoint as this concept will have been developed throughout the lesson.
Lesson 2: *Constructing Prisms* **SCO SS4**	***Explore*** The first activity in this section is a good one for students to explore. They work with different shapes in the pattern block set to create prisms by stacking congruent shapes on top of one another. Help them identify which polygon is the *base,* and then name each of their prisms. ***Show and Share:*** Omit ***Connect:*** Omit ***Practice*** **Q1 & 2: Omit** **Q3**. Do orally. **Q3: *Numbers, Pictures, Words*** An alternative to using modeling clay would be to display an assortment of 3-D shapes (some geometric and some "real world") and have students select ones that fit the criteria. Create labels for each category: ***Has 6 congruent square faces***; ***Has 2 congruent triangle faces and 3 congruent rectangle faces; Has 3 pairs of congruent rectangle faces.*** **Q4:** This is similar to the previous one and should be done together. **Q5**: Students can do this one on their own. Make sure they understand that they can divide the number line into whatever fraction they wish as long as the parts are equal. ***Reflect*** This will be a good assessment checkpoint. Make sure they have strips of paper to fold.
Lesson 3: *Exploring Nets* **SCO SS4**	***Explore*** This will be difficult and frustrating activity for some students whose little hands may not be able to manipulate the paper nets. Move directly to the **Connect** where you can cut a few boxes (Kleenex box, *Toblerone* candy bar box, or other small boxes) along the edges so that they can be opened up flat, but still in one piece. These are *nets* for these boxes. Students can predict what the prism will be. This could also be an activity to send home with children to do with their parents (caution about using sharp tools). ***Practice*** **Q1:** Rather than copying this for students and having them colour the congruent faces, draw it on the board or chart paper, or cut a box apart to make a net. Remind students that this is a *net* for a

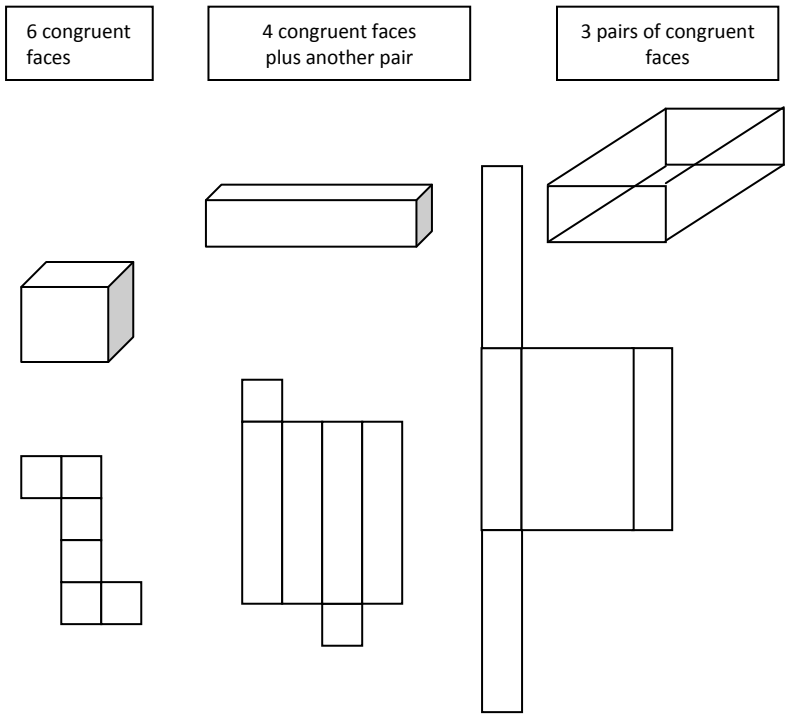
prism which shows all the faces laid out flat. Review term *congruent* and ask students to identify congruent faces in the net.

Q2: As a strictly spatial reasoning activity, this question may be quite challenging for many students. They should have squares that they can physically manipulate. One idea would be to cut out squares (about 5cm x 5cm) and give 18 squares and pieces of tape to each pair of students. (use a cutting board and pieces of construction paper or card stock as an alternative to paper).Students can build the nets as in a), b) and c) by taping the edges of adjacent faces, predicting which ones will form a cube, and then folding up the faces to confirm.

Q3*: Numbers, Pictures, Words*
Do all three parts of this question together. The folding in part b) will require additional support.

Q4: Do orally with the class.

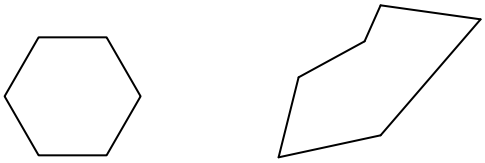
Reflect
Modify this question by providing a net of a rectangular prism so students can see the 6 rectangles. **Note:** In the ***Sample Solutions*** for this item, it states "…with no two congruent rectangles side by side." This is an incorrect idea. Rectangular prisms do not have to have three different pairs of congruent faces. They could have 6 faces that are all congruent, 4 faces that are all congruent and another pair of congruent faces, or 3 pairs of congruent faces.



Lesson 4: *Strategies Toolkit*	**Explore** Allow students to work in pairs with 36 snap cubes to build a rectangular prism. Ask different groups to describe their prisms (eg. 2 cubes wide by 18 cubes long). Record the dimensions for each prism in a chart on the board and ask students to see if there are any other prisms that can be constructed from 36 cubes. Use the chart to initiate a discussion about the patterns that emerge. **Connect** This section of the lesson simply outlines a strategy that many students will work out on their own. You may wish to skip over this section depending on how well your students get along. **Practice** **Q1:** Omit **Q2:** Students should be able to do this question independently. ***Reflect:*** Optional

Lesson 5: *Symmetrical Shapes* **SCO SS5**	***Explore*** Spend time discussing the term *symmetrical*. (If you "Google" symmetrical designs and click on images you will find lots of example to share with students to help them understand this concept)  ***Line symmetry, fold symmetry*** and ***mirror image*** are all phrases that can be used when discussing this concept. Provide copies of line master 6.23 and miras or small mirrors (from the science topic *Light*). Show students how to use this tool to find lines of symmetry in each figure (if any). They can trace the line of symmetry by running their pencil along the edge of the mirror or mira. ***Connect*** This is a review of the concept **of *line*** or ***fold symmetry*** just explored. ***Practice*** **Q1 Do orally** **Q2:** For consistency, run off a sheet of the letters of the alphabet (Arial font, 72 pt.) for each student and provide miras or small mirrors. Students can then check the letters that spell their first name to see which ones have line symmetry. Do some letters have more than one line of symmetry? Early finisher can investigate the symmetry of other letters in the alphabet. **Q3:** Some students may think that there are other lines of symmetry in flags ***a, c***, and ***d*** along the diagonals. Provide small rectangles (similar to the flag shape) for students to fold along the diagonals. They will see that the two halves do not lie on top of each other. **Q4: *Numbers, Pictures, Words*** This is a good assessment checkpoint. Be sure to provide grid paper for students to use. ***Reflect*** This is an interesting question for students to reflect on and write about in their journals.
Lesson 6: *Line Symmetry* **SCO SS5**	***Explore*** This should be an easy task for students to work on. To keep the designs from becoming too large, provide only 8-10 blocks to start with. Tell students to place the mira along the fold line and replicate the design. Alternatively, students could try to replicate the design without the use of a mira.

	Connect This is an important activity and should probably be done earlier on in this unit. It will help students grasp the idea of fold symmetry more easily. ***Practice*** **Q1:** Students can work with pattern blocks and miras to discover lines of symmetry. **Q2:** Make an overhead transparency of LM 6.24 and do this activity together. Students could come up and draw lines of symmetry as appropriate. **Q3:** Omit **Q4:** Good activity for students to work on independently. Watch for those who have a lot of difficulty copying the design in their books onto the dot paper. These students will be challenged by symmetry concepts. **Q5: Optional**. Consider the two games on p.v; ***Fun With Symmetry*** and ***Symmetrical Masterpiece.*** **Q6:** Optional **Q7:** Optional **Q8:** Nice activity that encourages creative exploration. **Q9:** Much of the work students will do on this will be trial and error. Provide paper rectangles (about 5cm x 10cm) for students to fold and experiment with. Remind or explain to them that a ***quadrilateral*** is <u>any</u> 4-sided closed figure, a ***pentagon*** is <u>any</u> 5-sided closed figure and a ***hexagon*** is <u>any</u> 6-sided closed figure. These figures may or may not look like the shapes they have seen in pattern blocks or other geometric models. ***Reflect*** Optional
Lesson 7: *Sorting by Lines of Symmetry* **SCO SS5**	***Explore*** Optional . ***Connect*** Provide paper copies of a rhombus, similar to the green figure in the students' text, for them to fold. The other figures should be more obvious to students. ***Practice*** **Q1:** Make a large copy of figure a) out of chart paper. Ask a student to fold it along the line of symmetry as shown in the book to prove that it is symmetrical. Do parts b) and c) together. **Q2**: Students should have a mira to answer this question. **Q3:** These figures are called ***regular*** polygons because their sides

	and angles are congruent. For example: Both of the figures below are hexagons, but only the one on the left is a ***regular*** hexagon.  **Note:** In the 7-sided polygon (heptagon) the 7 lines of symmetry run from each vertex to the midpoint of the opposite side **Q4**: **Omit or reconstruct** this task. Creating a figure with a specified number of sides and specified number of lines of symmetry is much more difficult than recognizing lines of symmetry in given shapes which is what the curriculum outcome specifies. **Q5: *Numbers, Pictures, Words*** Look at the design in the students' book (p.244) and discuss why it is not symmetrical. How could it be made symmetrical? What colours need to be changed? Give students 1cm sq. grid paper and have them use a ruler to draw a line that will become the line of symmetry for a design they are going to create. Ask them to use markers or colouring pencils to shade in 20-25 squares on one side of the line using different colours of markers or colouring pencil. They then exchange papers and and create the mirror image of their partner's design. ***Reflect*** Optional **At Home Connection** This could be a good activity to actively engage parents and their children in the search for pictures that have line symmetry.
GAME: *What's My Rule?*	An alternative to this game would be to make several copies of master 6.26. Cut out each figure and distribute to students so that each person has at least three different cards. Draw circles on the board or chart paper and label each as: O lines of symmetry 1 line of symmetry 2 lines of symmetry 3 lines of symmetry 4 lines of symmetry
Unit 6: *Show What You Know*	**Q1:** Omit **Q2**: Good review question **Q3 & 4:** Omit **Q5 & 6:** Do **Q7: Optional.** Copying these designs exactly will be a very time consuming and difficult task for some students. The next question

	is a more appropriate assessment task. **Q8:** Do **Q9:** Instead of having students draw each figure and then lines of symmetry, just ask them to say how many lines of symmetry they see in each figure. **Q10:** Omit
Unit Problem: *Building Castles*	This project should be considered optional. However, if you do choose to do it, omit Part 3

Unit 7: Data Analysis

Preparing to Teach This Unit	There are 4 lessons in this unit designed to help students understand how scales are used in data displays, specifically pictographs and bar graphs, where the numbers are too large to be represented as single entries. The ***Unit Rubric*** on p.27 and ***Ongoing Observations: Data Analysis*** on p.28 in the Teacher's Guide assess understanding of ***concepts, procedures, problem solving skills*** and ***communication*** as they relate to outcomes **SP1** and **SP2**. As well, the ***Assessment for Learning*** sections which are found on the last page of each lesson in the Teacher's Guide are very helpful in providing examples of what to look for when assessing learning and what to do when you don't see it.
Launch*: Using Data to Answer Questions*	P.252 and 253 of the student text has good examples of data displayed in various ways including pictograph, bar graph, and tables. The use of tally marks is illustrated in one of these tables. Students should be able to describe and answer questions about the data in each of the displays
Lesson 1: *Reading Pictographs and Bar Graphs*	The concepts explored in this lesson will not be new to students. In the APEF Mathematics Curriculum, concrete and "picture graphs" were first introduced in Kindergarten. This work was extended into grade 1 and in grade 2 bar graphs were introduced. In the restructured PEI Mathematics Curriculum, pictographs and bar graphs are introduced in grades 2 and 3 respectively. The terms ***vertical*** and ***horizonta***l should be reviewed in this context as this will support discussion around the vertical and horizontal ***axes*** later in the unit. ***Practice*** **Q1:** Do orally **Q2:** Before asking questions about this graph, draw students attention to the scale showing the number of days. Discuss why only some numbers are shown and what numbers are represented by the "ticks" between numbers. One of the biggest challenges for students will be deciding on what intervals to include on their graphs when they are working with large numbers. Do a) to f) together. **Q3**: Do Orally **Q4**: ***Numbers, Pictures, Words*** Good assessment question for students to work on independently. Help struggling readers read the players names, the labels on the graph, and each question about the information in the two graphs. **Q5:** a) Allow students time to study the graph and think about the information displayed. Discuss as a group, then read each question in b) to e) and assign. Support struggling readers. ***Reflect:*** Omit.

Lesson 2: Drawing Pictographs

Explore:
Read over this section with students, but instead of having them work two-sided counters, explain that you gathered data from 8 trials and you want them to create a pictograph displaying the results. Explain that a pictograph usually includes "pictures" of the object that the information is about. In this case you used round (or square) counters, so students could draw circles or squares to represent the counters.

Trial	Number of Red Counters
1	8
2	12
3	7
4	15
5	16
6	5
7	9
8	14

Connect:
Go over this with students so that they understand the idea of using a key.

Practice:
Q1: Do together. Students will need support in learning how to select an appropriate key for their data.

Q2: Introduce this task. What information is in the table? What would be a good symbol to use for students? What will each student symbol be equal to, i.e., the scale? Provide 1.5 or 2cm squared graph paper and allow time for students to construct their graphs. Omit parts b) and c).

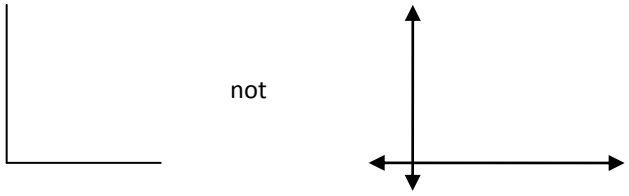
Q3: Assign. Omit the *exchange* part of this task.

Q4: *Numbers, Pictures, Words*
Excellent task for students to work on.

Reflect
Good assessment response for journals.

Lesson 3: *Drawing Bar Graphs*

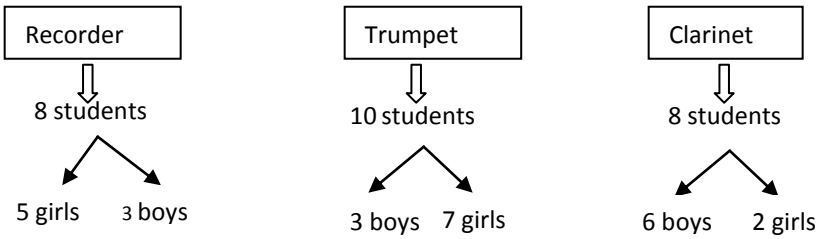
Explore and ***Connect***
Since this is primarily a review of the previous lesson, discuss the information as a group. Decide on a suitable scale for each table and model how you would represent this information as a bar graph. Use a ruler to draw a *horizontal* and *vertical* axis on grid paper (transparency or chart paper modified with vertical lines to create a grid). The axes should only show one quadrant.



Discuss a suitable scale to use for each set of data.

Practice

	Q1: Assign **Q2:** Review how tally marks are used. Draw some tallies on the board or chart and ask students to tell you the number represented each time. Also, write some 2-digit numbers on the board and have students use dry erase boards to make the tally marks that represent each number. Assign part a) to half the class and part b) to the other half. Do part c) together. **Q3:** a) Distribute 1cm grid paper for students to make their bar graphs. Explain to them that time is usually shown along the horizontal axis. b) omit c) Do students recognize that this question can be answered by looking at either the table or the graph? d) Discuss this together. **Q4**: Assign **Q5: *Numbers, Pictures, Words*** Discuss b), c), & d) together, then assign a) ***Reflect*** Optional
Lesson 4: *Comparing Pictographs and Bar Graphs*	***Explore and Connect*** Go over these sections together. Students should have little difficulty with the material. ***Practice*** **Q1-3:** Do these two questions as an oral review. **Q4**: ***Numbers, Pictures, Words*** Good assessment checkpoint. Students might choose a scale other than 1 symbol = 20 days for part d). 1 = 10 would be acceptable and perhaps others as well. **Q5: Optional** **Reflect:** Good question for students to reflect on and write about in their journals.
Lesson 5: *Strategies Toolkit*	For students to develop efficient problem solving strategies they must be presented with tasks within their level of ability and they must hear and see their teachers modeling good problem solving strategies. The adage, *students should be allowed to struggle, but not suffer*, is most applicable when they encounter complex problem situations such as those in this lesson. In order to build confidence and develop problem solving abilities, the teacher will need to lead by example. ***Explore:*** The “problem” presented in this section is too challenging for most students to do on their own. Help them understand how you might approach the problem using the information provided. • There are 26 in the class, 12 boys and 14 girls. • 8 students play the clarinet. 5 are girls, so 3 must be boys.

	• 8 students play the Clarinet. Since there are 26 in the class and 8 play the recorder and 8 play the clarinet; that leaves 10 to play the trumpet. • We know 3 boys play the trumpet, so 7 girls must play the trumpet also. • Now we know that 6 boys play either the recorder or the trumpet. That leaves 6 boys to play the clarinet. 2 girls must also play the clarinet.  ***Connect:*** Show how a table can be used to organize the information in this problem. ***Practice:*** **Q1 & 2:** Discuss with students how you might set up a table to organize the information about the weather in September and October, and the kinds of animals seen on Mr. Chu's field trip. Draw the table on chart paper or on the chalk board. Have students copy it into their notebooks, and then enter the information given in the problem. Help them use this information to answer the question. ***Reflect:*** Optional
Unit 7: *Show What You Know*	**Q1:** Assign Q2: Assign part a) only Q3: Assign parts a), c) & d) only Q4: Very good question to assign
Unit Problem: *Using Data to Answer Questions*	**This problem should be considered optional.**

Unit 8: Multiplying and Dividing Larger Numbers

Preparing to Teach This Unit	The lessons in this unit are divided into two clusters, one for multiplication and the other for division of 2- and 3- digit numbers by single digit numbers. The ***Unit Rubric*** on p.51 and ***Ongoing Observations: Multiplying and Dividing Larger Numbers*** on p.52 in the Teacher's Guide assess understanding of ***concepts, procedures, problem solving skills*** and ***communication*** as they relate to outcomes **N6, N7, PR1, PR5** and **PR6**. As well, the ***Assessment for Learning*** sections which are found on the last page of each lesson in the Teacher's Guide are very helpful in providing examples of what to look for when assessing learning and what to do when you don't see it. The 4 additional activities described on p.v are worth introducing to your students after they have had some experience with the underlying concepts. ***Spinning Products*** develops and reinforces estimation skills and the use of a calculator to check the actual answer. A special student recording sheet has been prepared and can be found at the end of this supplement***. Make the Greater Product*** should be introduced with small groups or the whole class. The teacher could hold up a numeral card from 0-9 and students enter it into their grids with the goal of arranging the factors to yield the greatest possible product. ***The Remainder Game*** reinforces "think addition" as a strategy for division. Since this activity involves 2-digit by 1 digit divisors, it's really the multiplication facts up to 9 x 9, although 99 ÷ 9 is a possibility but not beyond the ability of grade 4 students. There is a special student recording sheet for this activity as well. The directions for ***Race to 0*** are not as clear as they should be. Students pick a number on the 0-99 chart as their starting number and put a marker on it to begin. He or she then rolls a number cube (1-6) and writes the two numbers as a division statement. For example, the student picks 45 as the starting number and then rolls a 4. He or she writes these two numbers as 45 ÷ 4 = and then counts backwards in groups of 4 to zero. The partner helps by keeping track of the number of *jumps of* or *groups of* 4. In this example, there are 11 groups of 4 and 1 number left over (remainder). The student writes 11 R1 as the quotient to 45 ÷ 4. This game will also help to reinforce the relationship between multiplication and division.
Launch*: At the Garden Centre*	This activity reinforces the important concept that there are many ways to think about and work with numbers, and that counting, adding and multiplying are just different ways to get a total number. Multiplication is usually the fastest and easiest.
Lesson 1: *Exploring Multiplication Patterns*	Multiplying a number by 10 or by 100 should not be a difficult concept for students who can count by tens and hundreds. Multiplying a number by a ***multiple*** of 10 or 100 is only slightly more difficult, and can be easily mastered if the teacher uses the language of ***place value*** and connects students' knowledge of ***basic facts*** to computations such as 60 x 4= or 400 x 4= . The Grade 4 Mental Math Guide introduces multiplication by 10 and 100 beginning on p.45 with an emphasis on how place values change when multiplying by these numbers. This will help students understand the patterns they will later see when a number is multiplied by a decimal tenth or hundredth; eg. 45 ÷ 0.01 =.

	Explore & Connect A calculator is probably not necessary for students to understand what happens when a number is multiplied by 10 or 100. The teacher should begin with examples of multiplying by 10 and see if students are able to give the product. For example, write 5 x 10 on the board and say "Five groups of 10. How much is that?" Someone will invariably say, *"All you have to do is add zero."* Since that doesn't work when multiplying by 0.10, it's probably best to reinforce the "counting by tens" idea. Follow the same procedure for multiplying by 100. After students have has some practice in multiplying by one ten, introduce the idea of multiplying by 3 tens or 5 tens or 8 tens. A strategy they can use for these problems is to think "basic multiplication facts" and add the appropriate place value. For example, for 2 x 30, students should think 2 x 3 tens = 6 tens, so that's 60. For 4 x 80, think 4 x 8 tens is 32 tens, so that's 320. A good warm up drill for this type of thinking is to make a series of cards (or write each one on the board) and call on students to convert the number of tens into a 2-or 3-digit number. For example: 3 tens 5 tens 7 tens 8 tens 10 tens 11 tens 13 tens 20 tens 25 tens 30 tens 38 tens 40 tens ***Practice*** **Q1, 2, 3, 5, & 7: Mental Math!** Use the dry erase boards for these questions. Have students keep their books closed. Write each item (and others that you create) on the board or chart paper and ask students to hold up their answers when you ask for them. When you have finished with these practice items, ask students to open their books to p. 280-281 and work on Q4, 8, 9, 10, 11, & 12. Introduce each problem one at a time and allow students time to work out an answer. Some students may choose to use addition or some other strategy that makes the most sense to them. Allow them to do this and include their strategies during discussion ***Math Link:*** Remind students that they learned about area in Unit 4 and that area refers to the number of square units it takes to cover something. In the example in the student book, the total number of square units could be figured out by counting them all by ones (very slow), counting by 6s (difficult), counting by 10s (easy) or multiplying the length by the width 6 x 10 (easiest). ***Reflect:*** This might be a good assessment checkpoint to have students write about in their journals. It will reveal who can explain multiplying by 10 and 100 in terms of patterns and who works from an *"add zeros"* strategy.
Lesson 2: *Estimating Products*	***Explore & Connect:*** Refer to the section on estimation in the Grade 4 Mental Math Guide beginning on p.49. We want students to realize that there are times in our daily lives outside of school when an exact answer is required and

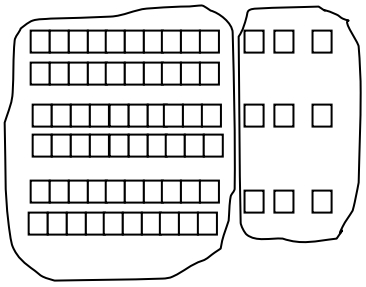
	times when an approximate answer or estimate is suitable. And within estimation itself, there is a wide range of estimates that are acceptable, depending on the situation and the needs of the person doing the estimating. The skill (and habit) of estimating is best developed if it is constantly modeled by the teacher. The question in the students' text about the number of passengers 8 planes will carry should be modeled using a "think-aloud" strategy. If it is merely assigned for students to do on their own, chances are that they will use multiplication to get an exact answer and then change the product to make it look more like an estimate. "Each plane holds about 20 passengers and there are 8 planes so that would be 20 + 20 + 20 + 20 … or 8 x 20. So that would be about 160 passengers. We could also estimate 10 x 20 which would be a bit farther from the actual answer but perhaps good enough depending on your need. For example, if you're trying to estimate whether or not you have enough air sickness bags, it's probably better to estimate a little high. ***Practice:*** ***Mental Math!*** Use the dry erase boards *to* help students develop estimation skills. Individual items in **Q1, 3, & 5** should be written on the board or chart. **Q2, 4, & 6** provide a context for students to think about. Discuss each situation one at a time with students and have them use the white boards to record their estimations **Q7** & **8** are a little different and should be done as a group. ***Reflect*** Optional
	Mental Math should be part of every day. Mental computation drills will reinforce concepts introduced in this unit. Students have been learning how to multiply by 10 and 100 as well as by multiples of these numbers. They have also been developing their estimation skills which draw upon their understanding of the type of multiplication just described. For example to estimate 22 x 8 we want students to think 20 x 8 which is multiplying by a multiple of 10.
Lesson 3: *Using Models to Multiply*	***Explore*** Ask students to think about the number of eggs in 6 trays and to use numbers, pictures and/or words to show how they would work out an answer to this question. You may continue to see a variety of counting strategies and even multiplication by some students. Watch for students who seem to show that they understand something about the distributive property. They may use addition or some other method to think about 24 as 20 and 4 and thinks six twenties plus 24. ***Connect:*** Introduces base ten blocks and grid paper as array models for multiplication. Model how base ten blocks can be used to represent multiplication sentences such as 3 x 12, 4 x 15, 2 x 26. It would be very useful if students had base ten materials at their desk so that they could model number sentences such as these. The answer to each multiplication can be found in various ways. *Refer to the Elaboration for N6 in the Grade 4 Mathematics Curriculum Guide.* It is critical that students understand how two numbers can be multiplied by first finding *partial products* and then adding them together to get the overall product.

Follow a similar procedure for introducing and working with grid paper arrays for multiplication. Model first, then have students model at their desks

Practice
Q1: Do orally

Q2: Introduce a) by saying, “Draw an array on your grid paper that shows 5 groups of 41”. Assign the rest.

Q3 & 4: These two questions are trying to get students to the symbolic stage whereby they will use the distributive property to determine partial products. You will need to model several of these problems as examples to support students in applying this property. Continue to use the base ten blocks or the grid paper to make arrays, but begin to connect what you do with the model to what happens when we work just with numbers. For example:



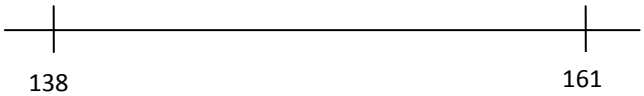
23 x 3 =

“Twenty times three (20 x 3 = 60). And three times three (3 x 3) is 9; so 60 + 9 = 69.”

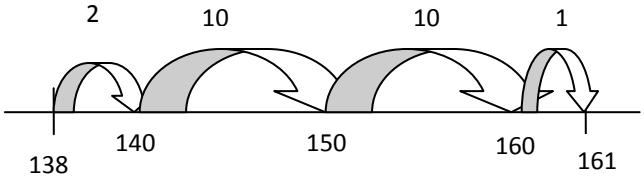
Q5: The mental math strategy that Eva uses is called ***Compensation.*** You should model it with materials first to show **3 x 30 (*three groups of 30***) and then to show how **3 x 29 *(three groups of twenty nine)*** is one less in each group. That’s why Eva said she **subtracted 3,** but it will not be easy for most students to understand this right away. It’s worthwhile to explore this idea as another way to think about the numbers, but do not expect your students to be able to apply it with ease.

Q6: Students should be able to do this on their own.

Q7: Ask students to find the two products. Do they understand what the question is asking? Do they know that *“How much greater?”* is asking them to figure out the difference between the two numbers? You could place both products on an open number line to help them think about the difference.



Starting at 138 and working your way up to 161 would be the easiest way to get the difference between these two numbers using this number line, and there at least a couple of ways to do it.

	A jump of 2 puts me at 140, then a jump of 20 (or two jumps of 10) puts me at 160, and then a jump of 1. So that's 2 + 20 (10 + 10) + 1 = 23. ***You could model this thinking and draw the jumps as arcs with the size of the jump written above each arc.***  ***Another way to do it would be to start at 138, and then take two jumps of 10 (148, 158) and then a jump of 2 and a final jump of 1. So that would be 10 + 10 + 2 + 1 = 23*** **Q8 - 11:** Assign ***Reflect*** Optional ***At Home*** This would be a good exercise for students and parents to engage in. Since many parents will simply write the problem down and perform R to L multiplication, it would be especially effective if students could perform the same computation in another way; eg. 70 x 5 plus 4 x 5 or the other way around. Some may be able to use *compensation* and multiply 5 x 75 (150 +150 +75) = 375 – 5 = 370. There will be a number of parents who will use one of these strategies or some other strategy that works well for them.
Lesson 4: *Strategies Toolkit*	**NOTE:** You may want to defer the Strategies Toolkit lesson until later in the unit since Lesson 5, *Other Strategies for Multiplication,* is really about using the ***distributive property*** which students were using in Lesson 3. ***Explore and Connect*** Go over these two sections together. Think aloud and show how you might use a drawing or a table or a simpler problem to help make sense of each problem situation. Wherever possible, allow students to copy tables or diagrams that you have started and complete them on their own. ***Practice*** **Q1& 2:** Students should be able to do these two questions on their own. Make colour tiles, snap cubes, or square counters available for students to use in Q1. **Reflect:** Omit

Lesson 5: *Other Strategies for Multiplication*	This is a continuation of the work on the distributive property in Unit 3. Much of these exercises should be done as part of Mental Math. Present 2-digit by 1-digit multiplication problems on the board or on the chart and have students write partial products on their dry erase boards and then add the two products together to get the overall product. Story problems **(Q4-7)** should be read, discussed and solved one-at-a- time. ***Reflect:*** It would be interesting to see is students are flexible enough in their thinking to be able to solve a given computation in more than one way.

Lesson 6: *Using Patterns to Multiply*	***Explore & Connect*** This is the compensation strategy that Eva used in question 5 of Lesson 3. In one example, you compensate by subtracting and in the other you add on. Students need to see 6 x 80 modeled (grid paper) so that they can understand how 6 x 79 is one less in each of the six groups. 8 x 40 can be modeled in the same way so that 8 x 42 will add 2 to each of the 8 groups. ***Practice*** **Q1:** Do these together to ensure students understand how to apply the strategy. For each one say something like, *"Two groups of 100 is 200. So what would two groups of 99 be? Will it be more or less? How many less?"* **Q2:** Students should be allowed to use whatever strategy works for them. They might simply prefer to use the distributive property to multiply 43 x 8 instead of using a compensation strategy. **Q3**: Assign **Q4:** Optional ***Q5: Numbers, Pictures, Words*** Omit **Q6:** Assign ***Reflect:*** Omit
Lesson 7: *Multiplying a 3-Digit Number by a 1-Digit Number*	***Explore & Connect*** If you do not have enough Base Ten blocks to allow students to model 136 x 2 at their desks, use grid paper instead. Discuss ways to figure out the total product and connect this type of computation to 2-digit by 1-digit multiplication. ***Practice*** **Q1:** Model and discuss a) and b) and assign c) and d). What would a reasonable estimate for each one be? **Q2 & 3:** Same procedure as for Q1

	Q4 & 5: Assign **Q6:** Students should choose just one equation to create a story problem **Q7: *Numbers, Pictures, Words*** This should be a reasonable task for students. **Q8:** Help students to break this problem up into parts. *How many people can ride on the roller coaster at once? How many rides do they have in one hour? (6) How many people would that be? Would 800 people be able to ride in one hour?* **Q9:** Assign each chart one at a time. For each one, model the language of multiplication to help students *think* about the size of the groups. For example, *"Three groups of 172 is 516. What would three groups of 173 be? Would it be more? How much more Why? Three groups of 174 is 522. What would 3 groups of 175 be?"* Ask students to do the same for the other rows in the chart. **Note: on the second row, they should think, 4 groups of 173 is 692. What would 4 groups of 172 be?** ***Reflect:*** Good assessment checkpoint.

Lesson 8: *Estimating Quotients*	***Explore & Connect*** Help students use their understanding of the multiplication facts to help them with division problems. ***Practice*** **Q1, 2 & 3:** Do orally with students to help them get into the habit of using multiplication to solve division problems. For example, for 27 ÷ 4, ask, *"What can we multiply 4 by to get as close as we can to 27? (7 x 4 = 28. That's a little too much, but it's very close."* **Q4, 6 & 8:** Assign **Q5:** Do together **Q7:** Do together **Q9:** Optional **Q10:** Omit ***Reflect:*** Omit
Lesson 9: *Division With Remainders*	***Explore:*** Make materials available for students to use as models to think with. Ask them to use numbers, pictures and/or words to solve the problem with the orange baskets. ***Connect:*** Provide counters for students to model the problem at their tables***.***

	Practice: **Q1 & 2:** Do a) together in each question and assign the rest. **Q3:** Ask students to use numbers, pictures and words to solve this problem. **Q4:** Do together **Q5:** Assign (Use numbers, pictures and words) **Q6:** Do the first two using "think multiplication". 14 ÷ 7, *"Seven times what is 14? 7 x 2"* 15 ÷ 7, *"Seven times what is 15? 7 x 2 is 14 and then one more, so 15 ÷ 7 = 2 R 1.* **Q7 & 8:** These are two good questions to assign to students. **Q9:** This is a good problem as well. Students have to think about the situation described. An answer of 5 R2 would not work since "remainder 2 cartons" doesn't make any sense. A similar question might be *"Each car holds 5 people. How many cars are needed to transport 17 people to the hockey game?"* ***Reflect:*** Optional
Lesson 10: *Using Base Ten Blocks to Divide*	***Explore*** Make materials available for students to use as models to think with. Ask them to use numbers, pictures and/or words to solve the problem with the boxes and books. ***Connect*** Distribute base ten blocks for students to model 36 ÷ 3. Have them do others as well; eg. 24 ÷ 2, 44 ÷ 4, 66 ÷ 3. The example on p.306 is the same kind of process, but it involves regrouping and a remainder. Provide an easier example that does not have a remainder. For example, 57 ÷ 3. Students will still need to regroup or exchange their blocks, but there is no remainder to deal with. Provide similar examples and then include 57 ÷ 4 and other division involving remainders. ***Practice*** **Q1:** Use base ten blocks for all of these. **Q2:** Assign **Q3:** Remind students that they can "think multiplication" to solve these division problems. They do not have to use base ten blocks if they know a more efficient way. **Q4: *Numbers, Pictures, Words*** Good assessment checkpoint. Have students write a response in their journals. **Q5 & 6:** Same as for Q3 **Q7:** Assign ***Reflect:*** Good assessment question.

Lesson 11: *Another Strategy for Division*	This lesson attempts to connect the use of base ten blocks for division problems to the "short division algorithm". Students should become comfortable in using the model to think about division situations, but it is not an expectation of the grade 4 mathematics curriculum that they use the short division algorithm. Using their understanding and knowledge of multiplication to solve division problems, including those with remainders, deserves the most emphasis in this lesson. For all exercises, help students see how multiplication can be used.
Game: *Less is More*	If you do not have a 0-9 die, use the numeral cards on LM8.10. Make several copies to create a "deck" of cards for students to use.
Unit 8: *Show What You Know* Unit Problem: *At the Garden Centre* **Investigation: *Circle Patterns***	**Q1:** Assign **Q2:** Do together. For 5 x = 300, ask, *"Five times what is 30? Five times what is 300?* **Q3:** Assign **Q4:** Orally **Q5:** Assign **Q6:** Assign. Do not ask students to write about the strategy they used. However, you should have that conversation with some of your students as a way to assess their understanding. Q7 & 8: Review how students thought about the size of the groups back in lesson 7. Q9: Assign Q10: Orally Q11: Remind students to use multiplication to think about division. Q12: Assign Q13 & 14: Assign. Do not require a formal algorithm, but look for evidence that multiplication is being used. These are all reasonable problems for students to think about. Rather than assigning them all to every student, let children choose one that they would like to do. Tell them to use numbers, pictures and words to show their work and explain how they solved each problem. Early finishers or those who need a challenge could be asked to do additional problems. This is a good activity for students to try. Review the meaning of multiples and require that they use rulers to draw straight lines.

List of Grade 4 Specific Curriculum Outcomes

Number (N)

N1 Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.
N2 Compare and order numbers to 10 000.
N3 Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by: using personal strategies for adding and subtracting, estimating sums and differences, and solving problems involving addition and subtraction.
N4 Explain the properties of 0 and 1 for multiplication and the property of 1 for division.
N5 Describe and apply mental mathematics strategies, such as: skip counting from a known fact, using doubling or halving, using doubling or halving and adding or subtracting one more group, and using patterns in the 9s facts to determine basic multiplication facts to 9 x 9 and related division facts.
N6 Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by using personal strategies for multiplication (2- or 3-digit by 1-digit) to solve problems by using personal strategies for multiplication with and without concrete materials, using arrays to represent multiplication, connecting concrete representation to symbolic representations and estimating products.
N7 Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by using personal strategies for dividing with and without concrete materials, estimating quotients and relating division to multiplication.
N8 Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to name and record fractions for the parts of a whole or a set, compare and order fractions, model and explain that for different wholes, two identical fractions may not represent the same quantity and to provide examples of where fractions are used.
N9 Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically.
N10 Relate decimals to fractions (to hundredths).
N11 Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by using compatible numbers, estimating sums and differences and using mental math strategies to solve problems.

Patterns and Relations (PR)

(Patterns)

PR1 Identify and describe patterns found in tables and charts, including a multiplication chart.
PR2 Reproduce a pattern shown in a table or chart using concrete materials.
PR3 Represent and describe patterns and relationships using charts and tables to solve problems.
PR4 Identify and explain mathematical relationships using charts and diagrams to solve problems.

(Variables and Equations)

PR5 Express a given problem as an equation in which a symbol is used to represent an unknown number.
PR6 Solve one-step equations involving a symbol to represent an unknown number.

Shape and Space (SS)

(Measurement)

SS1 Read and record time using digital and analog clocks, including 24-hour clocks.
SS2 Read and record calendar dates in a variety of formats.
SS3 Demonstrate an understanding of area of regular and irregular 2-D shapes by recognizing that area is measured in square units, selecting and justifying referents for the units cm^2 or m^2, estimating area by using referents for cm^2 or m^2, determining and recording area (cm^2 or m^2), constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area.

(3-D Objects and 2-D Shapes)

SS4 Describe and construct rectangular and triangular prisms.

(Transformations)

SS5 Demonstrate an understanding of line symmetry by identifying symmetrical 2-D shapes, creating symmetrical 2-D shapes and drawing one or more lines of symmetry in a 2-D shape.
SS6 Demonstrate an understanding of congruency, concretely and pictorially.

Statistics and Probability (SP)

(Data Analysis)

SP1 Demonstrate an understanding of many-to-one correspondence.
SP2 Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.

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