

## Prince Edward Island Mathematics Curriculum

Education and Early Childhood Development English Programs


## Mathematics



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## Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base in its creation. From examining the curriculum proposed throughout Canada to securing the latest research in the teaching of mathematics, the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) Common Curriculum Framework for K-9 Mathematics (2006) has been adopted as the basis for a revised mathematics curriculum in Prince Edward Island. The Common Curriculum Framework was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP and the Principles and Standards for School Mathematics (2000), published by the National Council of Teachers of Mathematics (NCTM).

## > Essential Graduation Learnings

Essential graduation learnings (EGLs) are statements describing the knowledge, skills and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work and study today and in the future. Essential graduation learnings are cross-curricular, and curriculum in all subject areas is focused to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will be able to demonstrate knowledge, skills and attitudes in the following essential graduation learnings:

- Respond with critical awareness to various forms of the arts and be able to express themselves through the arts.
- Assess social, cultural, economic and environmental interdependence in a local and global context.
- Use the listening, viewing, speaking and writing modes of language(s), and mathematical and scientific concepts and symbols to think, learn and communicate effectively.
- Continue to learn and to pursue an active, healthy lifestyle.
- Use the strategies and processes needed to solve a wide variety of problems, including those requiring language, and mathematical and scientific concepts.
- Use a variety of technologies, demonstrate an understanding of technological applications and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

## > Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate high expectations for students in mathematics education to all educational partners. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM Principles and Standards for School Mathematics, 2000).

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning; and
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks; and
- exhibit curiosity.


## Conceptual Framework for K - 9 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes:


The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into four strands, namely: Number, Patterns and Relations, Shape and Space, and Statistics and Probability. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connection among concepts both within and across strands. Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
- Problem solving, reasoning and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.


## > Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to:

- communicate in order to learn and express their understanding of mathematics; [Communications: C]
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines; [Connections: CN]
- demonstrate fluency with mental mathematics and estimation; [Mental Mathematics and Estimation: ME]
- develop and apply new mathematical knowledge through problem solving; [Problem Solving: PS]
- develop mathematical reasoning; [Reasoning: R]
- select and use technologies as tools for learning and solving problems; [Technology: T] and
- develop visualization skills to assist in processing information, making connections and solving problems. [Visualization: V]


## Communication [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing and modifying ideas, knowledge, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written and mental representations of mathematical ideas.

## Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

## Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need - more than ever before - with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001). Mental mathematics "provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers" (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision making process as described below:

(NCTM)

## Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you. . . ?" or "How could you. . . ?" the problem-solving approach is being modeled. Students develop their own problem-solving strategies by being open to listening, discussing and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not
a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modeled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- use estimation
- guess and check
- look for a pattern
- make an organized list or table
- use a model
- work backwards
- use a formula
- use a graph, diagram or flow chart
- solve a simpler problem
- use algebra


## Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

## Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations; and
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in $\mathrm{K}-3$ to enrich learning, it is expected that students will meet all outcomes without the use of technology.

## Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, to determine when to estimate and to know several estimation strategies (Shaw \& Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.

## > The Nature of Mathematics

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this document. These components include: change, constancy, number sense, patterns, relationships, spatial sense and uncertainty.

## Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence $4,6,8,10,12, \ldots$ can be described as:

- skip counting by 2 s , starting from 4 ;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.


## Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS-Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:

- the area of a rectangular region is the same regardless of the methods used to determine the solution;
- the sum of the interior angles of any triangle is $180^{\circ}$; and
- the theoretical probability of flipping a coin and getting heads is 0.5 .

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

## Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (The Primary Program, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

## Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions, and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

## Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally or in written form.

## Spatial Sense

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example:

- knowing the dimensions of an object enables students to communicate about the object and create representations;
- the volume of a rectangular solid can be calculated from given dimensions; and
- doubling the length of the side of a square increases the area by a factor of four.


## Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of
probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

## Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking and critical thinking and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

## > Connections across the Curriculum

There are many possibilities for connecting Grade 5 mathematical learning with the learning occurring in other subject areas. Making connections between subject areas gives students experiences with transferring knowledge and provides rich contexts in which students are able to initiate, make sense of, and extend their learnings. When connections between subject areas are made, the possibilities for transdisciplinary inquiries and deeper understanding arise. When making such connections, however, teachers must be cautious not to lose the integrity of the learning in any of the subjects.

## > Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should reduce some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a parent will have a clearer understanding of the mathematics curriculum and the progress of his or her child in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

## $>$ Diversity in Student Needs

Every classroom comprises students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters but should be designed to help all students, whether strong, weak or average, to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson, from which all students come away with a better understanding of what the solution to an equation really means.

## > Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean that not only should enrolments of students of both genders and various cultural backgrounds in public school mathematics courses reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

## > Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English proficiency and cultural differences must not be a barrier to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and coordinated assessment.

The Principles and Standards for School Mathematics (NCTM, 2000) emphasizes communication "as an essential part of mathematics and mathematics education (p.60)." The Standards elaborate that all students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers that will facilitate "communicating to learn mathematics and learning to communicate mathematically (NCTM, p.60)."

To this end:

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counselors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated, with appropriate language support, to both students and parents; and
- to verify that barriers have been removed, educators should monitor enrollment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.


## $>$ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development - such as poverty alleviation, human rights, health, environmental protection and climate change - into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental and economic perspective and explores how those factors are inter-related and inter-dependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database

Resources for Rethinking, found at http://r4r.ca/en. It provides teachers with access to materials that integrate ecological, social and economic spheres through active, relevant, interdisciplinary learning.

## Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, whether teaching has been effective or how best to address student learning needs. The quality of the assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated and how results are communicated send clear messages to students and others.

## > Assessment

Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as:

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children's learning;
- providing information to teachers on the effectiveness of their teaching, the program and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources including:

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment

This balanced approach for assessing mathematics development is illustrated in the diagram below.


There are three interrelated purposes for classroom assessment: assessment as learning, assessment for learning and assessment of learning. Characteristics of each type of assessment are highlighted below.

Assessment as learning is used:

- to engage students in their own learning and self-assessment;
- to help students understand what is important in the mathematical concepts and particular tasks they encounter;
- to develop effective habits of metacognition and self-coaching;
- to help students understand themselves as learners - how they learn as well as what they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment for learning is used:

- to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
- to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
- to provide feedback to students about how they are doing and how they might improve;
- to provide feedback to other professionals and to parents about how to support students' learning.


## Assessment of learning is used:

- to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
- to facilitate reporting;
- to provide the basis for sound decision-making about next steps in a student's learning.


## > Evaluation

Evaluation is the process of analysing, reflecting upon and summarizing assessment information and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires:

- student learning;
- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information; and
- using a high level of professional judgment in making decisions based upon that information.


## $>$ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes and phone calls.

## > Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration and use of assessments must be followed. The document Principles for Fair Student Assessment Practices for Education in Canada (1993) articulates five fundamental assessment principles as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.
- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.
- Assessment reports should be clear, accurate and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that:

- the best interests of the student are paramount;
- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes; and
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he/she knows and can do.

## Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island mathematics curriculum are organized into four strands across the grades K-9. They are Number, Patterns and Relations, Shape and Space, and Statistics and Probability. These strands are further subdivided into sub-strands, which are the general curriculum outcomes (GCOs). They are overarching statements about what students are expected to learn in each strand or sub-strand from grades K-9.

| Strand | General Curriculum Outcome (GCO) |
| :--- | :--- |
| Number (N) | Number: Develop number sense. |
| Patterns and Relations (PR) | Patterns: Use patterns to describe the world and solve <br> problems. |
|  | Variables and Equations: Represent algebraic expressions in <br> multiple ways. |
|  | Measurement: Use direct and indirect measure to solve <br> problems. |
|  | 3-D Objects and 2-D Shapes: Describe the characteristics of <br> 3-D objects and 2-D shapes, and analyze the relationships <br> among them. |
|  | Transformations: Describe and analyze position and motion of <br> objects and shapes. |
|  | Data Analysis: Collect, display, and analyze data to solve <br> problems. |
|  | Chance and Uncertainty: Use experimental or theoretical <br> probabilities to represent and solve problems involving <br> uncertainty. |

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific outcome.

The first two pages for each outcome contain the following information:

- the corresponding strand and General Curriculum Outcome;
- the Specific Curriculum Outcome(s) and the mathematical processes which link this content to instructional methodology
- the scope and sequence of concept development related to this outcome(s) from grades 4-6;
- an elaboration of the outcome;
- a list of achievement indicators

Students who have achieved a particular outcome should be able to demonstrate their understanding in the manner specified by the achievement indicators. It is important to remember, however, that these indicators are not intended to be an exhaustive list for each outcome. Teachers may choose to use additional indicators as evidence that the desired learning has been achieved.

The last two pages for each outcome contain lists of instructional strategies and strategies for assessment.

The primary use of this section of the guide is as an assessment for learning (formative assessment) tool to assist teachers in planning instruction to improve learning. However, teachers may also find the ideas and suggestions useful in gathering assessment of learning (summative assessment) data to provide information on student achievement.

The second half of this curriculum guide contains a supplement which provides suggestions and recommendations for using Math Makes Sense 5 as the primary resource for addressing curriculum outcomes. A glossary of common mathematical models (manipulatives) is also provided.

NUMBER

## SCO: N1: Represent and describe whole numbers to 1000000.

[C, CN, V, T]

| $[$ [C] Communication | [PS] Problem Solving | [CN] Connections | [ME] Mental Math |
| :--- | :--- | :--- | :--- |
| [T] Technology | [V] Visualization | [R] Reasoning | and Estimation |

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :--- | :--- | :--- |
| N1 Represent and describe <br> whole numbers to 10 000, <br> concretely, pictorially and <br> symbolically. | N1 Represent and describe <br> whole numbers to 1000000. | N1 Demonstrate an understanding <br> of place value for numbers <br> - greater than one million |
| - less than one thousandth. |  |  |

## Elaboration

Students will continue to use whole numbers as they perform computations or measurements and as they read and interpret data. To have a better understanding of large numbers, such as a million, students need opportunities to investigate problems involving these numbers.

Students should have many opportunities to:

- read numbers several ways. For example, 879346 is read eight hundred seventy-nine thousand, three hundred forty-six but might also be renamed as 87 ten thousands, 9 thousands, 346 ones (other examples may include: 8 hundred thousands 79 thousands, 34 tens and 6 ones or 879 thousands, 3 hundreds 30 tens and 16 ones);
- record numbers. For example, ask students to write eight hundred thousand sixty; a number which is eighty less than one million; as well as write numbers in expanded notation (741 $253=700000+40$ $000+1000+200+50+3)$, and standard form.
- establish personal referents to develop a sense of larger numbers.

Through these experiences, students will develop flexibility in identifying and representing numbers up to 1000000 . It is also important for students to gain an understanding of the relative size (magnitude) of numbers through real life contexts that are personally meaningful. Students should establish personal referents to think about large numbers. Benchmarks that students may find helpful are multiples of 100, 1000,10000 and 100000 , as well as 250000,500000 , and 750000 (quarter, half, and three quarters of a million).

Include situations in which students use a variety of models, such as:

- base ten blocks (e.g., recognize that 1000 large cubes would represent 1000000 .)
- money (e.g., How many $\$ 100$ bills are there in $\$ 9347$ ?)
- place value charts.

| Millions |  |  | Thousands |  |  | Ones |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | O | H | T | O | H | T |  |

The focus of instruction should be on ensuring students develop a strong sense of number. The development of this outcome should be ongoing throughout the year.

## SCO: N1: Represent and describe whole numbers to 1000000.

[C, CN, V, T]

## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Write a given numeral using proper spacing without commas, e.g., 934567.
- Describe the pattern of adjacent place positions moving from right to left.
- Describe the meaning of each digit in a given numeral.
- Provide examples of large numbers used in print or electronic media.
- Express a given numeral in expanded notation, e.g., $45321=(4 \times 10000)+(5 \times 1000)+(3 \times 100)+$ $(2 \times 10)+(1 \times 1)$ or $40000+5000+300+20+1$.
- Write the numeral represented by a given expanded notation.
- Read a given numeral without using the word "and," e.g., 574321 is five hundred seventy-four thousand three hundred twenty-one, NOT five hundred AND seventy-four thousand three hundred AND twenty one. Note: The word "and" is reserved for reading decimal numbers.


## SCO: N1: Represent and describe whole numbers to 1000000. [C, CN, V, T]

## Instructional Strategies

Consider the following strategies when planning lessons:

- Use large numbers from students' experiences, such as populations and professional sport salaries.
- Use visual models based on the cubic centimetre and cubic metre.
- Share children's books to explore number concepts, such as How Much Is A Million? by David Schwartz.
- Provide students with frequent opportunities to read, write, and say numbers in standard and expanded form. Note: insist that students use proper spacing (not commas) when writing large numbers and reserve the use of "and" for reading decimal numbers.
- Discuss how large numbers can represent either a large amount or a small amount depending on the context used.
- Use various manipulatives (number cubes, spinners, number cards, etc.) to generate six-digit numbers. Students can then be asked to explore these numbers in many different ways.


## Suggested Activities

- Have students locate large numbers in newspapers or magazines. Ask them to read, write, and represent the numbers in different ways.
- Collect, as a class, some type of object with the objective of reaching a specific quantity. For example, 100000 buttons, pieces of junk mail or pop can tabs. If collecting is not possible, students could start a project where they draw a specific number of dots each week until the objective is reached.
- Identify how many $\$ 100$ bills it would take to make $\$ 1000000$.
- Identify how long a line of 1 million unit cubes would be.
- Ask students questions about the reasonableness of numbers, such as "Have you lived 1 million hours yet?" "Are there 1 million people in any Prince Edward Island city?" Have students explain their thinking.
- Create 2 page spreads for a class book about 1 million. Each spread could begin: "If you had a million $\qquad$ , it would be $\qquad$ ." Alternatively the sentences could start, "I wish I had a million $\qquad$ , but I would not want a million $\qquad$ ."
- Ask students to create six-digit numbers by rolling a number cube six times and order the numbers.
- Have students explore the way numbers have been expressed in examples of whole numbers found in various types of media and personal conversations, and discuss why variations in saying and writing numbers might occur.
- Ask students to compare 10000 steps to 10000 metres. If you walked 10000 steps per day, in how many days would you have walked 1 million steps?
- Ask students to list three non-consecutive numbers between 284531 and 285391.
- Have students place counters on a place value chart to represent a number stated orally. The digital form can be written once the chart is filled in, and the number can be read back.


## SCO: N1: Represent and describe whole numbers to 1000000.

[C, CN, V, T]

## Assessment Strategies

- Ask students to record a series of numbers that have been read to them. Ensure students include correct spacing without commas. Have students express those same numbers in expanded notation.
- Ask, "How does a million compare to 1000, 10000,100000 ?"
- Have students record a number that is 100000 more than a given number (or variations of this such as 20000 less, etc.).
- Ask the students to use newspapers or catalogues to find items that would total $\$ 1$ million.
- Ask the student how she/he knows that 1000000 is the same as 1000 thousands.
- Tell students you bought a car with 50 hundred dollar bills, 20 thousand dollar bills, 100 ten dollar bills and 46 loonies. Ask students to determine the cost of the car.
- Have students explain at least three things they know about a number with 7 digits.
- Ask a student to describe when 1000000 of something might be a big amount? A small amount?

```
SCO: N2: Use estimation strategies including:
    - front-end rounding
    - compensation
    - compatible numbers
    in problem-solving contexts.
    [C, CN, ME, PS, R, V]
```

| [C] Communication | [PS] Problem Solving | [CN] Connections | [ME] Mental Math |
| :--- | :--- | :--- | :--- |
| [T] Technology | [V] Visualization | [R] Reasoning | and Estimation |

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :---: | :---: | :---: |
| N3 Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4digit numerals) by: <br> - using personal strategies for adding and subtracting <br> - estimating sums and differences solving problems involving addition and subtraction. | N2 Use estimation strategies including: <br> - front-end rounding <br> - compensation <br> - compatible numbers in problemsolving contexts. |  |

## Elaboration

Students need to recognize that estimation is a useful skill in their lives. To be efficient when estimating sums and differences mentally, students must be able to access a strategy quickly and they need a variety from which to choose. Students should be aware that in real-life estimation contexts overestimating is often important.

The context and the numbers and operations involved affect the estimation strategy chosen.

- Rounding - There are a number of things to consider when rounding to estimate for a multiplication calculation. If one of the factors is a single digit, consider the other factor carefully. For example, when estimating $8 \times 693$, rounding 693 to 700 and multiplying by 8 is a much closer estimate than multiplying 10 by 700. Explore rounding one factor up and the other one down, even if it does not follow the "rule". For example, when estimating 77 by 35 , compare $80 \times 30$ and $80 \times 40$ to the actual answer of 2695.
- Compensation - In this case, compensation refers to increasing one value and decreasing the other. For example, $35+57$ might be estimated as $30+60$ (rather than $40+60$ ) as this is a more accurate estimation.
- Compatible numbers or "nice numbers" - Clustering compatible (or near compatible) numbers is useful for addition. For example, to solve $134+55+68+46$, the 46 and 55 together make about 100; the 134 and 68 make about another 200 for a total of 300 . Look for compatible numbers when rounding for a division estimate. For $4719 \div 6$, think " $4800 \div 6$ ". For $3308 \div 78$, think " $3200 \div 80$ ".

Students and teachers should note that multiplication and division estimations are typically further from the actual value because of the nature of the operations involved.

SCO: N2: Use estimation strategies including:

- front-end rounding
- compensation
- compatible numbers
in problem-solving contexts.
[C, CN, ME, PS, R, V]


## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Provide a context for when estimation is used to:
make predictions
check reasonableness of an answer
determine approximate answers.
- Describe contexts in which overestimating is important.
- Determine the approximate solution to a given problem not requiring an exact answer.
- Estimate a sum or product using compatible numbers.
- Estimate the solution to a given problem using compensation and explain the reason for compensation.
- Select and use an estimation strategy for a given problem.
- Apply front-end rounding to estimate:
sums, e.g., $253+615$ is more than $200+600=800$
differences, e.g., $974-250$ is close to $900-200=700$
products, e.g., the product of $23 \times 24$ is greater than $20 \times 20(400)$ and less than $25 \times 25$ (625)
quotients, e.g., the quotient of $831 \div 4$ is greater than $800 \div 4$ (200).

```
SCO: N2: Use estimation strategies including:
- front-end rounding
- compensation
- compatible numbers
in problem-solving contexts.
[C, CN, ME, PS, R, V]
```


## Instructional Strategies

Consider the following strategies when planning lessons:

- Support students in exploring personal strategies for estimation, but then guide students toward more efficient and accurate strategies as needed.
- Have students share personal strategies. Begin with the least efficient strategies and then share progressively more complex as this encourages participation from all and does not discourage others.
- Accept a range of estimates, but focus on "good" estimate.
- Provide real-world contexts for estimations, as most situations require estimations and not precise answers.
- Practice strategy selection and explain the choice for estimation.


## Suggested Activities

- Tell the student that $\square 834 \div 6$ is about 300 . Ask the student to decide what digit should go in the box.
- Ask the students to find two numbers with a difference of about 150 and a sum of about 500 or two numbers with a difference of about 80 and a sum of about 200.
- Ask the student to estimate what one might subtract in each case below so that the answer is close to, but not exactly, fifty: 384 - $\qquad$ , 219 - $\qquad$ 68- $\qquad$
- Have students describe a real world situation when overestimating is appropriate.
- Ask students if their age in days would be closer to 400,4000 , or 40000 days and to explain their thinking.


## SCO: N2: Use estimation strategies including:

- front-end rounding
- compensation
- compatible numbers
in problem-solving contexts.
[C, CN, ME, PS, R, V]


## Assessment Strategies

- Ask: Which pair of factors would you choose to estimate $37 \times 94$ ? Explain why.

$$
30 \times 90 \quad 40 \times 100 \quad 35 \times 95 \quad 40 \times 95 \quad 40 \times 90
$$

- Have students estimate each sum and explain their strategies: $1976+345669423+21097$
- Have students estimate each difference and explain their strategies: 99 764-17368 5703-755
- Have students add $6785+1834$. Explain how they know their answer is reasonable.
- Have students solve problems that require an estimate; for example: "Jeff has 138 cans of soup. He wants to collect 500 cans for the food bank. About how many more does he need to collect?"
- Ask students for an estimate if a number between 300 and 400 is divided by a number between 60 and 70.
- Say to students, "A bus holds 58 students. How would you estimate how many buses are needed to transport 3000 students?"
- Tell students that you have multiplied a 3-digit number by a 1-digit number and the answer is about 1000. Ask them to write three possible pairs of factors.

SCO: N3: Apply mental mathematics strategies and number properties, such as:

- skip counting from a known fact
- using doubling or halving
- using patterns in the 9s facts
- using repeated doubling or halving
to determine answers for basic multiplication facts to 81 and related division facts. [C, CN, ME, R, V]

| [C] Communication | [PS] Problem Solving | [CN] Connections | [ME] Mental Math |
| :--- | :--- | :--- | :--- |
| [T] Technology | [V] Visualization | [R] Reasoning | and Estimation |

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :---: | :---: | :---: |
| N5 Describe and apply mental mathematics strategies, such as: <br> - skip counting from a <br> - known fact <br> - using doubling or halving <br> - using doubling or halving and adding or subtracting one more group <br> - using patterns in the 9s facts <br> - using repeated doubling <br> to determine basic multiplication facts to $9 \times 9$ and related division facts. | N3 Apply mental mathematics strategies and number properties, such as: <br> - skip counting from a known fact <br> - using doubling or halving <br> - using patterns in the 9s facts <br> - using repeated doubling or halving <br> to determine answers for basic multiplication facts to 81 and related division facts. | N3 Demonstrate an understanding of factors and multiples by: <br> - determining multiples and factors of numbers less than 100 identifying prime and composite numbers <br> - solving problems involving multiples. |

## Elaboration

This is an extension of the grade 4 outcomes, N4 and N5. The goal for grade 5 is automaticity, which means that students are able to recall multiplication facts with little or no effort. The facts have been committed to memory as a result of extensive use of strategies.

Students need to understand and use the relationship between multiplication and division. Students should recognize that multiplication can be used to solve division situations. Contextual problems are key to emphasizing this connection.

Along with understanding why multiplication by 0 produces a product of 0 , students must be able to explain why division by $\mathbf{0}$ is undefined or not possible. It is not possible to make a set of zero from a given group, nor is it possible to make zero sets from a given group. When demonstrated as repeated subtraction, removing groups of zero will never change your dividend. Rather than telling students these properties, pose problems involving 0.

In grade 4, students will have become proficient at doubling ( $4 \times 3=(2 \times 3) \times 2)$. This idea is extended in grade 5 to include repeated doubling. For example, to solve $8 \times 6$, students can think $2 \times 6$ $=12 ; 4 \times 6=24$, so $8 \times 6=48$. The same principle applies to halving and repeated halving. For example, for $36 \div 4$, think $36 \div 2=18$; so $18 \div 2=9$.

Skip counting up or down from a known fact reinforces the meanings of multiplication and division as students must be thinking about the addition or subtraction of "groups". For example, for $8 \times 7$, think $7 \times 7=49$ and then add another group of $7 ; 49+7=56$.

```
SCO: N3: Apply mental mathematics strategies and number properties, such as:
    - skip counting from a known fact
    - using doubling or halving
    - using patterns in the 9s facts
    - using repeated doubling or halving
    to determine answers for basic multiplication facts to 81 and related division facts.
    [C, CN, ME, R, V]
```


## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Describe the mental mathematics strategy used to determine a given basic fact, such as:
- skip count up by one or two groups from a known fact, e.g., if $5 \times 7=35$, then $6 \times 7$ is equal to $35+$ 7 and $7 \times 7$ is equal to $35+7+7$
- skip count down by one or two groups from a known fact, e.g., if $8 \times 8=64$, then $7 \times 8$ is equal to $64-8$ and $6 \times 8$ is equal to $64-8-8$
- doubling, e.g., for $8 \times 3$ think $4 \times 3=12$, and $8 \times 3=12+12$
- patterns when multiplying by 9 , e.g., for $9 \times 6$, think $10 \times 6=60$, and $60-6=54$; for $7 \times 9$, think $7 \times 10=70$, and $70-7=63$
- repeated doubling, e.g., if $2 \times 6$ is equal to 12 , then $4 \times 6$ is equal to 24 and $8 \times 6$ is equal to 48
- repeated halving, e.g., for $60 \div 4$, think $60 \div 2=30$ and $30 \div 2=15$.
- Explain why multiplying by zero produces a product of zero.
- Explain why division by zero is not possible or undefined, e.g., $8 \div 0$.
- Recall multiplication facts to 81 and related division facts.

```
SCO: N3: Apply mental mathematics strategies and number properties, such as:
    - skip counting from a known fact
    - using doubling or halving
    - using patterns in the 9s facts
    - using repeated doubling or halving
    to determine answers for basic multiplication facts to 81 and related division facts.
    [C, CN, ME, R, V]
```


## Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Introduce and practise strategies. When students are proficient at more than one strategy, have them explain why one strategy may be better than another in a given situation.
- Have students start with what they know.
- Allow students to use counters or base ten blocks as they continue to develop strategies.
- Have students skip count aloud.
- Play games to practise strategies that lead to fact recall.
- Use a problem solving context to practice facts.
- Ensure students understand why strategies work. Fact strategies should not become "rules without reasons" (Van de Walle \& Lovin, vol. 2, 2006; p. 90).
- Avoid using drill until students have mastered a strategy. Unless students have mastered a strategy, drills are not effective.


## Suggested Activities

- Use counters to model $6 \times 6$ in an array. Add another row or column to demonstrate a related fact.
- Ask students, "If you didn't know the answer to $6 \times 8$, how could you figure it out?" (see Van de Walle 3-5)
- Ask students, "Jennifer reads a chapter of a novel each day. How many chapters will she have read in 8 weeks. Describe your strategy.
- Have students agree or disagree with this statement, "There are more than 2 ways to figure out any multiplication fact". Have students use a fact of their choice.
- Provide students with food labels showing 0 g of fat per serving. Ask them how many grams of fat are in 4 servings.
- Ask students if they agree or disagree with this statement, "If you know your multiplication facts, you already know your division facts." Have students provide a rationale.
- Provide small groups with a square piece of paper. Have them fold the paper in half and record how many sections they have. Have them fold it again and record how many sections. Have them continue until they see a pattern of doubling. Relate this to halving.
- Have students fill in all the facts they know in a multiplication table. Ask them to work with a partner to identify what strategies they could use to fill in the rest of the table.
- Use sets of "loop cards" (I have $\qquad$ , who has $\qquad$ ) where the answer for one card answers the question on another to form a loop of questions and answers. For example, one card could read, "I have 24 . Who has $3 \times 4$ ?"

SCO: N3: Apply mental mathematics strategies and number properties, such as:

- skip counting from a known fact
- using doubling or halving
- using patterns in the 9s facts
- using repeated doubling or halving
to determine answers for basic multiplication facts to 81 and related division facts.
[C, CN, ME, R, V]


## Assessment Strategies

- Ask students to list three multiplication facts they can use to help calculate $5 \times 8$, and explain how they can use each fact.
- Ask, "If you buy muffins in boxes of 6, how many muffins are in 7 boxes? How would the number of muffins change if you bought 9 boxes? If you needed 36 muffins for a party, how many boxes would you buy?"
- Ask students how they could use multiplication to find the perimeter of a square.
- Tell the students that you have eight boxes, each of which holds six markers, and one other box that has only five markers in it. Ask the students to describe at least two ways one could find the total number of markers, and to explain which way they would prefer and why.


## SCO: N4: Apply mental mathematics strategies for multiplication, such as: <br> - annexing then adding zero <br> - halving and doubling <br> - using the distributive property. <br> [C, ME, R]

| [C] Communication | [PS] Problem Solving | [CN] Connections | [ME] Mental Math |
| :--- | :--- | :--- | :--- |
| [T] Technology | [V] Visualization | [R] Reasoning | and Estimation |

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :--- | :--- | :--- |
| N5 Describe and apply mental | N4 Apply mental mathematics |  |
| mathematics strategies, such | strategies for multiplication, such |  |
| as: | as: |  |
| - skip counting from a known | l annexing then adding zero |  |
| fact | - halving and doubling |  |
| - using doubling or halving | - using the distributive property. |  |
| - using doubling or halving |  |  |
| - adding or subtracting one |  |  |
| - more group |  |  |
| using patterns in the 9s <br> facts using repeated <br> doubling |  |  |
| to determine basic |  |  |
| multiplication facts to $9 \times 9$ and |  |  |
| related division facts. |  |  |

## Elaboration

A mental computation is one that produces the actual answer, not an estimate. In grade 5 , students are extending the strategies learned in grade four to multiply mentally. It is important to recognize that these strategies develop and improve over the years with regular practice. This means that mental mathematics must be a consistent part of instruction in computation from primary through the elementary and middle grades. Mental strategies must be taught both explicitly as well as being embedded in problem solving situations. Sharing of computational strategies within the context of problem-solving situations is essential.

Students should perform and discuss the following types of mental multiplication on a regular basis:

- Annexing then adding zero for multiplication by 10, 100 and 1000 and multiplication of single-digit multiples of powers of ten (e.g., for $30 \times 400$, students should think "Tens times hundreds is thousands. How many thousands? $3 \times 4$ or 12 thousands.")
- Halving and doubling: For example, to solve $4 \times 16$, students can change it to $2 \times 32$ or $8 \times 8$.
- Distributive property: The ability to break numbers apart is important in multiplication. For example, to multiply $5 \times 43$, think $5 \times 40(200)$ and $5 \times 3(15)$ and then add the results. This principle also applies to multiplication questions in which one of the factors ends in a nine (or eight or seven). For such questions, one could use a compensating strategy - multiply by the next multiple of ten and compensate by subtracting to find the actual product. For example, when multiplying 39 by 7 mentally, one could think, " 7 times 40 is 280 , but there were only 39 sevens so I need to subtract 7 from 280 which gives an answer of 273."

Whenever presented with problems that require computations, students should be encouraged to first check to see if it can be done mentally. Students should select an efficient strategy that makes sense to them.

SCO: N4: Apply mental mathematics strategies for multiplication, such as:

- annexing then adding zero
- halving and doubling
- using the distributive property. [C, ME, R]


## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Determine the products when one factor is a multiple of 10,100 or 1000 by annexing zero or tacking on zeros, e.g., for $3 \times 200$ think $3 \times 2$ hundreds which equals six hundreds (600).
- Apply halving and doubling when determining a given product, e.g., $32 \times 5$ is the same as $16 \times 10$.
- Apply the distributive property to determine a given product involving multiplying factors that are close to multiples of 10 , e.g., $98 \times 7=(100 \times 7)-(2 \times 7)$

```
SCO: N4: Apply mental mathematics strategies for multiplication, such as:
    - annexing then adding zero
    - halving and doubling
    - using the distributive property.
        [C, ME, R]
```


## Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students with many experiences to construct a personal strategy and then be guided to use the most efficient strategy available. Mental strategies encourage students to think about the whole number and not just the digits.
- Provide students with frequent opportunities to share their mental strategies.
- Provide problem-based situations that support the use of mental strategies.
- Use materials and pictorial representations to demonstrate mental strategies.
- Introduce a strategy with the use of materials, practice the strategy, and continue to introduce and practice new strategies. When students have two or more strategies, it is important to encourage them to choose the most efficient strategy for the student.
- Encourage students to visualize the process for the strategy they are using.
- Place students in pairs to practise strategies as well as strategy selection.
- Avoid timed tests until students have developed and practised specific mental strategies in other contexts.
- Ask students to keep track of when they use their mental math strategies outside of the classroom and to write about these experiences.
- Ask the students to keep a list of mental math strategies that they use regularly.


## Suggested Activities

- Use two recipe cards and have students write a series of mental math questions. Students take the cards home to have a "race" with a parent/guardian. The student can then "teach" the strategy being practiced at home.
- Ask students to explain how they could calculate $23 \times 8$ if the "eight" key on the calculator was broken.
- Mix cards with number sentences that can be solved using two or more strategies into a single package. Prepare simple pictures or labels for the strategies in the package. Have students sort the problems and then solve them using the appropriate strategy.
- Ask the student to use square tiles to show that if the length of a rectangle is halved and the width is doubled, the area remains the same.
- Ask the student to provide an explanation and examples for how to multiply a 1 -digit number by 99 mentally.

```
SCO: N4: Apply mental mathematics strategies for multiplication, such as:
    - annexing then adding zero
    - halving and doubling
    - using the distributive property.
    [C, ME, R]
```


## Assessment Strategies

- Tell the student that when asked to multiply $36 \times 11$, Kelly said, "I think $360+36=396$." Ask the student to explain Kelly's thinking.
- Ask: Why is it easy to calculate the questions below mentally?

$$
48 \times 20
$$

$$
50 \times 86
$$

- Ask the student why Lynn multiplied $11 \times 30$ to find $22 \times 15$.
- Provide students with a problem situations to solve, such as:

1. Fourteen students raised $\$ 20$ each in pledges for "Save the Wetlands Walk". How much money was raised? How much money would be raised if the pledges were increased to $\$ 50$ each?
2. A hotel has 7 floors with 39 windows on each floor. How many windows are in the hotel? Explain how you know.

- Explain how you know that $48 \times 50$ is the same as $24 \times 100$.


## SCO: N5: Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems. <br> [C, CN, PS, V]

| [C] Communication <br> [T] Technology | [PS] Problem Solving | [CN] Connections <br> [R] Visualization | [ME] Mental Math <br> and Estimation |
| :--- | :--- | :--- | :--- |


| Grade Four | Grade Five | Grade Six |
| :--- | :--- | :--- |
| N6 Demonstrate an | N5 Demonstrate an | N8 Demonstrate an |
| understanding of multiplication | understanding of multiplication | understanding of <br> (2- or 3-digit by 1-digit) to solve <br> (2-digit by 2-digit) to solve <br> problems by: using personal <br> strategies for multiplication with <br> problems. |
| and without concrete materials; |  | of decimals (1-digit whole <br> using arrays to represent <br> multiplication; connecting |
| 1-digit natural number and |  |  |
| concrete representations to |  |  |
| symbolic representations; |  |  |
| estimating products. |  |  |

## ELABORATION

Strategies for multiplication can be more complex than those for addition and subtraction. Students need to be flexible in the way they think about the factors, and should be thinking about numbers, not just digits. Students should have many opportunities to practise and share their ideas.

Model multiplying two 2-digit numbers concretely:

- Model the product as the area of a rectangle with the dimensions of the two numbers. This can be done using base-ten blocks and grid paper. Students should relate the model to an algorithm. The symbolic steps should be recorded and related to each physical manipulation.
- When the students understand the area model, they may choose to use a grid-paper drawing as an explanation, but it is important to record the process. A standard algorithm might be presented, but it is important that an explanation with models be provided, not just procedural rules.
The distributive property of multiplication allows students to record partial products. For example: $43 \times 24=(40+3) \times(20+4)$


The commutative property of multiplication means the order in which you multiply does not matter. The above example closely resembles what students may already think of as front end multiplication.

- As always, students should be given the choice of using a standard algorithm. If, however, students are using inefficient algorithms, they should be guided to select more appropriate ones.
- As for all computational questions, students should estimate before calculating. Immediate recall of basic multiplication facts is a necessary prerequisite not only for paper-and-pencil algorithmic procedures, but also for estimation and mental computation.


## SCO: N5: Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems. <br> [C, CN, PS, V]

## ACHIEVEMENT INDICATORS

Students who have achieved this outcome(s) should be able to:

- Illustrate partial products in expanded notation for both factors, e.g., for $36 \times 42$, determine the partial products for $(30+6) \times(40+2)$.
- Represent both 2-digit factors in expanded notation to illustrate the distributive property, e.g., to determine the partial products of $36 \times 42,(30+6) \times(40+2)=(30 \times 40)+(30 \times 2)+(6 \times 40)+(6 \times 2)$ $=1200+60+240+12=1512$.
- Model the steps for multiplying 2-digit factors using an array and base ten blocks, and record the process symbolically.
- Describe a solution procedure for determining the product of two given 2-digit factors using a pictorial representation, such as an area model.
- Solve a given multiplication problem in context using personal strategies and record the process.


## SCO: N5: Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems. <br> [C, CN, PS, V]

## Instructional Strategies

## Consider the following strategies when planning lessons:

- Model multiplication concretely (base ten blocks, grid paper).
- Develop the symbolic representation from the model.
- Introduce the traditional algorithm only after students have conceptual understanding.
- Use place value language. (e.g., $24 \times 62$ is twenty $\times 62+$ four $\times 62$ )
- Have students estimate the product first.
- Encourage frequent use of mental math strategies.
- Guide students to use efficient strategies.
- Use the language of multiplication, such as factor, product, distributive and commutative property.


## Suggested Activities

- Provide students with a large rectangle (e.g., $24 \mathrm{~cm} \times 13 \mathrm{~cm}$ ). Have students fill the rectangle with base ten materials to find the area. Have them write the related multiplication equation.
- Use known facts and combinations of facts that students know to solve more complex computations. For example, provide students with $31 \times 24$ and use $31 \times 10,31 \times 4,30 \times 24,1 \times 24$ to solve.
- Ask students to explore the pattern in these products: $15 \times 15,25 \times 25,35 \times 35$, etc. Have them describe the pattern and tell how the pattern could be used to predict $85 \times 85$ or $135 \times 135$. They might then test their predictions using a calculator. Alternatively, students might explore the pattern in these products: $19 \times 21,29 \times 31,39 \times 41$, and use it to make a prediction for $79 \times 81$ and $109 \times 111$.
- Find the product of $25 \times 25$. How can the product of $25 \times 25$ be used to help find the products of $25 \times 24,25 \times 50$ and $25 \times 75$ ?
- Discuss multiplication strategies. Have students share which strategies they prefer for particular situations and why.
- Ask students to explore the following: $24+35$ is the same as $25+34$. Is $24 \times 35$ the same as $25 \times 34$ ? Have students provide an explanation.


## SCO: N5: Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems. <br> [C, CN, PS, V]

## Assessment Strategies

- Have the student use a model to show how to find the total money collected for photos if 43 students each bring in \$23.
- Ask the students to explain why the product of two different 2-digit numbers is always greater than 100.
- Have students draw an array to represent $32 \times 16$.
- Present problems such as the following to students:
- "Hardcover books are being sold at a book sale for $\$ 26$ each. If 48 hardcover books were sold, how much did they cost?"
- "If a cheetah can run 29 m per second, how far can it run in 1 minute?"
- Show students the following:

31
$\times 25$
124
$\underline{62}$
186
Ask students to identify the error and explain how to correct it.

# SCO: N6: Demonstrate, with and without concrete materials, an understanding of division (3digit by 1-digit) and interpret remainders to solve problems. <br> [C, CN, PS] 

| [C] Communication <br> [T] Technology | [PS] Problem Solving | [CN] Connections <br> [V] Visualization | [ME] Mental Math <br> and Estimation |
| :--- | :--- | :--- | :--- |

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :--- | :--- | :--- |
| N7 Demonstrate an | N6 Demonstrate, with and | N8 Demonstrate an |
| understanding of division (1-digit | without concrete materials, an | understanding of |
| divisor and up to 2-digit | understanding of division | multiplication and division |
| dividend) to solve problems by: | (3-digit by 1-digit) and interpret | of decimals (1-digit whole |
| - using personal strategies for | remainders to solve problems. | number multipliers and |
| $\quad$ dividing with and without |  | 1-digit natural number divisors). |
| concrete materials |  |  |
| - estimating quotients |  |  |
| - relating division to |  |  |
| $\quad$ multiplication. |  |  |

## Elaboration

Division problems can involve either sharing or finding how many groups.

| 151 | "4 hundreds shared among 3, each gets |  | 100 | "Make 100 sets of 3 , using 300; 153 left. Make |
| :---: | :---: | :---: | :---: | :---: |
| $3 \longdiv { 4 5 3 }$ | 1 with 1 hundred left. | $3 \longdiv { 4 5 3 }$ |  |  |
| $3 \longdiv { 4 5 3 }$ | Trade 1 hundred for | 3) 453 |  | left...etc" The number of |
| -3 | 10 tens; now 15 tens | -300 |  | sets at each stage tends |
| 15 | to share, each gets 5 | 153 |  | to be a multiple of 10 or |
| -15 | tens, etc" | -150 | 50 | 100 to facilitate |
| 3 |  |  |  | ion. |

Division should be connected to estimation and multiplication to check the reasonableness of the answer. It is necessary that division be modelled as "equal sharing" using base ten materials. Recording the process symbolically will support student's understanding of division. The traditional long-division algorithm, whether modelled with base-ten blocks or not, is best described using "sharing words" as seen above.

Students also need to know that the answer for a division sentence is the quotient and the number to be divided is the dividend.

The context of remainders must be discussed with students. They must understand why the number of units leftover after the sharing must be less than the divisor. Models help to clarify this idea. Students need many opportunities to explore the interpretation of the remainder in problem solving situations to decide if it should be ignored, rounded up, expressed as a fraction or a decimal. A common mistake of students is to write a remainder as a decimal when the divisor is other than 10 (e.g., a remainder of 7 is written as ". 7 "). This should be addressed through a discussion of remainders and the meaning of tenths.

Students should have many opportunities to solve and create word problems for the purpose of answering real-life questions of personal interest. These opportunities provide students with a chance to practice their computational skills and clarify their mathematical thinking.

## SCO: N6: Demonstrate, with and without concrete materials, an understanding of division (3digit by 1 -digit) and interpret remainders to solve problems. <br> [C, CN, PS]

## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Model the division process as equal sharing using base ten blocks and record it symbolically.
- Explain that the interpretation of a remainder depends on the context:
- ignore the remainder, e.g., making teams of 4 from 22 people
- round up the quotient, e.g., the number of five passenger cars required to transport 13 people
- express remainders as fractions, e.g., five apples shared by two people
- express remainders as decimals, e.g., measurement and money.
- Solve a given division problem in context using personal strategies and record the process.

SCO: N6: Demonstrate, with and without concrete materials, an understanding of division (3digit by 1 -digit) and interpret remainders to solve problems.
[C, CN, PS]

## Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students the opportunity to solve division problems using base ten materials.
- Present division questions in a problem solving situation.
- Provide regular practice and discussion of estimation strategies to support division.
- Have students create and share problems involving division.


## Suggested Activities

- Ask students to write a word problem involving division by a 2-digit number for each of the following:
a) a situation in which the remainder would be ignored.
b) a situation in which the remainder would be rounded up.
c) a situation in which the remainder would be part of the answer.
- Ask the student to tell what division is being modelled below and to provide a word problem that would apply to the model.

$$
100 \div 32=3 R 4
$$



SCO: N6: Demonstrate, with and without concrete materials, an understanding of division (3digit by 1-digit) and interpret remainders to solve problems. [C, CN, PS]

## Assessment Strategies

- Ask the student to use materials to model how to divide 489 by 7.
- Ask: At the T-Shirt Shop, you can buy t-shirts in packages of 8. One package costs \$130. At "Big Deals", a t-shirt costs \$18. Does "Big Deals" have the better price? How do you know?
- Ask: Jenna solved a problem by dividing 288 by 4. She said the answer was 72 . What could the problem have been?


## SCO: N7: Demonstrate an understanding of fractions by using concrete and pictorial representations to: <br> - create sets of equivalent fractions <br> - compare fractions with like and unlike denominators. <br> [C, CN, PS, R, V]

[C] Communication
[PS] Problem Solving
[CN] Connections
[ME] Mental Math
[R] Reasoning
and Estimation

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :--- | :--- | :--- |
| N8 Demonstrate an | N7 Demonstrate an <br> understanding of fractions less <br> understanding of fractions <br> by using concrete and <br> concrete and pictorial | N4 Relate improper fractions to <br> mixed numbers. |
| representations to: | pictorial representations to: <br> - create sets of equivalent <br> name and record fractions for <br> fractions |  |
| the parts of a whole or a set |  |  |
| - compare fractions with like and |  |  |
| - model and explain that for |  |  |
| different wholes, two identical |  |  |
| fractions may not represent |  |  |
| the same quantity denominators. |  |  |
| provide examples of where |  |  |
| fractions are used. |  |  |

## Elaboration

Developing number sense with fractions takes time and is best supported with a conceptual approach and the use of materials. Using a variety of manipulatives helps students understand properties of fractions and realize that the relationship between the two numbers in a fraction is the focus. A fraction does not say anything about the size of the whole.

Students should continue to use conceptual methods to compare fractions. These methods include: i) comparing each to a reference point; ii) comparing the two numerators when the fractions have the same denominator; and iii) comparing the two denominators when the fractions have the same numerator. A common error made by students at this level is to think, for example, that $\frac{4}{7}$ is greater than
$\underline{4}$ because of their experience comparing whole numbers.
$\overline{6}$
Considerable time needs to be spent on activities and discussion to develop number sense of fractions. Provide students with a variety of experiences using different models (number lines, pattern blocks, counters, etc.) and different representations of the whole with the same model. Students should recognize that a fraction can name part of a set as well as part of a whole and the size of these can change. Students also need to understand that fractions can only be compared if they are parts of the same whole. Half of a cake can not be compared to half a brownie. When comparing one half and one quarter, the whole is the "unit" (1).

It is important that students are able to visualize equivalent fractions as the naming of the same region or set partitioned in different ways. At this stage, a rule about multiplying numerators and denominators to form equivalents should not be offered. Such a rule could be confirmed if students observe it; however, the explanation should be connected to manipulatives.

## SCO: N7: Demonstrate an understanding of fractions by using concrete and pictorial representations to: <br> - create sets of equivalent fractions <br> - compare fractions with like and unlike denominators. <br> [C, CN, PS, R, V]

## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Create a set of equivalent fractions and explain why there are many equivalent fractions for any given fraction using concrete materials.
- Model and explain that equivalent fractions represent the same quantity.
- Determine if two given fractions are equivalent using concrete materials or pictorial representations.
- Formulate and verify a rule for developing a set of equivalent fractions.
- Identify equivalent fractions for a given fraction.
- Compare two given fractions with unlike denominators by creating equivalent fractions.
- Position a given set of fractions with like and unlike denominators on a number line and explain strategies used to determine the order.


## SCO: N7: Demonstrate an understanding of fractions by using concrete and pictorial representations to: <br> - create sets of equivalent fractions <br> - compare fractions with like and unlike denominators. <br> [C, CN, PS, R, V]

## Instructional Strategies

Consider the following strategies when planning lessons:

- Provide the students with a variety of activities that include the three interpretations of fractions: 1) part of a whole (e.g., part a chocolate bar); 2) part of a set (e.g., part of 30 marbles); and 3) part of a linear measurement (e.g., part of a 4 m baseboard).
- Provide many opportunities for students to model fractions both concretely and pictorially, using a variety of models such as, pattern blocks, grid paper, fraction pieces, fraction towers, Cuisenaire ${ }^{\circledR}$ rods, counters, egg cartons, number lines, etc.
- Point out to the student that to rename $\frac{6}{8}$ as $\frac{3}{4}$, you can "clump" the 8 sections of the whole into 2 s . There are then four groups of 2 sections; three of the four groups are shaded.
- Use number lines and other models to compare fractions.



## Suggested Activities

- Have the student make a diagram and identify the "clump size" that should be used to show that $\frac{10}{15}=\frac{2}{3}$. Ask how one might predict the "clump size" without drawing the diagram.
- Fold a piece of paper into fourths. Colour $\frac{1}{4}$. Fold the paper again. What equivalent fraction is represented? Fold the paper again. What equivalent fraction is shown? Discuss the pattern.
- Have students prepare a poster showing all the equivalent fractions they can find using a set of no more than 30 pattern blocks.
- Give students a sheet with 4 squares. Have them shade $\frac{3}{4}$ on each square vertically. Have them subdivide each square with a different number of horizontal lines. Use the resulting pictures to find possible equivalent fractions for $\frac{3}{4}$.

- Ask the student to use his/her fingers and hands to show that $\frac{1}{2}$ and $\frac{5}{10}$ are equivalent fractions. Alternatively, the student might be asked to choose a manipulative of choice to show this or some other equivalence.


## SCO: N7: Demonstrate an understanding of fractions by using concrete and pictorial representations to: <br> - create sets of equivalent fractions <br> - compare fractions with like and unlike denominators. <br> [C, CN, PS, R, V]

## Assessment Strategies

- Have students create a diagram to show why $\frac{4}{8}=\frac{1}{2}$ are equivalent.
- Give students an equation expressing equivalence between two fractions but with one of the terms missing. Find the missing term and explain your solution.

$$
\frac{4}{10}=\frac{\square}{5}
$$

- Have students place the following fractions on a number line: $\frac{1}{2}, \frac{9}{10}, \frac{4}{5}, \frac{1}{5}$.
- Have students write two equivalent fractions for the following diagram.



## SCO: N8: Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically. <br> [C, CN, R, V]

| [C] Communication | [PS] Problem Solving | [CN] Connections | [ME] Mental Math |
| :--- | :--- | :--- | :--- |
| [T] Technology | [V] Visualization | [R] Reasoning | and Estimation |

Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :--- | :--- | :--- |
| N9 Describe and represent <br> decimals (tenths and <br> hundreths) concretely, <br> pictorially and symbolically. | N8 Describe and represent <br> decimals (tenths, hundredths, <br> thousandths) concretely, <br> pictorially and symbolically. |  |
|  |  |  |

## Elaboration

Students need to understand that anything to the right of the decimal represents a quantity less than one. Students should continue to use physical materials to represent or model decimals. In this way, they can better see the relationship between hundredths and thousandths.

0.57

0.236

Alternatively, base ten blocks might be used to illustrate the relationship.


Students can also represent thousandths using a metre stick or tape measure since $1 \mathrm{~mm}=0.001 \mathrm{~m}$. For example, 0.423 m can be represented as 423 mm or 42.3 cm ("a little more than 42 cm ").

```
SCO: N8: Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically.
[C, CN, R, V]
```


## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Write the decimal for a given concrete or pictorial representation of part of a set, part of a region or part of a unit of measure.
- Represent a given decimal using concrete materials or a pictorial representation.
- Represent an equivalent tenth, hundredth or thousandth for a given decimal using a grid.
- Express a given tenth as an equivalent hundredth and thousandth.
- Express a given hundredth as an equivalent thousandth.
- Describe the value of each digit in a given decimal.

```
SCO: N8: Describe and represent decimals (tenths, hundredths, thousandths) concretely,
    pictorially and symbolically.
    [C, CN, R, V]
```


## Instructional Strategies

Consider the following strategies when planning lessons:

- Writing decimals using place value language and expanded notation can help explain equivalence of decimals.
\(\left.\begin{array}{l}0.4=4 tenths <br>
0.40=4 tenths+0 hundredths <br>

0.400=4 tenths+0 hundredths+0 thousandths\end{array}\right\}\)| Since adding zeros have |
| :--- |
| no effect, 0.4 must equal |
| 0.40 and 0.400 |

- Use the same sized tenth, hundredths and thousandths grid squares to draw equivalent decimals.
- Help students extend the place value system to decimals by focusing on the basic pattern of ten. While building on their understanding of tenths and hundredths from grade 4, students need to know that it takes 1000 equal parts (thousandths) to make one whole.
- Vary the representation of the whole. Use a cube, flat, or rod. Students will have a fixed notion of what these models represent and it is important to reinforce the idea that a decimal relates a part to a whole the same way that fractions do.
- Provide opportunities for students to read decimals in context. Saying decimals correctly will help students make the connection between decimals and fractions (N9) 3.147 should be read as "three and one hundred forty-seven thousandths" not "three point one four seven".


## Suggested Activities

- Present a riddle to the class such as, "I have 25 hundredths and 4 tenths. What am I?" Have students use a model of their choice to represent the solution to the riddle.
- Make sets of cards showing decimals in different forms including expanded form, pictorial representations and equivalent decimals. Students can play matching games or decimal snap.
- Provide opportunities for students to find and share how large numbers are represented in newspapers and magazines. For example, a CEO's salary may be written as 4.5 million dollars.
- Place five different displays of combinations of base-ten blocks. Ask the students to visit the centre and record the five decimals displayed.
- Provide students with two hundredths disks, each of a different color. Cut each disk along one radius so they can be fit together. Students can use these to model given decimals, or to write decimals from a given model.
- Give students three number cubes. Have them make the greatest and least possible decimals using the numbers rolled as the digits. Have students read the decimal numbers aloud.
- Use the calculator to "count". Enter $+0.1=,=\ldots$ when the display shows 0.9 have students predict what number will be next. Extend this to use 0.01 and 0.001 to demonstrate the relative magnitude of hundredths and thousandths.
- Give students a drawing of an irregular shape and have them shade in approximately 0.247 of it.

```
SCO: N8: Describe and represent decimals (tenths, hundredths, thousandths) concretely,
    pictorially and symbolically.
    [C, CN, R, V]
```


## Assessment Strategies

- Ask the student to express 0.135 in at least three different ways.
- Tell the student that gasoline is priced at 83.9 C per litre. Ask: What part of a dollar is this?
- Ask the student to explain why newspapers might record a number as 2.5 million rather than as 2500 000. Ask him/her to discuss whether or not this is a good idea.
- Ask the students to write 10 different decimal numbers that have tenths, hundredths and/or thousandths. Have them make base-ten block pictures that would represent their numbers.
- Ask students use hundredths and thousandths grids or base ten blocks to model equivalent decimals.
- Ask the student to write the numerals for "two hundred fifty-six thousandths" and "two hundred and fifty-six thousandths". Ask the student to explain why watching and listening for "and" is important when interpreting numbers.

SCO: N9: Relate decimals to fractions (to thousandths).
[CN, R, V]
N10: Compare and order decimals (to thousandths) by using:

- Benchmarks
- place value
- equivalent decimals.
[CN, R, V]

| [C] Communication | [PS] Problem Solving | [CN] Connections <br> [T] Technology | [R] Visualization Reasoning |
| :--- | :--- | :--- | :--- |$\quad$| [ME] Mental Math |
| :---: |
| and Estimation |

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :--- | :--- | :--- |
| N10 Relate decimals to fractions <br> (to hundredths). | N9 Relate decimals to fractions <br> and fractions to decimals (to |  |
|  | thousandths). |  |
|  | N10 Compare and order decimals |  |
|  | (to thousandths), by using: |  |
| - benchmarks |  |  |
| - place value |  |  |
| - equivalent decimals. |  |  |
|  |  |  |

## Elaboration

Decimals are simply another way of writing fractions. Students should continue to build their conceptual understanding of the relationship of decimals to fractions as they explore numbers to the thousandths. One thousandth (0.001) can be written as $\frac{1}{1000}$. Students should be encouraged to read decimals as
fractions. e.g., 0.246 is read as 246 thousandths and can be written as $\frac{246}{1000}$. Measurement contexts provide valuable learning experiences for decimal numbers because any measurement can be written in an equivalent unit that requires decimals e.g., one metre is $\frac{1}{1000}$ of a kilometre, ( $1 \mathrm{~m}=0.001 \mathrm{~km}$ ). To develop decimal and fractional number sense, it is essential to discuss the magnitude of the number, such as 493 thousandths is about one half, and 1.761 is about $1 \frac{3}{4}$. By using number lines with benchmarks such as $\frac{1}{4}(0.25), \frac{1}{2}(0.5), \frac{3}{4}(0.75)$ we can help to create a visual for students.
Students should be able to determine which of two decimal numbers is greater by comparing the whole number parts first and then the amounts to the right of the decimals. It is important that students understand decimal numbers do not need the same number of digits after the decimal to be compared. For example, one can quickly conclude that $0.8>0.423$, without converting 0.8 to 0.800 , because the former is much more than half and the latter is less than half. Other common misconceptions include students thinking 0.101 is greater than 0.11 because 101 is larger than 11; others thinking it is less just because it has thousandths while the other number has only hundredths. These same students would also say 0.101 is less than 0.1 because it has thousandths while 0.1 has only tenths. Such misconceptions are best dealt with by having students make base ten block representations of numbers that are being compared. Decimal equivalence becomes apparent when the connection is made between decimals and fractions e.g., $0.3=\frac{3}{10}$ or $\frac{30}{100}$ or $\frac{300}{1000}$

## SCO: N9: Relate decimals to fractions (to thousandths). <br> [CN, R, V]

N10: Compare and order decimals (to thousandths) by using:

- Benchmarks
- place value
- equivalent decimals.
[CN, R, V]


## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

## N9

- Write a given decimal in fractional form.
- Write a given fraction with a denominator of 10,100 or 1000 as a decimal.
- Express a given pictorial or concrete representation as a fraction or decimal, e.g., 250 shaded squares on a thousandth grid can be expressed as 0.250 or $\frac{250}{1000}$.


## N10

- Order a given set of decimals by placing them on a number line that contains benchmarks, $0.0,0.5$, 1.0.
- Order a given set of decimals including only tenths using place value.
- Order a given set of decimals including only hundredths using place value.
- Order a given set of decimals including only thousandths using place value.
- Explain what is the same and what is different about $0.2,0.20$ and 0.200 .
- Order a given set of decimals including tenths, hundredths and thousandths using equivalent decimals.


## SCO: N9: Relate decimals to fractions (to thousandths).

[CN, R, V]
N10: Compare and order decimals (to thousandths) by using:

- Benchmarks
- place value
- equivalent decimals.
[CN, R, V]


## Instructional Strategies

## Consider the following strategies when planning lessons:

- Have students place only decimal tenths on a number line, then repeat for decimal hundredths and thousandths.

- Have students use thousandths grids to model the equivalency of tenths, hundredths and thousandths (e.g., $0.3,0.30,0.300$ ) and explain what is the same and what is different.
- Have students order a given set of decimals including tenths, hundredths and thousandths using equivalent decimals. For example, to order $0.402,0.39$ and 0.4 , students should be encouraged to think of them as thousandths ( $0.402,0.390,0.400$ ).
- Have students begin to explore the relationship between fraction and decimal benchmarks. For example, 0.5 is another name for $\frac{1}{2} ; 0.25$ is another name for $\frac{1}{4} ; 0.75$ is another name for $\frac{3}{4}$.
- Represent decimals in a variety of ways. For example: 0.452 is $\frac{452}{1000}$ and can be expressed as $0.4+0.05+0.002$ or $\frac{4}{10}+\frac{5}{100}+\frac{2}{1000}$
- Provide a variety of models, stressing the magnitude of the number, e.g., 0.452 could be modeled using a number line (about one half), base 10 blocks, thousandths grids, and place value chart.


## Suggested Activities

- Have students express given numbers as fractions and decimals e.g., sixty-four hundredths $\left(\frac{64}{100}, 0.64\right)$
- Ask students to investigate where in the media fractions and decimals are used, and to write a report on their findings.
- Give students a "number of the day" and have them express this number in as many ways as they can. For example: 0.752 could be shown as: $\frac{752}{1000}$ or $\frac{7}{10}+\frac{5}{100}+\frac{2}{1000}$; about $\frac{3}{4}$; plotted on a number line; modelled with base ten materials on a place value chart; shown on a thousandths grid; or described in a variety of ways ("It's 0.248 less than one whole", etc.).
- Give each student a different irregular shape and ask him/her to tear off about 0.256 of that shape. Have students explain how they estimated 0.256 , and why pieces may not be the same size or shape.

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SCO: N9: Relate decimals to fractions (to thousandths).
    [CN, R, V]
    N10: Compare and order decimals (to thousandths) by using:
    - Benchmarks
    - place value
    - equivalent decimals.
    [CN, R, V]
```


## Assessment Strategies

- Have students compare and order decimals tenths, hundredths and thousandths and express them as fractions.
- Have students model decimal thousandths using base ten materials.

2.044
- Have students place decimals and fractions on a number line, such as: $\frac{3}{4}, 0.31, \frac{6}{10}, \frac{102}{1000}$.
- Tell students that they have properly placed 796 pieces of the 1000 -piece jigsaw puzzle. Ask what part (fractional and decimal) of the puzzle has been completed? What part of the puzzle has yet to be finished? $\left(\frac{204}{1000}, 0.204\right)$



## SCO: N11: Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths). <br> [ $\mathrm{C}, \mathrm{CN}, \mathrm{PS}, \mathrm{R}, \mathrm{V}$ ]

| $[$ C] Communication | [PS] Problem Solving | [CN] Connections | [ME] Mental Math |
| :--- | :--- | :--- | :--- |
| [T] Technology | [V] Visualization | and Estimation |  |

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :--- | :--- | :--- |
| N11 Demonstrate an <br> understanding of addition and <br> subtraction of decimals (limited <br> to hundredths) by: <br> - using compatible numbers <br> - estimating sums and <br> differences using mental math <br> strategies <br> to solve problems. | N11 Demonstrate an <br> understanding of addition <br> and subtraction of decimals <br> (limited to thousandths). | N8 Demonstrate an <br> understanding of multiplication <br> and division of decimals (1-digit <br> whole number multipliers and 1- <br> digit natural number divisors). |

## Elaboration

It is essential that students recognize that all of the properties established and strategies developed for the addition and subtraction of whole numbers apply to decimals. For example, adding or subtracting tenths (e.g., 3 tenths and 4 tenths are 7 tenths) is similar to adding or subtracting quantities of other items (e.g., 3 apples and 4 apples are 7 apples). This could be extended to addition with tenths that total more than one whole (e.g., 7 tenths and 4 tenths are 11 tenths or 1 and 1 tenth). The same is true with hundredths and thousandths. Rather than simply telling students to line up decimals vertically, or suggesting that they "add zeroes," they should be directed to think about what each digit represents and what parts go together. For example, to add 1.625 and 0.34 , a student might think using front end addition, 1 whole, $9(6+3)$ tenths and $6(2+4)$ hundredths, and 5 thousandths or 1.965 .

Base-ten blocks and hundredths grids continue to be useful models. If a flat represents one whole unit, then $3.7+1.54$ would be modeled as:


Students need to recognize that estimation is a useful skill in their lives. To be efficient when estimating sums and differences mentally, students must be able to access a strategy quickly and they need a variety from which to choose. An example may be using front estimation e.g., for $9.65+8.106$, think of $9+8$, so the sum is greater than 17. Situations must be provided regularly to ensure that students have sufficient practice with mental math strategies and that they use their skills as required. When a problem requires an exact answer, students should first determine if they are able to calculate it mentally; this should be an automatic response.

## SCO: N11: Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths). <br> [C, CN, PS, R, V]

## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Place the decimal point in a sum or difference using front-end estimation, e.g., for $6.3+0.25+306.158$, think $6+306$, so the sum is greater than 312 .
- Correct errors of decimal point placements in sums and differences without using paper and pencil.
- Explain why keeping track of place value positions is important when adding and subtracting decimals.
- Predict sums and differences of decimals using estimation strategies.
- Solve a given problem that involves addition and subtraction of decimals, limited to thousandths.

> SCO: N11: Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths). [C, CN, PS, R, V]

## Instructional Strategies

Consider the following strategies when planning lessons:

- Provide opportunity for students to model and solve addition and subtraction questions involving tenths, hundredths and thousandths concretely, pictorially, and symbolically (e.g., thousandths and hundredths grids, base ten blocks, and number lines.)
- Present addition and subtraction questions both horizontally and vertically to encourage alternative computational strategies. For example, for $1.234+1.990$, students might calculate:
$1.234+2=3.234$ followed by $3.234-0.01=3.224$.
- Have students investigate the relationship between adding decimals numbers and whole numbers. For example, $356+232=588$; this looks similar to $0.356+0.232=0.588$.
- Provide problem solving situations that require students to add or subtract decimals using a variety of strategies.
- Have students estimate first when asked to solve problems involving adding and or subtracting decimals.


## Suggested Activities

- Provide base ten blocks or thousandths grids. Have the student choose addition or subtraction questions involving decimal numbers to represent with the models.
- Model 4.23 and 1.359 with base ten blocks or thousandths grids. Ask the student to use the materials to explain how to find the difference between the two numbers.
- Provide students with the batting averages of some baseball players. Have them calculate the spread between the player with the highest average and the one with the lowest. Have students create problems using the averages on the list.
- Request that the students provide examples of questions in which two decimal numbers are added and the answers are whole numbers.
- Tell students that you have added three numbers, each less than 1 , and the result is 2.4. Ask if all the decimal numbers could be less than one half and to explain why or why not. Once students realize the numbers cannot all be less than one-half, ask them how many could be less.
- Have students find situations in which decimals are added and subtracted beyond the classroom and present their findings to the class.


## SCO: N11: Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths). <br> [C, CN, PS, R, V]

## Assessment Strategies

- Ask the students to fill in the boxes so that the answer for each question is 0.4 . The only stipulation is that the digit 0 cannot be used to the right of the decimal points.

- Present the following situation in which Jane made an error when she subtracted. Ask the students what could be said to Jane to help her understand why the answer is incorrect: $5.23-1.453=3.783$.
- Provide students with addition and subtraction questions in which the decimal is missing from the sum or difference. Have students place the decimal in the correct position.
- Use an example to explain why it is important to keep track of place value positions when adding and subtracting decimal numbers. (This can be written in a journal.)
- Have students solve problems such as (ensure that students provide an estimate):
- John needs 2 kg of hamburger for a recipe. He has a 0.750 kg package. How much more does he need to buy?
- Sasha bought two books at the book fair. One was $\$ 6.95$ and the other was $\$ 7.38$. How much change will she get from a $\$ 20$ bill?


## PATTERNS AND RELATIONS

## SCO: PR1: Determine the pattern rule to make predictions about subsequent elements.

 [C, CN, PS, R, V][C] Communication
[T] Technology
[PS] Problem Solving
[V] Visualization
[CN] Connections
[R] Reasoning
[ME] Mental Math and Estimation

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :--- | :--- | :--- |
| PR1 Identify and describe <br> patterns found in tables and <br> charts, including a multiplication <br> chart. | PR1 Determine the pattern rule to <br> make predictions about <br> subsequent elements. | PR1 Demonstrate an <br> understanding of the relationship <br> within tables of values to solve <br> problems. |
|  |  | PR2 Represent and describe <br> patterns and relationships using <br> graphs and tables. |

## Elaboration

Patterns are key to understanding mathematical concepts. The ability to create, recognize and extend patterns is essential for making generalizations, seeing relationships, and understanding the order and logic of mathematics (Burns, 2007; p.144). These skills provide the foundation for algebraic reasoning and inquiry.

Patterns represent identified regularities based on rules describing the patterns' elements. Unless a pattern rule is provided there is no single way to extend a pattern (e.g., 1, 3, 5, 7 might be an odd number sequence, or a repeating sequence $1,3,5,7,1,3,5,7 \ldots)$.

Patterns can be used to represent a situation and to solve problems. They can be extended with and without concrete materials and can be described using mathematical language. When discussing a pattern, students should be encouraged to determine how each step in the pattern is different from the preceding step.


Tables and charts provide an opportunity to display patterns and see relationships. For most students, these tables and charts make it easier to see the patterns from one step to the next. When a chart has been constructed, the differences from one step to the next can be written by it. Students will probably first observe the pattern from one step to the next; however, using the chart to find the twentieth or hundredth step is not reasonable. If a rule or relationship can be discovered, any table entry can be determined without building or calculating all of the intermediate entries. Some students might recognize that the rule can be described as a mathematical expression. For example, in the above pattern, the rule could be described as $3 n$. The $20^{\text {th }}$ step would be $20 \times 3$ (60).

## SCO: PR1: Determine the pattern rule to make predictions about subsequent elements.

$$
[\mathrm{C}, \mathrm{CN}, \mathrm{PS}, \mathrm{R}, \mathrm{~V}]
$$

## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Extend a given pattern with and without concrete materials, and explain how each element differs from the preceding one.
- Describe, orally or in writing, a given pattern using mathematical language, such as one more, one less, five more.
- Write a mathematical expression to represent a given pattern, such as $r+1, r-1, r+5$.
- Describe the relationship in a given table or chart using a mathematical expression.
- Determine and explain why a given number is or is not the next element in a pattern.
- Predict subsequent elements in a given pattern.
- Solve a given problem by using a pattern rule to determine subsequent elements.
- Represent a given pattern visually to verify predictions.

SCO: PR1: Determine the pattern rule to make predictions about subsequent elements. [C, CN, PS, R, V]

## Instructional Strategies

## Consider the following strategies when planning lessons:

- Have students practice extending patterns with materials and drawings and then translate the pattern elements to a table or T-chart. Ask them to describe what is happening as the pattern grows, and how the new step is related to the previous one.
- Have students describe, using mathematical language (e.g., one more, seven less) and symbolically (e.g., $r+1, p-7$ ), a pattern represented concretely, pictorially, or from a chart.
- Have students verify whether or not a particular number belongs to a given pattern.
- Have students solve problems and make decisions based upon the mathematical analysis of a pattern and other contributing factors.


## Suggested Activities

- Show students the first three or four steps of a pattern. Provide them with appropriate materials and/or grid paper and have them extend the patterns recording each step, and explain why their extension follows the pattern. Have them determine the pattern rule.



- Have students examine number sequences to determine subsequent terms and explain their extensions. Ask students to determine the pattern rule.
$1,4,7,10,13, \ldots$
$20,19,16,11, \ldots$
$0,2,6,14,30, \ldots$
$1,2,5,11,23, \ldots$
- Ask students to work in pairs to explore the many patterns on a multiplication chart (e.g., square numbers on the diagonal, sums of rows and columns, adjacent square patterns, doubling between columns, such as the 2's, 4's, and 8's).
- Provide students with a growing pattern and have them extend it. They should make a table showing how many items are needed to make each step of the pattern. Have them predict the number of items in the tenth or twentieth step of the pattern. For example, four people can sit at one table, six people can sit at two tables pushed together, eight people can sit at three tables. How many can sit at ten tables? Twenty? How many tables are needed for 24 people?


| Number of tables | 1 | 2 | 3 | 4 | $\ldots$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Number of seats | 4 | 6 | 8 | $?$ | $\ldots$ |

This pattern could be displayed on a T-chart:

| Tables | Seats |
| :--- | :--- |
|  |  |

SCO: PR1: Determine the pattern rule to make predictions about subsequent elements.
[C, CN, PS, R, V]

## Assessment Strategies

- Have students fill in missing elements from number sequences and identify pattern rules.
e.g., 1, 4, $\qquad$ , 16, $\qquad$ 36

2, 5, 11, 23, $\qquad$ , $\qquad$
2.4, 2.7, $\qquad$ , $\qquad$ , 3.6

- Give the students an input/output machine and ask them to provide different possible rules.


What's My Rule?
Some possibilities: Add 7; $3 n+1 ; 4 n-2$

- Have students extend patterns concretely or pictorially, then complete tables and identify pattern rules.
- Have students solve real-world problems that require identifying a pattern rule to determine subsequent elements. For example, to bake cookies for a school bake sale, the quantities of ingredients in the recipe must be determined for multiple batches of cookies. If $\frac{2}{3}$ cups of sugar, and $1 \frac{1}{2}$ cups flour is needed for one batch, how much is needed for 4 batches? 7 batches?

SCO: PR2: Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.
[C, CN, PS, R]

| [C] Communication | [PS] Problem Solving | [CN] Connections | [ME] Mental Math |
| :--- | :--- | :--- | :--- |
| [T] Technology | [V] Visualization | [R] Reasoning | and Estimation |

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :--- | :--- | :--- |
| PR5 Express a given problem | PR2 Solve problems involving | PR3 Represent generalizations <br> as an equation in which a <br> symbol is used to represent an <br> unknown number. |
| single-variable, one-step <br> equations with whole number <br> PRolve one-step equations <br> involving a symbol to represent <br> an unknown number. | aricients and whole number <br> relationships using |  |
| solutions. | equations with letter variables. <br> PR4 Demonstrate and explain <br> the meaning of preservation of <br> equality concretely, pictorially <br> and symbolically. |  |

## Elaboration

Exploring patterns leads to algebraic thinking. Algebra is a system that allows us to represent and explain mathematical relationships. Students are thinking algebraically when they solve open number sentences like $5+\square=13$, first using boxes or open frames, then using letters, $5+n=13$. When letters or open frames are used in mathematics, they are called variables. Students usually progress from the use of open frame to letters. It is useful for students to think of variables as numbers that can be operated on and manipulated like other numbers.

An equation is a mathematical sentence with an equal sign. An expression does not include an equal sign and is used most frequently to describe a pattern rule. A coefficient is a quantity (usually a numerical constant), which is multiplied by another quantity following it in an expression or an equation; e.g., in the algebraic equation $4 p+2=10$, the coefficient is 4 .

In order to solve an equation, we need to find the value of the variable to make the equation true. Using the balance concept on a regular basis will help students develop a visual image for solving equations.


Students have been exploring the concept of equality since grade two. It is important for students to recognize that the equal sign indicates that both sides of the equation are balanced and does not simply mean "the answer is".

## SCO: PR2: Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions. <br> [C, CN, PS, R]

## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Explain the purpose of the letter variable in a given addition, subtraction, multiplication or division equation with one unknown; e.g., $36 \div n=6$.
- Express a given pictorial or concrete representation of an equation in symbolic form.
- Express a given problem as an equation where the unknown is represented by a letter variable.
- Create a problem for a given equation with one unknown.
- Solve a given single-variable equation with the unknown in any of the terms; e.g., $n+2=5,4+a=7$, $6=r-2,10=2 c$.
- Identify the unknown in a problem; represent the problem with an equation; and solve the problem concretely, pictorially or symbolically.

SCO: PR2: Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.
[C, CN, PS, R]

## Instructional Strategies

Consider the following strategies when planning lessons:

- Build on the students' knowledge from the previous grades to write addition, subtraction, multiplication and division equations. Connect the concrete (use models such as counters and balance scales) with pictorial and symbolic representations consistently as the students develop and demonstrate understanding of equations.
- Use everyday contexts for problems to which the students can relate so that they can translate the meaning of the problem into an appropriate equation using a letter to represent the unknown number.
- Have the students create problems for a variety of number sentences using the four operations.
- Explain that if the same variable, or unknown, is used repeatedly in the same equation, then there is only one possible solution for that variable or unknown; e.g., for $n+n=20$ can be written as $2 n=20$.
- Have students complete tables such as:

| $n$ | $3 n$ |
| :---: | :---: |
| 3 | 9 |
| 8 |  |
|  | 30 |
| 12 |  |

## Suggested Activities

- Have students play "Solve for my variable" I subtract 6 from $n$ and have 13 left. What is $n$ ?
Four more than $p$ is 37 . What is $p$ ?
Possible extension: Two more than $3 x$ is 23 . What is $x$ ? or One less than $4 k$ is 27 . What is $k$ ?
- Provide simple story problems and ask students to write equations. Include stories for all four operations. For example:
I had birthday money and spent $\$ 6.25$. I now have $\$ 8.75$. ( $n-\$ 6.25=\$ 8.75$ or $\$ 6.25+\$ 8.75=n$ )
There are 3 full boxes of pencils. There are 36 pencils in all. $(3 a=36)$
- Provide one-step single-variable equations and have students create story problems.
- Have students write equations for balances such as the ones below, using letters for the variables.


SCO: PR2: Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.
[C, CN, PS, R]

## Assessment Strategies

- Have students solve single-variable, one-step equations such as $18+n=31 ; 9=43-p ; 8 x=56$; $m \div 6=7$
- Write word problems that could be represented by each of the equations in the previous task.
- Have students complete tables using whole number coefficients.
- Have students write equations to describe the balance representations, such as the following:



## SHAPE AND SPACE

SCO: SS1: Design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions.
[C, CN, PS, R, V]
[C] Communication
[PS] Problem Solving
[CN] Connections
[R] Reasoning
[ME] Mental Math and Estimation

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :---: | :---: | :---: |
| SS3 Demonstrate an understanding of area of regular and irregular 2-D shapes by <br> - recognizing that area is measured in square units <br> - selecting and justifying referents for the units $\mathrm{cm}^{2}$ or $m^{2}$ <br> - estimating area by using referents for $\mathrm{cm}^{2}$ or $\mathrm{m}^{2}$ <br> - determining and recording area <br> - constructing different rectangles for a given area ( $\mathrm{cm}^{2}$ or $\mathrm{m}^{2}$ ) in order to demonstrate that many different rectangles may have the same area. | SS1 Design and construct different rectangles, given either perimeter or area, or both (whole numbers), and draw conclusions. | SS3 Develop and apply a formula for determining the: <br> - perimeter of polygons <br> - area of rectangles <br> - volume of right rectangular prisms. |

## Elaboration

Area is the measure of the space inside a region or how much it takes to cover a region; perimeter is the distance around a region. Students in grade five often do not make the distinction between area and perimeter and may calculate the area instead of perimeter or vice versa. Therefore, it is important that they have many opportunities to construct rectangles of different areas and perimeters concretely and pictorially.

Area and perimeter involve measuring length. Formulas may be part of what comes out of the activities but is not an essential learning at this point. When the students are able to measure efficiently and effectively using standard units, their learning experiences can be directed to situations that encourage them to construct measurement formulas. When determining the area of a rectangle, students may realize as they count squares that it would be quicker to find the number of squares in one row and multiply this by the number of rows. When finding perimeters of rectangles, students may discover more efficient methods instead of adding all four sides to find the answer (e.g., add the length and width and double).

It is important that students learn about area and perimeter together. Through explorations, students will:

- discover that it is possible for a rectangle of a certain area to have different perimeters
- discover that is possible for rectangles with the same perimeter to have different areas.
- discover that the closer the shape is to a square, the larger the area will be.


SCO: SS1: Design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions.
[C, CN, PS, R, V]

## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Construct or draw two or more rectangles for a given perimeter in a problem-solving context.
- Construct or draw two or more rectangles for a given area in a problem-solving context.
- Illustrate that for any given perimeter, the square or shape closest to a square will result in the greatest area.
- Illustrate that for any given perimeter, the rectangle with the smallest possible width will result in the least area.
- Provide a real-life context for when it is important to consider the relationship between area and perimeter.


## SCO: SS1: Design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions. <br> [C, CN, PS, R, V]

## Instructional Strategies

Consider the following strategies when planning lessons:

- Have students use geoboards to construct rectangles with specified perimeters and discuss the areas.
- Assign specific areas (e.g., 12 square units) and have students use colour tiles to create rectangles and find the possible perimeters.
- Ask the students to use dot paper to compare the areas of rectangles with the following dimensions: $2 \mathrm{~cm} \times 3 \mathrm{~cm}, 4 \mathrm{~cm} \times 3 \mathrm{~cm}, 6 \mathrm{~cm} \times 3 \mathrm{~cm}$. Ask what they observe and have them give another set of dimensions that follows the same pattern and draw conclusions.


## Suggested Activities

- Ask the student to explain why the perimeter of rectangles with whole number side lengths is always even. Have them use words, drawings, and/or numbers in their explanation.
- Have students relate perimeters to areas. For example, give pairs of students 24 colour tiles and ask them to find different rectangles, each with the area of 24 square units, but with different perimeters. Ask them to find a way to keep track of their rectangles and perimeters. What rectangle has the largest perimeter? The smallest? Have students draw conclusions.
- Ask the students to draw three different rectangles with the same perimeter.
- Construct rectangles on grid paper with a given perimeter, and then compare the side lengths and the areas. Have students discuss their findings and conclusions with regard to side length and area.
- Use a $2 \times 3$ rectangle, have students predict what would happen to the area and perimeter if the side lengths were doubled, halved? Have students draw conclusions.


## SCO: SS1: Design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions. <br> [C, CN, PS, R, V]

## Assessment Strategies

- Have students compare and contrast a given pair of rectangles with the same perimeter.
- Ask students to choose the dimensions of the rectangle with the largest area and the smallest area from a set of rectangles with the same perimeter.
- Have students construct (concretely or pictorially) and record the dimensions of two or more rectangles with a specified perimeter. Have students select and justify dimensions that would be most appropriate in a particular situation.
- Have students construct (concretely or pictorially) and record the dimensions of as many rectangles as possible with a specified area and select, with justification, the rectangle that would be most appropriate in a particular situation.
- Ask students to identify situations relevant to self, family, or community where the solution to problems would require the consideration of both area and perimeter, and solve the problems.

SCO: SS2: Demonstrate an understanding of measuring length (mm) by:

- selecting and justifying referents for the unit mm
- modeling and describing the relationship between mm and cm units, and between mm and m units.
[ $\mathrm{C}, \mathrm{CN}, \mathrm{ME}, \mathrm{PS}, \mathrm{R}, \mathrm{V}$ ]

| [C] Communication |
| :--- | :--- | :--- | :--- |
| [T] Technology |$\quad$| [PS] Problem Solving |
| :--- | :--- | :--- |
| [V] Visualization |$\quad$| [CN] Connections |
| :--- |
| [R] Reasoning |$\quad$| [ME] Mental Math |
| :---: |
| and Estimation |

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :---: | :---: | :---: |
| SS3 Demonstrate an understanding of area of regular and irregular 2-D shapes by <br> - recognizing that area is measured in square units <br> - selecting and justifying referents for the units $\mathrm{cm}^{2}$ or $m^{2}$ <br> - estimating area by using referents for $\mathrm{cm}^{2}$ or $\mathrm{m}^{2}$ <br> - determining and recording area <br> - constructing different rectangles for a given area ( $\mathrm{cm}^{2}$ or $\mathrm{m}^{2}$ ) in order to demonstrate that many different rectangles may have the same area. | SS2 Demonstrate an understanding of measuring length ( mm ) by: <br> - selecting and justifying referents for the unit mm <br> - modeling and describing the relationship between mm and cm units, and between mm and $m$ units. |  |

## Elaboration

Measurement is fundamentally about making comparisons. At this point in their learning, students are able to compare two objects directly by accurately using standard units of length such as millimetres, centimetres, and metres.

Students will choose a personal referent for one millimetre and explain their choice. They should continue to use their referents for one centimetre and one metre developed in grade three. For example, one millimetre is about the thickness of a dime, one centimetre is about the width of your baby finger, and one metre is about the height of the doorknob.

Students need to learn how to choose the appropriate unit or combination of units for the task at hand. This choice depends on the magnitude of the length to be measured and the level of precision required by the task (Small, 2008; p. 379). For example, millimetres can be used to measure small objects or to measure larger objects with more precision. Students should recognize that 1 metre is 100 centimetres or 1000 millimetres and 1 centimetre is 10 millimetres and change from the smaller unit to the larger unit. Flexibility with using the different measurements is in the developmental stage and needs to be supported with a variety of materials. Students need to be able to rename measurements such as a pencil that is 11 cm long could also be described as 110 mm or 0.11 m , but also able to identify which unit is the most appropriate.

Using rulers, metre sticks, Cuisenaire ${ }^{\circledR}$ rods and base ten materials will provide students with benchmarks when estimating lengths. Students should be encouraged to estimate measurements before actually verifying them using a measuring device.

SCO: SS2: Demonstrate an understanding of measuring length (mm) by:

- selecting and justifying referents for the unit mm
- modelling and describing the relationship between mm and cm units, and between mm and m units.
[C, CN, ME, PS, R, V]


## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Provide a referent for one millimetre and explain the choice.
- Provide a referent for one centimetre and explain the choice.
- Provide a referent for one metre and explain the choice.
- Show that 10 millimetres is equivalent to 1 centimetre using concrete materials, e.g., ruler.
- Show that 1000 millimetres is equivalent to 1 metre using concrete materials, e.g., metre stick.
- Provide examples of when millimetres are used as the unit of measure.

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SCO: SS2: Demonstrate an understanding of measuring length (mm) by:
    - selecting and justifying referents for the unit mm
    - modeling and describing the relationship between mm and cm units, and
    between mm and m units.
    [C, CN, ME, PS, R, V]
```


## Instructional Strategies

Consider the following strategies when planning lessons:

- Help the students develop mental images of various measurement standards. To provide estimation practice, involve students in activities such as, "Show me (with hands or arms): 75 centimetres; 20 millimetres; 0.5 metres".
- Have students use the relationships between standard metric units to rename measurements when comparing them.
- Share a short paragraph describing the measurements of a variety of classroom items. Ask the students to insert the appropriate unit for each. For example, the table was 1524 $\qquad$ long. On it was a pencil that was 0.17 $\qquad$ long.
- Encourage students to think of their ruler, as well as a metre stick or base ten blocks, when estimating length. Most rulers are 30 cm (or 300 mm ) long and serve as good benchmarks. For example, 62 cm can be thought of as the length of about 2 rulers.


## Suggested Activities

- Ask the students to show, with fingers or arms, the following lengths: $550 \mathrm{~mm}, 60 \mathrm{~cm}, 0.25 \mathrm{~m}$. Have them describe the length using another unit of measure.
- Ask the student to rewrite 2.3 m using other metric units.
- Ask: If you change metres to centimetres, will the numerical value become greater or less? Why?
- Have students measure objects that do not measure exact centimetres, thus stressing the importance of millimetres when striving for precision of measurement.
- Hold a "Measurement Scavenger Hunt" in the classroom. Students should estimate the length of objects first, and then measure for accuracy.

```
SCO: SS2: Demonstrate an understanding of measuring length (mm) by:
    - selecting and justifying referents for the unit mm
    - modelling and describing the relationship between mm and cm units, and
        between mm and m units.
    [C, CN, ME, PS, R, V]
```


## Assessment Strategies

- Have students choose and use referents for $1 \mathrm{~mm}, 1 \mathrm{~cm}, 1 \mathrm{~m}$ to determine approximate linear measurements in situations relevant to self, family, or community and explain the choice.
- Have students generalize measurement relationships between $\mathrm{mm}, \mathrm{cm}$, and m from explorations using concrete materials (e.g., $10 \mathrm{~mm}=1 \mathrm{~cm}, 0.01 \mathrm{~m}=1 \mathrm{~cm}$ ).
- Have students provide examples of situations that are relevant to their life, family, or community in which linear measurements would be made and identify the standard unit ( $\mathrm{mm}, \mathrm{cm}$, or m ) that would be used for that measurement and justify the choice (e.g., heights of people, crafts).
- Ask students to draw, construct, or physically act out a representation of a given linear measurement.
- Have students pose and solve problems that involve hands-on linear measurements using either referents or standard units.

```
SCO: SS3: Demonstrate an understanding of volume by:
    - selecting and justifying referents for cm}\mp@subsup{}{}{3}\mathrm{ or m}\mp@subsup{}{}{3}\mathrm{ units
    - estimating volume by using referents for cm}\mp@subsup{}{}{3}\mathrm{ or m
    - measuring and recording volume (cm or m}\mp@subsup{}{}{3}
    - constructing rectangular prisms for a given volume.
    [C, CN, ME, PS, R, V]
```

| [C] Communication | [PS] Problem Solving | [CN] Connections | [ME] Mental Math |
| :--- | :--- | :--- | :--- |
| [T] Technology | [V] Visualization | [R] Reasoning | and Estimation |

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :---: | :---: | :---: |
|  | SS3 Demonstrate an understanding of volume by: <br> - selecting and justifying referents for $\mathrm{cm}^{3}$ or $\mathrm{m}^{3}$ units <br> - estimating volume by using referents for $\mathrm{cm}^{3}$ or $\mathrm{m}^{3}$ <br> - measuring and recording volume ( $\mathrm{cm}^{3}$ or $\mathrm{m}^{3}$ ) <br> - constructing rectangular prisms for a given volume. | SS3 Develop and apply a formula for determining the: <br> - perimeter of polygons <br> - area of rectangles <br> - volume of right rectangular prisms. |

## Elaboration

Volume and capacity are both terms used for measuring the size of three-dimensional regions. Although these two concepts are related, we will concentrate on volume for this specific outcome. Volume typically refers to the amount of space that an object takes up. Volume is measured with cubic centimetres and cubic metres (Van de Walle \& Lovin, vol. 2, 2006; p. 265).
Volume can also be used to refer to the capacity of a container.
Students should develop personal referents for cubic centimetres and cubic metres. The use of personal referents helps students establish the relationships between the units. Students should realize that a cubic centimetre is the size of a cube 1 cm on a side and a cubic metre the size of a cube 1 m on an edge. Being able to estimate the volume of various containers and then to measure in the appropriate unit is important as students begin to construct rectangular prisms of various sizes.
Students should be given opportunities to explore the size of a million using visualization. By building a cubic metre with metre sticks, they will to have a good mental image of $1 \mathrm{~m}^{3}$.

Students should have a sense of which volume or capacity unit is more appropriate to use in various circumstances.

## SCO: SS3: Demonstrate an understanding of volume by:

- selecting and justifying referents for $\mathrm{cm}^{3}$ or $\mathrm{m}^{3}$ units
- estimating volume by using referents for $\mathrm{cm}^{3}$ or $\mathrm{m}^{3}$
- measuring and recording volume ( $\mathrm{cm}^{3}$ or $\mathrm{m}^{3}$ )
- constructing rectangular prisms for a given volume.
[C, CN, ME, PS, R, V]


## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Identify the cube as the most efficient unit for measuring volume and explain why.
- Provide a referent for a cubic centimetre and explain the choice.
- Provide a referent for a cubic metre and explain the choice.
- Determine which standard cubic unit is represented by a given referent.
- Estimate the volume of a given 3-D object using personal referents.
- Determine the volume of a given 3-D object using manipulatives and explain the strategy.
- Construct a rectangular prism for a given volume.
- Explain that many rectangular prisms are possible for a given volume by constructing more than one rectangular prism for the same given volume.

SCO: SS3: Demonstrate an understanding of volume by:

- selecting and justifying referents for $\mathrm{cm}^{3}$ or $\mathrm{m}^{3}$ units
- estimating volume by using referents for $\mathrm{cm}^{3}$ or $\mathrm{m}^{3}$
- measuring and recording volume ( $\mathrm{cm}^{3}$ or $\mathrm{m}^{3}$ )
- constructing rectangular prisms for a given volume.
[C, CN, ME, PS, R, V]


## Instructional Strategies

Consider the following strategies when planning lessons:

- Identify the cube as the most efficient unit for measuring volume and explain why.
- Provide a referent for a cubic centimetre and explain the choice.
- Provide a referent for a cubic metre and explain the choice.
- Determine which standard cubic unit is represented by a given referent.
- Estimate the volume of a given 3-D object using personal referents.
- Determine the volume of a given 3-D object using manipulatives and explain the strategy.
- Construct a rectangular prism for a given volume.
- Explain that many rectangular prisms are possible for a given volume by constructing more than one rectangular prism for the same given volume.


## Suggested Activities

- Measure the volume of small rectangular prisms by counting the number of centimetre cubes it takes to build a duplicate of it.
- Use base ten blocks or linking cubes to build several different structures each with a set volume. Discuss the different dimensions of the rectangular prisms.
- Provide students with a pair of small boxes, exactly one block and a ruler. Have students decide which box has the greater volume. Students use words, drawings and numbers to explain their conclusions. Sample box dimensions: $(6 \times 3 \times 4)(5 \times 4 \times 4)(6 \times 6 \times 2)$.
- Have students build a cubic metre using metre sticks or other materials. Keep a model to use as a referent for $\mathrm{m}^{3}$.
- Use centimetre grid paper to make nets for two prisms that have the same volume, but different shapes.
- Have students research the volumes of moving trucks. Ask what is a reasonable estimate for the volume of all the furniture in a school or in a house?


## SCO: SS3: Demonstrate an understanding of volume by:

- selecting and justifying referents for $\mathrm{cm}^{3}$ or $\mathrm{m}^{3}$ units
- estimating volume by using referents for $\mathrm{cm}^{3}$ or $\mathrm{m}^{3}$
- measuring and recording volume ( $\mathrm{cm}^{3}$ or $\mathrm{m}^{3}$ )
- constructing rectangular prisms for a given volume.
[C, CN, ME, PS, R, V]


## Assessment Strategies

- Ask students to calculate the volume of each size of base-ten blocks.

- Ask the student to estimate the volume of the classroom in cubic metres and give an explanation as to how the estimate was determined.
- Tell the student that you need a box with a volume of 400 cubic centimetres to hold a gift you have purchased. Ask: What might that gift be?
- Give students the volume of a rectangular prism and have them construct it.
- Ask students to describe the strategy they would use to estimate the volume of certain common rectangular prisms such as lunchboxes, pasta boxes, tissue boxes, etc.

SCO: SS4: Demonstrate an understanding of capacity by:

- describing the relationship between mL and L
- selecting and justifying referents for mL or L units
- estimating capacity by using referents for mL or L
- measuring and recording capacity ( mL or L ).
[C, CN, ME, PS, R, V]

| [C] Communication | [PS] Problem Solving | [CN] Connections <br> [T] Technology | [V] Reasoning |
| :--- | :--- | :--- | :--- | | [ME] Mental Math |
| :---: |
| and Estimation |

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :--- | :--- | :--- |
|  | SS4 Demonstrate an <br> understanding of capacity by: <br> - describing the relationship <br> between $m L$ and $L$ <br> e <br> - selecting and justifying <br> referents for $m L$ or $L$ units <br> - estimating capacity by using <br> referents for $m L$ or $L$ <br> - measuring and recording <br> capacity $(m L$ or $L$ ). |  |

## Elaboration

Volume and capacity are both terms for measuring the size of three-dimensional regions. Although these two concepts are related, this specific outcome is focused on capacity. It is useful for students to recognize the difference between volume (the amount of space occupied by a three-dimensional object) and capacity (the amount a container is capable of holding). Capacity units introduced in grade 5 are millilitres ( mL ) and litres (L). Capacity units are usually associated with measures of liquid (e.g., litres of milk, juice and gasoline).

Students should develop personal referents for units. The use of personal referents helps students establish the relationships between the units (e.g., the small cube in the base-ten blocks is $1 \mathrm{~cm}^{3}$ and would hold 1 mL and the large cube is $10 \mathrm{~cm} \times 10 \mathrm{~cm} \times 10 \mathrm{~cm}$ and would hold 1 L ).

Students should have a sense of which volume or capacity unit is the more appropriate to use in various circumstances.

SCO: SS4: Demonstrate an understanding of capacity by:

- describing the relationship between mL and L
- selecting and justifying referents for mL or $L$ units
- estimating capacity by using referents for $m L$ or $L$
- measuring and recording capacity ( mL or L ).
[C, CN, ME, PS, R, V]


## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Demonstrate that 1000 millilitres is equivalent to 1 litre by filling a 1 litre container using a combination of smaller containers.
- Provide a referent for a litre and explain the choice.
- Provide a referent for a millilitre and explain the choice.
- Determine which capacity unit is represented by a given referent.
- Estimate the capacity of a given container using personal referents.
- Determine the capacity of a given container using materials that take the shape of the inside of the container, e.g., a liquid, rice, sand, beads, and explain the strategy.


## SCO: SS4: Demonstrate an understanding of capacity by:

- describing the relationship between mL and L
- selecting and justifying referents for mL or L units
- estimating capacity by using referents for mL or L
- measuring and recording capacity ( mL or L ).
[C, CN, ME, PS, R, V]


## Instructional Strategies

Consider the following strategies when planning lessons:

- Have students discuss a variety of situations where they need to choose the unit of measurement that would be used for each. Have students compare their answers and defend their choices. For example, the unit of measure to find the capacity of cough syrup bottles, water bottles, juice boxes, yogurt containers, bathtub, gas tank, etc.
- Invite groups of students to investigate the capacities of various beverage containers to determine which size container is found most often.
- Provide ample opportunity for students to measure the capacity of different shaped and sized containers. Have students predict which unit of measure will be used.
- Have students find appropriate personal referents for litres and millilitres.


## Suggested Activities

- Have students measure the capacity of several different containers and record the most common capacities.
- Provide students with a pair of containers and ask them to predict which has the largest capacity (which holds more). Have them verify their predictions.
- Have students compare several cereal bowls to see how much a typical bowl can hold.
- Have students estimate the number of beans to fill a litre container.
- Have students suggest containers for fixed capacities. (e.g., What kind of containers would hold $500 \mathrm{~mL}, 1 \mathrm{~L}$, or 250 mL ?)

SCO: SS4: Demonstrate an understanding of capacity by:

- describing the relationship between mL and L
- selecting and justifying referents for mL or L units
- estimating capacity by using referents for mL or L
- measuring and recording capacity ( mL or L ).
[C, CN, ME, PS, R, V]


## Assessment Strategies

- Tell the student that a container holds 1.5 L . Ask if it is large enough to make a jug of orange juice, if the concentrate is 355 mL and you have to use the concentrate can to add three full cans of water.
- Ask the student how he/she could use a 1 L milk carton to estimate 750 mL of water.
- Ask students to describe the strategy they would use to estimate the capacity of certain common containers such as water bottles, bathtub, various milk containers, etc.

```
SCO: SS5: Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D
    shapes that are:
    - parallel
    - intersecting
    - perpendicular
    - vertical or horizontal.
    [C, CN, R, T, V]
```

| [C] Communication | [PS] Problem Solving | [CN] Connections | [ME] Mental Math |
| :--- | :--- | :--- | :--- |
| [T] Technology | [V] Visualization | [R] Reasoning | and Estimation |

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :--- | :--- | :--- |
| SS4 Describe and construct | SS5 Describe and provide | SS4 Construct and compare |
| rectangular and triangular | examples of edges and faces of | triangles, including: |
| prisms. | 3-D objects and sides of 2-D | - scalene |
|  | shapes that are: | - isosceles |
|  | - parallel | - equilateral |
|  | - intersecting | - right obtuse |
|  | - perpendicular | - vertical or horizontal. |
|  | in different orientations. |  |

## Elaboration

There is a gradual progression from identifying and describing two- and three-dimensional objects in students' own words to identifying and describing them in the formal language of geometry. It is important that students become familiar with the vocabulary associated with describing the attributes of 2-D shapes and 3-D objects such as parallel, intersecting, perpendicular, vertical and horizontal.


Students will also be expected to compare and describe 2-D shapes by relating their attributes and will also be expected to compare and describe 3-D objects in the same way. When given a set of attributes, students should be able to construct or draw the 2-D shape or 3-D object that corresponds to the description.

SCO: SS5: Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are:

- parallel
- intersecting
- perpendicular
- vertical or horizontal.
[C, CN, R, T, V]


## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Identify parallel, intersecting, perpendicular, vertical and horizontal edges and faces on 3-D objects.
- Identify parallel, intersecting, perpendicular, vertical and horizontal sides on 2-D shapes.
- Provide examples from the environment that show parallel, intersecting, perpendicular, vertical and horizontal line segments.
- Find examples of edges, faces and sides that are parallel, intersecting, perpendicular, vertical and horizontal in print and electronic media, such as newspapers, magazines and the Internet.
- Draw 2-D shapes or 3-D objects that have edges, faces and sides that are parallel, intersecting, perpendicular, vertical or horizontal.
- Describe the faces and edges of a given 3-D object using terms, such as parallel, intersecting, perpendicular, vertical or horizontal.
- Describe the sides of a given 2-D shape using terms, such as parallel, intersecting, perpendicular, vertical or horizontal.

SCO: SS5: Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are:

- parallel
- intersecting
- perpendicular
- vertical or horizontal.
[C, CN, R, T, V]


## Instructional Strategies

Consider the following strategies when planning lessons:

- Provide opportunities for students to manipulate 2-D shapes and 3-D objects and become familiar with the vocabulary associated with describing the attributes of 2-D shapes and 3-D objects such as parallel, intersecting, perpendicular, vertical and horizontal.
- Encourage students to use the formal language of geometry as they describe the attributes of classroom or real-life 2-D shapes and 3-D objects (e.g., the opposite walls in the classroom are parallel).


## Suggested Activities

- Have students draw and construct on a geoboard 2-D shapes with specific attributes (e.g., construct a shape with a pair of parallel sides).
- Have students draw or construct (with toothpicks and marshmallows) 3-D objects and have them describe their attributes.
- Have students sort 2-D shapes and 3-D objects according to their attributes and justify their sorting scheme.
- Compare and describe the faces and edges of two prisms or two pyramids with different bases (e.g., triangular prisms and rectangular prisms.
- Go on a walk to look at 2-D shapes and 3-D objects in the environment. Have students discuss the attributes of shapes and objects in their environment using geometric language.

```
SCO: SS5: Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D
    shapes that are:
    - parallel
    - intersecting
    - perpendicular
    - vertical or horizontal.
    [C, CN, R, T, V]
```


## Assessment Strategies

- Provide students with several different 2-D shapes and have them sort and justify their sorting scheme. Observe whether students use correct geometric terminology in their descriptions.
- Provide students with several different 3-D objects and have them sort and justify their sorting scheme. Observe whether students use correct geometric terminology in their descriptions.
- Have students draw 2-D shapes and 3-D objects that satisfy a given set of attributes. For example, draw a parallelogram with both parallel and perpendicular sides (rectangle).
- Complete a Venn or Carroll diagram focusing on the attributes of 2-D shapes and 3-D objects as shown below.

|  | 2-D object | 3-D shape |
| :--- | :--- | :--- |
| Has parallel sides, edges or <br> faces |  |  |
| Does not have any parallel <br> sides, edges or faces |  |  |

SCO: SS6: Identify and sort quadrilaterals, including:

- rectangles; squares
- trapezoids
- parallelograms
- rhombuses
according to their attributes.
[C, R, V]

| [C] Communication | [PS] Problem Solving | [CN] Connections | [ME] Mental Math |
| :--- | :--- | :--- | :--- |
| [T] Technology | [V] Visualization | [R] Reasoning | and Estimation |

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :---: | :--- | :--- |
|  | SS6 Identify and sort <br> quadrilaterals, including: <br> - Rectangles and squares <br> - trapezoids | SS5 Describe and compare the <br> sides and angles of regular and <br> i parregular polygons. <br> - rhombuses <br> according to their attributes. |

## Elaboration

Quadrilaterals are four-sided polygons. Although rectangles are the most common quadrilateral that you see in everyday life, students will soon discover that there are many classes of quadriaterals (Small, 2008; p. 295). Students will be exploring the attributes of various quadrilaterals such as rectangles, squares, trapezoids, parallelograms, rhombuses. They will compare the similarities and differences and sort them according to their attributes.

Common attributes will be side lengths, pairs of opposite sides parallel, and lines of symmetry.


SCO: SS6: Identify and sort quadrilaterals, including:

- rectangles; squares
- trapezoids
- parallelograms
- rhombuses
according to their attributes.
[C, R, V]


## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Identify and describe the characteristics of a pre-sorted set of quadrilaterals.
- Sort a given set of quadrilaterals and explain the sorting rule.
- Sort a given set of quadrilaterals according to the lengths of the sides.
- Sort a given set of quadrilaterals according to whether or not opposite sides are parallel.

SCO: SS6: Identify and sort quadrilaterals, including:

- rectangles; squares
- trapezoids
- parallelograms
- rhombuses
according to their attributes.
[C, R, V]


## Instructional Strategies

Consider the following strategies when planning lessons:

- Use models, drawings and real life examples of quadrilaterals, to identify and describe the characteristics of each and classify them. Have students explain their classification system.


## Suggested Activities

- Have students go on a "Quadrilateral Scavenger Hunt". Have them sort their quadrilaterals with similar attributes and explain their rules for sorting.
- Have students prepare property lists with headings: sides, parallel, perpendicular, symmetries. Using a collection of quadrilaterals made on recipe cards, have students describe the shapes using language such as: at least 2 lines of symmetry.
- Provide students with a list of attributes and have them construct a quadrilateral that has the set of attributes. Have students share and compare with the class.
- Prepare a "Guess What Quadrilateral I Am?" game with clues about their attributes.

SCO: SS6: Identify and sort quadrilaterals, including:

- rectangles; squares
- trapezoids
- parallelograms
- rhombuses
according to their attributes.
[C, R, V]


## Assessment Strategies

- Provide students with several different quadrilaterals to sort and have them justify their classification scheme.
- Have students draw quadrilaterals that satisfy a given set of attributes.


## SCO: SS7: Perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image. <br> [C, CN, T, V] <br> SS8: Identify a single transformation, including a translation, rotation, and reflection of 2-D shapes. <br> [C, T, V]

| [C] Communication <br> [T] Technology | [PS] Problem Solving | [CN] Connections | [ME] Mental Math |
| :--- | :--- | :--- | :--- |
| [V] Visualization |  |  |  |$\quad$| [R] Reasoning |
| :--- |

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :--- | :--- | :--- |
| SS5 Demonstrate an | SS7 Perform a single | SS6 Perform a combination of |
| understanding of line symmetry | transformation (translation, | translation(s), rotation(s) |
| by: | rotation, or reflection) of a 2-D | and/or reflection(s) on a single |
| - identifying symmetrical 2-D | shape (with and without | 2-D shape, with and without |
| shapes | technology) and draw and | technology, and draw and |
| - creating symmetrical 2-D | describe the image. | describe the image. |
| shapes | SS8 Identify a single | SS7 Perform a combination of |
| - drawing one or more lines of | transformation, including a | successive transformations of 2- |
| symmetry in a 2-D shape. | translation, rotation, and | D shapes to create a design, |
|  | reflection of 2-D shapes. | and identify and describe the |
| transformations. |  |  |

## Elaboration

There are three transformations that change the location of an object in space, or the direction in which it faces, but not its size or shape. The three types of transformations are: translations, reflections and rotations. These transformations result in images that are congruent to the original object. Students are expected to identify and perform these three types of transformations.

Translations move a shape left, right, up, down or diagonally without changing its orientation in any way. A real life example of a translation may be a piece moving on a chessboard.


Reflections can be thought of as the result of picking up a shape and turning it over. The reflection image is the mirror image of the original shape. A real life example of a reflection would be a pair of shoes.


Rotations move a shape around a turn centre. When students first start working with rotations they identify them as fractions of a circle (e.g., a quarter turn, half turn and three-quarter turn). Students are also expected to identify the direction of the rotation; clockwise and counterclockwise. It is also important to identify the turn centre which could be in the centre, outside of the shape or on the perimeter of the shape (Small, 2008; p. 349).


## SCO: SS7: Perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image. <br> [C, CN, T, V] <br> SS8: Identify a single transformation, including a translation, rotation, and reflection of 2-D shapes. <br> [C, T, V]

## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

## SS7

- Translate a given 2-D shape horizontally, vertically or diagonally, and describe the position and orientation of the image.
- Rotate a given 2-D shape about a point, and describe the position and orientation of the image.
- Reflect a given 2-D shape in a line of reflection, and describe the position and orientation of the image.
- Perform a transformation of a given 2-D shape by following instructions.
- Draw a 2-D shape, translate the shape, and record the translation by describing the direction and magnitude of the movement.
- Draw a 2-D shape, rotate the shape and describe the direction of the turn (clockwise or counterclockwise), the fraction of the turn and point of rotation.
- Draw a 2-D shape, reflect the shape, and identify the line of reflection and the distance of the image from the line of reflection.
- Predict the result of a single transformation of a 2-D shape and verify the prediction.


## SS8

- Provide an example of a translation, a rotation and a reflection.
- Identify a given transformation as a translation, rotation or reflection.
- Describe a given rotation by the direction of the turn (clockwise or counter-clockwise).

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SCO: SS7: Perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image.
[C, CN, T, V]
SS8: Identify a single transformation, including a translation, rotation, and reflection of 2-D shapes.
[C, T, V]
```


## Instructional Strategies

Consider the following strategies when planning lessons:

- Provide students many opportunities to concretely translate a given 2-D shape using a geoboard and grid paper by identifying the direction and the magnitude of the translation.
- Provide students many opportunities to concretely rotate a given 2-D shape using a geoboard and grid paper ensure that a point of rotation is identified and that the fraction of the turn is described.
- Provide students many opportunities to concretely reflect a given 2-D shape using a Mira ${ }^{\circledR}$ and grid paper ensure that a line of reflection is identified as well as the distance from the line of reflection is described.
- Provide students with samples of wallpaper and have them explore various types of transformations.
- Explore this concept in other curricular areas such as art and physical education. Have students create their own wallpaper patterns using different transformations or act out a transformation in the gym.
- Have students discuss their predictions prior to performing a given transformation to a shape.


## Suggested Activities

- Have students describe the direction as well as the magnitude of a given translation.
- Have students determine which transformation was performed on a given shape.
- Provide pattern blocks and have students practice each transformation and draw them on grid paper.
- Have students choose a pattern block, perform a transformation of their choice, draw the transformation on grid paper and have a partner describe the transformation that was performed.
- Ask students to perform a rotation given the direction of the turn (clockwise or counterclockwise), the fraction of the turn and the point of rotation.
- Ask students to perform a translation given the direction and magnitude of the movement.
- Ask students to perform a reflection given the line of reflection and the distance from the line of reflection.
- Ask students to create a shape on the geoboard, perform a transformation of their choice, and describe the transformation that was performed.
- Respond in their journal to the following prompts:

Explain using words and pictures if a translation can ever look like a reflection. Explain how you know if a figure and its image show a reflection, translation, or rotation. Use pictures and words in your explanation.

SCO: SS7: Perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image. [C, CN, T, V]
SS8: Identify a single transformation, including a translation, rotation, and reflection of 2-D shapes.
[C, T, V]

## Assessment Strategies

- Provide students with diagrams of different transformations and have them label each diagram with the type of transformation the diagram showed.
- Provide diagrams of rotations and ask, "Which picture shows a quarter turn? Half turn? Three-quarter turn." Have students identify the turn centre of the rotation.
- Provide a 2-D shape and have students show a rotation, reflection, or translation on grid paper of that shape.
- Have students draw a shape, translate it, and then describe and explain the direction and the magnitude of the translation.
- Ask students to use their hands to show the three different transformations.
- Ask students to explain the differences and similarities among the three different transformations.
- Explain using words and pictures if a translation can ever look like a reflection.
- Explain using words and pictures how you know if a figure and its image show a translation, reflection or rotation.


## STATISTICS AND PROBABILITY

SCO: SP1: Differentiate between first-hand and second-hand data.
[C, R, T, V]

| [C] Communication | [PS] Problem Solving | [CN] Connections | [ME] Mental Math |
| :--- | :--- | :--- | :--- |
| [T] Technology | [V] Visualization | [R] Reasoning | and Estimation |

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :--- | :--- | :--- |
| SP1 Demonstrate an <br> understanding of many-to-one <br> correspondence. | SP1 Differentiate between first- <br> hand and second-hand data. | SP2 Select, justify and use <br> appropriate methods of <br> collecting data, including: <br> questionnaires; experiments; <br> databases; electronic media. |

## Elaboration

Students are familiar with collecting and organizing data from previous grades. They will learn about firsthand data, that they collect themselves, and second-hand data that other people have collected. The focus will be on comparing the collection methods and communicating results.

First-hand data: Collecting first-hand data can be done using a variety of methods such as interviews, surveys, experiments and observations. Students will then analyze the data and use reasoning to draw conclusions.

Students will need to determine what data they want to collect, and then design a survey with an appropriate question that will provide the information they want. They also have to determine who to survey, and whether their choice will influence their results, e.g., surveying people at the hockey rink about their favourite sport.

Second-hand data: Some data is difficult to obtain first-hand, but can be found in print and electronic media. Students will need to create appropriate questions that can be answered using second-hand data, then use that data to communicate different conclusions.

This outcome provides an opportunity for students to work with large numbers in context (relating to outcome N 1 ), for example, comparing populations.

SCO: SP1: Differentiate between first-hand and second-hand data.
[C, R, T, V]

## Achievement Indicators

- Explain the difference between first-hand and second-hand data.
- Formulate a question that can best be answered using first-hand data and explain why.
- Formulate a question that can best be answered using second-hand data and explain why.
- Find examples of second-hand data in print and electronic media, such as newspapers, magazines and the Internet.


## SCO: SP1: Differentiate between first-hand and second-hand data.

[C, R, T, V]

## Instructional Strategies

Consider the following strategies when planning lessons:

- Provide questions and have students determine how best to collect the data in order for students to recognize the difference between first-hand and second-hand data.
- Have students generate which can best be answered using first-hand data, describe how that data could be collected.
- Have students generate questions that can best be answered using second-hand data, and describe how that data could be collected.


## Suggested Activities

- Provide examples of data relevant to self, family, or community and categorize the data, with explanation, as first-hand or second-hand data.
- Have students formulate questions that can best be answered using first-hand data (e.g., "What game will we play at home tonight?" "I can survey everyone at home to find out what games everyone wants to play.").
- Have students formulate a question related to self, family, or community, which can best be answered using second-hand data (e.g., "Which has the larger population - my community or my friend's community?"). Then students should describe how this data could be collected (e.g., find the data on the StatsCan website: http://www.statcan.gc.ca), and finally answer the question.
- Have students find examples of second-hand data in print and electronic media, such as newspapers, magazines, and the Internet, and compare different ways in which the data might be interpreted and used (e.g., statistics about health-related issues, sports data, or votes for favourite websites).


## SCO: SP1: Differentiate between first-hand and second-hand data.

[C, R, T, V]

## Assessment Strategies

- Ask students to write a question about preferred types of books that can be answered using first-hand data and explain why. How and from whom would the data be collected?
- Ask students to write a question about the populations of the cities in New Brunswick, and have them explain why the question is best answered using second-hand data. Where can the data be found?
- Have students work in groups to generate questions for which the data would be collected first- and second-hand.

SCO: SP2: Construct and interpret double bar graphs to draw conclusions.
[C, PS, R, T, V]
[C] Communication
[PS] Problem Solving
[CN] Connections
[T] Technology
[V] Visualization
[R] Reasoning
[ME] Mental Math and Estimation

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :--- | :--- | :--- |
| SP2 Construct and interpret | SP2 Construct and interpret | SP1 Create, label and interpret |
| pictographs and bar graphs | double bar graphs to draw | line graphs to draw conclusions. |
| involving many-to-one | conclusions. | SP3 Graph collected data and <br> analyze the graph to solve <br> correspondence to draw <br> conclusions. |
|  |  |  |

## Elaboration

Students should be aware that sometimes when two pieces of data are collected about a certain population, it is desirable to display both of them side by side, using the same scale. For example, census data often shows male and female data separately for different years. This is usually done using a double bar graph. A legend is used to help the reader interpret a double bar graph. An example is presented below. Five students in the class have been asked how many brothers and sisters they have.

This type of graph allows students to be compared not only in terms of how many brothers they have, or how many sisters they have, but also to compare the number of brothers versus the number of sisters.

It should be emphasized that students include titles, horizontal and vertical axis headings, legends and category labels. The pairs of bars should be separated. A common mistake made by students is to place the incremental numbers in the space rather than on the line.


|  | Brothers | Sisters |
| :--- | :---: | :---: |
| Student 1 | 1 | 1 |
| Student 2 | 2 | 0 |
| Student 3 | 1 | 2 |
| Student 4 | 0 | 1 |
| Student 5 | 2 | 1 |

The data may be displayed horizontally or vertically as shown below.


## SCO: SP2: Construct and interpret double bar graphs to draw conclusions.

[C, PS, R, T, V]

## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Determine the attributes (title, axes, intervals and legend) of double bar graphs by comparing a given set of double bar graphs.
- Represent a given set of data by creating a double bar graph, label the title and axes, and create a legend without the use of technology.
- Draw conclusions from a given double bar graph to answer questions.
- Provide examples of double bar graphs used in a variety of print and electronic media, such as newspapers, magazines and the Internet.
- Solve a given problem by constructing and interpreting a double bar graph.


## SCO: SP2: Construct and interpret double bar graphs to draw conclusions.

[C, PS, R, T, V]

## Instructional Strategies

Consider the following strategies when planning lessons:

- Have students determine when it is appropriate to display data in a double bar graph.
- Provide students with sets of data and have them determine appropriate scales.
- Provide students with two double bar graphs displaying the same data using a different scale, and have students determine which they prefer and why.
- Have students collect first-hand and second-hand data and create double bar graphs making sure to include appropriate title, axis labels, intervals and legend.
- Have students use second-hand data collected from sites, such as Statistics Canada that use large numbers. (http://www.statcan.gc.ca and Census at School: www.censusatschool.ca)
- Have students generate sets of questions that can be answered by reading various double bar graphs.
- Have students compare data in the double bar graph within and among the pairs.


## Suggested Activities

- Provide examples of double bar graphs from a variety of media sources, and ask students to bring in examples from similar sources.
- Have students examine double bar graph samples and determine the attributes (title, axes, legend, intervals). Ask them to compare and share the information displayed.
- Have students collect and graph first-hand data, such as girls' and boys' favourite activity in gym.
- Have students collect information on the length and mass of various animals and display the data in a double bar graph. Ask what conclusions they might draw.
- Have students create double bar graphs on subjects that are of personal interest, such as comparing hockey players' salaries from two different teams.

SCO: SP2: Construct and interpret double bar graphs to draw conclusions.
[C, PS, R, T, V]

## Assessment Strategies

- Ask the student to describe some data that would be appropriate to display using a double bar graph.
- Have students generate a double bar graph from given sets of data without the use of technology. Rubrics might include appropriate scales and labeling as well as accuracy.
- Have students draw conclusions from a given double bar graph to answer questions.

```
SCO: SP3: Describe the likelihood of a single outcome occurring using words, such as:
    - impossible
    - possible
    - certain.
    SP4: Compare the likelihood of two possible outcomes occurring using words, such as:
    - less likely
    - equally likely
    - more likely.
    [C, CN, PS, R]
```

| [C] Communication | [PS] Problem Solving | [CN] Connections | [ME] Mental Math |
| :---: | :---: | :---: | :---: |
| [T] Technology | [V] Visualization | [R] Reasoning | and Estimation |

## Scope and Sequence

| Grade Four | Grade Five | Grade Six |
| :---: | :---: | :---: |
|  | SP3 Describe the likelihood of a single outcome occurring using words, such as: <br> - impossible <br> - possible <br> - certain. <br> SP4 Compare the likelihood of two possible outcomes occurring using words, such as: <br> - less likely <br> - equally likely <br> - more likely. | SP4 Demonstrate an understanding of probability by: <br> - identifying all possible outcomes of a probability experiment <br> - differentiating between experimental and theoretical probability <br> - determining the theoretical probability of outcomes in a probability experiment <br> - determining the experimental probability of outcomes in a probability experiment <br> - comparing experimental results with the theoretical probability for an experiment. |

## Elaboration

The occurrence of a future event can be characterized along a continuum from impossible to certain. The key idea to developing chance or probability on a continuum is to help children see that some events are more likely than others. Before students attempt to assign numeric probabilities to events, it is important that they have the basic idea that some events are certain to happen, some are certain not to happen or are impossible, and others have different chances of occurring that fall between these extremes. (Van de Walle \& Lovin, vol. 2, 2006)

Students should be encouraged to use their reasoning skills to make predictions about outcomes, and to communicate the results using probability language. This introduction to the probability of an event gives students the opportunity to bring their own life experiences to the discussion.

Once students have mastered the concept of likelihood (probability) of a single outcome occurring, they can then begin to compare the likelihood of two outcomes occurring, using the comparative language less likely, equally likely, more likely.

Students will begin to design and conduct probability experiments for the likelihood of single outcomes occurring, as well as a comparison of two outcomes. They will be expected to record the outcomes and explain the results.

SCO: SP3: Describe the likelihood of a single outcome occurring using words, such as:

- impossible
- possible
- certain.

SP4: Compare the likelihood of two possible outcomes occurring using words, such as:

- less likely
- equally likely
- more likely.
[C, CN, PS, R]


## Achievement Indicators

Students who have achieved this outcome(s) should be able to:

- Provide examples of events that are impossible, possible or certain from personal contexts.
- Classify the likelihood of a single outcome occurring in a probability experiment as impossible, possible or certain.
- Plot the likelihood of a single outcome occurring in a probability experiment along a continuum.
- Design and conduct a probability experiment in which the likelihood of a single outcome occurring is impossible, possible or certain.
- Conduct a given probability experiment a number of times, record the outcomes and explain the results.
- Identify outcomes from a given probability experiment which are less likely, equally likely or more likely to occur than other outcomes.
- Design and conduct a probability experiment in which one outcome is less likely to occur than the other outcome.
- Design and conduct a probability experiment in which one outcome is equally as likely to occur as the other outcome.
- Design and conduct a probability experiment in which one outcome is more likely to occur than the other outcome.

```
SCO: SP3: Describe the likelihood of a single outcome occurring using words, such as:
    - impossible
    - possible
    - certain.
    SP4: Compare the likelihood of two possible outcomes occurring using words, such as:
    - less likely
    - equally likely
    - more likely.
    [C, CN, PS, R]
```


## Instructional Strategies

Consider the following strategies when planning lessons:

- Provide a list of single outcomes (events) ranging from impossible to certain and ask students to identify the likelihood of the event occurring using probability language.
- Have students plot outcomes on a probability continuum accompanied by an explanation as to their placement.

- Have students generate a list of events that would fall on the probability continuum.
- Have students conduct a given probability experiment in which the likelihood of a single outcome occurring is impossible, possible or certain, recording and explaining their results.
- Have students design and conduct probability experiments on a single outcome, recording and explaining their results.
- Give students frequent opportunities to identify outcomes from given probability experiments that are less likely, equally likely or more likely to occur than other outcomes.
- Have students design and conduct probability experiments in which one outcome is less likely, equally likely and more likely to occur than the other outcome.


## Suggested Activities

- Have the student design experiments for which a certain outcome is impossible, possible and certain.
- Have the student design experiments with two possible outcomes in which one of the outcomes is less likely, equally likely or more likely to occur.
- Ask the child to design a spinner so that spinning red is more likely than spinning green, but spinning red is less likely than spinning yellow.


## SCO: SP3: Describe the likelihood of a single outcome occurring using words, such as:

- impossible
- possible
- certain.

SP4: Compare the likelihood of two possible outcomes occurring using words, such as:

- less likely
- equally likely
- more likely.
[C, CN, PS, R]


## Assessment Strategies

- Tell the student he/she wins $\$ 1$ if the spinner lands on red and loses $\$ 1$ if it lands on blue. Ask: How would you like the spinner to be designed?
- Ask the child to think of an event that is possible, but not very likely, and another event that is very likely, but might not happen.


## Curriculum Guide Supplement Math Makes Sense 5 Unit Plans

## Grade 5 Mathematics Curriculum Pacing Guide

## Resource: Math Makes Sense 5

Dates and number of classes suggested are based on a typical school year and are estimates.

| September | October | November | December | January |
| :---: | :---: | :---: | :---: | :---: |
| Mental Math: Fact Learning, Mental Computation \& Estimation Should Be A Daily Routine |  |  |  |  |
| UNIT 2: Whole Numbers 20 Classes Sept. 13 - Oct. 8 | UNIT 1: Patterns and Equations 20 Classes Oct. 18 - Nov. 12 |  | Measurement <br> Classes - Christmas | UNIT 6: Geometry <br> 20 Classes Jan. 10 - Feb. 4 |
| Mental Math: Fact Learning, Mental Computation \& Estimation Should Be A Daily Routine |  |  |  |  |
| February | March | April | May | June |
|  | UNIT 5: Fractions and Decimals |  |  | UNIT 8: <br> Transformation |
| UNIT 3: M and Dividi Num 25 Cl Feb. 7 - |  | 5 Classes $\text { r. } 21 \text { - May } 6$ | UNIT 7: Statistics and Probability Outcomes can be integrated with the Science, Social Studies and Health Curriculum throughout the year. | 15 Classes May 30 - June 17 |

## Unit 1: Patterns and Equations

Teachers using Math Makes Sense as their primary resource for addressing the Grade 5 mathematics curriculum outcomes should consider the suggestions and recommendations outlined in this supplement.

Note: Each lesson ends with a Reflect. You may wish to use some of these prompts for math journals. The notation O.T.O. (On Their Own) suggests that most students should be able to do what is asked by the question.

| Notes, Suggestions, Recommendations |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Preparing to Teach This Unit | Review Specific Curriculum Outcomes PR1 \& 62 in your Curriculum Guide as these are the SCOs the unit is attempting to address. Think about these two guiding questions, "What do I want my students to learn? What do I want my students to understand and be able to do?" The Achievement Indicators and the Assessment Strategies for these outcomes will help you answer these questions and give you a better understanding of what it is you need to emphasize in you teaching. You should also refer to the Unit Rubric on p. 33 in the Teacher's Guide, the Ongoing Observations sheet on p. 34 and the Assessment for Learning sections which are components of each individual lesson. <br> Note: Pages from the student text can be projected onto a screen if you have access to a computer and LCD. Use the ProGuide DVD which came with your Grade 5 Teacher's Resource Binder to locate specific content for each unit. Some teachers find this strategy helpful when discussing diagrams, tables and other illustrations with the whole class. |  |  |  |
| Investigation: Building Patterns | You do not need to begin the unit with this investigation although it will allow students to revisit patterning concepts from the previous year. Most classes will not have enough pattern blocks as suggested in the student book, but students can refer to the pictures in each frame. Part 1: Work through this with the students. Draw a table on chart paper or the board. Review the meaning of perimeter and count the squares, triangles, and $p$ in frames 1 and 2 together. Let students do frame 3 on their own. Record the information in the table. After the first three frames, help students to see the pattern in each column and predict what it will be for the $4^{\text {th }}, 5^{\text {th }}$ and $6^{\text {th }}$ frames. |  |  |  |
|  | Frame | Squares | Triangles | Perimeter |
|  | 1 | 5 | 2 | 14 |
|  | 2 | 9 | 3 | 23 |
|  | 3 | 13 | 4 | 32 |
|  | 4 | 17 | 5 |  |
|  | 5 |  |  |  |
|  | 6 |  |  |  |
|  | Part 2: If you don't have enough squares and triangles for students to build frames 4 and 5 , let them use cm grid paper (or larger) to draw each one. The emphasis in this investigation is on students using the data from the table to make predictions about |  |  |  |


|  | subsequent designs. <br> Discuss the patterns that appear in the figures themselves and in the table. |
| :---: | :---: |
| Launch: Charity Fund-raising | After students have read through this launch, initiate a discussion about fund-raising and the Terry Fox run. Carly is going to be running on a track that is 400 m in length and she needs to run a total of 10 km . Show students a meter stick. Ask them to estimate the length of the classroom in meters. Have a volunteer do the actual measurement. Round off to the nearest m . Compare the length of the room to the length of the 400 m track that Carly is going to be running on. <br> Explain that a km is 1000 meters and ask how many meters will Carly have to run if she's going to run 10 km . Copy Carly's tables onto chart paper so you can refer to the patterns as you notice them. That way, everyone will be looking at the same thing. Leave enough room on your chart to continue the pattern. |
| Lesson 1: Number Patterns and Pattern Rules SCO PR 1 | This lesson uses the term "pattern rule" which was first introduced in grade 4. The key idea for students to understand is that pattern rules are different ways to describe what's happening to make the pattern. Different people will describe the pattern rule in different ways using different language. Growing patterns are the focus of this unit. <br> (TG 4) Explore <br> Do the four number patterns in the student book together with the kids. For example, in the first pattern ask, "What happened to the 3 to get to the next number? What happened to the 4 to get the next number? The 6 ..."etc. <br> In the next part of the Explore, the students are asked to use counters to show the pattern of one of the number strings. These might grow in an orderly fashion, or they might just be growing sets. <br> $3,4,6,9,13, \ldots$ (pattern is $+1,+2,+3,+4$ ) <br> OR <br> Connect (text p.7) <br> The pattern shown with counters does not need to look exactly like this as long as they are increasing by the right number. <br> Practice <br> Q1. a) students can do this on their own <br> b) make sure they understand that the amount being added each time increases by one. Do the first few together: $1,3,6,10,15, \ldots$ |


|  | Q2. Optional. The same information can be obtained from looking at the patterns in Q1. <br> Q3 \& 4. It is worthwhile doing both of these questions together with students. Help them discover that sometimes patterns go up and then down within the same pattern. <br> Q5. Quite a challenging question. You will need to help students understand that this means subtracting 8 seven times and that it's the same as subtracting $7 \times 8$ or $200-56=144$ (the $7^{\text {th }}$ term) <br> Q6. Explain and assign <br> Q7. Limit this to just two different patterns. <br> Reflect <br> Consider this as a journal writing prompt. |
| :---: | :---: |
| Lesson 2: Using Patterns to Solve Problems SCO PR1 | The lesson organizer suggests running off copies of a 2-column chart. Save paper by having students draw their own in their notebooks. <br> Explore <br> $\mathbf{1}^{\text {st }}$ bullet: Get students to use a ruler to draw a table like the one in their books so that they can enter the information for each hour that Sam baby-sits. <br> $\mathbf{2}^{\text {nd }}$ bullet: Ask how much Sam will earn in 10 hours, 20 hours, 21 hours. <br> $3^{\text {rd }}$ bullet: To explain how they know, students might say that the dollar amounts shown are not multiples of 6 or "they're not in my table." <br> $4^{\text {th }}$ bullet: Get students started on this. Review how much Sam makes for 10 hrs. 20 hrs., 30 hrs., 40 hrs. <br> $5^{\text {th }}$ bullet: Optional <br> Connect (student text p.10) <br> $\$ 17$ is a difficult multiple to work with. Change it to $\$ 15$. When they are asked to predict the cost of 20 books, help them see how multiplication can be used $\$ 15 \times 20=$. How many students use the associative property of multiplication to multiply $(15 \times 10)+(15$ $x$ 10)? If not many, then use this as an opportunity to reinforce that computation strategy. <br> Calculator. Use the "constant" feature on the calculator to explore number patterns. To add 17 consecutively, press $+17=,=,=,=$. Once students understand how this feature works, allow them to work on problem solving the question about buying $\$ 200$ worth of puzzle books. <br> Practice <br> Q1: Go over the number of blocks in the first 3 pictures. Give students linking cubes to make the $4^{\text {th }}$ object. Notice which students appear to be spatially confused. See if students can construct the $5^{\text {th }}$ object. <br> Students should copy the table into their notebooks and leave enough room so that it can be extended. <br> part e): Optional, but if you do decide to do it, let students extend |


|  | the table to see if any object will have 50 or 51 cubes. Don't expect them to offer the explanation suggested in the teachers guide. <br> Q2. a) \& b) Do these two parts together. Students may have difficulty phrasing the pattern rule. <br> c) O.T.O. <br> d) Omit <br> Q3. Good question. Students will be working with multiples of 25. <br> Q4. O.T.O. <br> Q5. (Numbers, Pictures, Words) <br> b) Review a strategy for adding a column of 2-digit numbers eg., adding the 10 s first and then the 1 s . <br> c) This challenging question is appropriate for more skilled students. <br> Q6. Omit |
| :---: | :---: |
| Lesson 3: Using a Variable to Describe a Pattern SCO PR1 | Explore <br> Use pattern block triangles or some other equilateral triangles on the overhead to model a growing pattern and resulting perimeter based on side length. Try adding just one triangle each time. Record and discuss the patterns in a table drawn on chart paper. Repeat by adding two triangles each time instead of one. <br> Connect (p.14) <br> This is the first time the terms variable and expression are used. Go over this section carefully with students to make sure they understand. <br> (p.15) Students may wonder why $\boldsymbol{f}$ was used to represent the variable in the first one and now $\boldsymbol{n}$ is being used. Explain that it doesn't matter which letter is used, but that $\boldsymbol{n}$ is very common in math because it stands for number. <br> Continue to help students understand that letters (variables) are being substituted for numbers when describing a pattern rule (expression). <br> Practice <br> Q1-3. Work through these three questions with students. <br> Q4. Make sure they understand that another name for pattern rule is "expression". See if they can come up with the correct expressions on their own. Check together afterwards. <br> Q6. In a number pattern like $15,16,17,18$ it is easy for students to see that the pattern increases by one each time starting at 15 and they may think that the expression is simply $n+1$. However, if 1 was the first term, the answer to $n+1$ would be 2 instead of 15 . Point this out to students so that they see the expression would need to be $\mathrm{n}+14$ to result in 15 being the first term of the pattern. <br> Q7. This may confuse students since it involves subtraction and the variable is the second term in each expression. |


|  | Reflect <br> Discuss this question together. Most of your students will not have had enough experience with variables to be able to phrase an acceptable response. <br> Game: Tic-Tac-Toe Challenge <br> The main advantages of allowing students to play this game are that it develops spatial and logical reasoning skills as well as problem-solving abilities. Consider including it in your math centers. |
| :---: | :---: |
| Lesson 4: Strategies Toolkit | Explore <br> This is quite a difficult task as described. Consider modifying it to achieve the same objective. Instead of using modeling clay, talk about cutting a strip of paper. Give students their own strips of paper and scissors and ask them to draw a table in their notebooks to record the results of the first four rounds. <br> Connect <br> The problem solving strategy being developed in this lesson is "Draw a Diagram", but this is a complicated problem that does not lend itself very well to a diagram - more like a table. Instead of the story in the student text, try the following: <br> Model Trains <br> Jenny's model train is set up on a circular track. Six <br> telephone poles are spaced evenly around the track. The engine of Jenny's train takes 10 seconds to go from the first pole to the third pole. How long would it take the engine to go all the way around the track? <br> Get students to create a diagram to model the situation in the problem and explain any answers (in writing) they come up with. Some may say that from pole 1 to pole 3 is one third of the way around, so three thirds would take 30 seconds. Another student might say that it's 5 seconds from pole to pole and $5 \times 6=30$. <br> Practice (p.19) <br> Q1. Make sure students understand that the direction must be forward. They can't zig-zag backwards. <br> Q2. \& Reflect <br> Both tasks are well worth doing. |
| Lesson 5: Using a Variable to Write an Equation SCO PR2 | Explore <br> Begin this lesson by having students keep their books closed so you can do the Explore together. Write the statement at the top of p. $20(3+7=10$, etc.) on the chalkboard. Discuss what an equation is and what = means. (= does not mean "the answer", It means balance) <br> For the activity involving game cards, write the equations on the chalk board and discuss what the symbol in each represents (the unknown, the number we're trying to figure out to make the expression true (balance). Match each expression with the correct word sentence. |


|  | Connect <br> Go through this page slowly and carefully with students. Many will fall behind right here if we go too quickly. <br> Practice <br> Q1-4. Discuss and work through each one together one-at -atime. <br> Q5 \& 6. Let students try these on their own. <br> Q7. Optional, depending on how your students are getting along. May be appropriate for some. <br> Reflect <br> Optional <br> In the Assessment for Learning section (Teachers Guide p.20) there is a good suggestion for helping students who are having difficulty writing expressions to represent problems (see What to Do If You Don't See It). In fact, the climate in your classroom should be such that using models such as counters are a normal part of doing mathematics for any students who wants to use them. |
| :---: | :---: |
| Lesson 6: Solving Equations Involving Addition and Subtraction SCO PR2 | Explore and Connect <br> The two problems on p. 23 of the student text can be solved in many ways. Allow students to work on these on their own. <br> p. 24 (Guess and Test) For $98=72+$ w, say, " 98 is equal to 72 plus something. Could it be 72 plus 10? No, that's only 82. Could it be $72+20$ ? No, that's only 92. Could it be $72+30$ ? No, that would be 102 (too much). So what would work?" <br> (By Inspection). Downplay this as a strategy. It implies that you don't have to look carefully at the numbers when you guess and check. We want students to look first to the numbers every time before deciding what to do with them. Review how think addition could be used to solve $98=72+\mathrm{w}$, starting with the smaller number. 72...82, 92, 93, 94, 95, 96, 97, 98 (that's 26 altogether) <br> Practice <br> In this section, don't force students into using a particular strategy. Let them do it in ways that make sense. Watch for students who have inefficient strategies or no real strategy at all. Let them do Q1 - Q3 .on their own. <br> Note: Q3. $13+p=36$ or $36-13=p$ <br> Q4. Do this together since 19 is "nineteen". <br> Q5. O.T.O. $24-c=11$ or $11+c=24$ <br> Q6. Together. $40-13=f$ or $f=40-13$ or $f+13=40$ <br> Q7. Good problem. Suggest they consider drawing a diagram since this was a problem solving strategy they used back in lesson 4. |


|  | Q8. Do the first two together as a class. Record some stories on the chalkboard or overhead. For example, for $30=a+5$, the story might be: <br> Mike had 30 golf balls to start the season. At the end of the season, he only had 5 golf balls left. How many did he lose? $\begin{aligned} & 30=a+5 \\ & 30-a=5 \end{aligned}$ <br> Q9. Numbers, Pictures, Words: <br> Use this as a formative assessment checkpoint. <br> In the Assessment for Learning section (Teachers Guide p.23) there is a suggestion that you provide addition charts for students whose addition and subtraction skills are weak (see What to Do If You Don't See It). This is a counter-productive suggestion that does little to support student learning. Instead, teachers should change the size of numbers that struggling students are working with so that they will experience success. |
| :---: | :---: |
| Lesson 7: Solving Equations Involving Multiplication and Division SCO PR2 | Explore \& Connect <br> Go over this problem with the whole class so that they understand what it is asking. Allow them to think about it. Ask for some possible equations and write them on the board. Discuss what each one means. $\begin{gathered} 48 \div 6=b \\ b=48 \div 6 \\ 48=6 \times b \\ 6 b=48 \end{gathered}$ <br> Practice <br> Q1-4. For each group, do the first one together, then allow students to work on b-h. Check each group as they are completed. <br> Q5. O.T.O. <br> Q6. Talk this through with students. Help them to see 2 different ways to represent the problem with an equation. <br> Q7-9. O.T.O. <br> Q10. Do one at a time as a class. <br> Q11. Numbers, Pictures, Words <br> Use this as a formative assessment checkpoint. <br> Reflect <br> Discuss this as a class. Help them to understand the relationship between multiplication and division. <br> Game Match It <br> This is a good activity, but it would be difficult for the teacher to monitor groups' answers and disagreements might take away from the intent of the activity. Student pairings will be an important consideration. As an alternative, perhaps 10 students can have the problem cards and 10 students the equation cards. A problem |


|  | card is read and the person with the matching equation stands up. |
| :--- | :--- |
| Unit 1:: Show What You Know | Do not expect your students to be able to complete this work <br> independently at the beginning of the year. Use the questions as <br> teaching activities to review and reinforce the kind of pattern <br> work students have been doing in this unit. Students should be <br> able to complete selected questions on their own, but it will be up <br> to the teacher to decide which ones are reasonable at this point in <br> the year. |

## Unit 2: Whole Numbers

| Notes, Suggestions, Recommendations |  |
| :---: | :---: |
| Preparing to Teach This Unit | Review Specific Curriculum Outcomes N1 \& N2 in your Curriculum Guide as these are the SCOs the unit is attempting to address. Think about these two guiding questions, "What do I want my students to learn? What do I want my students to understand and be able to do?" The Achievement Indicators and the Assessment Strategies for these outcomes will help you answer these questions and give you a better understanding of what it is you need to emphasize in you teaching. You should also refer to the Unit Rubric on p. 39 in the Teacher's Guide, the Ongoing Observations sheet on p. 40 and the Assessment for Learning sections which are components of each individual lesson. <br> Note: Pages from the student text can be projected onto a screen if you have access to a computer and LCD. Use the ProGuide DVD which came with your Grade 5 Teacher's Resource Binder to locate specific content for each unit. Some teachers find this strategy helpful when discussing diagrams, tables and other illustrations with the whole class. |
| Launch: Languages We Speak | This Launch introduces students to some larger numbers by listing the number of people who speak various aboriginal languages in Western Canada according to the 2001 Canadian census. Statistics Canada www.statcan.gc.ca also lists "Aboriginal identity by province" based on the 2006 census and includes aboriginal populations for PEI (1730), NB (17650) and NS (24 175). Teachers should include these population numbers during the Launch and acknowledge the contribution of the Malecite-Passamaquoddy and Mi'kmaq people to the heritage of the maritime provinces. <br> Go over the questions in the text p. 35 with students. For the first two questions, they may not pick up on the fact that the population numbers ending in zero or five are probably estimates (rounded off). The last four questions provide a good opportunity for you to assess students' understanding and ability to read and interpret information presented in a table, and to "read" larger numbers correctly. |
| Lesson 1: Numbers to 100000 SCO N 1 | In the Explore, save on photocopying by showing students how to use a ruler to draw a 5-column chart. <br> - Hold up a unit cube from the base ten set and ask, "How many of these units are there in a rod (hold up rod in other hand)? How many in this flat? How many in this large cube?" <br> - Hold up a rod and ask, "How many of these rods are there in a flat (hold up a flat)? How many are there in this large cube?" <br> - Hold up a flat and ask, "How many of these flats are there in this large cube?" |

Do the second part of the Explore together. Write the resulting place value pattern on the board as you discuss the different ways to represent 10000 with base ten blocks.

| 10000 |  |
| :---: | :---: |
| $10000 \times 1$ | "Ten thousand ones." |
| $1000 \times 10$ | "One thousand tens." |
| $100 \times 100$ | "One hundred hundreds." |
| $10 \times 1000$ | "Ten thousands." |

In the Show and Share, students are asked to, "Talk about how the numbers $10,100,1000$, and 10000 are related." Get them to write about this first (about 10 minutes) and then call on several to share. (As students are writing, circulate around the room to identify the students you are going to call on. Begin with the less sophisticated explanations first).
In the Connect, for each model shown, write the quantity in words, eg.
$10000=$ "ten thousands"
$10000=$ "one hundred hundreds"
$10000=$ "one thousand tens"
$10000=$ "ten thousand ones"

To help students understand how the value of a digit increases ten times moving from left to right, ask students, "How many ones are there in 3 tens? How many tens are there in 3 hundreds? How many hundreds are there in 3 thousands? How many thousands are there in 3 ten-thousands?"

## Practice

Q1. This is a good opportunity to create a classroom model for $100,1000,10000$, and 50000 which will give students a sense of the size of larger numbers. They can also refer to it in later lessons when the number 1000000 is explored.
Using the small dot arrays (provided) make 31 copies and distribute them to students. Confirm by counting that there are 2000 dots on each page. Organize the class into groups to represent different parts of the model. Use the wall or bulletin board to represent 100, 1000, 10000 and 50000 . Arrange these right to left with appropriate labels. You could also include a model for 10 and for 1 if you wish.

Q2. O.T.O. (students should be able to complete this question on their own)

Q3. O.T.O. but ask them to use numbers, pictures and words to explain.

Q4. Have students do $\boldsymbol{c}$ ) and $\boldsymbol{d}$ ) first as these two are easier.
Q5 \& 6. When asked, "How many tens are in 8000?"some students may just focus on the "tens place" in the number say, "zero". To help them think about how place value changes, ask questions like, "Ten times what equals 8000 ? One hundred times what equals 8000 ? One thousand times what equals 8000 ?" You might also use "eight thousand" and "eighty hundred" interchangeably to help students conceptualize the quantity.

| Q7. O.T.O. |  |
| :--- | :--- |
|  | Reflect <br> Most students will not be ready for this type of question. Use it to <br> enrich the more capable students in your class. <br> Game Aim for 100 000 |
| Introduce this game by playing it with a student volunteer. Draw |  |
| two charts on the board and fill them in as you go. Write the |  |
| standard form of the number for each roll off to the right of your |  |
| chart. Read each of your numbers aloud and have the student do |  |
| the same. Refer to each player's running total from time to time. |  |
| Provide each student with one copy of the score sheet (Master |  |
| $2: 20) ~ f o r ~ t h e i r ~ f i r s t ~ g a m e, ~ b u t ~ h a v e ~ t h e m ~ d r a w ~ t h e i r ~ o w n ~ c h a r t s ~ f o r ~$ |  |
| additional games. |  |
| RCO N1 Exploring One Million | Refer to the model you made for 50 000 using the dot arrays. <br> Ask, "What would we have to do to this model to make it <br> represent 100 000? (double it)" Consider doing this with your <br> students because, as a tenth of a million, it will provide a referent <br> for the larger number. <br> Explore |
| Talk through this section with students. Refer to the chart and the |  |
| "ten thousand" place. Ask how many of the previous thousands |  |
| cube are needed? (10) Do the same for the other places. (ten of |  |
| the ten-thousands make 100 000, and 10 of the hundred |  |
| thousands make 1 million.) The important idea for students to |  |
| grasp is that everything get ten times larger as we move from |  |
| right to left. (adding of zeros to numbers). |  |
| One part of the Explore asks students to sketch each block. This |  |
| task would be too difficult and time consuming for most students |  |
| and should probably be omitted. |  |
| Connect |  |
| Review what a cubic meter looks like. Use masking tape to make |  |


|  | Q2. Show students how to use the constant feature on a calculator. To count by 1000s, press + $1000=$. Each time the equals sign is pressed, the total will increase by 1000. Do part a) together with students. Have them do b) and $\boldsymbol{c}$ ) on their own using the constant feature on their calculators. <br> Q3. Together <br> Q4. Number, Pictures, Words. Talk through this question with students and create a table to organize the increasing sizes. Hold up a meter stick and refer to the numbers 1-100. Ask how long is 100 cm ? ( 1 m ) Ask how long 1000 cm would be ( 10 m ), 10000 cm ? $(100 \mathrm{~m}) 100000 \mathrm{~cm}$ ? ( 1000 m which is also 1 km ) How long would ten hundred thousand or one million cm be? ( 10000 m or 10 km ) <br> Q5 \& 6. Together <br> Q7-9. O.T.O. Make sure they notice the operation signs for $d$ ), e) and $f$ ) in number 9 . <br> Q10. Together. <br> 1 straw $=\sim 24 \mathrm{~cm}$, so 1000000 straws would be about 24000 000 cm which is the same as 240000 m or 240 km since there's 1000 m in a km. <br> Reflect <br> Students should write a response to this in their math notebooks or journals. |
| :---: | :---: |
| Lesson 3: Representing Numbers SCO N1 | Call on students to read the numbers in each of the headline at the top of p .43 in their texts. <br> Explore <br> Draw the table on the board and complete together. Refer to Ten thousands row and ask, "Ten thousands multiplied by what is equal to 350000 ? " (35). Repeat for $910000,280000,50000$, and 200000 . Use the same line of questions for the Thousands, Hundreds, Tens, and Ones rows. You may also want to write out $10000,1000,10$ and 1 if you think it will help students understand multiplication by these amounts. <br> Show and Share <br> Discuss any patterns that you notice in the completed chart. <br> Connect <br> Copy the 6-column place value chart onto the board and go over the meaning of each place value. <br> Practice <br> Q1. O.T.O. Ask students to use a ruler to draw and label a 6column place value chart into their notebooks. Have them record each number. <br> Q2. Together. Write 25630 on the board and ask questions such as, "How ones are in this number? How many hundreds? Tens? Thousands? Ten thousands?" |

Q3. Together
a) Begin by writing $600000+20000+300+50+7$ and discuss with students how it would be recorded in standard form. It might be helpful if students mentally group the thousands (620 thousands) so that the number can be written the way it is spoken.
The next step is to write $600000+20000+50+7$ on the board and discuss how the two large numbers are different.
b) Ask, "How would we write nine hundred fifty thousand?" Write 950000 on board and then ask, "How would we write nine hundred fifty thousand six?"
c) \& d). O.T.O.

Q4. O.T.O.
Q5. Work through this question with your students but have them do most of the work. Note that when writing larger numbers in expanded form, students may record numbers the way they are read. For example, 343246 could be written in expanded form as $300000+40000+3000+200+40+6$ or as $343000+200+40$ +6 . This last representation shows that the student is able to group the largest part of the number in a way that makes sense. It is a good opportunity to focus on this part of the number and reinforce the place value ideas of hundreds of thousands plus tens of thousands plus thousands.

Q6. O.T.O. Review the meaning and use of $<$ and $>$.
Q7 \& 8. O.T.O.
Q9. Instead of writing these numbers in many different ways, have them read each number aloud or write each one in words.

Q10. Do $\boldsymbol{f}$ ) as an example. Let them do the rest O.T.O.
Q11-13. O.T.O. Review what area is and a square km in particular. Relate it to as square meter.

Q14. This may be a difficult activity to control and supervise. It is best suited for students who have a good understanding of place value and can work independently and cooperatively.

Q15. O.T.O. but go over it together. For example, in a) 40 187, the teacher's guide suggests that 0 indicates that there are no thousands. However, when a student reads the number, she would say, "Forty thousand eight hundred seventeen." Perhaps a better answer is that there is nothing in the thousands place, but that there are 4 ten-thousands in the number.

Q16. a) \& b) O.T.O.
Do c) together
Q17. a) O.T.O.
b) Omit. Most students will have no way to think about this question other than to say, "a lot"

|  |
| :--- |
| Lesson 4: Estimating Sums |
| SCO N2 |



A more advanced estimation strategy introduced in the Connect is called compatible numbers. With this strategy, students consider not only the largest place values in each number, but also the value of the other parts of the number. This will usually result in an estimate that is closer to the actual answer. For example, $11090+10651=$.
Using just front-end rounding, the estimated answer is 21000.
But by looking at 090 and seeing that it is close to 100 and 651 is close to 650, a closer estimate would be 21 750. A student's skill in estimating develops as his or her number sense develops. Looking for compatible numbers should be introduced after students have had much success with front-end rounding.

On p. 50 of the student text, have students read the numbers aloud from the table of Olympic games athletes. Work through the examples with students focusing on rounding up and rounding down.

## Practice

Q1. O.T.O. Be sure they understand that there is more than one pair of numbers for each sum.

Q2. Together. Change this question by asking students to pick out pairs of numbers that are easy to add (compatible numbers, friendly numbers, nice number etc.) This will vary from student to student. Ask them to explain why they think the numbers they have chosen are easy to add.

Q3. a) O.T.O. Review what multiples are list multiples of 100 on the board in random order.
b) Instead of this question, ask students how they went about identifying numbers that add to multiples of 100. What did they look for? Did they look beyond the hundreds place for 2-digits that add to 100 ?

Q4. Do a few of these together, then pick one or two for students to estimate and write an explanation for.

Q5. Make sure students just do the ones < 10 000. Identify which

|  | ones these are and list them on the board. <br> Q6 \& 7. Together. For b) don't worry about them using <br> compensation or compatible numbers specifically as strategies. <br> Just ask them to explain how their estimates could be improved. <br> Q8 \& 9. O.T.O. <br> Q10. Together. This question involves pattern recognition and <br> estimation. There is more educational value in having a class <br> discussion than in having everyone working on it on their own. <br> Discuss the patterns in the table. For example, the dates are 10 <br> years apart. Estimate roughly how much the population has <br> increased from date to date and list these changes on the board. <br> Help students see that the trend slows down between 1991 and <br> 2001 and that this should be taken into consideration when <br> predictions are made about 2011. |
| :--- | :--- |
| Q11. a) \& b) O.T.O. Begin by calling on several students to read |  |
| the number of tickets sold each day. |  |
| c) Do this part together after you've gone over the answers to a) |  |
| and b) with students. |  |
| Q12. O.T.O. |  |
| Lesson 5: Using Benchmarks to |  |
| Estimate |  |
| SCO N2 |  |


|  | Q9. O.T.O. <br> Q10. Numbers Pictures, Words: Omit <br> Q11. O.T.O. <br> Q12. Omit <br> Reflect <br> Written response |
| :---: | :---: |
| Lesson 6: Estimating Differences SCO N2 | Explore <br> Go over the information about the number of lift tickets sold. Read 1368 as "Thirteen hundred sixty eight" and 1155 as "Eleven hundred fifty five." <br> Students need considerable support with estimation involving subtraction. There are couple of different strategies introduced in this lesson. Again, it is important that students develop skill in producing broad estimates before they are expected to produce more refined, estimates. Teachers should refer to the strategies and practice items provided in the estimation section of Mental Math: Fact Learning, Mental Computation, Estimation Grade 5 Teachers Guide. <br> Practice <br> Q1. Together <br> Q2. O.T.O. Allow 5-10 minutes for students to complete and then check together. <br> Q3. O.T.O. Make sure they understand what happens with frontend rounding. <br> Q4. Ask students to write the second number in standard form. Review how a 6- or 7-column place value chart can help. <br> Q5. Numbers, Pictures, Words. This is a good assessment checkpoint that students can complete O.T.O. It may take about 20 minutes to complete. <br> Q6. O.T.O. <br> Q7. Together. Copy table onto board. <br> Q8. Optional <br> Reflect <br> Optional <br> Math Link <br> Do this as a subtraction using "think addition" on an open number line. Make sure students understand that subtraction is asking us to find out the difference between two number, in this case, the difference between the years 1875 and 1997 |



|  | Reflect Omit |
| :---: | :---: |
| Lesson 8: Strategies Toolkit | Specific problem solving strategies such as the one in this lesson (Guess and Check) should be introduced and facilitated by the teacher. <br> Explore <br> Read the distances on the map as a group. Be sure to explain that you have to take into consideration the return trip. For example, the distance from Vancouver to London return is 7596 $\mathrm{km}+7596 \mathrm{~km}$. Guide students through the use of "Guess and Test" as one problem solving strategy. |
| Unit 2: Show What You Know | Use the Show What You Know as a review and formative assessment and not as a summative test. Use the items as teaching activities to review and reinforce the kind of work students have been doing in this unit. Students should be able to complete some questions on their own, but it will be up to the teacher to decide which ones are reasonable. <br> Q1. Draw a chart on the board or refer to the class place value chart. <br> Q2. Do a) and b) together, but have students complete $\boldsymbol{c}$ ) and $\boldsymbol{d}$ ) O.T.O. <br> Q3. Do a) Together. <br> b) and c) O.T.O. <br> Q4. O.T.O. <br> Q5. a), b) \& c) Together <br> d), e) \& f) O.T.O. <br> Q6. O.T.O. <br> Q7. Addition O.T.O.; Subtraction together <br> Q8 \& 9. O.T.O. <br> Q10. Review benchmarks numbers with students. Draw 3 number lines on board and ask students to tell you the numbers to include when considering the closest 100,1000 and 10 for 6526 . <br> Q11. Together <br> Q12. Together. Use "open number lines" to help students work out the difference using think addition. <br> Q13 a). O.T.O. <br> b) Compute the difference for the ferris wheel tickets together, then have students do the others O.T.O. |


| Unit Problem: Languages We Speak | Make a transparency of the table on 68 of the student text. <br> Review and discuss the data as a group. <br>  <br>  <br>  <br>  <br>  <br> Q1-3. Together <br> Q4. Together <br> Q5. O.T.O. <br>  <br>  <br> Q6. Together <br> Q7. O.T.O. <br>  <br>  |
| :--- | :--- |

## Unit 3: Multiplying and Dividing Whole Numbers

| Notes, Suggestions, Recommendations |  |
| :---: | :---: |
| Preparing to Teach This Unit | Review Specific Curriculum Outcomes N1, N2, N3, N4, N5, N6 \& PR1 in your Curriculum Guide as these are the SCOs the unit is attempting to address. Think about these two guiding questions, "What do I want my students to learn? What do I want my students to understand and be able to do?" The Achievement Indicators and the Assessment Strategies for these outcomes will help you answer these questions and give you a better understanding of what it is you need to emphasize in you teaching. You should also refer to the Unit Rubric on p. 53 in the Teacher's Guide, the Ongoing Observations sheet on p. 54 and the Assessment for Learning sections which are components of each individual lesson. <br> In addition to Math Makes Sense, the resource Mental Math: Fact Learning, Mental Computation, Estimation should play an important role in your daily planning and teaching. The Grade 5 teacher's guide reviews the variety of thinking strategies introduced in Grade 4 to help students master the multiplication facts. <br> Understanding of division as an operation, mastery of division facts, and mental computation involving multiplication and division will be a major focus in Grade 5. However, that does not mean that practice and reinforcement of addition and subtraction skills is no longer important. Mental Math time in your class should include daily, brief practice and reinforcement of basic addition and subtraction facts. As well, practice in applying mental computation strategies such as Front-End Addition, Compensation, Compatibles, and Make 10, 100, should be ongoing so that both components support one another. <br> Note: Pages from the student text can be projected onto a screen if you have access to a computer and LCD. Use the ProGuide DVD which came with your Grade 5 Teacher's Resource Binder to locate specific content for each unit. Some teachers find this strategy helpful when discussing diagrams, tables and other illustrations with the whole class. |
| Launch: On the Dairy Farm | This Launch provides a context which will be familiar to many Island students and is a good opportunity for the teacher to ask the kinds of questions and model the type of thinking required to make sense of many word problems. The numbers involved are relatively easy for students to work with and there will be a variety of ways for them to think about and answer the three questions. |
| Lesson 1: Patterns in Multiplication and Division SCO N3, PR1 | The Explore section asks students to write "related facts". Most students will not know what this means. In the "Get Started" section of the teachers guide (Lesson 2, p.8) there is a good suggestion for using an array model to review the turnaround property of multiplication and connect the idea of division as a related operation. You should probably go over this before you |

ask students to write the related facts in the Explore on p. 72 in the text

## Connect

The Mental Math Grade 5 Teachers Guide introduces and develops strategies for helping students master multiplication facts and teachers should be reinforcing these ideas regularly in

## class.

The thinking strategy introduced in the Connect (p.73) is to use a known fact to help you figure out a fact that you don't know. However, the number lines in the student text don't do much to help students think because the products are all given and they simply count ahead or back the correct number of jumps and then read the answer. A better strategy is to use "groups of" language. To follow the example in the book, if you know 6 groups of 6 ( $6 \times 6$ ) is 36 , then 6 groups of $8(6 \times 8)$ will be 2 more in each group, so that's 12 more. The answer to $6 \times 8$ is $36+12$ which is 48 . Provide some more examples like this and call on students to explain their thinking. Even if a student knows that $7 x 8$ is 56 , he or she should be still required to explain how knowing some other fact, say $7 x 7$, could help with $7 \times 8$. In their daily work, however, any students who have mastered particular facts are not using any strategy at all They 'just know them."

The strategy of "think multiplication" for the division facts is probably the most important one for students to become comfortable with. Every time they see a division fact such as $45 \div$ 5 , they should immediately think, " 5 times what is equal to 45 ?" In this way, they are using their knowledge of the multiplication facts to help with the related division facts. The terms divisor, dividend, and quotient should be added to your word-wall along with a horizontal and vertical example.

The last part of the Connect introduces multiplying and dividing by zero. For $8 \times 0$, have students use cubes or counters to model 8 groups of $3(8 \times 3=24)$ then 8 groups of $2(8 \times 2=16), 8$ groups of 1 $(8 \times 1=8)$ and finally 8 groups of zero $(8 \times 0=0)$. Do the same thing when zero is the first number because zero groups of 8 is harder to think about. Start with 2 groups of 8 , then 1 group of 8 , and then zero groups of 8 to help students get a feel for the concept of zero in a multiplication sentence.

Dividing zero by a number and dividing zero by a number are probably best introduced in the way suggested in the student text - related multiplication facts. Note that dividing a number by zero (eg. $5 \div 0$ ) is "not possible". Be sure students understand why this is true.

## Practice

Q1. These should probably be done orally. For each one, call on a student to provide a fact that might be used to help and explain how it could be used. Use numbers, picture, words to record the explanation so that other students can see and hear how the strategy would work. The discussion is the most important part of this exercise, not completing 10 multiplication facts.

|  | Q2. Review with the class this concept first introduced in the <br> Connect. <br> Q3. O.T.O. <br> Q4. Review with the class this concept first introduced in the <br> Connect. <br> Q5 \& 6 O.T.O. Get them to write a response to Q6 to explain their <br> thinking and discuss as a group afterwards. <br> Q7, 8, \& 9. O.T.O. <br> Q10. You should do this orally with students to help develop the <br> Qthink multiplication" strategy. For each one, ask students to "think <br> aloud". For example, for 45 9 9, a student would say, "9 times <br> what is 45? 9 times 5 is 45, so 45 -9 is 5."There are only four <br> division facts presented in this question so it would be a good <br> idea to add some extra division facts to this mental math <br> exercise. <br> Q11. Numbers, Pictures, Words This is a good question for |
| :--- | :--- |
| students to write about in their math journals. It will assess |  |
| understanding of the "groups of" and "helping fact" idea in |  |
| multiplication. |  |



Lesson 3: Multiplying With Multiples of 10
SO N3

## Explore

Rather than give students calculators to multiply by multiples of 10, review the following. (See Mental Math Grade 5 Teachers Guide p. 45 for explanation and practice items)
$10 \times 1=10$ so $10 \mathrm{~s} \times 1 \mathrm{~s}=10 \mathrm{~s}$
Eg. $30 \times 6$ is 3 tens by 6 ones; 18 tens; 180)
$10 \times 10=100$, so $10 \mathrm{~s} \times 10 \mathrm{~s}=100 \mathrm{~s}$
Eg. $40 \times 70$ is 4 tens by 7 tens; 28 tens; 280
$10 \times 100=1000$, so $10 s \times 100 s=1000 s$
Eg. $40 \times 500$ is 4 tens by 5 hundreds; 20 thousands; 20000
You might even present images such as these and ask students to describe each using place value language:


Ones by tens equals tens


Tens by hundreds equals thousands

Students should be able to calculate the products on p. 80 using single digit multiplication facts and then applying the appropriate place value as outlined above. The only exception will be $12 \times 8$ which is not a basic fact. Help students apply this strategy.

## Practice

Q1- 7. Do all of these questions together as a group; discuss thinking strategies as you go. Review multiplying by 10, 100 and 1000 using a place-value change strategy. (See Mental Math Grade 5 Teachers Guide p. 46 for explanation and practice items)

Q8. Omit.
Q9. O.T.O.
Q11. Ask the group how they could figure this out. Some students may recognize that they have already worked out this answer in Q10 (216 000)

Q12. Together. $20 \times 500$ is tens by hundreds, so the answer

|  | should be thousands. $5 \times 2$ is 10, so 10 thousands is written as <br> 10 000. <br> Q13. O.T.O. <br> Reflect <br> Omit |
| :--- | :--- |
| Lesson 4: Estimating Products to <br> Solve Problems <br> SCO N2 | Review the Mental Math Grade 5 Teachers Guide p.56-60 for <br> other estimation strategies. <br> Note: Students do not need to remember the names of every <br> strategy (compatible numbers, compensation etc.). The goal is for <br> them to develop estimation skills which can be refined with <br> regular practice and as their number sense becomes more <br> sophisticated. <br> Connect <br> Help students round numbers to the nearest multiple of 10. Use a <br> number line as a model. When they round 48 8 8 to 50 x 8, they <br> can use single digit multiplication facts (5x8) and then apply the <br> appropriate place value. <br> Practice <br> Q1. Together. Change this question by asking "How should we <br> think about these numbers to help us estimate?" |
|  | Q2. O.T.O.. <br> Q3 \& 4. Together. Talk through the estimation process with <br> students for each one. |
| Q5 - 7. O.T.O. |  |
| Q8. Go over this question carefully with the class before they |  |
| work on it. Break it down and ask questions such as, "How many |  |
| days do they sell tickets? How many tickets do they sell each |  |
| day? About how many tickets will that be? How did you work that |  |
| out? How many tickets were they hoping they could sell? How did |  |
| they do? Are they close? " |  |
| Q9. Omit. |  |

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Lesson 5: Using Mental Math to
Multiply
SCO N4
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## Connect

There are three different ways for students to think about multiplication in this section. They should be introduced one at a time and you should perhaps spend a lesson or two developing and reinforcing each strategy. The first method relies on an understanding of the Distributive Property for multiplication. A grid paper transparency would probably be an easier model for the teacher to work with instead of loose counters. In the first example, a $15 \times 7$ rectangle ( 15 rows of 7 ) could be drawn. This rectangle can then be subdivided into two smaller rectangles, $10 \times 7$ and $5 \times 7$. Students will need help decomposing multiplication problems into parts.
Eg. $35 \times 12$ is the same as $(30+5) \times(10+2)=(30 \times 10)+(30 \times$ 2) $+10 \times 5)+(2 \times 5)$.

The second method is called halving and doubling and is almost the same strategy introduced in lesson 2 . The only difference is that if you halve one factor and double the other factor before you multiply you don't have to double the product. Students should see how and why both methods work and they can use the one they like the best. It's just a question of when you want to double.

## Practice

Q1. Together.
a) Help your students see that the whole array is a $13 \times 8$ rectangle (13 rows of 8 ) but that it has been subdivided into 2 smaller rectangles. The larger of the two is 10 rows of 8 or $10 \times 8$ (80) and the smaller rectangle is 3 rows of 8 or $3 \times 8$ (24)
b) The rectangle drawn in the students' book is $6 \times 20$ and was drawn to help with $6 \times 19$. ("if I know 6 groups of 20 is 120 , then 9 groups of 20 is 6 less, 114"). You might want to just draw the rectangle for $6 \times 19$ on the board and help students see the 19 as 10 and 9 , so they can work with the partial products $10 \times 6$ plus 9 x 6 .

Q2. Use several of these multiplication questions to model how to make arrays on grid paper or with base ten blocks using the "rows of" language. Have students do some as well.

Q3. Do this question quickly together.
Q4. O.T.O.
Q5. Together. Put each question on the board. Ask, "Is one of the factors even? (yes) Can we use the halve-double strategy here? (yes) How will it work?"

Q6. O.T.O.

Q7. Write each one of these down on the board or chart. Ask students to solve them mentally if they can and to explain their strategies. Students should be allowed to jot down parts of their thinking as they go to support short term memory. There may be more than one strategy for certain problems or there may some

|  | problems for which no one has a strategy. Work on these together. <br> Q8. You may want to include this game at one of your math centers. Struggling students might be limited to 2 -digit by 1 -digit numbers. <br> Q9. Omit <br> Q10. Numbers Pictures, Words: O.T.O. <br> Q11. O.T.O. <br> Q13. O.T.O. <br> Reflect <br> Written response. Change the question to, "Which mental math strategies for multiplication do find easiest to use? Why?" |
| :---: | :---: |
| Lesson 6: Multiplying 2-Digit Numbers SCO N5 | In this lesson, students will be modeling multiplication sentences using base ten block arrays, rectangular arrays on grid paper, and then rectangular arrays on plain paper. It represents a progression from the concrete, through the pictorial, to the symbolic. Teachers should spend an entire class (or more) on each model until students are comfortable with the representations. When creating base ten arrays, teachers will need to model first and then have students try some using their own base ten materials. For example, the teacher could write 12 $x 8$ on the board and then proceed to build an array that is 12 rows of 8 . Students could then make arrays for $10 \times 13,12 \times 15$, $22 \times 12$ etc. How can we figure out the total by finding partial products? <br> Connect <br> Arrays With Base Ten Blocks <br> To model $21 \times 13$ ( 21 rows of 13 ) start by making 1 row of 13 by using a rod and 3 unit cubes. Then make 2 rows of 13 , etc. When you get to 10 rows of 13 ask if there's and easier way to build this other than sing 10 rods? (use a flat instead of 10 rods) What about all the unit cubes? Is there an easier way to show these? (use a rod for every 10 cubes). Continue until the $21 \times 13$ array or rectangle is completed. Look for groupings that involve 10 ( 10 x $10,10 \times 3,1 \times 10$ etc.) to calculate partial and total products. <br> Arrays With Grid Paper <br> Working with grid paper is similar to using base ten blocks. Follow the same procedure. Model several examples on an overhead transparency, and then have students try some on their own. Look for groupings that involve 10 to calculate partial and total products. <br> Arrays on Plain Paper <br> This model is a bit different. It doesn't matter if students rectangles are not the same size as long as they can subdivide them into the smaller rectangles given the numbers involved. Again, students should identify groupings that involve 10 to |

calculate partial and total products.

## Practice

Q1. O.T.O
Q2. O.T.O. Allow students to use whatever method they want including the traditional algorithm. However, they must also do the multiplication in another way to check to see if they get the same answer. They could use mental math (describe the steps), expanded notation using the distributive property, arrays, or some other efficient method for multiplication. Help students see that the factors in each pair are the same so the products will be the same.

Q3. Do a) and b) together and have students do c) and d) on their own.

Q4. Do a) and b) together and then assign the rest. Note that mental math, despite what the teacher's guide says, does not work well for c) and h). Encourage students to use some of the other strategies that you've been working on in class even though many of them may prefer to use the traditional algorithm.

Q5 \& 6.Combine these two questions. Ask students to do a \& b any way they wish, but to do the remaining ones using expanded notation, mental math and a sketch of an array. They can choose which ones they want to do in a particular way.

Q7. O.T.O.. It will be interesting to see how students respond to this $w=q u e s t i o n$. Do they use place value language to describe the computation or do they describe it moving from right to left using "write the zero, multiply, and carry, single-digit language"?

Q8. Use grid paper to build a $25 \times 25$ rectangle. Challenge students to describe how $25 \times 26,24 \times 25,50 \times 25$, and $75 \times 25$ are different from $25 \times 25$.

Q9. O.T.O. Good problem solving activity

## Q10. Numbers, Pictures, Words.

Together. Use "think aloud". For $45 \times 23$, think, $(40+5) \times(20+3)$, so for $40 \times 20$ I could use $4 \times 2$, for $40 \times 3$ I could use $4 \times 3$ etc.

Q11. Do the estimates together. Review the strategies that could be used as you go along. Record the steps for other to see.
Have students do the actual computations for those with products $>3000$.

Q12. O.T.O.
Q13. Estimate together. Review strategies. Record the steps for other to see.

Q14. O.T.O. Students' explanations will reveal a great deal about their understanding of place value.

|  | Reflect <br> Optional. Depends on the numbers involved. <br> At Home <br> Connect the work you've been doing in class with arrays and rectangles to finding the area of a rectangle and recording it as square units. Be sure students understand this before assigning it for homework. |
| :---: | :---: |
| Lesson 7: Estimating Quotients to Solve Problems SCO N2 | Note: This lesson will take at least two classes <br> Explore <br> Start students off on this question and then circulate to help as needed. Ask questions such as, "Would they drive 100 km in an hour? (No) Why? (Because that would be 800 km in 8 hrs. and it says 675 km .) Would it be 50 km in an hr.? (No, that would be 400 km in 8 hrs.) What about 600 km ? Are we getting close? <br> Connect <br> Remind students that the answer to a division problem is called the quotient. <br> Since this involves division, it will be important for the teacher to go slowly with the examples and to use "think aloud" frequently as an instructional strategy. For the first number \$873, ask "About how much is this? <br> In the second bullet in the Connect, the image is misleading since it doesn't show baskets with 4 grapefruits each. <br> Practice <br> Q1 \& 2. Do all of these orally with students. Make sure they understand that because you're estimating, you're trying to round the dividend to a number that works well with the divisor. For example, if I'm working with $238 \div 3$, I might round 238 to 240 because I know 24 and 3 work well together ( $3 \times 8=24$ so $3 \times 80$ is 240) <br> Q3. O.T.O. <br> Q4. Do together. The numbers involved are difficult to work with at this stage. <br> Q5. O.T.O. <br> Q6. O.T.O. Clarify the problem and get students started in their thinking. <br> Q7. Together. Help them reason with the information presented in the problem. Ask questions such as, "Will they get 50 tokens each? (That would be 450 tokens in all) How about 100 tokens each? (That would be 900). Are we getting close? <br> Q8. Numbers, Pictures, Words Omit |


|  | Q9 -12. O.T.O. <br> Reflect Optional |
| :---: | :---: |
| Lesson 8: Dividing a 3-Digit Number By a 1-Digit Number | This lesson will take several days since there are two different methods for division that students will be trying out. <br> Explore <br> Let students struggle (but not suffer) with this problem. Be sure to spend time with the Show and Share to hear how different students approached the task. <br> Connect <br> Division using base ten blocks and repeated subtraction are the two methods that students should become familiar with in this lesson. Introduce Base ten blocks first, and begin with 2-digit by 1-digit division with no remainder to start off. For example $45 \div 3$, then $53 \div 4$, then $124 \div 3$, etc. Students need to work with the blocks at their own desks as they model the division problems you put on the board. The focus at this stage is on modeling the operation and recording the quotient. <br> Repeated subtraction for division connects with the think multiplication strategy for division facts. We multiply the divisor by multiples of 10 (because that's easy) and subtract from the dividend. We keep track as we go. "Think aloud" will be an important instructional strategy for helping students come to understand how repeated subtraction works. <br> Practice <br> Q1. The numbers in these problems are too large for students to solve using base ten blocks. Most classrooms would not have enough materials for everyone. However, every class should have at least one larger set (home-made) that could be used for demo purposes. Call on individual students to come up and model the division problems using this set of materials. <br> Q2. Some of the numbers involved in these problems are cumbersome to work with. Make the following changes: <br> a) $155 \div 5$ <br> c) $132 \div 4$ <br> d) $127 \div 3$ <br> e) $116 \div 4$ <br> f) $106 \div 4$ <br> g) $126 \div 4$ <br> h) $122 \div 3$ <br> Call on students to come up and model them with the blocks. <br> Q3. O.T.O. <br> Q4. Have students use repeated subtraction for all of these. The teacher should model the first one and have students do the next one. Continue in this manner until all problems have been completed. |


|  | Q5. O.T.O. <br> Q6. Model repeated subtraction for a) and b) and assign the rest. <br> For the question about dividing by 5, review the patterns in a <br> hundreds chart that result when you multiply by 5 (the products all <br> end in zero or in 5) |
| :--- | :--- |
| Q7 \& 8. O.T.O. |  |
| Q9. Numbers, Pictures, Words |  |
| Discuss this question as a group. Refer to the connection |  |
| between multiplication and division. |  |
| Q10 \& 11. O.T.O. Students could use repeated addition, repeated |  |
| subtraction or some form of multiplication (guess and check) to |  |
| solve these problems. |  |
| Q12. Together |  |
| Q13. Optional |  |
| Q14. Omit |  |
| Reflect |  |
| Consider discussing these questions with students as a group. |  |
| Provide example for them to think about or elicit examples from |  |
| students themselves. |  |


| Lesson 9: Other Strategies for Dividing Whole Numbers | Connect <br> This introduces the process of division using base ten blocks and connects it to the traditional algorithm which most students don't understand because they have learned a rote procedure. The Connect also introduces another way to use the distributive property so that the dividend is broke up into parts that are easily divisible by the divisor. <br> Practice <br> Q1 \& 2. The division problems presented here can be done in a variety of ways depending on the numbers involved. Think multiplication using basic facts, repeated subtraction, base ten blocks (long division), and using the distributive property. Do about 12 of the problems from these first two questions with students and assign several for them to do on their own. Have them apply some of the different methods. <br> Q3. Omit <br> Q4-7. O.T.O. <br> Q8.Together <br> Q9. Optional <br> Q10. Numbers, Pictures, Words O.T.O. Start students off with the first couple of combinations. <br> Reflect <br> Omit |
| :---: | :---: |
| Lesson 10: Solving Problems (Problem Solving) | This lesson will provide students with experience in solving word problems that have more than one step. Because of the structure of these problems and the amount of text involved, it is recommended that teachers pick and choose which problems are right for their students and then guide them through the problem solving process. The following suggestions may be helpful. <br> Practice <br> Q1. a) O.T.O. <br> b) \& c) omit <br> Q2. Together <br> Q3 \& 4. O.T.O. Clarify each question before they begin. <br> Q5. Omit <br> Q6. O.T.O. Check for understanding before they begin. |


| Lesson 11: Strategies Toolkit <br> (Make an Organized List) | This is a very important problem solving strategy that is <br> introduced in this lesson. Review all of the problem solving <br> strategies listed in the side bar on p. 12 of the student text. The <br> strategy in this lesson is highlighted. Teachers will need to show <br> how to make an organized list and use it to solve problems. Call |
| :--- | :--- |
| on students for help in adding new information until is is |  |
| completed and the problem solved. Use the story problems in the |  |
| Connect and Practice to further develop and reinforce this |  |
| strategy. |  |

## Unit 4: Measurement

## Notes, Suggestions, Recommendations

Preparing to Teach This Unit
Review Specific Curriculum Outcomes SS1, SS2, SS3, \& SS4 in your Curriculum Guide as these are the SCOs the unit is attempting to address. Think about these two guiding questions, "What do I want my students to learn? What do I want my students to understand and be able to do?" The Achievement Indicators and the Assessment Strategies for these outcomes will help you answer these questions and give you a better understanding of what it is you need to emphasize in you teaching. You should also refer to the Unit Rubric on p. 49 in the Teacher's Guide, the Ongoing Observations sheet on p. 50 and the Assessment for Learning sections which are components of each individual lesson.
Note: Pages from the student text can be projected onto a screen if you have access to a computer and LCD. Use the ProGuide DVD which came with your Grade 5 Teacher's Resource Binder to locate specific content for each unit. Some teachers find this strategy helpful when discussing diagrams, tables and other illustrations with the whole class.

When teaching measurement, it is important to distinguish between students going about a measurement process and students conceptually understanding what they are doing as they measure. In general, an instructional plan for measurement should lead students through three phases of understanding. 1. Various attributes of objects can be measured through comparison activities to determine longer/shorter, heavier/lighter, more/less etc. Once students understand the attribute, there is no further need for comparison activities.
2. Using physical models of measuring units to fill, cover, or match the attribute with the unit produces a number we call a measure. In Grade 5, it is appropriate to begin with informal units (paper clips, marbles, popsicle sticks, index cards etc.) and progress to the direct use of standard units (cm cubes, meters) when appropriate and certainly before using formulas or measuring tools.
3. By carefully comparing standard measurement tools with the individual units that they have been using, students will understand how the formal tool (eg. ruler, tape measure, meter stick) performs the same function and they will be better prepared to use these tools correctly.

Throughout this unit, students are encouraged to develop referents for a variety of measures including $\mathrm{mm}, \mathrm{cm}$, and m , and use these to estimate various lengths. For example, a piece of dried spaghetti is about 1 mm wide; the thickness of a CD case is about 1 cm ; the distance from the floor to the door knob is about 1 m . Using referents helps students think in a more concrete way about the actual size of each unit. Instead of just thinking that a mm is really small and a meter is pretty big, students can relate these measures to more familiar objects and visualize the actual length of a particular measure.

| Launch: At the Zoo | In this Launch, students are invited to identify measurements in the picture. Begin by asking, "What are some length measurements in this picture? Height measurements? Width measurements? What other kinds of measures do you notice?" At the end of this unit, students will have an opportunity to design their own petting zoo using some of the measurement attributes and skills that these lessons will develop. |
| :---: | :---: |
| Lesson 1: Measuring Length SCO SS1, SS2 | The Explore section asks students to use estimation for a scavenger hunt to locate items in the classroom that are different lengths in mm . This would be a good activity for pairs of students and you might require them to find two or three different objects for each length specified You could also include some additional lengths for them to identify. Do not pass out the rulers or tape measures until after they have identified various objects. <br> Connect <br> Students are shown how a measure in mm can also be recorded as a decimal number involving cm and tenths of a cm (eg. 62 mm $=6.2 \mathrm{~cm}$ ). It would be a good idea to review what one tenth means, why it is used here, and how it is recorded and read as a decimal number ( 6.2 is read as " 6 and two tenths", not " 6 point 2" or "6 decimal 2") These concepts were first introduced in grade 4. <br> Practice <br> Q1. Rather than have students copy these tables, the teacher should sketch each one on the chalkboard and enter the data that students provide. For each table, complete in random fashion. <br> For 1b) begin by saying, "We might measure something really small and find that it's only 9 mm long. How much of a cm is that? Is it more than a cm or less?" (It's nine tenths of a cm) "How do we write nine tenths?" (0.9) "What does the zero mean? What if it was 12 mm long? How many tenths of a cm is that?" (Twelve tenths) "Is that more or less than a cm?" (More because 10 tenths is 1 cm and there would be 2 extra tenths, so that's one and two tenths of a centimetre.) "How would we write that number?" (1.2) <br> For 1c), show students a meter stick to give them a sense of the length of 100 cm and 1000 mm . <br> Q2. This should be obvious from the work you just did with the three tables. Discuss the patterns you find. <br> Q3 \& 4. O.T.O. Circulate to see if everyone understands the questions. Allow students to use rulers or tape measures if they wish. <br> Q5. Do this question together. Ask leading questions. Hold up a meter stick and ask how long it is ( $1 \mathrm{~m}, 100 \mathrm{~cm}, 1000 \mathrm{~mm}$ ). "So how long would 2000 mm be?" Etc. <br> Q6. Encourage students to think of something that is about each measure ( $\mathrm{cm}, \mathrm{mm}, \mathrm{m}$ ). If you just ask them to "name another referent" and then "explain ...", they probably won't understand what you mean. |


|  | Q7. Optional <br> Q8. This will be a better "visualizing" activity if students don't use grid paper. Grid paper will change it into a "count the number of squares" activity. <br> Q9. O.T.O. <br> Q10 \& 11. These two questions could be used as a formative assessment checkpoint to see where students are in their understanding. <br> Q12. Together <br> Q13. Numbers, Pictures, Words This is a good question for students to write about in their math journals. <br> Q14 \& 15. O.T.O. <br> Q16. Before asking students to do this question, draw two similar, but longer, line segments on the chalk board and model by thinking aloud. "I'm going to estimate the length of this line in cm first. Then I will multiply by 10 to get the number of mm because I know that there are 10 mm in each $\mathbf{~ c m}$." <br> Reflect <br> This is a good prompt for students to respond to in their journals. It could also be a homework assignment if they were asked to draw a sketch for each object. |
| :---: | :---: |
| Lesson 2: Strategies Toolkit Use a Pattern \& Make a Table | Review the problem solving strategies listed in the side bar on $p$. 126 of the student text. The strategies in this lesson are highlighted and this is a good opportunity to remind students that we often use more than one strategy when solving problems. <br> Explore <br> Teachers will need to help students get started in making the table and recording the dimensions of each garden. Rough sketches of each garden (squares) will help connect the related concepts of side length, perimeter, and area and should be used for the Connect and Practice sections of this lesson. <br> Practice <br> Q1. Change the number of tiles from 72 to 24 . Start by modeling how you could make a $1 \times 24$ rectangle with tiles. <br> Q2. O.T.O. Encourage students to draw sketches of each rectangle or use grid paper. |
| Lesson 3: Exploring Rectangles With Equal Perimeters SCO SS1, SS2 | Explore <br> This is a difficult explore and should be done as a group. If you are going to use a geoboard for this activity, be sure that it has a $10 \times 10$ array of pegs. <br> Begin by asking students, "What do we know for sure?" (The pen is a rectangular shape and Simon has 22 m of wire mesh to go |

around the whole pen). Draw a rectangle on the board and outline the perimeter to show where the 22 m of wire will go. "What are some possible dimensions for the pen?" Use guess and test as a problem solving strategy. For any dimensions that yield a perimeter of 22 m , transfer the rectangle onto a grid paper transparency so that the area can be seen as square meters. Create a table like the one in the student book or like the one below:

| P | L | L | W | W | A |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 22 | 10 | 10 | 1 | 1 | $10 \mathrm{~m}^{2}$ |
| 22 | 5 | 5 | 6 | 6 | $30 \mathrm{~m}^{2}$ |
| 22 |  |  |  |  |  |
|  |  |  |  |  |  |

The "big idea" that we hope students come to understand is this: When rectangles have the same perimeter, those with the least width have the least area. Rectangles that are closest to square have the largest area.

## Practice

Q1. Since there will be many opportunities for students to draw rectangles throughout this lesson, it is important to ensure that they understand some of the strategies that can be used to change a given rectangle to one with different side lengths but the same perimeter. For that reason, teachers should do a) and b) with students one at a time. Use an overhead transparency of grid paper and replicate the $2 \times 4$ rectangle in the student book. Demonstrates what happens when we change one of the side dimensions and then do the opposite to the other dimension. For example, a $2 \times 4$ rectangle can be changed to a $3 \times 3$ rectangle or to a $1 \times 5$ rectangle. What happens to the perimeter in each case? What about the area?

Do the same for the $6 \times 4$ rectangle, and then let students do the $5 \times 3$ rectangle on their own.

Q2 \& 3. O.T.O.

## Q4. Numbers, Pictures, Words

Good formative assessment question for students to complete on their own.

## Q5. Optional

Q6. The use of a $10 \times 10$ geo-board for this question is a valuable hands-on experience for students. If some kids seem to be struggling, or if their methods seem inefficient or random, consider modeling how you might think of the area of a rectangle and its dimensions in the same way that you think of a product and its factors. For example, if we know that the area of a rectangle is 32 , we might ask, "What times what is 32 ?" We would come up with $1 \times 32,2 \times 16$, and $4 \times 8$. These could be the dimensions of the rectangle, but if we also know that the perimeter has to be a certain length, then only one set of dimensions will work. In this example, if the perimeter is 24 , then

|  | the dimensions of the rectangle must be $4 \times 8$ because $4+4+8$ $+8=24$. <br> Q7. O.T.O. Just make sure students understand that Xavier's flower garden is square. <br> Q8. Optional. There are many different rectangles that could be made. <br> Reflect <br> Good assessment checkpoint. <br> Game: Who Can Fill the Page? <br> Optional. This game can be time consuming with quite a bit of "wait time" between plays depending on the numbers rolled. For example, if Player A rolls two 6s for a perimeter of 36, Player B would have to wait until 9 different rectangles were drawn on the grid paper. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Lesson 4: Exploring Rectangles With Equal Areas SCO SS1, SS2 | Explore <br> This is a good activity for pairs of students to explore. After each pair has made one rectangle with the 36 tiles, ask for volunteers to share with the whole group. Show students how to record the information in the chart. Allow them to continue with the rest of this activity. <br> Connect <br> Help students see that each rectangle they made, regardless of the shape and perimeter, had an area of 36 square units. <br> Connect the dimensions of each rectangle and its area to factors and products. <br> Practice <br> Q1. Do a) and b) together. Ask, "What are some possible dimensions for rectangles if we know the area is $8 \mathrm{~cm}^{2}$ ? What times what is the same as 8 ?" etc. Let students do $\mathbf{c}$ ) and d) on their own. <br> Q2. O.T.O. Just make sure students understand that the area of each dog pen must be 48 square units. <br> Q3. Numbers, Pictures, Words <br> Do Together. Since square meters are involved, draw $1 \mathrm{~m}^{2}$ on the chalkboard as a referent for students to envision the size of the $64 \mathrm{~m}^{2}$ garden. Generate all the possible rectangles that would yield an area this size. |  |  |  |  |
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|  | Q4. O.T.O. Do a) together to get them started. Review how to find all the possible side lengths and then choose the one that matches the given perimeter. <br> Q5. a) and b) O.T.O. c) optional <br> Q6. Good formative assessment checkpoint. <br> Reflect <br> This would be a good question for students to think, write, and draw about. By now they should understand that if you know the area of a rectangle, you can find all the possible dimensions, but you can't know for sure which one is the exact shape. |
| :---: | :---: |
| Lesson 5: Exploring Volume SCO SS3, SS4 | When most students hear the word volume, they will immediately think about the "loudness" of something, particularly TV and music. Grade 5 is the first time they will study volume in a mathematical context and it is important that you begin with informal units such as blocks, marbles, styrofoam "packing peanuts" etc., so that students begin to develop an understanding of volume as the number of identical units that can fit inside an object. They will soon recognize that cube-shaped objects provide the most accurate measure of volume because the cubes fit together without as many gaps between or around them. This leads to the introduction of the cubic centimeter $\left(\mathrm{cm}^{3}\right)$ and cubic metre $\left(\mathrm{m}^{3}\right)$ as the "standard" unit for measuring volume. <br> Explore <br> In this activity, students are asked to estimate the number of objects that a small box can hold (consider using something smaller than a shoe box). To save time and ensure that everyone has a chance to think about volume in this context, collect a variety of different sized objects (or have a few students bring collections from home) that can be used to fill the box. Involve students in estimating, and in filling and counting the number of objects each time. Use the word about to describe the measure because of the spaces between objects. <br> Practice <br> Q1-6. Together as a group. The teacher should gather readily available materials to allow students to make estimates and check estimates. This is similar to the work done in the Explore. In Q6 ask why the term about is used to describe the volume of each item. <br> Reflect <br> Optional |
| Lesson 6: Measuring Volume in Cubic Centimeters SCO SS3, SS4 | Explore <br> The grid paper specified for this activity (Master 4.21) is the wrong size. Each square is only .5 cm instead of a full centimetre. A better idea is to give students some linking cubes and ask them to build a rectangular prism. How many cubes did they use altogether? Record these as "cubic units". If no one builds a prism in which there are cubes that can't be seen (eg., $3 \times 4 \times 3$ ) display |


|  | one that you made and ask them to think about the number of blocks that were used and how they know. <br> Connect <br> Instead of beginning with centimetre cubes, start with larger cubes such as linking cubes or wooden blocks if you have them and refer to them as "cubic units". Display a variety of open rectangular prisms such as the top or bottom of a shoe box etc., and have pairs of students fill the containers by stacking cubes inside. The containers may not exactly accommodate the material (could be a little more or a little less) so, again, it's important to describe the volume as "about 40 cubic units (units ${ }^{3}$ )". <br> There is also an example of an object with an irregular shape. We often see irregular shapes in commercial packaging and it's important to remember that volume refers to the amount of space an object takes up as measured in cubic units. <br> Practice <br> Q1-3. For the purpose of these questions, linking cubes (eg. cube-a-links) can be used in place of the smaller centimetre cubes and refered to as "cubic units". However, because a linking cube is $2 \mathrm{~cm} \times 2 \mathrm{~cm} \times 2 \mathrm{~cm}$, you could convert the number of linking cubes to centimetre cubes by multiplying by 2 and use $\mathrm{cm}^{3}$ as the notation. <br> Q4 \& 5. Optional <br> Q6. O.T.O. <br> Q7. Omit <br> Q8. O.T.O. This is a very good question for students to answer. Make sure they understand that Ogi only has a few cubes to work with, not enough to fill the box. <br> Q9. Numbers, Pictures Words <br> Distribute cubes for students to problem-solve with. <br> Q10. Together. Using the larger cube-a-links, the text book is about $14 \times 11 \times 1$ cubic units or $308 \mathrm{~cm}^{3}$. <br> Q11. Omit <br> Reflect <br> Optional |
| :---: | :---: |
| Lesson 7: Constructing Rectangular Prisms With a Given Volume SCO SS3, SS4 | Explore \& Connect <br> Each student or pair of students will need 24 cubes to construct prisms. <br> Practice <br> Q1. Build each prism for students to see and handle if they wish. <br> Q2. Do these as a group and record the different ways prisms with the same volume can be built. For example, by asking students to build a rectangular prism with a volume of $36 \mathrm{~cm}^{3}$, they will see that this can be done in a variety of ways. |


|  | Q3 \& 4. O.T.O. <br> Q5. Optional <br> Q6. Numbers, Pictures, Words <br> Use Step-by-Step p. 64 of your guide to help students with this question. <br> Q7. Optional <br> Q8. O.T.O. <br> Reflect <br> Discuss as a group and then have students write a response. |
| :---: | :---: |
| Lesson 8: Measuring Volume in Cubic Meters SCO SS3, SS4 | Explore <br> A valuable activity is to have students actually construct a model of a cubic metre. However, rolled-up newspapers and masking tape may not be the best material to use. The Grade 3 teachers in your school may have something that you could borrow. A few years ago, they were given some red plastic rods of different lengths to use with their "Materials and Structures" unit in science. Some of the pieces are .5 m long and these can be joined with a linking cube to make 1 m lengths. These can then be joined to form a cubic metre. |
|  | Practice <br> Q1. a) Consider re-phrasing this question to, "Can you think of something that has a volume of about 1 cubic metre?" <br> b) It will be difficult for students to visualize the volume of these three objects. Look for examples in your class or around the school that students can actually see and spatially reason with. <br> For example, the gymnasium or library, a broom closet or storage |

$\left.\left.\begin{array}{|l|l|}\hline & \begin{array}{l}\text { room, the office. Include some examples that are a bit less than a } \\ \text { cubic metre as well. } \\ \text { Q2- 4. O.T.O. In Q4, volume can be determined in a few different } \\ \text { ways including counting by 1s, addition, and by multiplication. At } \\ \text { this level, students are not expected to memorize the standard } \\ \text { formula for finding the volume of a 3-D object (L x W H), but } \\ \text { many will use this method if it makes sense to them. }\end{array} \\ & \begin{array}{l}\text { Q5. Numbers, Pictures, Words } \\ \text { a) O.T.O. Be sure to give students cubes to model the crates. Ask } \\ \text { them to stack their "crates" in more than one way. List the various } \\ \text { dimensions for each method. } \\ \text { b) Optional. This will come up in the discussion of part a) }\end{array} \\ \hline \text { Lesson 9: Exploring Capacity: The } \\ \text { Litre } \\ \text { SCO SS3, SS4 } \\ \text { This could be used as a formative assessment checkpoint to see } \\ \text { if they have a sense of the size of 1m }{ }^{3} \text {. } \\ \text { In this lesson, students are introduced to the concept of capacity. } \\ \text { Initially, there may be some confusion about the difference } \\ \text { between volume and capacity since both terms seem to be } \\ \text { describing the same thing, that is, the 3-dimensional size of an } \\ \text { object. While volume refers to the space taken up by an object as } \\ \text { measured in cubic units, capacity refers to the amount of liquid or } \\ \text { other pourable substance a container can hold as measured by } \\ \text { milliliters (mL) and litres (L). Sometimes the word capacity is used } \\ \text { in other ways, too, such as, "The gym was filled to capacity." Or } \\ \text { "A whale has a large lung capacity." }\end{array}\right\} \begin{array}{l}\text { Q4 \& 5. Optional } \\ \text { Q6. O.T.O. To get students started, ask the question in this way; }\end{array}\right\}$

|  | "If 1L fills about 4 regular juice glasses, about how many glasses can you fill with 4L of apple juice? <br> Q7. O.T.O. The explanation is probably the best part of this question! <br> Q8. O.T.O. Get them started on this question by referring to the 8 glasses of water and asking them to say how many Litres that is. Q9. O.T.O. <br> Reflect Optional |
| :---: | :---: |
| Lesson 10: Exploring Capacity: The Millilitre <br> SCO SS3, SS4 | Explore <br> To prepare for this lesson, ask students to bring in some small containers including plastic measuring cups if you don't have any on hand. <br> Connect <br> An eyedropper is introduced here as a referent for $1 \mathrm{~mL}(\sim 10$ drops equals 1 mL ). On p. 154 the Math Link states that a mosquito's bite removes about $1 / 200$ of a mL of blood. That would be about $1 / 20$ of a single drop from the eyedropper! <br> Practice <br> Q1. Optional. Discuss when it would be appropriate to measure in mL . <br> Q2. O.T.O. If students have the eyedropper referent for 1 mL , this should not be a difficult question. Watch for those students who have not constructed this understanding. <br> Q3-5. O.T.O. Display the 1 L and 1 mL referents to help get students started. <br> Q6 \& 7. Together. This lesson has not really developed the idea that there are 1000 mL in 1 L . It first appears at the bottom of p.152, but it isn't as explicit as it needs to be in order for students to complete these two questions on their own. Measuring cups, graduated cylinders and everyday containers that list the capacity in mL will help students develop this understanding. <br> Q8 \& 9.O.T.O. <br> Reflect <br> This is a good prompt for formative assessment. Ask them to use numbers, pictures, and words to explain their thinking. |
| Lesson 11: Relating Capacity and Volume SCO SS3, SS4 | Explore and Connect <br> This section is about using the displacement of water as a method for measuring volume. Do this as class demo, but involve students in recording the data on the chart and in pouring and reading the levels. <br> Practice <br> Q1. The teacher should lead this exploration with the group. |


|  | Q2. O.T.O. Have a 1 L graduated cylinder(s) on hand filled to about the 400 mL mark. Students can come up to check the volume of their models. <br> Q3. Numbers, Pictures, Words <br> Optional <br> Q4. Do this activity as a group. Have 5 students each count out 20 cm cubes and put them into the cylinder. When you put 100 $\mathrm{cm}^{3}$ into the cylinder, the level will be almost to the 200 mL mark because of the space around each cube. <br> Q5 \& 6. Optional <br> Reflect <br> O.T.O. Formative assessment checkpoint. |
| :---: | :---: |
| Unit 4 Show What You Know | Use the items on these two pages as a review of the material in this unit. Questions $1,7,12 \& 15$ can be omitted or considered optional. |
| Assessment | Selected items from the Unit Quiz, Show What You Know p.158-159 in the student text should not be the only assessment tool you use to determine the degree to which students have achieved the curriculum outcomes. <br> > Review the Achievement Indicators and the Assessment Strategies in the Grade 5 Mathematics Curriculum Guide for SCOs SS1, 2, 3 \& 4. You may also use the Unit Rubric on P. 49 in the Teachers Guide in conjunction with the Ongoing Observations (p.50) and Assessment for Learning sections (for each lesson) as tools to help you gather relevant assessment information. Consider these three guiding questions: "What conclusions can be made from assessment information? How effective have my instructional approaches been? What are the next steps in instruction?" |
| Unit Problem: At the Zoo | This project could be introduced at the beginning of the unit and referred to as students are learning about the different measurement units; $\mathrm{cm}, \mathrm{cm}^{2}, \mathrm{~cm}^{3}, \mathrm{~m}, \mathrm{~m}^{2}, \mathrm{~m}^{3}, \mathrm{~mL}$, and L . Encourage creativity and suggest that they look for pictures in magazines that they could use. They might even write a newspaper article or advertisement inviting visitors to their petting $z o o$. |
| Investigation: Rep-Tiles | Optional |

## Unit 5: Fractions and Decimals

## Notes, Suggestions, Recommendations

| Preparing to Teach This Unit | Review Specific Curriculum Outcomes N7, N8, N9, N10 \& N11 in <br> your Curriculum Guide as these are the SCOs the unit is <br> attempting to address. Think about these two guiding questions, <br> "What do I want my students to learn? What do I want my <br> students to understand and be able to do?" The <br> Achievement Indicators and the Assessment Strategies for <br> these outcomes will help you answer these questions and give <br> you a better understanding of what it is you need to emphasize in <br> you teaching. You should also refer to the Unit Rubric on p.59 in <br> the Teacher's Guide, the Ongoing Observations sheet on p. 60 <br> and the Assessment for Learning sections which are <br> components of each individual lesson. <br> Note: Pages from the student text can be projected onto a screen <br> if you have access to a computer and LCD. Use the ProGuide <br> DVD which came with your Grade 5 Teacher's Resource Binder <br> to locate specific content for each unit. Some teachers find this <br> strategy helpful when discussing diagrams, tables and other <br> illustrations with the whole class. |
| :--- | :--- |
| For young students, the world of fractions and the world of <br> decimals are very distinct. Even adults tend to think of fractions <br> as subsets of a larger group or as part of a region, whereas we <br> usually think of decimals as being more like numbers. A <br> significant goal of instruction in decimal and fraction numeration <br> should be to help students see that both systems represent the <br> same concepts. In working towards this goal we can do three <br> things. <br> First, we can use familiar fraction concepts and models to explore |  |
| Launch: In the Garden |  |
| numbers that are easily represented by decimals: tenths, |  |
| hundredths, and thousandths. Second, we can help them see |  |
| how the base-ten system can be extended in both directions, to |  |
| include tiny values less than 1 as well as very large numbers. |  |
| Third, we can help children use models to help students make |  |
| meaningful transitions between fractions and decimals. |  |


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| Lesson 1: Equivalent Fractions |
| SCO N7 |

think addition to work out the exact difference between $\$ 2.08$ and $\$ 5.00$. "lt's $2 \phi$ to get to $\$ 2.10$, and then $90 \phi$ to get to $\$ 3.00$; Then $\$ 2.00$ to get to $\$ 5.00$, so the difference is $\$ 2.92$.

The last question involves estimation and an exact answer, but the question has two parts. To estimate, most students will know that 10 pkg . of seeds at $\$ 1.50$ is $\$ 15.00$. The zucchini seeda are about another dollar, so that's about $\$ 16$. To work out the exact answer, it's basically the same thing except you have to add on the 9 c to get $\$ 16.09$.
The concept of equivalent fractions can be a difficult concept for students. It is the first time in their experience that a fixed quantity can have multiple names. While all students should eventually be able to write an equivalent fraction for a given fraction, the rules should never be taught or used until the students understand what the result means. From that understanding students will recognize that there are patterns in the way equivalent fractions are written and from these patterns will come the standard algorithm. A serious instructional error is to rush too quickly to this rule. Be patient! Intuitive methods are always best at first.

## Explore

This activity will be too challenging for most students to work on independently. It will be less frustrating and more beneficial if the teacher directs the learning at the beginning of this lesson.

Begin by writing a fraction such as $1 / 6$ on the board and ask students to explain what it means. Do they know that the denominator indicates that something (a region, a length, or a set) is divided up into six equal parts and that the numerator tells us that we are considering just one of those parts? From this discussion, refer to the $4 \times 6$ grid in the student text and ask students to tell you the number of squares (24). If we want to make $1 / 6$ of these squares red, we have to divide the 24 squares into 6 equal groups and make one of the groups red. (Distribute congruent squares and ask students to use 24 of them to make a 6 equal groups. If they have two colours, ask them to make one of the 6 groups $(1 / 6)$ a different colour. Ask them to use the tiles to build a $4 \times 6$ array like the grid in their books. Ask them how many tiles there are in $1 / 6$ (4). Can the red tiles be anywhere in the rectangle? (yes, as long as there are only 4 of them)

For the second bullet, most students won't be able to describe $1 / 6$ of 24 in another way. Help shed some light on this idea by asking them to divide the 24 tiles into 12 equal groups. Each of these smaller groups is $1 / 12$. How many 12 ths are red? (two) So $2 / 12$ is the same as $1 / 6$. (write $1 / 6=2 / 12$ ). What if we divide the 24 tiles into 24 groups of 1 ?

The $3^{\text {rd }}$ bullet is a continuation. Tell students to use 12 tiles to show $1 / 3$ in different ways and 10 tiles to think about different names for $8 / 10$. Omit $5 / 8$ and $\frac{6}{12}$ as fractions to work with.

## Connect

For each array, ask questions such as the following. "How many equal groups are shown in this array? How many of these groups



|  | Q8. Students can do the same thing they did in Q6. The only difference this time is that not all pairs of fractions are equivalent. Advise them that, for each pair of fractions, they should start with the larger region ( $1 / 6,3 / 5,4 / 7$, etc) because it's easier to divide the rectangle into fewer parts at the beginning. <br> Q9. Numbers, Pictures, Words. O.T.O. <br> Q10.Do a) together. Use the board to model multiplication of top and bottom number to produce an equivalent fraction from the first one. <br> b) O.T.O. <br> Reflect <br> Formative assessment checkpoint. |
| :---: | :---: |
| Lesson 2: Comparing and Ordering Fractions <br> SCO N7 | To help students think and reason with fractions, write pairs of fractions on the chalkboard that have the same denominator (3/5, $1 / 5,7 / 5$ etc.) and ask them to say which is greater and to explain why. The next step is to write fractions that have different denominators but the same numerators (4/5, 4/6, 4/10 etc.) Again, ask students to name the larger fraction and to explain their thinking. If there are any students who struggle with this type of task, they are missing a basic understanding of what fractions are and what the numerator and denominator represent. You will need to provide some hands-on activities to help these students apply the idea of fair shares to fraction concepts.. <br> Explore <br> Pass out paper strips of the same length for students to fold and compare (remind students that they don't have to use the paper strips if they already know which is the greater fraction in a pair.) <br> For each pair of fractions, students fold two different strips of paper and mark the fold lines with a pencil so that they are easier to read. For the first pair, a strip of paper is folded into halves and another is folded into thirds. Then one half and one third are compared to see which is greater. There will be some trial and error, especially when folding to make thirds and fifths. Note that the second pair of fractions includes 24ths which is awkward to work with. Change this to something like 8/12. <br> Connect <br> Give students paper strips to fold into fourths, fifths and eighths so they can compare the fractions written in the book, $3 / 4,3 / 5,5 / 8$. Folding paper strips is an effective strategy that develops conceptual understanding of fractions and students need practice in doing this. Be sure to point out to them that we can compare the first two fractions just by thinking about the size of them because the numerators are the same. So $3 / 4$ is greater than $3 / 5$ because fourths are larger than fifths! <br> The fraction circles in the student book are self explanatory, although you might also want to ask students if they could use paper folding to compare fourths and eighths. |


|  | For the number line, show students how to draw a line labeled 0 <br> at one end and 1 on the other. |
| :--- | :--- |
|  | Ask students how you could divide the line into halves. Label it $1 / 2$ <br> .Do the same thing for fourths and label each section. Write $2 / 4$ <br> under $1 / 2$ <br> students need lhe same for eighths. As with paper opportunities to divide line segments into <br> equal sized fractional parts. They are using their spatial intuition, <br> and may not yet be confident or skilled in doing so. <br> The last part of the Connect (p.172) is about using the standard <br> procedure of multiplying the numerator and denominator by the <br> same number until you have fractions with the same denominator <br> or the same numerator so that they can be compared. Be careful <br> not to send the message to students that this is the best or only <br> way. Like all standard algorithms, it can be used if there isn't an <br> easier way. You want your students to "look to the numbers first" <br> to see if that easier way is obvious. <br> Practice <br> Q1. O.T.O. They can compare these fractions just by looking at <br> them. <br> Q2. Do a) with the class and model how to draw and partition a <br> number line. Let them do b) and c) O.T.O. <br> Q3 \& 4. O.T.O. |
| Lesson 3: Strategies Toolkit |  |
| Q5. Together |  |


|  | Practice <br> While this lesson highlights Use a Model as a problem solving strategy, Draw a Diagram (make a picture) and Make a Table are all strategies that make the most sense in this section. <br> Q1. Draw a picture of a log and compare it to a number line which students have already worked with. Show them that Brenna needs to make 2 cuts to show thirds and that this takes her 10 minutes. Let students finish the problem. <br> Q2. Draw a scaled $10 \mathrm{~m} \times 10 \mathrm{~m}$ "garden" on the board and divide it into tenths vertically. Discuss how much is planted in corn and in squash and what that looks like as shaded areas. For beans, discuss how they are going to show hundredths and then let them work on the rest of the problem. <br> Q3. Start a table on the board similar to this. <br> Reflect <br> This is a good question that teachers can use as a formative assessment checkpoint. |
| :---: | :---: |
| Lesson 4: Relating Fractions to Decimals SCO N8, N9, N10 | This lesson will use the $10 \times 10$ hundredths grid as the model to develop understanding of tenths and hundredths as base-ten fractions and as decimal numbers. Money is introduced as one application of decimal numeration, but since it is basically a twoplace system, it has limited usefulness as a model for decimal tenths, hundredths and thousandths. Both of these concepts were first introduced in grade 4. <br> A much more effective model, and probably the best length model, is the metre stick. Each decimeter is one-tenth of the whole stick, each centimetre is one-hundredth, and each millimeter is one-thousandth. <br> Explore <br> Optional. <br> Connect <br> Display 4 images of a square, one as a whole, one divided into tenths, another into hundredths and the last one into fourths or quarters. Discuss different fractional amounts represented by each type. Challenge students to find equivalencies among the 4 grids. Look at 1 tenth. How many hundredths is that? Look at two fourths. What is that amount the same as? <br> Students should now be prepared to make sense of the Jake and |

Willa's flower garden and the shaded grid which follows.
You should also re-introduce money as a model for tenths and hundredths and reinforce the idea that a dollar is 100 cents. One fourth can also be called a quarter and on a hundredths grid, it's easy to see that a quarter is 25 squares. In money, a quarter is 25 cents; one fourth of a dollar.

The last part of the Connect (p.178) reinforces some of the equivalent fraction ideas for tenths and hundredths. The third model of a $10 \times 10$ array of circles illustrates fiftieths as a fraction. Students may have difficulty recognizing this and you may wish to leave it for another time.

## Practice

Q1.a) O.T.O. Students should do lots of questions like this one.
b) Omit.

Q2 \& 3. Do as a group. Give 1 flat and some rods and unit cubes to each student or pair of students so that they can model each fractional amount. Compare this model to the hundredths grid which students have been using. Read each decimal number as a fraction and write it on the board as you go; 0.3 (three tenths) $3 / 10,0.07$ (7 hundredths) ${ }^{7} / 100$. Instruct students to represent each fraction/decimal by placing rods and unit cubes on top of the flat which is the whole in this case. Don't bother to have them "sketch" the blocks they used. You should be able to see which students clearly understand and which ones are struggling.

Q4 \& 5. O.T.O.
Q6. O.T.O. Review the connection to money that you made during the Connect.

Q7. Together. Say, "One twentieth of a dollar; that's $1 / 20$ of 100 cents or pennies. That means that 100 pennies are divided into 20 equal groups. How big would each group be?" Let students think about this and then use grid paper to outline the 20 equal groups. This may look different from student to student, but each group will have 5 squares.

## Q8. Optional

Q9. Together. For a) and b), have students reason about these fractions by looking at a hundredths grid. For c) and d) review how multiplication was used to find equivalent fractions. In this case, we want to know what $3 / 50$ is as a fraction of $100(3 / 50=\overline{100})$.

Q10. O.T.O. Let students use hundredths grids instead of counters if they need to.

## Q11. Numbers, Pictures, Words

Together. Each student should have a hundredths grid. Point to 0.35 and ask, "How do we read this number? (35 hundredths). Shade in 35 hundredths on your grid."

|  | Point to the denominator in the fraction $3 / 5$ and say, "This tells us that the square is divided into 5 equal groups, fifths. Can you see where fifths are on this hundredths grid? Shade in 3 fifths. How many hundredths are there in the 3 fifths that you just shaded? (60). So, are 3 fifths and 35 hundredths the same amount? Which is more?" <br> Reflect <br> Very good formative assessment checkpoint. Require that they use numbers, pictures and words. |
| :---: | :---: |
| Lesson 5: Fraction and Decimal Benchmarks SCO N8, N9, N10 | Explore <br> By now, students will be familiar with the number line as a length model for representing various fractions such as thirds, fourths, fifths, sixths, eighths, tenths and twelfths. They should also be able to write the decimal names for some common fractions such as halves and tenths. Fourths and fifths will be more problematic. For these fractions, help struggling students recall the hundredths grid and what one fourth and one fifth looked like and how many hundredths were in each fraction. <br> Connect <br> Students are asked to compare 0.25 and 0.7. Ask them to read each number aloud as a fraction. ( 25 hundredths and 7 tenths) Show a hundredths grid and ask them to tell you what 7 tenths looks like. "How many hundredths is that the same as?" (70) So which is greater, 25 hundredths or 7 tenths? <br> Encourage students to use the same reasoning for the remainder of the Connect on p. 181. <br> Practice <br> Q1-3. Students shouldn't need to use a number line for these three questions. If they read each decimal as a fraction, they should know how to put them in order. ${ }^{4} / 5$ and $1 / 4$ may cause some difficulty. <br> Q4. O.T.O. <br> Q5. Do a) together. Discuss the idea that every tenth can also have a hundredth name and every hundredth name that ends in zero has a tenth name. 6 tenth $=60$ hundredths and 40 hundredths $=4$ tenths. Using a metre stick as model or a hundredths grid will help to illustrate why this is so. <br> Q6. O.T.O. The symbols > and < were first introduced in grade 3. For students who have not understood completely, it may be helpful for them to think of < and > as the jaws of a hungry alligator who always eats the larger number! <br> Q7. O.T.O. <br> Q8. Omit <br> Reflect <br> Omit |

Lesson 6: Exploring Thousandths
SCO N8, N9, N10

## Explore

For tenths and hundredths, the base ten flat was used as the model for the whole. Now, for thousandths, it will be the large cube from the base-ten set. Show this cube to your class and explain that it is going to be the whole. Then show them one of the flats and ask them what this will be in relation to the whole. (one tenth). Continue for the rod (one hundredth) and the unit cube (one thousandth). Most class will not have enough base ten materials for the Explore as outlined on p. 183. However, every school will have the basic materials needed to introduce the concept. The thousandths grid will be the primary model for developing understanding of place value to thousandths.


The fraction ${ }^{70} / 100$ may be more difficult for students to identify on the thousandths grid than others. Keep bringing them back to the meaning of the denominator first, and then the numerator. In this example, when they see ${ }^{70} / 100$, it means that the thousandths grid is divided into 100 equal groups. What do these groups look like? How large are they? Show me one on these groups ( $1 / 100$ ). Where would 70 of these groups be?

## Connect

When base-ten fractions are later written as decimals, they are usually read as a single fraction; 0.65 is read as sixty-five hundredths. But to understand them in terms of place value (Lesson 6), the same number must be thought of as 6 tenths and 5 hundredths. A mixed number such as $5{ }^{13} / 100$ is usually read as five and thirteen hundredths. For purposes of place value, it should also be understood as $5+\frac{1}{10}+\frac{3}{100}$.

Use this type of reasoning to extend understanding of place value from whole numbers to amounts less than 1.

There are few opportunities in the elementary grades to measure thousandths and it may be difficult to help students understand why we would need to do it in the first place. We rarely see it in our daily lives. One place we do see it, however, is in the realm of sport and, in particular, olympic racing events such as Short Track and Luge. At the 2010 Olympics in Vancouver, Canada won the Gold and Bronze medals in the Men's 500 m Short Track race. The times for the top three racers were
40.77 (Canada) Heather Moyse, from Summerside PEI was a member of this team!
40.821 (S. Korea)
41.326 (Canada).

In Men's Luge Singles, the times were
48.17 (Germany)
48.330 (Germany)
48.459 (Italy)

Canada finished in $7^{\text {th }}$ place with a time of 48.373 sec .
Use results such as these to help students develop a better understanding of how close these races were to be determined by thousandths of a second!

## Practice

Q1. Together. Supplement the base-ten models shown in the text with physical models of your own that the class can see.

Q2. O.T.O. Before assigning this to students, have them read each decimal number aloud as a fraction in standard and in expanded form. Shading the thousandths grid will be easier to do after.

Q3. O.T.O. Have students read the numbers in the chart aloud in standard and expanded form before answering the place value questions.

Q4. First part O.T.O. then the second part through class discussion.

Q5. a) Together. Use an overhead transparency of a thousandths grid. The rest O.T.O.

Q6. O.T.O.

|  | Q7. a - c together e - g O.T.O. <br> Q8-11. O.T.O. <br> Q12. O.T.O. Limit the number of ways they have to write this number to two. <br> Q13. Omit <br> Reflect <br> Omit |
| :---: | :---: |
| Lesson 7: Comparing and Ordering Decimals SCO N8, N9, N10 | Explore <br> Decimal numbers are often comprised of many digits, and for students who are just beginning to develop an understanding of numeration beyond the whole number system it's critical that they are able to think about fractional amounts in relation to whole numbers. Mount Logan, is 5.959 km high. If this is read as "five point nine five nine km" high, it is more difficult to reference this number to the real world. However, if students understand anything about decimal notation, they will realize that this mountain is 5 km high for sure and there's almost enough in the fractional part for another whole km . Can students tell you how close this is to 6 km ? ( 41 m !) Discuss these fundamental ideas with your students as part of the Explore. <br> Before students work to put the data in the table on p. 187 in order, ask questions that encourage reasoning and understanding. "Which mountains are less than 1 km high? Which of these is closest to 1 km in height? Where is the highest mountain located? What's it called?" <br> Connect <br> The data in this table has shifted from the use of a very large unit $(\mathrm{km})$ to a very small one ( mm ). Conceptually, this will not be an easy transition for students so it is important that you help them establish some new referent for the mm , or perhaps recall one such as "the thickness of a dime" that they found useful back in Lesson 1 of Unit 4 (Measurement). Discuss the size of the microorganisms in this context and review how numbers written as hundredths, as for Tardigrade (0.15) and Vorticella (0.11) can also have "thousandths names". <br> The place-value chart helps students understand the value of each of the digits in the decimal number, but because it is going in the opposite direction to whole number values, it initially holds little meaning for students. The big idea in all place value, regardless of the direction we are talking about, is the multiplicative nature of the system. Each place is ten-times larger (or smaller) than the place next to it. <br> Discuss what "zero in the ones place" means for each of the organisms. <br> Do the section on equivalent decimals together with students. Because of the size of the units involved, converting hundredths |


|  | to thousandths will do little to give students a better sense of the length of each organism, but it is an important skill to develop. <br> The last section wants students to use a number line to place decimal numbers. Many students will not find this model helpful and it's advisable to use another context to reinforce the relationship between hundredths and thousandths. <br> Practice <br> Q1. O.T.O. Reinforce the proper way to "read" each decimal number. Each group all have the same denominator and should not be difficult to put in order. <br> Q2. $\boldsymbol{a}-\boldsymbol{c}$ O.T.O. <br> $\boldsymbol{d}-\boldsymbol{f}$ Together <br> Q3.a Together b O.T.O. <br> Q4-8 O.T.O. (Q 6 \& 8 are good formative assessment checkpoints) <br> Q9. O.T.O. Display a metre stick or another 1 m length as a referent for the data in the table. <br> Q10. Together. Identify a referent for the mass (g). One gram is about equal to the mass of 1 mL of water, and back in Unit 4, a referent for 1 mL of water was about 10 drops from an eyedropper. <br> Q11. Numbers, Pictures, Words O.T.O. <br> Q12. a-c Together $\boldsymbol{d}-\boldsymbol{f}$ O.T.O. <br> Q13. O.T.O. <br> Q14. O.T.O. Have students read each decimal number aloud and to change 1.27 and 1.2 to their thousandths equivalent. Ask them to list them in order from least to greatest, but don't require that they use a number line. <br> Reflect <br> O.T.O. |
| :---: | :---: |
| Lesson 8: Using Decimals to Relate Units of Measure SCO N8, N9, N10 | Explore <br> This is a very easy and worthwhile activity that will engage students. Have them cut pieces of string that they think are about each length (estimate). They can check actual measurements later. The metre stick was one important model used in the Measurement unit to illustrate tenths, hundredths, and thousandths of 1 m . Hundredths and thousandths are the units being emphasized in this lesson. <br> Connect <br> The length of a hummingbird could reasonably be recorded in cm or in mm , but it is unlikely, or even very helpful in understanding |


|  | how small this bird is, that it would be described in relation to a metre. The purpose for including it here is to reinforce the idea of fractional equivalency, but remind students that it's not a practical way to measure very small measures of length. <br> The table at the top of p .192 should be discussed and written on the board as follows: <br> $1 \mathrm{~mm}=0.1 \mathrm{~cm}$ (one mm is one tenth of a cm so 1 cm is the same as 10 mm$) 1 \mathrm{~cm}=10 \mathrm{~mm}$ <br> $1 \mathrm{~cm}=0.01 \mathrm{~m}$ (one cm is one hundredth of a metre, so 1 metre is the same as 100 cm$) 1 \mathrm{~m}=100 \mathrm{~cm}$ <br> $1 \mathrm{~mm}=0.001 \mathrm{~m}$ (one mm is one thousandth of a m , so 1 m is the same as 1000 mm$) 1 \mathrm{~m}=1000 \mathrm{~mm}$. <br> Practice <br> Q1. Work through this conversion exercise together. <br> Q2. O.T.O. <br> Q3. a-c Together <br> $\boldsymbol{d} \boldsymbol{f}$ O.T.O. <br> Q4. Together <br> Q5 \& 6. O.T.O. <br> Q7. O.T.O. Require that they estimate and draw feathers first and then measure and draw to compare. <br> Q8. O.T.O. Tell students to pick any 2 that they want to write explanations for. Provide time for whole group share. <br> Q9. O.T.O. Ask, "Would we usually measure the length of something as long as a whale in cm ? Why or why not?" <br> Q10 \& 11. O.T.O. <br> Q12. O.T.O. Ask, "Will the width of the door change when it's measured in different ways? (no) What will be different?" (the numbers) <br> Reflect <br> O.T.O. (formative assessment checkpoint) |
| :---: | :---: |
| Lesson 9: Relating Fractions and Decimals to Division SCO N8, N9, N10 | Explore <br> It will be difficult for students to see the relationship between a division statement and a fraction from the sharing examples provided in the book. There may be more effective ways and better examples available. <br> Practice <br> Q1 - 4. O.T.O. <br> Q5 \& 6. Together <br> Q7 \& 8. Together. Students will have difficulty knowing how to |


|  | express the remainder and these two questions ask them to convert the remainder to a fraction and then write an equivalent fraction that can be recorded as a decimal. Q9 - 12. O.T.O. <br> Q13. Together <br> Reflect <br> Omit |
| :---: | :---: |
| Lesson 10: Estimating Sums and Differences SCO N11 | Estimation should be part of your daily mental math routine. The work students do in this lesson is an extension of the work they do on a regular he table of masses $(\mathrm{kg})$ for various fruit in the student book. Provide a referent for 1 kg to give students a sense of each mass. A 1 kg box of table salt (or some other item) would be good to bring into your class so that students can feel the "heaviness" of 1 kg . Two methods for estimating both the sum and the difference of two decimal numbers are provided. Students will have difficulty making sense of these if they are simply asked to read about them on their own. Demonstrate on the board how these methods work. <br> Practice <br> Q1 \& 2. Together. Integrate these practice items with work from the Grade 5 Mental Math Teachers Guide. Start with decimal 10ths and 100ths if necessary to ensure early success. <br> Q3. O.T.O. Discuss these heights. Display a metre stick or some other appropriate referent. <br> Q4. O.T.O. <br> Q5. Together. Discuss how this is a situation where it is better to estimate high because of safety concerns. <br> Q6 - 8. O.T.O. <br> Q9. Optional <br> Reflect. <br> Optional <br> At Home. <br> Good homework assignment |
| Lesson 11: Adding Decimals SCO N11 | Connect <br> There are probably not enough base-ten blocks in your school for every student to model addition of decimals. Adding the whole numbers first will give a better sense of what the final answer should be. The fractional parts are then added to see if there is enough to yield another whole number. <br> The third method illustrated on p. 202 is the standard algorithm for adding decimal numbers. When students rely on this procedure alone, they often have incorrect answers because they have not thought about what is reasonable and have misplaced the decimal point. |


|  | Practice <br> Q1 \& 2. O.T.O. <br> Q3. $\boldsymbol{a} \& \boldsymbol{b}$ Together <br> $\boldsymbol{c} \boldsymbol{f}$ O.T.O. <br> Q4. O.T.O. Be sure students understand that the two pieces of ribbon don't have to be equal in length. Ask for 3 different answers. Q5 - 9. О.T.O. <br> Reflect <br> Omit |
| :---: | :---: |
| Lesson 12: Subtracting Decimals SCO N11 | Explore <br> Display and discuss a referent for 1 m . Review the data in the table on p.205. "Which city gets the most snow? The least snow?" etc. <br> Connect <br> Reinforce the important idea that subtraction is asking us for the difference between two numbers and one way to find the difference is to start with the smaller number and keep track of how far it is to the larger number. This method is illustrated on p.207. An open number line is preferable to one which already has increments marked on it. For example, for 3.22 - 2.61 <br> Discuss the half way point between 2 and 3 and mark 2.50 as a reference. Ask students where 2.61 and 3.22 would be on this line. Students then take jumps on the number line until they reach 3.22. For example, a student might take a jump of 0.09 to get to 2.70, then a jump of 0.30 to get to 3 and finally a jump of 0.22 to get to 3.22. The jumps are then added together ( 9 hundredths plus thirty hundredths is 39 hundredths, plus 22 hundredths is 61 hundredths) and this is the difference between the two numbers, 0.61 <br> Practice <br> Q1. O.T.O. Work as a group to make reasonable estimates before students find actual answers. <br> Q2. O.T.O. Make estimates first. <br> Q3. O.T.O. Discuss equivalent decimals first. <br> Q4 - 8. O.T.O. Discuss table in Q5 together. Provide a referent. <br> Q9. Omit <br> Q10 \& 11. O.T.O. |


|  | Q12. Optional <br> Q13. O.T.O. <br> Reflect <br> O.T.O. <br> Math Link <br> Discuss as a group |
| :---: | :---: |
| Lesson 13: Adding and Subtracting Decimals <br> SCO N11 | Explore <br> Discuss the values in the table on p.211. What does thousandths of 1 million mean? (It means one million is divided into 1000 equal groups) How large is each group? (1000) So a number like 3.290 means 3 million and 290 thousand more. 0.029 means there is less than 1 million; there's 29 thousand. <br> Practice <br> Q1 \& 2. There are some difficult computations here (subtractions especially). Help students work through these. Discuss reasonable estimates for each one first. <br> Q3 - 7. O.T.O. <br> Q8. Omit <br> Q9 \& 10. O.T.O. (find one way in Q9) <br> Q11. Optional <br> Q12. O.T.O. <br> Q13. Omit <br> Reflect <br> Together |
| Unit 5 Show What You Know | Use the items on these two pages as a review and a re-teaching opportunity of the material you have just covered in the unit. Most of the questions are O.T.O. unless otherwise noted. <br> Q6. Do without a number line. Order from least to greatest. <br> Q7. Review the fractional size of each base-ten piece in relation to the large cube. <br> Q12. Do without a number line. <br> Q15. Together <br> Q18. Ask students to write an explanation for one of these. |


| Assessment | - Selected items from the Unit Quiz, Show What You Know p.216-217 in the student text should not be the only assessment tool you use to determine the degree to which students have achieved the curriculum outcomes. <br> > Review the Achievement Indicators and the Assessment Strategies in the Grade 5 Mathematics Curriculum Guide for SCOs N7-11. You may also use the Unit Rubric in the Teachers Guide in conjunction with the Ongoing Observations and Assessment for Learning sections (for each lesson) as tools to help you gather relevant assessment information. <br> > Consider these three guiding questions: "What conclusions can be made from assessment information? How effective have my instructional approaches been? What are the next steps in instruction?" |
| :---: | :---: |
| Unit Problem: In the Garden | Students will have learned about equivalent fractions and decimals in this unit and can design a garden following the directions in part 1, p.218. Encourage art work and other creative elements in their gardens. Parts 2 and 3 should be considered optional. |

Unit 6: Geometry

## Notes, Suggestions, Recommendations

| Preparing to Teach This Unit | Review Specific Curriculum Outcomes SS5 \& SS6 in your <br> Curriculum Guide as these are the SCOs the unit is attempting to <br> address. Think about these two guiding questions, "What do I <br> want my students to learn? What do I want my students to <br> understand and be able to do?" The Achievement Indicators <br> and the Assessment Strategies for these outcomes will help you <br> answer these questions and give you a better understanding of <br> what it is you need to emphasize in you teaching. You should <br> also refer to the Unit Rubric on p.39 in the Teacher's Guide, the <br> Ongoing Observations sheet on p.40 <br> and the Assessment for Learning sections which are <br> components of each individual lesson. |
| :--- | :--- |
|  | In the elementary mathematics curriculum, geometry instruction is <br> organized around two quite different yet related frameworks: <br> spatial reasoning and the specific content for each grade level. |
|  | Spatial sense is often defined as an intuition about shapes and <br> their relationships. It includes the ability to visualize mentally - to <br> turn objects around in your mind. People with spatial sense <br> appreciate geometric form in art, nature, and architecture and <br> they are able to use geometric ideas to describe and analyze <br> their world. Spatial sense is not an ability that people are born <br> with; it develops through rich experiences with shapes and spatial <br> relationships provided consistently over time from one grade to <br> the next. |
| Geometric content, as identified in the curriculum, includes a |  |
| study of the properties of shapes in both two and three |  |
| dimensions, as well as a study of the relationships between and |  |
| among shapes. Students use visualization to recognize shapes in |  |
| the environment, develop relationships between two- and three- |  |
| dimensional objects, and draw and recognize objects from |  |
| different perspectives. |  |



Q2. Students are asked to pick two shapes from Q1 and draw shapes "like them" on dot paper. Most students will simply make larger or smaller versions of the shapes they have selected. The intent of this question is to help students recognize and describe various attributes. Consider modifying this question by asking students to draw one shape that has parallel sides and one that does not or one shape that has 3 sides or vertices, and one that has more than three sides/vertices. They should use square or triangular dot paper and a ruler for these tasks. Call on a few students to share their work and talk about the attributes of the shapes that they drew.

## Q3. Numbers, Pictures, Words

This task is similar to the previous one. If you do not have geoboards, dot paper will suffice.

Q4. Do these together to review how shapes can be named. Make sure they understand that the order of the letters named does not matter as long as they follow the perimeter of the shape as for quadrilaterals.

Q5. O.T.O. Make sure students understand that intersecting lines cross each other and meet at a common point. Sometimes the intersecting lines cross each other all the way (street intersection), but still share a common point.

Common


Students could create a table for the information.

|  | Shape <br> RPQ | Shape <br> BCDE | Shape <br> HJKM |  |
| :--- | :--- | :--- | :--- | :--- |
| Sides that <br> intersect |  |  |  |  |
| Parallel sides |  |  |  |  |

Q6. This would be a good activity for partners to do together.

They could be asked to use numbers, pictures and words to explain their thinking on paper.

Q7. O.T.O. Most students will only be familiar with the regular hexagon that they've seen in school such as the yellow piece from the Pattern Blocks. Explain that any 6-sided polygon is a hexagon.

## Reflect

This is a good opportunity to assess whether or not students understand the concept of parallel sides. Have them use numbers, pictures, words in their explanation.

Lesson 2: Investigating
Perpendicular Sides
SCO: SS5, SS6

## Explore

The concept of angles will be new to most students. If asked to make a shape with a corner larger or smaller than the corner in a square they may think something like the following:


As an alternative, give each student a plastic drinking straw and introduce the idea of a square corner by bending your straw into an "L" shape. Refer to the two sides or "rays" and the point at which they intersect to form i.e., the corner or angle. Ask students to make a square corner with their straws. As them to describe the sides in relation to each other. They may say things like "straight up and down" or "shaped like an L". The idea of perpendicular is beginning to emerge.


Angles are sometimes described as the degree of "open-ness" Close your straw so that the two sides lie on top of each other and then slowly spread them apart to show how the angle between them gets bigger and bigger. Have students do the same with their straws.


Compare all of these angles to the right angle and help students see that each of them is smaller than a square corner. Continue to open the straw to form angles larger than a square corner.


|  |  |
| :--- | :--- |
|  | For each angle, ask students to describe the sides in relation to <br> each other. This discussion will lead into terms like vertical, <br> horizontal and perpendicular when talking about right angles. <br> Practice <br> Q1. It will be difficult for students to refer in writing to the parts of <br> the structure that they think have the attributes listed. If possible,, <br> project this image onto a screen to facilitate discussion. |
| Q2. Review what perpendicular means and then select a shape <br> such as $f$ f) to do together. Let students do the rest O.T.O. |  |
| Q3. O.T.O. In part c) there are more intersecting sides than those <br> given in the teacher's guide. Any two sides that meet to form an <br> angle are said to be intersecting sides. |  |
| Q4. O.T.O. Use either the geoboards or the dot paper. |  |


|  | even larger group called quadrilaterals. <br> Connect <br> Properties of various quadrilaterals, including squares, rectangles, rhombuses, parallelograms, trapezoids and kites are introduced in this section. It is not an expectation of the grade 5 mathematics curriculum that students memorize definitions for these shapes. Instead, they are expected to be able to compare the differences and similarities of these shapes and sort them according to their attributes. For example, "Sort this set of quadrilaterals according to side lengths" or "Sort this set of quadrilaterals into two groups; those with opposite sides parallel and those without this attribute." <br> Practice <br> Q1. O.T.O. Limit this to just 3 parallelograms, with or without the geoboard. <br> Q2. Together. Use geoboards and have students make each shape one at a time. Ask students to hold up their shapes each time to illustrate a variety of solutions. <br> Q3. Together. Review the attributes of the shapes on p. 231. <br> Q4. O.T.O. (Tell students that there are two shapes that share these attributes) <br> Q5-7. O.T.O. <br> Q8. Optional <br> Reflect <br> Optional |
| :---: | :---: |
| Lesson 4: Other Attributes of Quadrilaterals <br> SCO: SS5, SS6 | Explore <br> This would be a good partner activity. Begin with a review of the attributes of quadrilaterals, rhombuses, parallelograms, trapezoids and kites as illustrated on p.231-2. Students can refer to these pages when they are doing the Explore. Remind partner groups that either student may be called on to justify the choice they made in naming a shape. <br> Connect <br> Students encountered lines of symmetry (reflective symmetry, mirror symmetry) last year in grade 4 but you will need to spend some time reviewing this concept. Pass out some simple paper shapes (rectangles, equilateral triangles, parallelograms) so that students can try folding them into two or more equal parts to see if they have line symmetry. |


Qune quadrilaterals have all four sides equal in length

|  | diagram. <br> Q8. O.T.O. Help students construct the Carroll diagram and then discuss the attributes that might be used to fill each cell. Let them sort according to the attributes they have selected. <br> Q9. Omit <br> Q10 \& 11. O.T.O. <br> Q12. Numbers, Pictures, Words <br> O.T.O. <br> Reflect <br> Optional |
| :---: | :---: |
| Lesson 5: Strategies Toolkit | This lesson should be considered optional. Although "Solve a Simpler Problem" is a common problem solving strategy, this lesson involving tangrams does not support its development. Solving a simpler problem generally involves modifying or simplifying the quantities in a problem so that the resulting task is easier to understand. By solving the easier problem, the hope is to gain insights that can then be used to solve the original, more complex problem. |
| Lesson 6: Exploring Faces and Edges of Objects SCO: SS5 | At some point in your class, the question may arise, "What is the difference between a pyramid and a triangular prism?" The following explanation comes from Ask Dr. Math http://mathforum.org/dr/math/ <br> Whether you are making a prism or a pyramid, you start with some plane figure - a triangle, a square, etc. To make a prism, just make a second copy of this figure and move it somewhere outside the plane, keeping it parallel. If you slide it vertically, straight up, so it's exactly over the base, we call it a right prism, meaning it goes up at right angles to the base. If you slide it up at an angle, it's an oblique prism. <br> For example, draw a triangle on your desk. Put an identical triangle on top of it and lift the second triangle straight up above the desk. Now imagine connecting each vertex of the bottom triangle to the corresponding vertex of the top triangle. You've just made a right triangular prism. |


|  | To make a pyramid, you start with a base as before. But this time, instead of putting a copy of the base above it, just make a single point somewhere up in the air, and connect every vertex of the base to that one point. <br> Prisms have 2 congruent bases and 2 rectangular faces. Pyramids have 1 base and triangular faces that meet at one vertex. <br> Explore \& Connect <br> Many of the concepts that have been introduced for 2-dimensional figures also apply to 3 -dimensional objects. Parallel, horizontal, vertical, congruent, intersecting, perpendicular, vertex/vertices are all mathematical terms with which students should be familiar. Sides will be called edges and shapes will be referred to as faces in the 3-D world. <br> Practice <br> Students should be able to complete any of the questions teachers might like to ask in this portion of the lesson. |
| :---: | :---: |
| Lesson 7: Drawing Objects SCO: SS5 | A common misconception among students (and many adults) is that the base of a prism is the face upon which it sits. Because prisms are names for their bases (rectangular prism, triangular prism), it is important to consider how bases are identified. If a prism has two opposite faces that are congruent and parallel to one another, then the shape of this face gives the prism its name. <br> The first two prisms are rectangular prisms because opposite faces are congruent and parallel rectangles, so the base of each |


|  | shape is a rectangle. The shape on the right, of course, is commonly called a cube made up of 6 congruent squares. In this context, however, it's important to remember that a square is a special kind of rectangle, so technically it is a rectangular prism. <br> The triangular prism is so named because the two triangles are opposite each other, congruent, and parallel. While this prism does have three congruent rectangular faces, they are adjacent and not opposite and are not parallel to one another. Therefore, the rectangle cannot be considered the base of the prism. <br> Connect <br> Step 1 on P. 248 names the figure drawn for the base as a parallelogram. Because the side lengths appear to be equal, some students may argue that it is actually a rhombus. <br> Note: A square based pyramid can also be drawn on both square and triangular dot paper starting with a square as the base. The "viewing angle" or perspective will change depending on the materials used. <br> Practice <br> Q1-3. O.T.O. The discussion from the beginning of Lesson 6 about the difference between prisms and pyramids should help students create the drawings in these questions. <br> Q4. Discuss together. <br> Q5. Optional <br> Q6. Numbers, Pictures, Words O.T.O. Provide triangular and square dot paper for students to complete this work. <br> Q7. O.T.O. Just ask students to use triangular dot paper to draw a triangular prism. <br> Q8.O.T.O. Give students square dot paper. Ask them to copy the rectangle onto the paper and then complete the picture to make a rectangular prism. <br> Reflect <br> Good formative assessment question. |
| :---: | :---: |
| Unit 6 Show What You Know | Most of the questions are O.T.O. unless otherwise noted. <br> Q4. Tell students that they can look back through this for help with identifying the names of some quadrilaterals. <br> Q5. Together. It would probably be time consuming to have students re-sort the shapes. <br> Q8. Together |
| Unit Problem: Building Bridges | This might be a project that some students would be interested in doing. However, it's probably not for everyone. Advise those who are interested to use popsicle sticks and white glue as described in the contest for Engineering Month. (See notes re. Launch at the beginning of this unit plan) |


| Asessment | Selected items from the Unit Quiz, Show What You <br> Know p.250-251 in the student text should not be the <br> only assessment tool you use to determine the <br> degree to which students have achieved the |
| :--- | :--- |
| curiculum outcomes. |  |
| Review the Achievement Indicators and the Assessment |  |
| Strategies in the Grade 5 Mathematics Curriculum Guide |  |
| for SCOs SS5 \& SS6. You may also use the Unit Rubric |  |
| in the Teachers Guide in conjunction with the Ongoing |  |
| Observations and Assessment for Learning sections |  |
| (for each lesson) as tools to help you gather relevant |  |
| assessment information. |  |
| Consider these three guiding questions: "What |  |
| conclusions can be made from assessment |  |
| information? How effective have my instructional |  |
| approaches been? What are the next steps in |  |
| instruction?" |  |

## Unit 7: Statistics and Probability

| Notes, Suggestions, Recommendations |  |
| :---: | :---: |
| Preparing to Teach This Unit | Review Specific Curriculum Outcomes SP1, SP2, SP3 \& SP4 in your Curriculum Guide as these are the SCOs the unit is attempting to address. Think about these two guiding questions, "What do I want my students to learn? What do I want my students to understand and be able to do?" The Achievement Indicators and the Assessment Strategies for these outcomes will help you answer these questions and give you a better understanding of what it is you need to emphasize in you teaching. You should also refer to the Unit Rubric on p. 41 in the Teacher's Guide, the Ongoing Observations sheet on p. 42 and the Assessment for Learning sections which are components of each individual lesson. <br> Note: Pages from the student text can be projected onto a screen if you have access to a computer and LCD. Use the ProGuide DVD which came with your Grade 5 Teacher's Resource Binder to locate specific content for each unit. Some teachers find this strategy helpful when discussing diagrams, tables and other illustrations with the whole class. <br> Data, especially in the form of various types of graphs, play a significant role in the information we receive every day in newspapers, magazines, and on television. In elementary school, students are expected to develop an understanding of graphs and how graphs depict information. Students should come to learn that the primary purpose of data, either in graphical form or in numeric form, is to answer questions about the population from which the data are drawn. <br> Throughout this unit, students should be given opportunities to generate their own questions, decide on appropriate data to help answer these questions, and determine methods for collecting the data. Avoid gathering data simply to make a graph! When students formulate the questions they want to ask, the data they gather become more and more meaningful. <br> In your initial discussions with students about data collection, you may want to share the story of how one class of students in an intermediate school gathered data about which cafeteria foods were most often thrown in the garbage. As a result of these efforts, certain items were removed from the regular menu. The activity illustrated to students the power of organized data. <br> There are many possibilities for connecting Grade 5 mathematics with the learning occurring in other subject areas. Outcomes related to Statistics and Probability, in particular, can be addressed throughout the year in other subject areas The following are just a few of the ways to make these connections. |

## Health

As a part of their Grade 5 Health Education, students are to analyze their personal eating practices with respect to information that is available to them such as food labels and food guides. A strong connection can be made between this learning and mathematics outcomes SP1 and SP2 in which students learn to differentiate between first- and second-hand data and to construct and interpret double bar graphs. The resulting graphs can be used by the students to analyze their eating practices.
Students could also use personal and researched data regarding the eating practices of themselves and family members or friends to reflect on the likelihood of engaging in different eating practices (SP3, SP4).

## Science

Mathematics and Science have many common topics and features, such as the recognition and description of patterns, sorting and categorizing, measurement, and the use of multiple representations. Opportunities for connections occur in the Grade 5 Science topics of Properties and Changes of Materials, Forces and Simple Machines, and Weather. In Mathematics class, students can use observations and measurements from Science to make predictions about the probabilities of different weather occurrences. Second-hand data can also be researched and compared to the collected first-hand data and analyzed through the use of double-bar graphs.

## Social Studies

Social Studies and Mathematics often connect through the investigation of patterns and trends and in the representation of data.

| Lesson 1: First-Hand Data and Second-Hand Data SCO: SP1, SP2 | Gathering data can mean using data that have been collected by others. For example, newspapers, almanacs, sports record books, and various government publications <br> (http://www.statcan.gc.ca/start-debut-eng.html) are sources of data that may be used to answer student questions. Students may be interested in facts about another country as a result of a social studies unit. Olympic records in various events over the years or data related to space flight are other examples of topics around which student questions may be formulated. <br> In Connect (p.259) students are provided with second-hand data about the average annual precipitation for five cities in Western Canada. The table below, courtesy of the Meteorological Service of Canada, provides additional precipitation data for other cities including Charlottetown. Note: 1 cm of snow represents about 1 mm of water, so for Charlottetown the average precipitation includes about 339 mm as snow and the rest ( 862 mm ) as rainfall. |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Annual Average |  |  |  |
|  |  | Snowfall | Total precipitation | Wet days |
|  |  | cm | mm | number |
|  | St. John's | 322.1 | 1,482 | 217 |
|  | Charlottetown | 338.7 | 1,201 | 177 |
|  | Halifax | 261.4 | 1,474 | 170 |
|  | Fredericton | 294.5 | 1,131 | 156 |
|  | Québec | 337.0 | 1,208 | 178 |
|  | Montréal | 214.2 | 940 | 162 |
|  | Ottawa | 221.5 | 911 | 159 |
|  | Toronto | 135.0 | 819 | 139 |
|  | Winnipeg | 114.8 | 504 | 119 |
|  | Regina | 107.4 | 364 | 109 |
|  | Edmonton | 129.6 | 461 | 123 |
|  | Calgary | 135.4 | 399 | 111 |
|  | Vancouver | 54.9 | 1,167 | 164 |
|  | Victoria | 46.9 |  |  |
|  | Practice <br> Students should be able to distinguish between first- and secondhand data and answer all of the practice questions on their own. |  |  |  |
| Lesson 2: Interpreting Double Bar | Explore \& Connect <br> Review the types of graphs presented on p. 261-262. Discuss the headings used and ask questions that can be answered by interpreting the information displayed. <br> The double bar graph, "Breakfast Foods", uses a scale of 1 |  |  |  |
| Graphs SCO: SP1, SP2 |  |  |  |  |


|  | square = 10 students. This should be indicated in the legend. <br> Students were introduced to the use of scales in data displays in <br> grade 4. <br> Practice <br> Q1 \& 2. Together. Project p. 263 of the student text onto a screen <br> to facilitate whole class discussions. <br> Q3-6. O.T.O. <br> Reflect <br> Good formative assessment question. Ask students to use <br> numbers, pictures, words to explain their ideas. |
| :--- | :--- |

Lesson 3: Constructing Double Bar
Graphs
SCO: SP1, SP2

Technology: Using Census at School to Find Second-Hand Data SCO: SP1, SP2

## Explore

Can students determine the scale used in the graph at the top of p.266? Generate a list of things students "know" from reading this graph.

## Practice

Q1. Together.
Q2-4. O.T.O.

Q5. Together.
Q6. Optional.
Reflect. Optional. This is similar to the Reflect in the previous lesson.
Census at School, a site developed and maintained by Statistics Canada, is an international online project that engages students from grades 4 to 12 in statistical enquiry. The project began in the United Kingdom in 2000 and now includes participation from schools in Australia, Canada, New Zealand and South Africa.

Census at School aims to:

- provide real data for data management activities across the curriculum;
- show how information and communications technology (ICT) can be used effectively in teaching and learning;
- raise awareness of the national census, how it gathers data, and its benefits to society;
- develop statistical literacy skills in students.

Classroom participation in this project is completely voluntary.
Students in each participating country anonymously fill in an online survey in class. They answer non-confidential questions about topics such as their height, the time it takes to travel to school, and their favourite subject at school. The responses become part of a national database, which is later added to an

|  | international database that is maintained in the United Kingdom. <br> This project combines fun with learning, to the delight of hundreds of thousands of students around the world who have already participated. They discover how to use and interpret data about themselves as part of their classroom learning in math, social sciences or information technology. They also learn about the importance of the national census in providing essential information for planning education, health, transportation and many other services. <br> Census at School offers students a golden opportunity to be involved in the collection and analysis of their own data and to experience what a census is like. <br> Teacher Comments <br> "I have been participating in the Census at School program for the last two years, and love it! I use it to do an entire unit on data analysis. My students look back on this as one of their most favourite parts of the year in Math. They use the data to learn about population, sample, survey design, and then analyze the data for mean, median, mode, range. They also draw graphs of all sorts, exploring the relationships between data and sharing the conclusions that can be drawn from that type of information." <br> - Julie Hearn, Grade 6 and 7 teacher, Maple Ridge, B.C |
| :---: | :---: |
| Lesson 4: The Language of Probability SCO: SP3, SP4 | In today's world, references to probability are all around us. The weather forecaster predicts a $60 \%$ chance of snow or rain; medical researchers predict people with certain diets have a high chance of heart disease; your chance of winning the lottery is 1 in 10000000. <br> In Grade 5, students begin to develop a better understanding of some of the big ideas in probability such as the difference between long-run and short-run results or the way one event can affect another. The instructional emphasis is on exploration rather than rules and formal definitions, and if done well, these informal experiences will provide a useful background from which more formal ideas can be developed in later years. <br> Explore \& Connect <br> Go over the material on p.272-3 together. <br> Practice <br> Q1-3. O.T.O. <br> Q4. Together. Call on a student volunteer to roll the die and another to keep the tally. Discuss the results. <br> Q5. Together <br> Q6. Numbers, Pictures, Words O.T.O. <br> Q7. Together |


|  | Reflect and/or At Home <br> Either of these two questions would be good to assign to students. |
| :---: | :---: |
| Lesson 5: Using Spinners to Compare Liklihoods SCO: SP3, SP4 | This lesson further develops the language of probability. Words and phrases such as possible, impossible, certain, likely, less likely, equally likely all help to describe the probability of an event occurring. <br> Practice <br> Q1-3. O.T.O. <br> Q4. Optional <br> Q5. O.T.O. Do a) quickly together "The spinner can land on either of the pictures of fruit. <br> b) Create one statement as an example for students, "It is more likely that the spinner will land on the pear than on the apple because the pear section is a larger area." <br> Q6. Discuss together <br> Q7. O.T.O. <br> Reflect <br> Optional |
| Lesson 6: Conducting Experiments SCO: SP3, SP4 | Explore <br> This is best done as a class with the teacher filling a paper bag with 4 different colours of counters, snap blocks, or some other suitable material. For the second part ( 50 times) call on students to draw counters from the bag and keep a tally on the board. Discuss the results. <br> Connect <br> Discuss the results from the experiment involving 100 trials. <br> Practice <br> Q1. Do this as a group so that you have only one coin being tossed 40 times. <br> Q2. O.T.O. <br> Q3. Numbers, Pictures, Words Together <br> Q4-7. O.T.O. <br> Reflect <br> Formative assessment question to determine level of understanding of probability. |
| Lesson 7: Designing Experiments SCO: SP3, SP4 | Explore <br> Use paper bags and two different colours of counters if you do not have coloured paper clips. <br> Practice <br> Q1-5. O.T.O. These questions are "hands-on" and will reveal the level of understanding in your students. |


|  | Q6. Together <br>  <br> Q7. O.T.O. <br> Reflect <br> Optional <br> Game "Sum Fun". <br> This is a good game for students to play. When they list all the <br> possible outcomes for a roll of 2 dice, see if they are systematic in <br> their approach or rather random. For example: <br> 6+6=12, 6+5=11, 6+4=10, $6+3=9,6+2=8,6+1=7$ |
| :--- | :--- |
|  | Review the problem solving strategies listed in the side-bar on <br> p.288. This lesson attempts to illustrate Work Backward From <br> the Solution as a strategy. Typically, Work Backwards problems <br> involve arithmetic operations and, given the solution, the task is to <br> find the starting point. The context of probability experiments, as <br> in the textbook, does little to help students think about the inverse <br> operations. A more appropriate process problem would be <br> something like the following: |
| Lesson 8: Strategies Toolkit |  |
| SCO: SP3, SP4 |  |
| Judy found an old plank on the beach. She cut off 12 |  |
| centimeters where the plank was broken. Then she cut the |  |
| remaining plank into 3 equal pieces. Each piece is now 20 |  |
| centimeters long. How long was the plank she found on the |  |
| beach? |  |

Asessment

Selected items from the Unit Quiz, Show What You Know p.290-291 in the student text should not be the only assessment tool you use to determine the degree to which students have achieved the curriculum outcomes.
> Review the Achievement Indicators and the Assessment Strategies in the Grade 5 Mathematics Curriculum Guide for SCOs SP1, SP2, SP3 \& SP4. You may also use the Unit Rubric in the Teachers Guide in conjunction with the Ongoing Observations and Assessment for Learning sections (for each lesson) as tools to help you gather relevant assessment information.
$>$ Consider these three guiding questions: "What conclusions can be made from assessment information? How effective have my instructional approaches been? What are the next steps in instruction?"

## Unit 8: Transformations

## Notes, Suggestions, Recommendations

| Preparing to Teach This Unit |
| :--- |
|  |
|  |

Lesson 1: Translations
SCO: SS7, SS8
Review Specific Curriculum Outcomes SS7 \& SS8 in your Curriculum Guide as these are the SCOs the unit is attempting to address. Think about these two guiding questions, "What do I want my students to learn? What do I want my students to understand and be able to do?" The Achievement Indicators and the Assessment Strategies for these outcomes will help you answer these questions and give you a better understanding of what it is you need to emphasize in you teaching. You should also refer to the Unit Rubric on p. 39 in the Teacher's Guide, the Ongoing Observations sheet on p. 40 and the Assessment for Learning sections which are components of each individual lesson.

Note: Pages from the student text can be projected onto a screen if you have access to a computer and LCD. Use the ProGuide DVD which came with your Grade 5 Teacher's Resource Binder to locate specific content for each unit. Some teachers find this strategy helpful when discussing diagrams, tables and other illustrations with the whole class.

Transformational geometry is also referred to as motion geometry. The size or shape of the object being moved does not change. In elementary school, three kinds of transformations are commonly explored: slides (translations), flips (reflections), and turns (rotations). Transformations and symmetry are closely related concepts.

## Explore \& Connect

The first transformation introduced in this unit is a translation. The terms orientation, translation image and translation arrow are used to describe the movement. An overhead transparency of square grid paper and some shapes that the teacher can slide around will help to illustrate how a translation is "mapped" (i.e. "4 squares to the right and 3 squares down").

## Practice

Q1. Do a) together, using the overhead transparency. Use a ruler to draw a triangle on the grid paper as shown in the student book. Place a smaller piece of transparency plastic over the shape and "trace" it using a ruler. You can now slide this copy of the triangle to a new position on the grid paper. For example, you might slide it to the right and then down. Help students to count the number of squares you moved in each direction. Mark the position of the triangle on the grid paper and use a ruler to copy it. Draw an arrow from one vertex on the original triangle to the corresponding vertex on the triangle you just drew.
b) \& c) O.T.O.

Q2. Discuss as a group.
Q3. a) Together
b) \& c) O.T.O.

|  | Q4. O.T.O. <br> Q5. Numbers, Pictures, Words O.T.O. <br> Q6. Optional <br> Q7. O.T.O. <br> Reflect <br> This would be a good homework assignment and one that would assess students' understanding of translations. Ask them to use numbers, pictures and words in their answer. |
| :---: | :---: |
| Lesson 2: Strategies Toolkit SCO: SS7, SS8 | Review the problem solving strategies in the side bar on p.300. The strategy in this lesson is "Draw a Picture". The problems presented here are good examples of where this kind of strategy can be helpful. Work through the Explore and Connect with students and call on volunteers to sketch pictures or diagrams that represent the context of each problem. In the Practice section, read each problem with students so that they understand what is being asked and then allow them to work on their own. |

## Lesson 3: Reflections SCO: SS7, SS8

In grade 4, students learned about fold symmetry or mirror symmetry. They learned that if an image can be folded so that both halves fit exactly on top of one another, the shape is said to have line (fold) symmetry. This lesson extends the concept of symmetry by exploring how a shape and its reflected image lie on either side of a Line of Reflection.

## Connect

Project this page from the student book onto a screen or create a similar example using an overhead transparency. Review perpendicular and the use of hash marks from Unit 6 Geometry.

## Practice

Q1. a) Together. Model how to mark points for the reflection that correspond to each vertex of the image. Use a ruler to connect the points and complete the reflection.
b) - d) O.T.O.

Q2 \& 3. O.T.O. (Remind students what a translation is from the previous lesson)

Q4. Omit
Q5. O.T.O.

Q6. Optional. If you choose to do this question, provide students with copies of the alphabet in upper case. Create these using a font such as Arial and at least 36 pt. size.

## Q7. Numbers, Pictures, Words O.T.O.

## Reflect

This would be a good formative assessment checkpoint. Ask students to use numbers, pictures and words in their answer.

| Lesson 4: Rotations SCO: SS7, SS8 | Explore \& Connect <br> The most difficult idea about rotations for students to come to understand has to do with the point of rotation. Help them to see that any point on any shape can be the point around which the shape is turned; it becomes the center of an imaginary circle. <br> Cut out several opaque or transparent shapes and use the overhead projector to show how you can rotate the shape around any vertex. <br> Practice <br> Q1. a) Together <br> b) \& c) O.T.O. <br> Q2-4. Discuss together. <br> Q5. O.T.O. <br> Q6. Together <br> Q7. Numbers, Pictures, Words O.T.O. <br> Q8. Optional <br> Reflect <br> Ask students to use numbers, pictures and words to describe the differences between reflections, translations, and rotations. |
| :---: | :---: |
| Lesson 5: Exploring Different Points of Rotation SCO: SS7, SS8 | This lesson is optional. |
| Unit 7 Review: Show What You Know | Use the items on these two pages as a review and a re-teaching opportunity of the material you have just covered in the unit. |


|  | Q1.a) - c) O.T.O. <br> d) Omit |
| :--- | :--- |
|  | Q2. O.T.O. |
| Q3. Omit |  |$\quad$| Q4. O.T.O. |  |
| :--- | :--- |
|  | Q5 \& 6. Optional |
|  | Q7 \& 8. Omit |

## GLOSSARY OF MODELS

Mathematical models, often referred to as "manipulatives", have a variety of uses at different grade levels and are referenced throughout the mathematics curriculum and in other resources. Many comprehensive reviews of the research into the use of mathematical models have concluded that student achievement is increased as a result of long term exposure to mathematical models. It is important to remember, however, that it depends on how the models are used in the classroom. In themselves, mathematical models do not teach but, in concert with good teaching, make a great deal of difference. The purpose of this glossary is to provide a visual reference for each model and a brief description of it. It is the responsibility of individual schools to maintain and enhance their inventory of available mathematical models.

| Name | Picture | Description |
| :---: | :---: | :---: |
| Area Model | To model $12 \times 23$ : | - Use base ten blocks to represent the parts of each number that is being multiplied. <br> - To find the answer for the example shown, students can add the various parts of the model: $200+30+40+6=276$. <br> - This model can also be used for fraction multiplication. |
| Arrays and Open Arrays | To model $4 \times 6$ : 000000 000000 000000 000000 <br> To model $7 \times 36$ : 36 | - Use counters arranged in equal rows or columns or a Blackline Master with rows and columns of dots. <br> - Helpful in developing understanding of multiplication facts. <br> - Grids can also be used to model arrays. <br> - Open arrays allows students to think in amounts that are comfortable for them and does not lock them into thinking using a specific amount. These arrays help visualize repeated addition and partitioning and ultimately using the distributive property. |
| Attribute Blocks |  | - Sets of blocks that vary in their attributes: 5 shapes circle, triangle, square, hexagon, rectangle 2 thicknesses 2 sizes 3 colours |
| Balance (pan or beam) scales |  | - Available in a variety of styles and precision. <br> - Pan balances have a pan or platform on each side to compare two unknown amounts or represent equality. Weights can be used on one side to measure in standard units. <br> - Beam balances have parallel beams with a piece that is moved on each beam to determine the mass of the object on the scale. Offer greater accuracy than a pan balance. |



| Dice (Number Cubes) |  | - Standard type is a cube with numbers or dots from 1 to 6 (number cubes). <br> - Cubes can have different symbols or words. <br> - Also available in: <br> - 4-sided (tetrahedral dice) <br> - 8 -sided (octahedral dice) <br> - 10 -sided (decahedra dice) <br> - 12-sided, 20 -sided, and higher <br> - Place value dice |
| :---: | :---: | :---: |
| Dominoes |  | - Rectangular tiles divided in two-halves. <br> - Each half shows a number of dots: 0 to 6 or 0 to 9 . <br> - Sets include tiles with all the possible number combinations for that set. <br> - Double-six sets include 28 dominoes. <br> - Double-nine sets include 56 dominoes. |
| Dot Cards |  | - Sets of cards that display different number of dots ( 1 to 10 ) in a variety of arrangements. <br> - Available as free Blackline Master online on the "Teaching Student-Centered Mathematics K-3" website (BLM 3-8). |
| Decimal Squares ${ }^{\circledR}$ |  | - Tenths and hundredths grids that are manufactured with parts of the grids shaded. <br> - Can substitute a Blackline Master and create your own class set. |
| Fraction Blocks |  | - Also known as Fraction Pattern blocks. <br> - 4 types available: pink "double hexagon", black chevron, brown trapezoid, and purple triangle. <br> - Use with basic pattern blocks to help study a wider range of denominators and fraction computation. |
| Fraction Circles |  | - Sets can include these fraction pieces: $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}$ <br> - Each fraction graduation has its own colour. <br> - It is helpful to use ones without the fractions marked on the pieces for greater flexibility (using different piece to represent 1 whole). |


| Fraction Pieces |  | - Rectangular pieces that can be used to represent the following fractions: $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}$ <br> - Offers more flexibility as different pieces can be used to represent 1 whole. <br> - Each fraction graduation has its own colour. <br> - Sets available in different quantities of pieces. |
| :---: | :---: | :---: |
| Five Frames Ten Frames | $O \operatorname{OD}$ | - Available as a Blackline Master in many resources or you can create your own. <br> - Use with any type of counter to fill in the frame as needed. |
| Geoboards |  | - Available in a variety of sizes and styles. $5 \times 5$ pins $11 \times 11$ pins <br> Circular 24 pin <br> Isometric <br> - Clear plastic models can be used by teachers and students on an overhead. <br> - Some models can be linked to increase the size of the grid. |
| Geometric Solids |  | - Sets typically include a variety of prisms, pyramids, cones, cylinders, and spheres. <br> - The number of pieces in a set will vary. <br> - Available in different materials (wood, plastic, foam) and different sizes. |
| Geo-strips |  | - Plastic strips that can be fastened together with brass fasteners to form a variety of angles and geometric shapes. <br> - Strips come in 5 different lengths. Each length is a different colour. |
| Hundred Chart |  | - $10 \times 10$ grid filled in with numbers $1-100$ or 0-99. <br> - Available as a Blackline Master in many resources or you can create your own. <br> - Also available as wall charts or "Pocket" charts where cards with the numbers can be inserted or removed. |


| Hundred Grid |  | - $10 \times 10$ grid. <br> - Available as Blackline Master in many resources. |
| :---: | :---: | :---: |
| Hundredths Circle | Percent Circles | - Also known as "percent circles". <br> - Two circles can be cut out on different coloured card stock and overlapped to represent tenths and hundredths. |
| Mira ${ }^{\left({ }^{\text {® }}\right.}$ |  | - Clear red plastic with a bevelled edge that projects reflected image on the other side. <br> - Other brand names include: Reflect-View and Math-Vu ${ }^{\text {TM }}$. |
| Number Lines (standard, open, and double) |  | - Number lines can begin at 0 or extend in both directions. <br> - Open number lines do not include pre-marked numbers or divisions. Students place these as needed. <br> - Double number lines have numbers written above and below the line to show equivalence. |


| Pattern Blocks |  | - Standard set includes: <br> Yellow hexagons, red trapezoids, blue parallelograms, green triangles, orange squares, beige parallelograms. <br> - Available in a variety of materials (wood, plastic, foam). |
| :---: | :---: | :---: |
| Pentominoes |  | - Set includes 12 unique polygons. <br> - Each is composed of 5 squares which share at least one side. <br> - Available in 2-D and 3-D in a variety of colours. |
| Polydron |  | - Geometric pieces snap together to build various geometric solids as well as their nets. <br> - Pieces are available in a variety of shapes, colours, and sizes: <br> Equilateral triangles, isosceles triangles, right-angle triangles, squares, rectangles, pentagons, hexagons <br> - Also available as Frameworks (open centres) that work with Polydrons and another brand called G-O-Frames ${ }^{\text {TM }}$. |
| Power Polygons ${ }^{\text {TM }}$ |  | - Set includes the 6 basic pattern block shapes plus 9 related shapes. <br> - Shapes are identified by letter and colour. |
| Math Rack (Rekenrek®) |  | - Counting frame that has 10 beads on each bar: 5 white and 5 red. <br> - Available with different number of bars (1, 2, or 10). |


| Spinners |  | - Create your own or use manufactured ones that are available in a wide variety: <br> - number of sections; <br> - colours or numbers; <br> - different size sections; <br> - blank. <br> - Simple and effective version can be made with a pencil held at the centre of the spinner with a paperclip as the part that spins. |
| :---: | :---: | :---: |
| Tangrams |  | - Set of 7 shapes (commonly plastic): <br> - 2 large right-angle triangles <br> - 1 medium right-angle triangle <br> - 2 small right-angle triangles <br> - 1 parallelogram <br> - <br> 1 square <br> - 7-pieces form a square as well as a number of other shapes. <br> - Templates also available to make sets. |
| Trundle Wheel |  | - Tool for measuring longer distances. <br> - Each revolution equals 1 metre usually noted with a click. |
| Venn Diagram |  | - Used for classification of different attributes. <br> - Can be one, two, or three circles depending on the number of attributes being considered. <br> - Attributes that are common to each group are placed in the interlocking section. <br> - Attributes that don't belong are placed outside of the circle(s), but inside the rectangle. <br> - Be sure to draw a rectangle around the circle(s) to show the "universe" of all items being sorted. <br> - Similar to a Carroll Diagram. |

## List of Grade 5 Specific Curriculum Outcomes

## Number (N)

N1 Represent and describe whole numbers to 1000000.
N2 Use estimation strategies including front-end rounding, compensation, and compatible numbers in problem-solving contexts.

N3 Apply mental mathematics strategies and number properties, such as skip counting from a known fact, using doubling or halving, using patterns in the 9s facts, and using repeated doubling or halving to determine answers for basic multiplication facts to 81 and related division facts.

N4 Apply mental mathematics strategies for multiplication, such as annexing then adding zero, halving and doubling, and using the distributive property.
N5 Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems.
N6 Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems.

N7 Demonstrate an understanding of fractions by using concrete and pictorial representations to create sets of equivalent fractions and compare fractions with like and unlike denominators.
N8 Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically.
N9 Relate decimals to fractions (to thousandths).
N10 Compare and order decimals (to thousandths) by using benchmarks, place value, and equivalent decimals.
N11 Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).

## Patterns \& Relations (PR) <br> (Patterns)

PR1 Determine the pattern rule to make predictions about subsequent elements.

## (Variables and Equations)

PR2 Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.

## Shape and Space (SS) <br> (Measurement)

SS1 Design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions.
SS2 Demonstrate an understanding of measuring length (mm) by selecting and justifying referents for the unit mm and modelling and describing the relationship between mm and cm units, and between mm and m units.
SS3 Demonstrate an understanding of volume by selecting and justifying referents for $\mathrm{cm}^{3}$ or $\mathrm{m}^{3}$ units, estimating volume by using referents for $\mathrm{cm}^{3}$ or $\mathrm{m}^{3}$, measuring and recording volume $\left(\mathrm{cm}^{3}\right.$ or $\left.\mathrm{m}^{3}\right)$ and constructing rectangular prisms for a given volume.
SS4 Demonstrate an understanding of capacity by describing the relationship between mL and L , selecting and justifying referents for $m L$ or $L$ units, estimating capacity by using referents for $m L$ or $L$ and measuring and recording capacity ( mL or L).

## (3-D Objects and 2-D Shapes)

SS5 Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are parallel, intersecting, perpendicular; vertical, and horizontal
SS6 Identify and sort quadrilaterals, including rectangles, squares, trapezoids, parallelograms, and rhombuses according to their attributes.

## (Transformations)

SS7 Perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image.

SS8 Identify a single transformation including a translation, a rotation and a reflection of 2-D shapes.

## Statistics and Probability (SP) <br> (Data Analysis)

SP1 Differentiate between first-hand and second-hand data.
SP2 Construct and interpret double bar graphs to draw conclusions.
SP3 Describe the likelihood of a single outcome occurring using words, such as impossible, possible, and certain.

## (Chance and Uncertainty)

SP4 Compare the likelihood of two possible outcomes occurring using words, such as less likely, equally likely, and more likely.

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