Prince Edward Island

CANADA

Education and Early Childhood Development

English Programs

Prince Edward Island Mathematics Curriculum

CURRICULUM

Mathematics

MAT621A

CANADA

2013
Prince Edward Island
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Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base. Developers have examined the curriculum proposed throughout Canada and secured the latest research in the teaching of mathematics, and the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for Grades 10-12 Mathematics* (2008) has been adopted as the basis for a revised high school mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories, and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP, and on the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➢ Essential Graduation Learnings

Essential Graduation Learnings (EGLs) are statements describing the knowledge, skills, and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills, and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work, and study today and in the future. Essential graduation learnings are cross curricular, and curriculum in all subject areas is focused to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will demonstrate knowledge, skills, and attitudes expressed as essential graduation learnings, and will be expected to

- respond with critical awareness to various forms of the arts, and be able to express themselves through the arts;
- assess social, cultural, economic, and environmental interdependence in a local and global context;
- use the listening, viewing, speaking, and writing modes of language(s), and mathematical and scientific concepts and symbols, to think, learn, and communicate effectively;
- continue to learn and to pursue an active, healthy lifestyle;
- use the strategies and processes needed to solve a wide variety of problems, including those requiring language, and mathematical and scientific concepts;
- use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

➢ Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate to all educational partners high expectations for students in mathematics education. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM, *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning;
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will

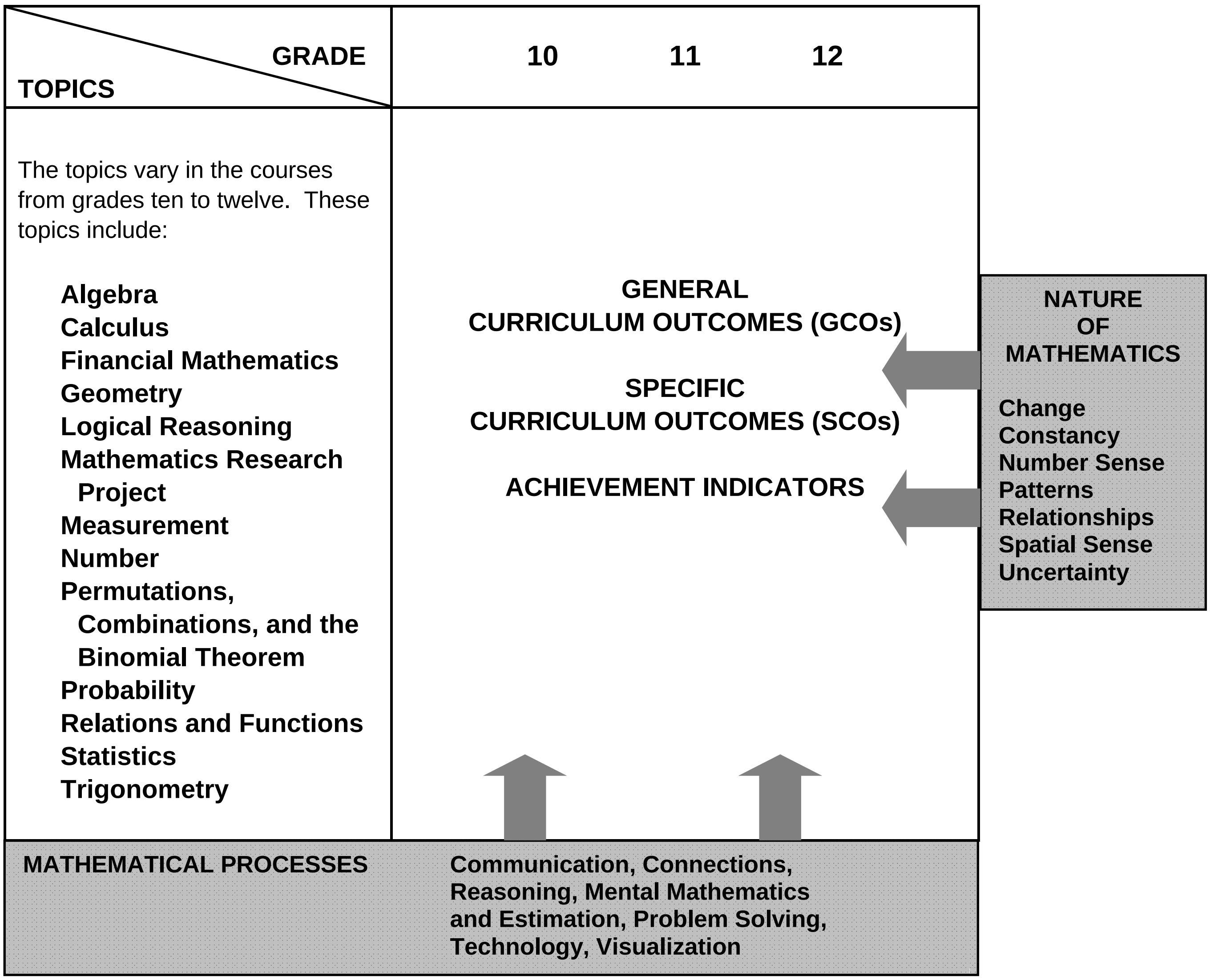
- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks;
- exhibit curiosity.

➢ Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the understanding of mathematical concepts by students and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, physical education, and other subject areas. Efforts should be made to make connections and use examples drawn from a variety of disciplines.

Conceptual Framework for 10-12 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



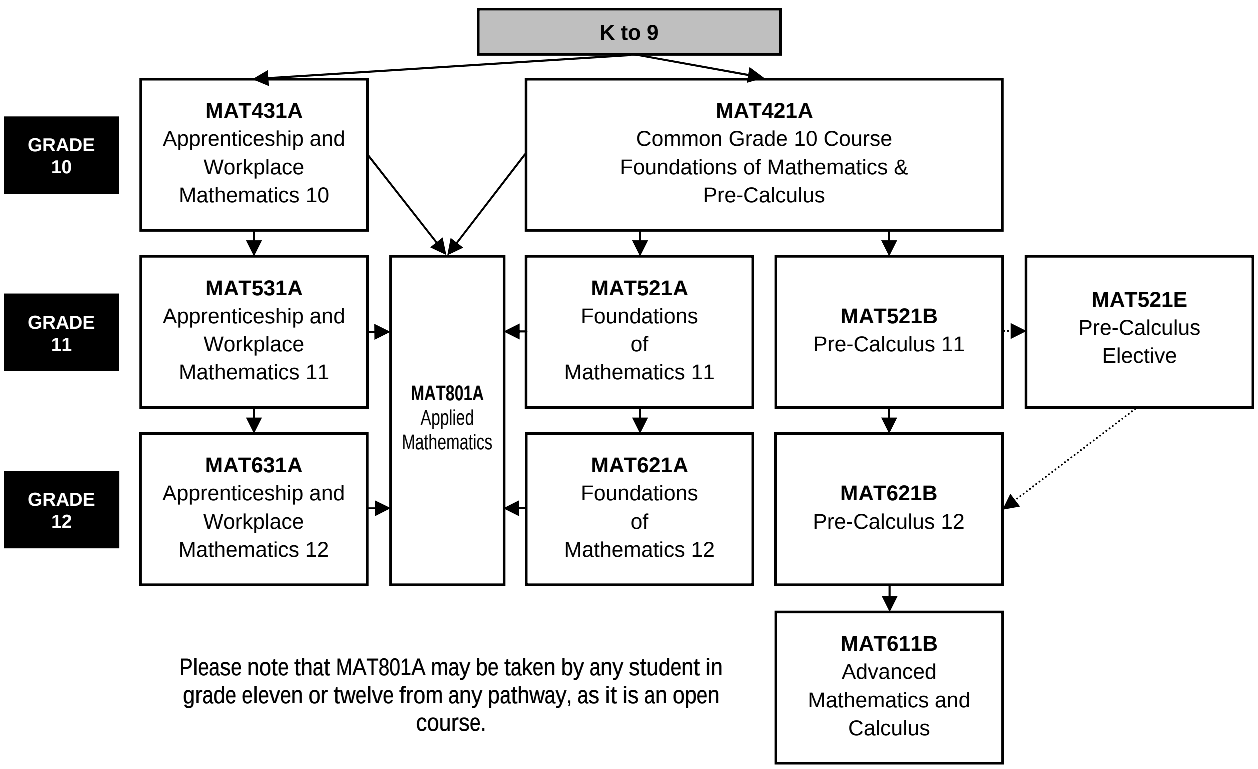
The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into a number of topics, as described above. These topics are not intended to be discrete units of instruction. The integration of outcomes across topics makes mathematical experiences meaningful. Students should make the connections among concepts both within and across topics. Consider the following when planning for instruction:

- Integration of the mathematical processes within each topic is expected.
- Decreasing emphasis on rote calculation, drill, and practice, and the size of numbers used in paper and pencil calculations makes more time available for concept development.

- Problem solving, reasoning, and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➢ Pathways and Topics

The Prince Edward Island 10-12 mathematics curriculum includes pathways with corresponding topics rather than strands, which are found in the Prince Edward Island K-9 mathematics curriculum. Three pathways are available: **Apprenticeship and Workplace Mathematics**, **Foundations of Mathematics**, and **Pre-Calculus**. A common grade ten course (Foundations of Mathematics and Pre-Calculus, Grade 10) is the starting point for the Foundations of Mathematics pathway and the Pre-Calculus pathway. Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings. These pathways are illustrated in the diagram below:



The goals of all three pathways are to provide the prerequisite knowledge, skills, understandings, and attitudes for specific post-secondary programs or direct entry into the work force. All three pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. Each pathway is designed to provide students with the mathematical understandings, rigour and critical-thinking skills that have been identified for specific post-secondary programs of study or for direct entry into the work force. When choosing a pathway, students should consider their interests, both current and future. Students, parents and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

Apprenticeship and Workplace Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into the majority of trades and for direct entry into the work force. Topics include algebra, geometry, measurement, number, statistics, and probability.

Foundations of Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of theoretical calculus. Topics include financial mathematics, geometry, measurement, algebra and number, logical reasoning, relations and functions, statistics, probability, and a mathematics research project.

Pre-Calculus

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Topics include algebra and number, measurement, relations and functions, trigonometry, combinatorics, and introductory calculus.

➢ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]**
- develop visualization skills to assist in processing information, making connections, and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

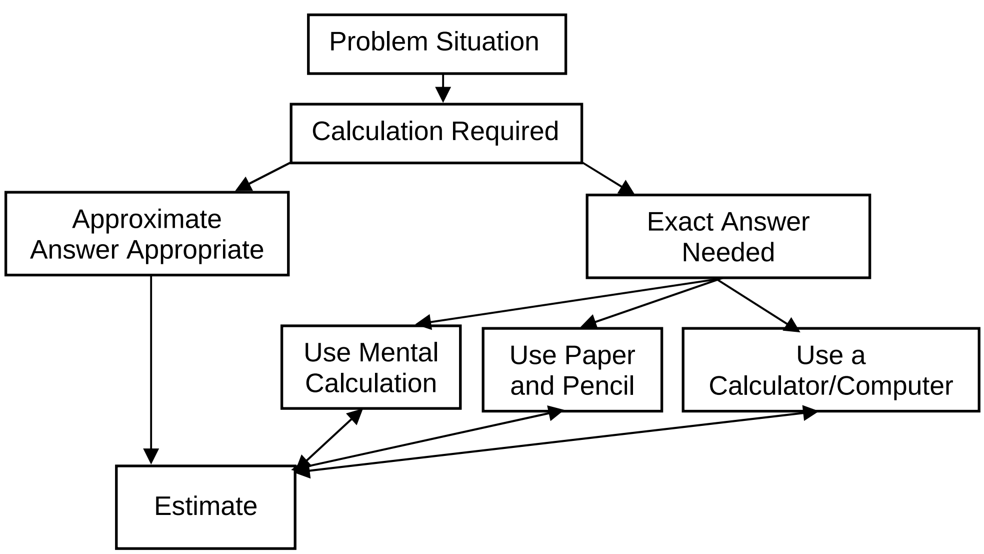
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves calculation without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need - more than ever before - with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein, 2001). Mental mathematics “provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers” (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know when to estimate, what strategy to use, and how to use it. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you. . . ?" or “How could you. . . ?” the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- using estimation
- guessing and checking
- looking for a pattern
- making an organized list or table
- using a model
- working backwards
- using a formula
- using a graph, diagram, or flow chart
- solving a simpler problem
- using algebra.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations;

- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K-3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform, and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and knowledge of several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

➢ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics which are woven throughout this document. These components include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180^0.

- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (*The Primary Program*, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, and flexible and intuitive with numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all topics in mathematics and it is important that connections are made among topics. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collecting and analysing of data, and describing relationships visually, symbolically, orally, or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to interpret representations of 2-D shapes and 3-D objects, and identify relationships to mathematical topics. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 2-D shapes and 3-D objects.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to use dimensions and make predictions about the results of changing dimensions.

- Knowing the dimensions of an object enables students to communicate about the object and create representations.
- The volume of a rectangular solid can be calculated from given dimensions.
- Doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Young children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Their mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Initial problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➢ Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding, and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should replace some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill-oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a teacher can give a parent clearer understanding of the mathematics curriculum and of the child's progress in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

➢ Diversity in Student Needs

Every class has students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters, but should be designed to help all students (whether strong, weak or average) to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson from which all students come away with a better understanding of what the solution to an equation really means.

➢ Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean not only that enrolments of students of both genders and various cultural backgrounds in public school mathematics courses should reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➢ Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English deficiencies and cultural differences must not be barriers to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and co-ordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication "as an essential part of mathematics and mathematics education" (p.60). The *Standards* elaborate that all students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading, and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers and will facilitate “communicating to learn mathematics and learning to communicate mathematically” (NCTM, p.60).

To this end,

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counsellors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not on other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated with appropriate language support to both students and parents;
- to verify that barriers have been removed, educators should monitor enrolment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➢ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development - such as poverty alleviation, human rights, health, environmental protection, and climate change - into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental, and economic perspective, and exploring how those factors are interrelated and interdependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database *Resources for Rethinking*, found at **http://r4r.ca/en**. It provides teachers with access to materials that integrate ecological, social, and economic spheres through active, relevant, interdisciplinary learning.

➢ Inquiry-Based Learning and Project Based Learning

Inquiry-based learning (IBL) allows students to explore, investigate, and construct new meaning from prior knowledge and from new information that is retrieved from other sources. It is not linear in nature, but promotes a continual looping back and forth throughout the process as students gather and process new information, redirect their inquiries, and continue through the process. Mathematical inquiry will require students to practise and refine their critical and creative-thinking skills. The terms *inquiry* and *research* are often used interchangeably within an educational context. While research often becomes the end-result of an inquiry process, it is the process itself that should be emphasized within an educational context. More information regarding the development of a mathematics research project is included in the appendix at the end of this document.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, teaching has been effective, or how best to address student learning needs. The quality of assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others.

➢ Assessment

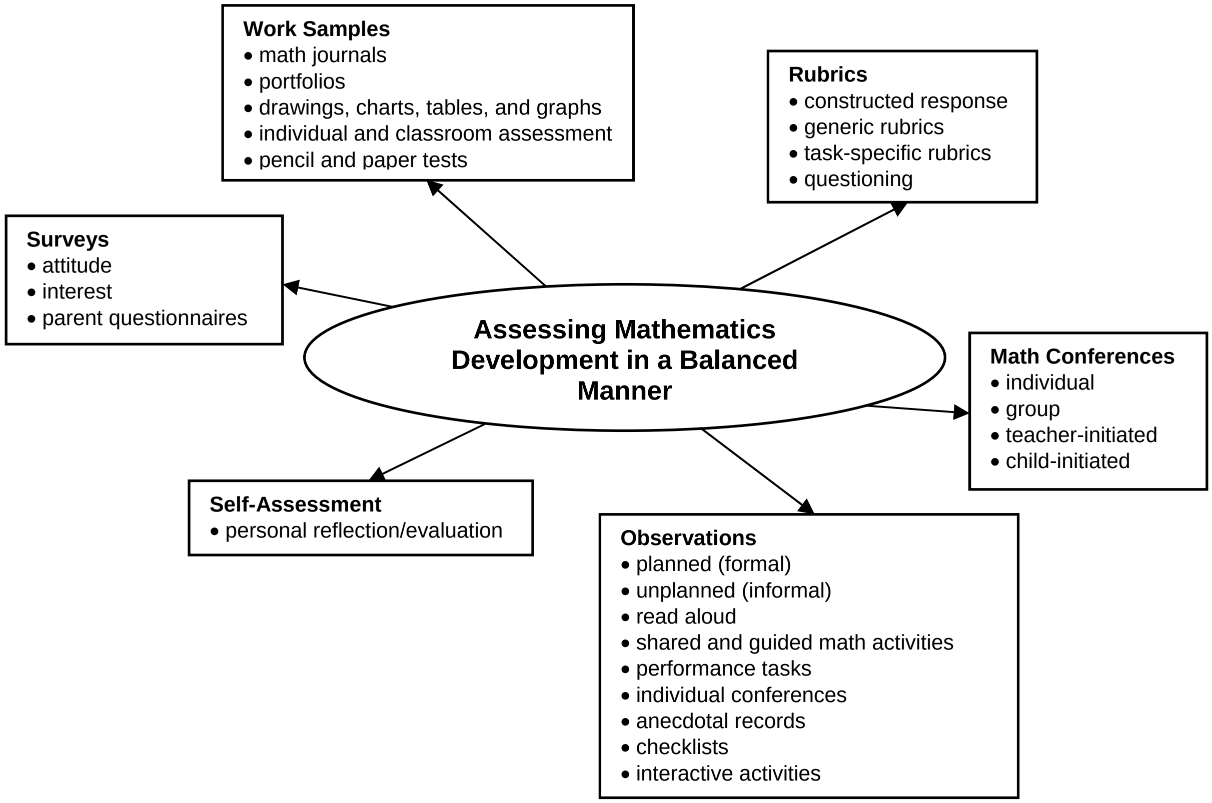
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children's learning;
- providing information to teachers on the effectiveness of their teaching, the program, and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources, including

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment.

This balanced approach for assessing mathematics development is illustrated in the diagram on the next page.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning, and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used

- to engage students in their own learning and self-assessment;
- to help students understand what is important in the mathematical concepts and particular tasks they encounter;
- to develop effective habits of metacognition and self-coaching;
- to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used

- to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
- to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
- to provide feedback to students about how they are doing and how they might improve;
- to provide feedback to other professionals and to parents about how to support students' learning.

Assessment *of* learning is used

- to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
- to facilitate reporting;

- to provide the basis for sound decision-making about next steps in a student's learning.

➢ Evaluation

Evaluation is the process of analysing, reflecting upon, and summarizing assessment information, and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires

- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information;
- using a high level of professional judgment in making decisions based upon that information.

➢ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes, phone calls, and electronic methods.

➢ Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration, and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles, as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes, or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.
- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.
- Assessment reports should be clear, accurate, and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that

- the best interests of the student are paramount;

- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes;
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he or she knows and can do.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island high school mathematics curriculum are organized into a number of topics across the grades from ten to twelve. Each topic has associated with it a general curriculum outcome (GCO). They are overarching statements about what students are expected to learn in each topic from grades ten to twelve.

Topic	General Curriculum Outcome (GCO)
Algebra (A)	Develop algebraic reasoning.
Algebra and Number (AN)	Develop algebraic reasoning and number sense.
Calculus (C)	Develop introductory calculus reasoning.
Financial Mathematics (FM)	Develop number sense in financial applications.
Geometry (G)	Develop spatial sense.
Logical Reasoning (LR)	Develop logical reasoning.
Mathematics Research Project (MRP)	Develop an appreciation of the role of mathematics in society.
Measurement (M)	Develop spatial sense and proportional reasoning. **(*Foundations of Mathematics and Pre-Calculus*)**
	Develop spatial sense through direct and indirect measurement. **(*Apprenticeship and Workplace Mathematics*)**
Number (N)	Develop number sense and critical thinking skills.
Permutations, Combinations and Binomial Theorem (PC)	Develop algebraic and numeric reasoning that involves combinatorics.
Probability (P)	Develop critical thinking skills related to uncertainty.
Relations and Functions (RF)	Develop algebraic and graphical reasoning through the study of relations.
Statistics (S)	Develop statistical reasoning.
Trigonometry (T)	Develop trigonometric reasoning.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific curriculum outcome.

In this curriculum guide, each specific curriculum outcome (SCO) is presented in a two-page format, and includes the following information:

- its corresponding topic and general curriculum outcome;
- the scope and sequence of the specific curriculum outcome(s) from grades eleven to twelve which correspond to this SCO;
- the specific curriculum outcome, with a list of achievement indicators;

- a list of the sections in *Foundations of Mathematics 12* which address the SCO, with specific achievement indicators highlighted in brackets;
- an elaboration for the SCO.

In the second half of this document, a curriculum guide supplement is presented which follows the primary resource, *Foundations of Mathematics 12*. As well, an appendix is included which outlines the steps to follow in the development of an effective mathematics research project.

FINANCIAL MATHEMATICS

SPECIFIC CURRICULUM OUTCOMES

FM1 – Solve problems that involve compound interest in financial decision making.

FM2 – Analyse costs and benefits of renting, leasing and buying.

FM3 – Analyse an investment portfolio in terms of:
- **interest rate;**
- **rate of return;**
- **total return.**

MAT621A – Topic: Financial Mathematics (FM)

GCO: Develop number sense in financial applications.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
	FM1 Solve problems that involve compound interest in financial decision making.

SCO: **FM1 – Solve problems that involve compound interest in financial decision making.** [C, CN, PS, T, V]

Students who have achieved this outcome should be able to:

A. Solve problems that involve simple interest.

B. Explain the advantages and disadvantages of compound interest and simple interest.

C. Identify situations that involve compound interest.

D. Determine, given the principal, interest rate and number of compounding periods, the total interest of a loan.

E. Graph and compare, in a given situation, the total interest paid or earned for different compounding periods.

F. Determine the principal or present value of an investment, given the future value and compound interest rate.

G. Graph and describe the effects of changing the value of one of the variables in a situation that involves compound interest.

H. Determine, using technology, the total cost of a loan under a variety of conditions, *e.g.*, different amortization periods, interest rates, compounding periods and terms.

I. Compare and explain, using technology, different credit options that involve compound interest, including bank and store credit cards, and special promotions.

J. Solve a contextual problem that involves compound interest.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.1 (A)

1.2 (B C)

1.3 (D J)

1.4 (E F G J)

2.1 (H J)

2.2 (I J)

2.3 (I J)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: FM1 – Solve problems that involve compound interest in financial decision making. [C, CN, PS, T, V]

Elaboration

Simple interest is determined only on the principal of an investment. The value of an investment that earns simple interest over time is a linear function. The accumulated simple interest over time is also a linear function. Since the interest is paid at the end of each period, the growth is not continuous. The amount of simple interest earned on an investment can be determined using the formula

$$I = Prt$$

where I is the interest, P is the principal, r is the annual interest rate, expressed as a decimal, and t is the time, in years. The future value or amount, A, of an investment that earns simple interest can be determined using the formula

$$A = P\left(1 + rt\right)$$

where P is the principal, r is the interest rate, expressed as a decimal, and t is the time, in years.

Compound interest is determined by applying the interest rate to the sum of the principal and any accumulated interest. Previously earned interest is reinvested over the course of the investment. If the same principal is invested in a compound interest account, with the same interest rate for the same term, the compound interest will grow faster (non-linear) than the corresponding simple interest (linear). The future value of an investment that earns compound interest can be determined using the compound interest formula

$$A = P\left(1 + i\right)^n$$

where A is the future value, P is the principal, i is the interest rate per compounding period, expressed as a decimal, and n is the number of compounding periods. When using the compound interest formula, use an exact value for i. For example, for an annual interest rate of 5%, compounded monthly, use the exact value of $\frac{0.05}{12}$ for i instead of the decimal approximation 0.00416, when inputting values in a calculator.

Four common compounding frequencies are given in the table below. The table shows how to calculate the interest rate per compounding period (i) and the number of compounding periods (n) for each frequency.

COMPOUNDING FREQUENCY	**TIMES PER YEAR**	**INTEREST RATE PER COMPOUNDING PERIOD (*i*)**	**NUMBER OF COMPOUNDING PERIODS (*n*)**
annually	1	$i = \text{annual interest rate}$	$n = \text{number of years}$
semi-annually	2	$i = \frac{\text{annual interest rate}}{2}$	$n = 2 \times \left(\text{number of years}\right)$
quarterly	4	$i = \frac{\text{annual interest rate}}{4}$	$n = 4 \times \left(\text{number of years}\right)$
monthly	12	$i = \frac{\text{annual interest rate}}{12}$	$n = 12 \times \left(\text{number of years}\right)$

The present value of an investment that earns compound interest can be determined using the formula

$$P = \frac{A}{\left(1 + i\right)^n}$$

where P is the present value (or principal), A is the amount (or future value), i is the interest rate per compounding period, expressed as a decimal, and n is the number of compounding periods.

MAT621A – Topic: Financial Mathematics (FM)

GCO: Develop number sense in financial applications.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
	FM2 Analyse costs and benefits of renting, leasing and buying.

SCO: **FM2 – Analyse costs and benefits of renting, leasing and buying.** [CN, PS, R, T]

Students who have achieved this outcome should be able to:

A. Identify and describe examples of assets that appreciate or depreciate.

B. Compare, using examples, renting, leasing and buying.

C. Justify, for a specific set of circumstances, if renting, leasing or buying would be advantageous.

D. Solve a problem involving renting, leasing or buying that requires the manipulation of a formula.

E. Solve, using technology, a contextual problem that involves cost-and-benefit analysis.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.4 (A B C D E)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: FM2 – Analyse costs and benefits of renting, leasing and buying. [CN, PS, R, T]

Elaboration

When deciding whether to rent, lease, or buy (with or without financing), each situation is unique. A cost and benefit analysis should take everything into account.

- Costs include initial costs and fees, short-term costs, long-term costs, disposable income, the cost of financing, depreciation and appreciation, penalties for breaking contracts, and equity.
- Benefits include convenience, commitments, flexibility, and personal needs or wants, such as how often a person wants to acquire a new car, for example.

Since each situation is unique, it is impossible to generalize about whether renting, leasing, or buying is best.

When renting, leasing or buying, payments often have to be made up front. Some payments go toward the overall cost, such as a down payment on a house, or a lease deposit and the first and last month's rent. Other deposits, such as a rental damage deposit, are refunded at a later date.

Appreciation and depreciation affect the value of a piece of property and should be considered when making decisions about renting, leasing, or buying, based on a particular situation. They are usually expressed as a rate per annum.

Equity can make buying a house a form of investment. This should also be considered when deciding to rent, lease, or buy.

MAT621A – Topic: Financial Mathematics (FM)

GCO: Develop number sense in financial applications.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
	FM3 Analyse an investment portfolio in terms of: • interest rate; • rate of return; • total return.

SCO: **FM3 – Analyse an investment portfolio in terms of:**

- **interest rate;**
- **rate of return;**
- **total return.**

[ME, PS, R, T]

Students who have achieved this outcome should be able to:

A. Determine and compare the strengths and weaknesses of two or more portfolios.

B. Determine, using technology, the total value of an investment when there are regular contributions to the principal.

C. Graph and compare the total value of an investment with and without regular contributions.

D. Apply the Rule of 72 to solve investment problems, and explain the limitations of the rule.

E. Determine, using technology, possible investment strategies to achieve a financial goal.

F. Explain the advantages and disadvantages of long-term and short-term investment options.

G. Explain, using examples, why smaller investments over a longer term may be better than larger investments over a shorter term.

H. Solve an investment problem.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.3 (D)

1.5 (B C)

1.6 (E F G H)

2.1 (A)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: FM3 – Analyse an investment portfolio in terms of:

- **interest rate;**
- **rate of return;**
- **total return.**

[ME, PS, R, T]

Elaboration

The Rule of 72 is a simple strategy for estimating the doubling time of an investment. It is most accurate when the interest is compounded annually. For example, $1000 invested at 3% interest, compounded annually, will double in value in about $72 \div 3$, or 24 years; and $1000 invested at 6%, compounded annually, will double in about $72 \div 6$, or 12 years.

For an investment that involves a series of equal deposits or payments made at regular intervals, the future value is the sum of all the regular payments plus the accumulated interest. The future value of an investment involving regular payments can be found by determining the sum of all the future values of each regular payment

$$A = R\left(1+i\right)^{0} + R\left(1+i\right)^{1} + R\left(1+i\right)^{2} + \cdots + R\left(1+i\right)^{n-1}$$

where A is the amount, or future value of the investment, R is the regular payment, i is the interest rate per compounding period, expressed as a decimal, and n is the number of compounding periods.

The future value of a single deposit has a greater future value than a series of regular payments of the same total amount. Small deposits over a long term can have a greater future value than large deposits over a short term because there is more time for compound interest to be earned.

An investment portfolio can be built from different types of investments, such as single payment investments (for example, Canada Savings Bonds and Guaranteed Investment Certificates) and investments involving regular payments. Some of these investments, such as Canada Savings Bonds, lock in money for specified periods of time, thus limiting access to the money, but offer higher interest rates. Other investments, such as savings accounts, are accessible at any time, but offer lower interest rates. Investments that involve greater principal amounts invested, or greater regular payment amounts when contracted tend to offer higher interest rates.

The rate of return is a useful measure for comparing investment portfolios. The factors that contribute to a larger return on an investment are time, interest rate, and compounding frequency. The longer that a sum of money is able to earn interest at a higher rate compounded more often, the more interest will be earned. For investments involving regular payments, the payment frequency is also a factor.

LOGICAL REASONING

SPECIFIC CURRICULUM OUTCOMES

LR1 – Analyse puzzles and games that involve numerical and logical reasoning, using problem-solving strategies.

LR2 – Solve problems that involve the application of set theory.

LR3 – Solve problems that involve conditional statements.

MAT621A – Topic: Logical Reasoning (LR)

GCO: Develop logical reasoning.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
LR2 Analyse puzzles and games that involve spatial reasoning, using problem-solving strategies.	**LR1** Analyse puzzles and games that involve numerical and logical reasoning, using problem-solving strategies.

SCO: **LR1 – Analyse puzzles and games that involve numerical and logical reasoning, using problem-solving strategies.** [CN, ME, PS, R]

Students who have achieved this outcome should be able to:

A. Determine, explain and verify a strategy to solve a puzzle or to win a game; *e.g.*:
- guess and check;
- look for a pattern;
- make a systematic list;
- draw or model;
- eliminate possibilities;
- simplify the original problem;
- work backward;
- develop alternate approaches.

B. Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.

C. Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

Note: It is intended this outcome be integrated throughout the course by using games and puzzles such as chess, Sudoku, Nim, logic puzzles, magic squares, Kakuro and cribbage.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

Integrated throughout the text.

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: LR1 – Analyse puzzles and games that involve numerical and logical reasoning, using problem-solving strategies. [CN, ME, PS, R]

Elaboration

It is intended this outcome be integrated throughout the course by using games and puzzles such as chess, Sudoku, Nim, logic puzzles, magic squares, Kakuro and cribbage. In each case, the student should be able to describe the strategy used to win the game or solve the puzzle after having gone through the process.

MAT621A – Topic: Logical Reasoning (LR)

GCO: Develop logical reasoning.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
	LR2 Solve problems that involve the application of set theory.

SCO: **LR2 – Solve problems that involve the application of set theory.** [CN, PS, R, V]

Students who have achieved this outcome should be able to:

A. Understand sets and set notation.

B. Provide examples of the empty set, disjoint sets, subsets and universal sets in context, and explain the reasoning.

C. Organize information such as collected data and number properties, using graphic organizers, and explain the reasoning.

D. Explain what a specified region in a Venn diagram represents, using connecting words (and, or, not) or set notation.

E. Determine the elements in the complement of a set, and the intersection or the union of two sets.

F. Explain how set theory is used in applications such as Internet searches, database queries, data analysis, games and puzzles.

G. Identify and correct errors in a given solution to a problem that involves sets.

H. Solve a contextual problem that involves sets, and record the solution, using set notation.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

3.1 (A B C)

3.2 (D)

3.3 (E)

3.4 (F G H)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

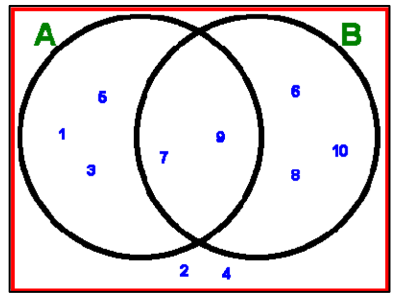
SCO: LR2 – Solve problems that involve the application of set theory. [CN, PS, R, V]

Elaboration

A set of elements can be represented by

- listing the elements; for example, $A = \{1, 2, 3, 4, 5\}$
- using words or a sentence; for example, $A = \{\text{all integers greater than 0 and less than 6}\}$
- using set builder notation, for example, $A = \{x \mid 0 < x < 6, x \in I\}$

The relationship between two sets can be illustrated by using a Venn diagram, as shown in the diagram at the right. As a rule, Venn diagrams do not usually show the relative sizes of the sets. The universal set can often be separated into subsets in more than one correct way.

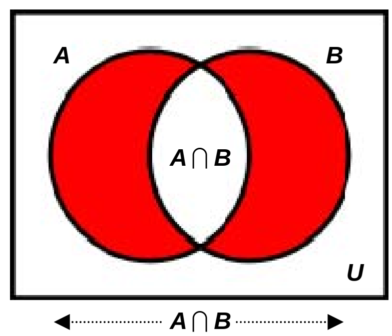


Sets are equal if they contain exactly the same elements, even if the elements are listed in different orders. Sometimes, it may not be possible to count the number of elements in very large or infinite sets, such as the set of real numbers.

The sum of the number of elements in a set, *A*, and its complement, A', is equal to the number of elements in the universal set, *U*.

$$n(A) + n(A') = n(U)$$

Each element in a universal set appears only once in a Venn diagram. If an element occurs in more than one set, it is placed in the area of the Venn diagram where the sets overlap. The union of two sets, denoted $A \cup B$, consists of all the elements that are in at least one of the sets. It is represented by the combined region of these sets on a Venn diagram. Union is indicated by the word *or*. The intersection of two sets, denoted $A \cap B$, consists of all the elements that are common to both sets. It is represented by the region of overlap on a Venn diagram. Intersection is indicated by the word *and*.

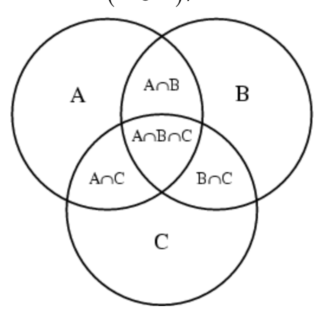


When two sets *A* and *B* are disjoint, the number of elements in *A* or *B*, denoted $n(A \cup B)$, is:

$$n(A \cup B) = n(A) + n(B)$$

If two sets, *A* and *B*, contain common elements, the number of elements in *A* or *B*, denoted $n(A \cup B)$, is:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$



Set theory is useful for solving many types of problems, including Internet searches, database queries, data analyses, games, and puzzles. To represent three intersecting sets with a Venn diagram, use three intersecting circles as shown in the diagram at the right. When solving a problem involving three sets, it is often best to start with the innermost section, where all three circles intersect, then work outward.

MAT621A – Topic: Logical Reasoning (LR)

GCO: Develop logical reasoning.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
LR1 Analyse and prove conjectures, using inductive and deductive reasoning, to solve problems.	**LR3** Solve problems that involve conditional statements.

SCO: LR3 – Solve problems that involve conditional statements. [C, CN, PS, R]

Students who have achieved this outcome should be able to:

A. Analyse an “if-then” statement, make a conclusion, and explain the reasoning.

B. Make and justify a decision, using “What if?” questions, in contexts such as probability, finance, sports, games or puzzles, with or without technology.

C. Determine the converse, inverse and contrapositive of an “if-then” statement; determine its veracity; and, if it is false, provide a counterexample.

D. Demonstrate, using examples, that the veracity of any statement does not imply the veracity of its converse or inverse.

E. Demonstrate, using examples, that the veracity of any statement does imply the veracity of its contrapositive.

F. Identify and describe contexts in which a biconditional statement can be justified.

G. Analyse and summarize, using a graphic organizer such as a truth table or Venn diagram, the possible results of given logical arguments that involve biconditional, converse, inverse or contrapositive statements.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

3.5 (A B C D F G)

3.6 (C D E G)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: LR3 – Solve problems that involve conditional statements. [C, CN, PS, R]

Elaboration

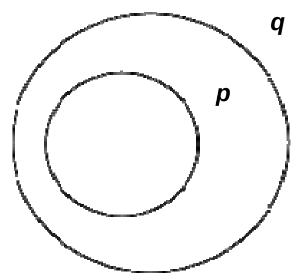
A conditional statement consists of a hypothesis, *p*, and a conclusion, *q*. Different ways to write a conditional statement include "If *p*, then *q*," "*p* implies *q*", and $p \rightarrow q$. To write the converse of a conditional statement, switch the hypothesis and the conclusion. In other words the converse of "If *p*, then *q*," is "If *q*, then *p*."

A conditional statement is either true or false. A truth table for a conditional statement, $p \rightarrow q$, can be set up as follows:

p	***q***	$p \rightarrow q$
T	T	T
F	F	T
F	T	T
T	F	F

A conditional statement is false only when the hypothesis is true and the conclusion is false. Otherwise, the conditional statement is true, even if the hypothesis is false.

A conditional statement can be represented using a Venn diagram with the inner oval representing the hypothesis and the outer oval representing the conclusion, as shown to the right. The statement "*p* implies *q*" means that *p* is a subset of *q*. Only one counterexample is needed to show that a conditional statement is false. If a conditional statement and its converse are both true, then they can be combined to create a biconditional statement using the phrase "if and only if."



The inverse of a conditional statement can be formed by negating the hypothesis and the conclusion. The contrapositive of a conditional statement can be formed by exchanging and negating the hypothesis and the conclusion. If a conditional statement is true, then its contrapositive is true, and vice versa. If the inverse of a conditional statement is true, then the converse of the statement is also true, and vice versa.

PROBABILITY

SPECIFIC CURRICULUM OUTCOMES

P1 – Interpret and assess the validity of odds and probability statements.

P2 – Solve problems that involve the probability of mutually exclusive and non-mutually exclusive events.

P3 – Solve problems that involve the probability of two events.

P4 – Solve problems that involve the fundamental counting principle.

P5 – Solve problems that involve permutations.

P6 – Solve problems that involve combinations.

MAT621A – Topic: Probability (P)

GCO: Develop critical thinking skills related to uncertainty.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
S1 Demonstrate an understanding of normal distribution, including: • standard deviation; • *z*-scores.	**P1** Interpret and assess the validity of odds and probability statements.

SCO: **P1 – Interpret and assess the validity of odds and probability statements.** [C, CN, ME]

Students who have achieved this outcome should be able to:

A. Provide examples of statements of probability and odds found in fields such as media, biology, sports, medicine, sociology and psychology.

B. Explain, using examples, the relationship between odds (part-part) and probability (part-whole).

C. Express odds as a probability and vice versa.

D. Determine the probability of, or the odds for and against, an outcome in a situation.

E. Explain, using examples, how decisions may be based on probability or odds, and on subjective judgments.

F. Solve a contextual problem that involves odds or probability.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.1 (A)

5.2 (B C D E F)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: P1 – Interpret and assess the validity of odds and probability statements. [C, CN, ME]

Elaboration

An event is a collection of outcomes that satisfy a specific condition. For example, when throwing a regular die, the event "throw an odd number" is a collection of the outcomes 1, 3, and 5. The probability of an event can range from 0 (impossible) to 1 (certain). A probability can be expressed as a fraction, a decimal, or a percent. Theoretical probability can be used to determine the likelihood that an event will happen.

Knowing the probability of an event is useful when making decisions. The experimental probability of event *A* is represented as

$$P(A) = \frac{n(A)}{n(T)}$$

where $n(A)$ is the number of times event *A* occurred and $n(T)$ is the number of trials, *T*, in the experiment. The theoretical probability of event *A* is represented as

$$P(A) = \frac{n(A)}{n(S)}$$

where $n(A)$ is the number of favourable outcomes for event *A* and $n(S)$ is the total number of outcomes in the sample space, *S*, where all outcomes are equally likely. A game is considered fair when all of the players are equally likely to win.

Odds express a level of confidence about the occurrence of an event. The odds in favour of event *A* occurring are given by the ratio

$$\frac{P(A)}{P(A')} \text{ or } P(A):P(A')$$

This corresponds to the ratio of favourable outcomes to unfavourable outcomes. The odds against event *A* occurring are given by the ratio

$$\frac{P(A')}{P(A)} \text{ or } P(A'):P(A)$$

This corresponds to the ratio of unfavourable outcomes to favourable outcomes. In both expressions, $P(A')$ is the probability of the complement of *A*, where

$$P(A') = 1 - P(A)$$

If the odds in favour of event *A* occurring are $m:n$, then the odds against event *A* occurring are $n:m$. Finally, if the odds in favour of event *A* occurring are $m:n$, then

$$P(A) = \frac{m}{m+n}$$

MAT621A – Topic: Probability (P)

GCO: Develop critical thinking skills related to uncertainty.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
S1 Demonstrate an understanding of normal distribution, including: • standard deviation; • *z*-scores.	**P2** Solve problems that involve the probability of mutually exclusive and non-mutually exclusive events.

SCO: **P2 – Solve problems that involve the probability of mutually exclusive and non-mutually exclusive events.** [CN, PS, R, V]

Students who have achieved this outcome should be able to:

A. Classify events as mutually exclusive or non-mutually exclusive, and explain the reasoning.

B. Determine if two events are complementary, and explain the reasoning.

C. Represent, using set notation or graphic organizers, mutually exclusive (including complementary) and non-mutually exclusive events.

D. Solve a contextual problem that involves the probability of mutually exclusive or non-mutually exclusive events.

E. Solve a contextual problem that involves the probability of complementary events.

F. Create and solve a problem that involves mutually exclusive or non-mutually exclusive events.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.4 (A B C D E F)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: P2 – Solve problems that involve the probability of mutually exclusive and non-mutually exclusive events. [CN, PS, R, V]

Elaboration

The favourable outcomes of two mutually exclusive events, *A* and *B*, can be represented as two disjoint sets. In this case, the probability that either *A* or *B* will occur is

$$P(A \cup B) = P(A) + P(B)$$

The favourable outcomes of two non-mutually exclusive events, *A* and *B*, can be represented as intersecting sets. In this case, the probability that either *A* or *B* will occur is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

This is called the Principle of Inclusion and Exclusion. An alternate formula is

$$P(A \cup B) = P(A \setminus B) + P(B \setminus A) + P(A \cap B)$$

where $A \setminus B$ refers to the elements of *A* that are not in *B*, and $B \setminus A$ refers to the elements of *B* that are not in *A*. When two events are mutually exclusive, both results are equivalent, because $n(A \cap B) = 0$, which results in $P(A \cap B) = 0$.

MAT621A – Topic: Probability (P)

GCO: Develop critical thinking skills related to uncertainty.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
S1 Demonstrate an understanding of normal distribution, including: • standard deviation; • *z*-scores.	**P3** Solve problems that involve the probability of two events.

SCO: **P3 – Solve problems that involve the probability of two events.** [CN, PS, R]

Students who have achieved this outcome should be able to:

A. Compare, using examples, dependent and independent events.

B. Determine the probability of an event, given the occurrence of a previous event.

C. Determine the probability of two dependent or two independent events.

D. Create and solve a contextual problem that involves determining the probability of dependent or independent events.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.5 (A B C D)

5.6 (B C D)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: P3 – Solve problems that involve the probability of two events. [CN, PS, R]

Elaboration

If the probability of one event depends on the probability of another event, then these events are called dependent events. For example, drawing a heart from a standard deck of 52 playing cards and then drawing another heart from the same deck without replacing the first card are dependent events. If event *B* depends on event *A* occurring, then the conditional probability that event *B* will occur, given that event *A* has occurred, can be represented by

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

If event *B* depends on event *A* occurring, then the probability that both events will occur can be represented by

$$P(A \cap B) = P(A) \bullet P(B \mid A)$$

If the probability of event *B* does not depend on the probability of event *A* occurring, then these events are called independent events. For example, tossing tails with a coin and drawing the ace of spades from a standard deck of 52 playing cards are independent events. The probability that two independent events, *A* and *B*, will both occur is the product of their individual probabilities:

$$P(A \cap B) = P(A) \bullet P(B)$$

A tree diagram is often used for modelling problems that involve both dependent and independent events.

MAT621A – Topic: Probability (P)

GCO: Develop critical thinking skills related to uncertainty.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
	P4 Solve problems that involve the fundamental counting principle.

SCO: **P4 – Solve problems that involve the fundamental counting principle.** [PS, R, V]

Students who have achieved this outcome should be able to:

A. Represent and solve counting problems, using a graphic organizer.

B. Generalize the fundamental counting principle, using inductive reasoning.

C. Identify and explain assumptions made in solving a counting problem.

D. Solve a contextual counting problem, using the fundamental counting principle, and explain the reasoning.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.1 (A B C D)

4.7 (D)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: P4 – Solve problems that involve the fundamental counting principle. [PS, R, V]

Elaboration

The Fundamental Counting Principle states that if one task can be performed in *a* ways, a second task can be performed in *b* ways, a third task in *c* ways, and so on, then all of these tasks can be performed in $a \bullet b \bullet c \bullet \cdots$ ways. The Fundamental Counting Principle applies when tasks are related by the word *and*.

The Fundamental Counting Principle does not apply when tasks are related by the word *or*. In this case, we apply one of the following formulas:

- If the tasks are mutually exclusive, they involve two disjoint sets, *A* and *B*, so $n(A \cup B) = n(A) + n(B)$.
- If the tasks are not mutually exclusive, they involve sets that are not disjoint, *C* and *D*, so $n(C \cup D) = n(C) + n(D) - n(C \cap D)$. In this case, the Principle of Inclusion and Exclusion must be used to avoid counting elements in the intersection of the two sets more than once.

Outcome tables, organized lists, and tree diagrams can also be used to solve counting problems. They have the added benefit of displaying all the possible outcomes, which can be useful in some problem situations. However, these strategies become difficult to use when there are many tasks involved and/or a large number of possibilities for each task.

MAT621A – Topic: Probability (P)

GCO: Develop critical thinking skills related to uncertainty.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
	P5 Solve problems that involve permutations.

SCO: **P5 – Solve problems that involve permutations.** [ME, PS, R, T, V]

Students who have achieved this outcome should be able to:

A. Represent the number of arrangements of *n* elements taken *n* at a time, using factorial notation.

B. Determine, with or without technology, the value of a factorial.

C. Simplify a numeric or algebraic fraction containing factorials in both the numerator and denominator.

D. Solve an equation that involves factorials.

E. Determine the number of permutations of *n* elements taken *r* at a time.

F. Determine the number of permutations of *n* elements taken *n* at a time where some elements are not distinct.

G. Explain, using examples, the effect of the total number of permutations of *n* elements when two or more elements are identical.

H. Generalize strategies for determining the number of permutations of *n* elements taken *r* at a time.

I. Solve a contextual problem that involves probability and permutations.

Note: It is intended that circular permutations not be included.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.2 (A B C D)

4.3 (E H)

4.4 (F G)

4.7 (H)

5.3 (I)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: P5 – Solve problems that involve permutations. [ME, PS, R, T, V]

Elaboration

A permutation is an arrangement of objects in a definite order, where each object appears only once in each arrangement. For example, the three letters A, B, and C can be listed in 6 different ordered arrangements or permutations: ABC, ACB, BAC, BCA, CAB, CBA.

The expression $n!$ is called "*n* factorial" and represents the number of permutations of a set of *n* different objects. It is calculated as

$$n! = n(n-1)(n-2)\cdots(3)(2)(1)$$

The expression $n!$ is defined only for the values that belong to the set of whole numbers, that is, $n \in \{0,\ 1,\ 2,\ 3,\ ...\}$. If $n = 0$, then $0!$ is defined as having a value of 1.

If order matters in a counting problem, then the problem involves permutations. The number of permutations from a set of *n* different objects, where *r* of then are used in each arrangement, can be calculated using the formula

$$_nP_r = \frac{n!}{(n-r)!}, \text{ where } 0 \le r \le n$$

When all *n* objects are used in each arrangement, *n* is equal to *r* and the number of arrangements is represented by $_nP_n = n!$. The number of permutations that can be created from a set of *n* objects, using *r* objects in each arrangement, where repetition is allowed, and $r \le n$, is n^r. All of these formulas are based on the Fundamental Counting Principle.

If a counting problem has one or more conditions that must be met,

- first, consider each case that each condition creates, as the solution is developed;
- then, add the number of ways each case can occur to determine the total number of outcomes.

There are fewer permutations when some of the objects in a set are identical compared to when all the objects in a set are different. This is because some of the arrangements are identical. The number of permutations of *n* objects, where *a* are identical, another *b* are identical, another *c* are identical, and so on, is

$$P = \frac{n!}{a!b!c!\cdots}$$

Dividing $n!$ by $a!$, $b!$, $c!$, and so on, deals with the effect of repetition caused by objects in the set that are identical. It eliminates arrangements that are the same and that would otherwise be counted multiple times.

MAT621A – Topic: Probability (P)

GCO: Develop critical thinking skills related to uncertainty.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
	P6 Solve problems that involve combinations.

SCO: **P6 – Solve problems that involve combinations.** [ME, PS, R, T, V]

Students who have achieved this outcome should be able to:

A. Explain, using examples, why order is or is not important when solving problems that involve permutations or combinations.

B. Determine the number of combinations of *n* elements taken *r* at a time.

C. Generalize strategies for determining the number of combinations of *n* elements taken *r* at a time.

D. Solve a contextual problem that involves combinations and probability.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.5 (A)

4.6 (B C)

4.7 (C)

5.3 (D)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: P6 – Solve problems that involve combinations. [ME, PS, R, T, V]

Elaboration

When order does not matter in a counting problem, combinations are being determined. For example, ABC, ACB, BAC, BCA, CAB, and CBA are the six different permutations of the letters A, B, and C, but they all represent the same single combination of letters. When all of the objects are being used in a combination, there is only one possible combination.

The number of combinations from a set of *n* different objects, where only *r* of them are used in each combination, can be denoted by ${}_nC_r$ or $\binom{n}{r}$, and is calculated by using the formula

$$ {}_nC_r = \frac{n!}{r!(n-r)!},\ \text{where } 0 \le r \le n $$

The formula for ${}_nC_r$ is the formula for ${}_nP_r$ divided by $r!$. Dividing by $r!$ eliminates the counting of the same combinations of *r* objects arranged in different orders. From a set of *n* distinct objects, the number of combinations is always less than or equal to the number of permutations when selecting *r* of these objects, where $r \le n$. When solving problems involving combinations, it may also be necessary to use the Fundamental Counting Principle.

Sometimes combination problems that have conditions can be solved using direct reasoning. To do this, follow these steps:

- Consider only cases that reflect the conditions.
- Determine the number of combinations for each case.
- Add the results of the previous step to determine the total number of combinations.

Sometimes combination problems that have conditions can be solved using indirect reasoning. To do this, follow these steps:

- Determine the number of combinations without any conditions.
- Consider only cases that do not meet these conditions.
- Determine the number of combinations for each case identified in the previous step.
- From the number of combinations in the previous step, subtract the number of combinations without any conditions.

RELATIONS AND FUNCTIONS

SPECIFIC CURRICULUM OUTCOMES

RF1 – Represent data, using polynomial functions (of degree $\leq$ 3), to solve problems.

RF2 – Represent data, using exponential and logarithmic functions, to solve problems.

RF3 – Represent data, using sinusoidal functions, to solve problems.

MAT621A – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
RF2 Demonstrate an understanding of the characteristics of quadratic functions, including: • vertex; • intercepts; • domain and range; • axis of symmetry.	**RF1** Represent data, using polynomial functions (of degree $\leq$ 3), to solve problems.

SCO: **RF1 – Represent data, using polynomial functions (of degree $\leq$ 3), to solve problems.** [C, CN, PS, T, V]

Students who have achieved this outcome should be able to:

A. Describe, orally and in written form, the characteristics of polynomial functions by analysing their graphs.

B. Describe, orally and in written form, the characteristics of polynomial functions by analysing their equations.

C. Match equations in a given set to their corresponding graphs.

D. Graph data and determine the polynomial function that best approximates the data.

E. Interpret the graph of a polynomial function that models a situation, and explain the reasoning.

F. Solve, using technology, a contextual problem that involves data that is best represented by graphs of polynomial functions, and explain the reasoning.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.1 (A)

6.2 (B C)

6.3 (D E F)

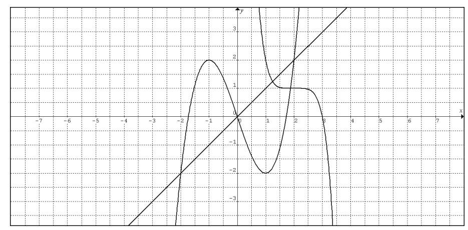
6.4 (D E F)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: RF1 – Represent data, using polynomial functions (of degree $\le$ 3), to solve problems. [C, CN, PS, T, V]

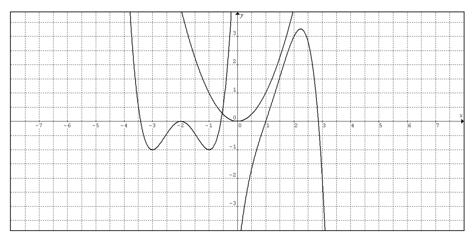
Elaboration

A polynomial function in one variable is a function that contains only the operations of multiplication and addition, with real-number coefficients, whole-number exponents, and two variables. The degree of the function is the greatest exponent of the function. For example, $f(x) = 6x^3 + 3x^2 - 4x + 9$ is a cubic polynomial function of degree 3.



Graphs of odd degree have the following characteristics:

- a graph that extends down into Quadrant III and up into Quadrant I when the leading coefficient is positive
- a graph that extends up into Quadrant II and down into Quadrant IV when the leading coefficient is negative
- a *y*-intercept that corresponds to the constant term of the function
- at least one *x*-intercept and up to a maximum of *n* *x*-intercepts, where *n* is the degree of the function
- a domain of $\{x \mid x \in R\}$ and a range of $\{y \mid y \in R\}$
- no maximum or minimum points
- an even number of turning points



Graphs of even degree have the following characteristics:

- a graph that extends up into Quadrant II and up into Quadrant I when the leading coefficient is positive
- a graph that extends down into Quadrant III and down into Quadrant IV when the leading coefficient is negative
- a *y*-intercept that corresponds to the constant term of the function
- from zero to a maximum of *n* *x*-intercepts, where *n* is the degree of the function
- a domain of $\{x \mid x \in R\}$ and a range that depends on the maximum or minimum value of the function
- an odd number of turning points

A scatter plot is useful when looking for trends in a given set of data. If the points on a scatter plot seem to follow a linear, quadratic, or cubic trend, then there may be a polynomial relationship between the independent and the dependent variable. If the points on a scatter plot do follow a trend, technology can be used to determine the line or curve of best fit. The method used by technology to find the line or curve of best fit is called regression, which results in an equation that balances the points in the scatter plot on both sides of the line or curve.

A line or curve of best fit can be used to predict values that are not recorded or plotted. Predictions can be made by reading values from the line or curve of best fit on a scatter plot by using the equation of the line or curve of best fit.

MAT621A – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
RF2 Demonstrate an understanding of the characteristics of quadratic functions, including: • vertex; • intercepts; • domain and range; • axis of symmetry.	**RF2** Represent data, using exponential and logarithmic functions, to solve problems.

SCO: **RF2 – Represent data, using exponential and logarithmic functions, to solve problems.**
[C, CN, PS, T, V]

Students who have achieved this outcome should be able to:

A. Describe, orally and in written form, the characteristics of exponential or logarithmic functions by analysing their graphs.

B. Describe, orally and in written form, the characteristics of exponential or logarithmic functions by analysing their equations.

C. Match equations in a given set to their corresponding graphs.

D. Graph data and determine the exponential or logarithmic function that best approximates the data.

E. Interpret the graph of an exponential or logarithmic function that models a situation, and explain the reasoning.

F. Solve, using technology, a contextual problem that involves data that is best represented by graphs of exponential or logarithmic functions, and explain the reasoning.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

7.1 (A)

7.2 (A B C)

7.3 (D E F)

7.4 (A B C)

7.5 (D E F)

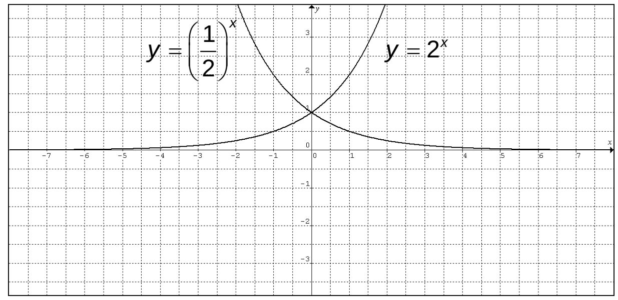
[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: RF2 – Represent data, using exponential and logarithmic functions, to solve problems.
[C, CN, PS, T, V]

Elaboration

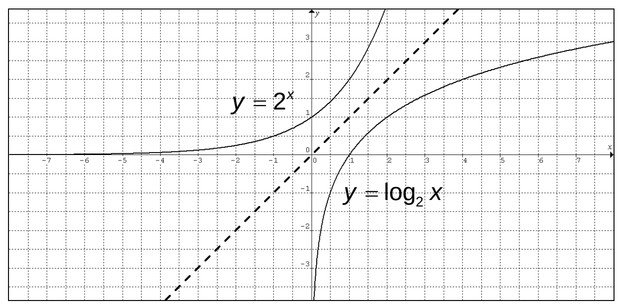
An exponential function of the form $y = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$, has the following characteristics:

- is increasing if $b > 1$
- is decreasing if $0 < b < 1$
- has a domain of $\{x \mid x \in R\}$
- has a range of $\{y \mid y > 0\}$
- has a y-intercept of 1
- has no x-intercept
- extends from Quadrant II to Quadrant I



The graphs of an exponential function and its inverse logarithmic function are reflections of each other in the line $y = x$, as shown below right. A logarithmic function of the form $y = \log_c x$, $c > 0$, $c \neq 1$, has the following characteristics:

- is increasing if $c > 1$
- is decreasing if $0 < c < 1$
- has a domain of $\{x \mid x > 0\}$
- has a range of $\{y \mid y \in R\}$
- has an x-intercept of 1
- has no y-intercept
- extends from Quadrant IV to Quadrant I if $c > 1$, or from Quadrant I to Quadrant IV if $0 < c < 1$



A scatter plot is useful when looking for trends in a given set of data. If the points on a scatter plot seem to follow an exponential or a logarithmic trend, then there may be an exponential or a logarithmic relationship between the independent and the dependent variable. If the points on a scatter plot do follow a trend, technology can be used to determine the curve of best fit. The method used by technology to find the curve of best fit is called regression, which results in an equation that balances the points in the scatter plot on both sides of the curve.

A curve of best fit can be used to predict values that are not recorded or plotted. Predictions can be made by reading values from the curve of best fit on a scatter plot by using the equation of the curve of best fit.

MAT621A – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
RF2 Demonstrate an understanding of the characteristics of quadratic functions, including: • vertex; • intercepts; • domain and range; • axis of symmetry.	**RF3** Represent data, using sinusoidal functions, to solve problems.

SCO: **RF3 – Represent data, using sinusoidal functions, to solve problems.** [C, CN, PS, T, V]

Students who have achieved this outcome should be able to:

A. Estimate and determine benchmarks for angle measure.

B. Describe, orally and in written form, the characteristics of sinusoidal functions by analysing their graphs.

C. Describe, orally and in written form, the characteristics of sinusoidal functions by analysing their equations.

D. Match equations in a given set to their corresponding graphs.

E. Graph data and determine the sinusoidal function that best approximates the data.

F. Interpret the graph of a sinusoidal function that models a situation, and explain the reasoning.

G. Solve, using technology, a contextual problem that involves data that is best represented by graphs of sinusoidal functions, and explain the reasoning.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

8.1 (A)

8.2 (B)

8.3 (B)

8.4 (C D)

8.5 (E F G)

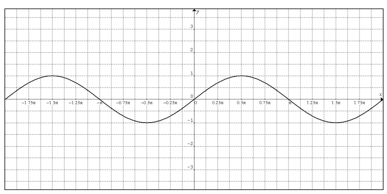
[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: RF3 – Represent data, using sinusoidal functions, to solve problems. [C, CN, PS, T, V]

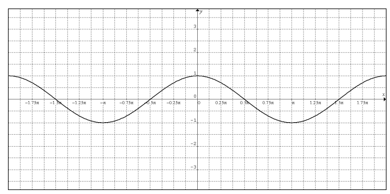
Elaboration

Radian measure is an alternative way to express the size of an angle. The measure of an angle can be expressed as a real number without units if radians are used. The central angle formed by one complete revolution in a circle is 360^0, or 2π, in radian measure. When converting between angle measures, 1 radian is equivalent to $\left(\frac{180}{\pi}\right)^0$ and 1^0 is equivalent to $\frac{\pi}{180}$ radians.

To sketch the graphs of $y = \sin x$ and $y = \cos x$, determine the coordinates of the key points representing the *x*-intercepts, maximum points and minimum points. Then, to get an accurate graph of the function, choose eight evenly-spaced points in each period of the function, and graph the results.



$y = \sin x$



$y = \cos x$

The following table highlights the characteristics of the graphs of each function:

	$y = \sin x$	$y = \cos x$
Maximum Value	1	1
Minimum Value	–1	–1
Amplitude	1	1
Period	2π or 360^0	2π or 360^0
***x*-intercepts**	$\pm\pi n,\ n \in I$	$\frac{\pi}{2} \pm \pi n,\ n \in I$
***y*-intercept**	0	1
Domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
Range	$\{y \mid -1 \le y \le 1\}$	$\{y \mid -1 \le y \le 1\}$
Midline	$y = 0$	$y = 0$

The characteristics of sinusoidal functions of the form $y = a \sin b(x - c) + d$ and $y = a \cos b(x - c) + d$ can be summarized as follows:

- The amplitude is represented by $|a|$. It can be found by using the formula $\text{amplitude} = \frac{\max - \min}{2}$.
- The period is can be calculated by using the formula $\text{period} = \frac{2\pi}{|b|}$, in radians, or $\text{period} = \frac{360^0}{|b|}$, in degrees.
- The horizontal translation is represented by *c*. It is a shift to the right if $c > 0$, and to the left if $c < 0$.
- The vertical displacement is represented by *d*. It is a shift up if $d > 0$, and a shift down if $d < 0$. As a result, the equation of the midline is $y = d$.

MATHEMATICS RESEARCH PROJECT

SPECIFIC CURRICULUM OUTCOMES

MRP1 – Research and give a presentation on a current event or an area of interest that involves mathematics.

MAT521A – Topic: Mathematics Research Project (MRP)

GCO: Develop an appreciation of the role of mathematics in society.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
MRP1 Research and give a presentation on a historical event or an area of interest that involves mathematics.	**MRP1** Research and give a presentation on a current event or an area of interest that involves mathematics.

SCO: **MRP1 – Research and give a presentation on a current event or an area of interest that involves mathematics.** [C, CN, ME, PS, R, T, V]

Students who have achieved this outcome should be able to:

A. Collect primary or secondary data (statistical or informational) related to the topic.

B. Assess the accuracy, reliability and relevance of the primary and secondary data collected by:
- identifying examples of bias and points of view;
- identifying and describing the data collection methods;
- determining if the data is relevant;
- determining if the data is consistent with information obtained from other sources on the same topic.

C. Interpret data, using statistical methods if applicable.

D. Identify controversial issues, if any, and present multiple sides of the issues with supporting data.

E. Organize and present the research project, with or without technology.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

Integrated throughout the text.

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: MRP1 – Research and give a presentation on a current event or an area of interest that involves mathematics. [C, CN, ME, PS, R, T, V]

Elaboration

See the appendix at the end of this document for specific details regarding the development of a mathematics research project.

Curriculum Guide Supplement

This supplement to the *Prince Edward Island MAT621A Mathematics Curriculum Guide* is designed to parallel the primary resource, *Foundations of Mathematics 12*.

For each of the chapters in the text, an approximate timeframe is suggested to aid teachers with planning. The timeframe is based on a total of 80 classes, each with an average length of 75 minutes:

CHAPTER	SUGGESTED TIME
Chapter 1 – Financial Mathematics: Investing Money	10 classes
Chapter 2 – Financial Mathematics: Borrowing Money	9 classes
Chapter 3 – Set Theory and Logic	11 classes
Chapter 4 – Counting Methods	10 classes
Chapter 5 – Probability	10 classes
Chapter 6 – Polynomial Functions	8 classes
Chapter 7 – Exponential and Logarithmic Functions	11 classes
Chapter 8 – Sinusoidal Functions	11 classes

Each chapter of the text is divided into a number of sections. In this document, each section is supported by a one-page presentation, which includes the following information:

- the name and pages of the section in the text;
- the specific curriculum outcome(s) and achievement indicator(s) addressed in the section (see the first half of the curriculum guide for an explanation of symbols);
- the student expectations for the section, which are associated with the SCO(s);
- the new concepts introduced in the section;
- other key ideas developed in the section;
- Suggested Problems in *Foundations of Mathematics 12*;
- possible instructional and assessment strategies for the section.

CHAPTER 1
FINANCIAL MATHEMATICS: INVESTING MONEY

SUGGESTED TIME

10 classes

Section 1.1 – Simple Interest (pp. 6-17)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **FM1 (A)** **After this lesson, students will be expected to:** • solve problems that involve simple interest **After this lesson, students should understand the following concepts:** • **term –** the contracted duration of an investment or loan • **interest –** the amount of money earned on an investment or paid on a loan • **fixed interest rate –** an interest rate that is guaranteed not to change during the term of an investment or loan • **principal –** the original amount of money invested or loaned • **simple interest –** the amount of interest earned on an investment or paid on a loan based on the original amount (the principal) and the simple interest rate; the formula for simple interest is $I = Prt$ where P is the principal, r is the annual interest rate, and t is the time, in years • **maturity –** the contracted end date of an investment or loan, at the end of the term • **future value –** the amount, A, that an investment will be worth after a specified period of time; the simple interest formula for future value is $A = P(1+rt)$ where P is the principal, r is the annual interest rate, and t is the time, in years • **rate of return –** the ratio of money earned (or lost) on an investment relative to the amount of money invested, usually expressed as a decimal or a percent **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 14-17:** #1-13	**Possible Instructional Strategies:** • Ask the class to research the origins of paying interest when money is borrowed or lent. **Possible Assessment Strategies:** • For each of the following, determine the future value of the loan, if simple interest is paid. a. $P = \$800$, $r = 4\%$, $t = 5$ years b. $P = \$1250$, $r = 8\frac{1}{2}\%$, $t = 6$ months c. $P = \$3800$, $r = 9\%$, $t = 60$ days • In addition to working and her family's contribution, Jane had to borrow \$8000 over the course of 6 years to complete her education. The simple interest is \$4046.40. Find the interest rate. • To take advantage of a going-out-of-business sale, the College Corner Furniture Store had to borrow some money. It paid back a total amount of \$150,000 on a 6-month loan at a simple interest rate of 12%. Find the principal. • To train employees to use new equipment. Williams Muffler Repair had to borrow \$4500 at a simple interest rate of $9\frac{1}{2}\%$. The company paid \$1282.50 in interest. Find the term of the loan.

Section 1.2 – Exploring Compound Interest (pp. 18-19)

ELABORATIONS & SUGGESTED PROBLEMS
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **FM1 (B C)** **After this lesson, students will be expected to:** • compare simple interest with compound interest **After this lesson, students should understand the following concept:** • **compound interest –** the interest that is earned or paid on both the principal and the accumulated interest **Suggested Problems in *Foundations of Mathematics 12*:** • **p. 19:** #1-3

Section 1.3 – Compound Interest: Future Value (pp. 20-33)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **FM1 (D J)**
- **FM3 (D)**

After this lesson, students will be expected to:

- determine the future value of an investment that earns compound interest

After this lesson, students should understand the following concepts:

- **compounded annually –** when compound interest is determined or paid yearly
- **compounding period –** the time over which interest is determined; interest can be compounded annually, semi-annually (every 6 months), quarterly (every 3 months), monthly, weekly, or daily
- **Rule of 72 –** a simple formula for estimating the doubling time of an investment; 72 is divided by the annual interest rate as a percent to estimate the doubling time of an investment in years; the Rule of 72 is most accurate when the interest is compounded annually

FORMULAS	
For the following formulas, P is the principal, i is the interest rate per compounding period, and n is the number of compounding periods.	
Compound Interest	$A = P(1+i)^n$
Interest Earned on an Investment	$I = A - P$ or $I = P\left[(1+i)^n - 1\right]$

Suggested Problems in *Foundations of Mathematics 12*:

- **pp. 30-32:** #1-13

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- It can be shown that the compound interest formula is an example of a geometric sequence, where n takes on whole number values.

Possible Assessment Strategies:

- Complete the following table:

	Annual Interest Rate	Frequency	***t***	***i***	***n***
a.	4%	annually	10 years		
b.	2%	quarterly	9 months		
c.	3%	monthly	18 months		

- For each of the following, determine the future value and the total interest earned.
  a. $200 invested for 9 years at 9% compounded monthly
  b. $750 invested for 12 years at 4% compounded quarterly
- Use the Rule of 72 to estimate the doubling time for $1250 invested at 6% compounded annually.
- Which is the better investment? Explain.
  - 4.5% compounded annually
  - 4.45% compounded quarterly
- A couple decides to set aside $5000 in a savings account for a second honeymoon trip. It is compounded quarterly for 10 years at 9%. Find the amount of money they will have in 10 years.
- In order to pay for college, the parents of a child invest $20,000 in a bond that pays 8% interest compounded semi-annually. How much money will the bond be worth in 19 years?
- A 25-year old woman plans to retire at age 50. She decides to invest an inheritance of $60,000 at 7% compounded quarterly. How much will she have at age 50?
- To pay for new machinery in 5 years, a company owner invests $10,000 at $7\frac{1}{2}\%$ compounded monthly. How much money will his investment be worth in 5 years?

Section 1.4 – Compound Interest: Present Value (pp. 34-42)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **FM1 (E F G J)** **After this lesson, students will be expected to:** • determine the principal or present value of an investment, given its future value and compound interest rate **After this lesson, students should understand the following concept:** • **present value –** the amount that must be invested now to result in a specific future value in a certain time at a given interest rate; the formula for present value is $$P = \frac{A}{(1+i)^n}$$ where A is the future value, i is the interest rate per compounding period, and n is the number of compounding periods **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 40-42:** #1-14	**Possible Instructional Strategies:** • Ensure that students are able to enter values into their calculators correctly in order to solve compound interest problems. **Possible Assessment Strategies:** • June, who is 25, would like to have $100,000 in an RRSP at age 55 when she retires. If her RRSP pays 5% per year compounded annually, how much money will she have to invest now in order to have her desired amount? • Karen has invested $20,000 in a Registered Education Savings Plan. She wants her investment to grow to $50,000 by the time her newborn enters university, in 18 years. What interest rate, compounded semi-annually, will result in a future value of $50,000? Round off the answer to two decimal places.

Section 1.5 – Investments Involving Regular Payments (pp. 46-57)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **FM3 (B C)** **After this lesson, students will be expected to:** • determine the future value of an investment that earns compound interest involving regular payments **FUTURE VALUE FORMULA** The regular payment future value formula is $A = R(1+i)^0 + R(1+i)^1 + \cdots + R(1+i)^{n-1}$ where R is the regular payment, i is the interest rate per compounding period, and n is the number of compounding periods. **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 55-57:** #1-17	**Possible Instructional Strategies:** • It can be shown that problems involving regular payments are examples of geometric series, where n takes on whole number values. **Possible Assessment Strategies:** • A husband and wife plan to save money for their daughter's college education in 8 years. They decide to purchase an annuity with an annual payment earning 5.5% compounded annually. Find the future value of the annuity in 8 years if the annual payment is $4000. • Joe invested $500 per month into an RRSP that earned 3% per year compounded monthly. How much will his investment be worth after 10 years? • When Benny turned 18, he stated that he had a goal of becoming a millionaire when he retired at age 65. To reach his goal, he decided to invest $1500 every year for each of the next 47 years. If his investment pays 10% per year compounded annually, will he reach his goal? If so, by how much will he exceed it?

Section 1.6 – Solving Investment Portfolio Problems (pp. 58-67)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **FM3 (E F G H)** **After this lesson, students will be expected to:** • analyse, compare, and design investment portfolios that meet specific financial goals **After this lesson, students should understand the following concept:** • **portfolio –** one or more investments held by an individual investor or by a financial organization **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 64-67:** #1-10	**Possible Instructional Strategies:** • Ask students to research the various types of investments that can make up an investment portfolio. **Possible Assessment Strategies:** • In 3 years, Luke wants to buy a new car that costs \$15,000. He plans to save \$300 per month in a savings account that earns 3% per year, compounded monthly. After 3 years, his parents agree to pay the remainder of the cost of the car. How much money will his parents give toward the purchase of the car?

CHAPTER 2

FINANCIAL MATHEMATICS: BORROWING MONEY

SUGGESTED TIME

9 classes

Section 2.1 – Analysing Loans (pp. 80-97)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **FM1 (H J)** • **FM3 (A)** **After this lesson, students will be expected to:** • solve problems that involve single payment loans and regular payment loans **After this lesson, students should understand the following concepts:** • **collateral –** an asset that is held as security against the repayment of a loan • **amortization table –** a table that lists regular payments of a loan and shows how much of each payment goes toward the interest charged and the principal borrowed, as the balance of the loan is reduced to zero • **mortgage –** a loan, usually for the purchase of real estate, with the real estate purchased used as collateral to secure the loan **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 92-96:** #1-18	**Possible Instructional Strategies:** • Discuss with the class examples of different types of loans, such as car loans, bank loans, and mortgages. **Possible Assessment Strategies:** • John borrowed $20,000 in student loans to help pay for university. His bank offered him an interest rate of 3% per year compounded annually. If he has 10 years to pay back the loan, a. How much will he have to pay back, in total? b. What will his monthly payment be? • Kayla's bank has approved a personal loan of $5000 at 6%, compounded monthly, so that she can do some minor home renovations. She wants finish paying the loan at the end of 5 years. Under those conditions, what will her monthly payment be? • Summer is negotiating with her bank for a mortgage on a house. She has been told that she will need to make a 15% down payment on the purchase price of $140,000. Then the bank will offer her a mortgage loan for the balance at 3%, compounded semi-annually, with a term of 25 years and with monthly mortgage payments. a. How much will each monthly payment be? b. How much interest will Summer end up paying by the time she has paid off the loan, in 25 years? c. How much will she pay altogether?

Section 2.2 – Exploring Credit Card Use (pp. 98-100)

ELABORATIONS & SUGGESTED PROBLEMS
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **FM1 (I J)** **After this lesson, students will be expected to:** • compare credit card options that are available to consumers **Suggested Problems in *Foundations of Mathematics 12*:** • **p. 100:** #1-4

Section 2.3 – Solving Problems Involving Credit (pp. 104-119)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **FM1 (I J)** **After this lesson, students will be expected to:** • solve problems that involve credit **After this lesson, students should understand the following concepts:** • **line of credit –** a pre-approved loan that offers immediate access to funds, up to a pre-defined limit, with a minimum monthly payment based on accumulated interest; a secure line of credit has a lower interest rate because it is guaranteed against the client's assets, usually property • **Bank of Canada prime rate –** a value set by Canada's central bank, which other financial institutions use to set their interest rates **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 114-118:** #1-15	**Possible Instructional Strategies:** • Discuss with the class examples of different types of credit. **Possible Assessment Strategies:** • Henry has a balance of $2000 on his credit card, which has an interest rate of 18%, compounded monthly. How much interest will he accumulate at the end of one month, when he makes his first payment? • Cindy has a credit card debt of $1500. The interest rate for the credit card is 18%, compounded monthly. She can only afford to pay the minimum payment each month, which is 4% of the balance, or $50, whichever is greater, until the balance is paid. a. How long will it take Cindy to pay off the credit card? b. How much will she pay back altogether? c. How much did she pay in interest?

Section 2.4 – Buy, Rent, or Lease? (pp. 120-133)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **FM2 (A B C D E)** **After this lesson, students will be expected to:** • solve problems by analysing renting, leasing, and buying options **After this lesson, students should understand the following concepts:** • **lease –** a contract for purchasing the use of property, such as a building or vehicle, from another, the lessor, for a specified period • **equity –** the difference between the value of an item and the amount still owing on it; can be thought of as the portion owned; for example, if a $25,000 down payment is made on a $230,000 home, $205,000 is still owing and $25,000 is the equity or portion owned • **asset –** an item or a portion of an item owned; also known as property; assets include such items as real estate, investment portfolios, vehicles, art, and gems • **appreciation –** increase in the value of an asset over time • **depreciation –** decrease in the value of an asset over time • **disposable income –** the amount of income that someone has available to spend after all regular expenses and taxes have been deducted **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 129-133:** #1-14	**Possible Instructional Strategies:** • Discuss with the class the advantages and disadvantages of buying, renting, and leasing. **Possible Assessment Strategies:** • Karen, a new teacher, is looking for a place to live for 10 months. She has two options: ➢ She can rent a room with a kitchenette at a hotel for $900 per month, which includes cleaning services and utilities. ➢ She can take a 10-month lease of a furnished apartment for $750 per month. As well, she would have to pay $225 per month in utilities. Which is the better option? Explain. • A new car that is bought for $25,000 depreciates in value 25% every year. What will be its resale value 4 years after it is bought?

CHAPTER 3
SET THEORY AND LOGIC

SUGGESTED TIME

11 classes

Section 3.1 – Types of Sets and Set Notation (pp. 146-158)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **LR2 (A B C)** **After this lesson, students will be expected to:** • understand sets and set notation **After this lesson, students should understand the following concepts:** • **set –** a collection of distinguishable objects; for example, the set of whole numbers is $W = \{0, 1, 2, 3, ...\}$ • **element –** an object in a set; for example, 3 is an element of *D*, the set of digits • **universal set –** a set of all the elements under consideration for a particular context (also called the sample space); for example, the universal set of digits is $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ • **subset –** a set whose elements all belong to another set; for example, the set of odd digits, $O = \{1, 3, 5, 7, 9\}$, is a subset of *D*, the set of digits; in set notation, the relationship is written as $O \subset D$ • **complement –** all the elements of a universal set that do not belong to a subset of it; for example, $O' = \{0, 2, 4, 6, 8\}$ is the complement of $O = \{1, 3, 5, 7, 9\}$, a subset of the universal set of digits, *D*; the complement is denoted with a prime sign, O' • **empty set –** a set with no elements; for example, the set of odd numbers divisible by 2 is the empty set; the empty set is denoted by $\{ \}$ or $\varnothing$ • **disjoint –** two or more sets having no elements in common; for example, the set of even numbers and the set of odd numbers are disjoint • **finite set –** a set with a countable number of elements; for example, the set of positive even numbers less than 10, $E = \{2, 4, 6, 8\}$ is finite • **infinite set –** a set with an infinite number of elements; for example, the set of natural numbers $N = \{1, 2, 3, ...\}$, is infinite • **mutually exclusive –** two or more events that cannot occur at the same time; for example, the sun rising and the sun setting are mutually exclusive events **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 154-158:** #1-19	**Possible Instructional Strategies:** • Ensure that the class understands the vocabulary related to sets, as these concepts will be new for all students. **Possible Assessment Strategies:** • a. Indicate the multiples of 3 and 6, from 1 to 60 inclusive, using set notation. b. List any subsets, and show the relationships among the sets and subsets in a Venn diagram. • A square number, such as 1, 4, 9, or 16, can be represented as a square array. a. Determine a pattern you can use to determine any square number. b. Determine how many natural numbers from 1 to 400 are: ➢ even and square ➢ odd and square ➢ not square c. How many numbers from 1 to 400 are square? • Joanne recorded the possible values that can occur in an outcome table when a six-sided die is rolled. a. Display the following sets in one Venn diagram. ➢ rolls that produce an odd number ➢ rolls that produce a multiple of 3 b. How many rolls can occur whose value is not an odd number nor a multiple of 3? • Organize the following information in a Venn diagram. ➢ $X = \{A, B, C, E\}$ ➢ $Y = \{A, B, D, F\}$ ➢ $Z = \{A, C, D, G\}$

Section 3.2 – Exploring Relationships Between Sets (pp. 159-161)

ELABORATIONS & SUGGESTED PROBLEMS
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **LR2 (D)** **After this lesson, students will be expected to:** • explore what the different regions of a Venn diagram represent **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 160-161:** #1-5

Section 3.3 – Intersection and Union of Two Sets (pp. 162-175)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **LR2 (E)**

After this lesson, students will be expected to:

- understand and represent the intersection and union of two sets

After this lesson, students should understand the following concepts:

- **intersection –** the set of elements that are common to two or more sets; in set notation, $A \cap B$ denotes the intersection of sets *A* and *B*; for example, if $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $A \cap B = \{3\}$
- **union –** the set of all the elements in two or more sets; in set notation, $A \cup B$ denotes the union of sets *A* and *B*; for example, if $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $A \cup B = \{1, 2, 3, 4, 5\}$
- **Principle of Inclusion and Exclusion –** the number of elements in the union of two sets is equal to the sum of the number of elements in each set, less the number of elements in both sets; using set notation, this is written as

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Suggested Problems in *Foundations of Mathematics 12*:

- **pp. 172-175:** #1-18

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Use a Venn diagram to visually explain what the expressions $A \cup B$ and $A \cap B$ represent.
- Ensure that when students use the Principle of Inclusion and Exclusion, they remember to subtract the number of elements that are in common from the total number of elements.

Possible Assessment Strategies:

- Let the universal set *U* be the set of natural numbers from 1 to 25. Consider each of the following sets:
  - ➢ $E = \{\text{even numbers from 1 to 25}\}$
  - ➢ $O = \{\text{odd numbers from 1 to 25}\}$
  - ➢ $P = \{\text{prime numbers from 1 to 25}\}$
  - ➢ $T = \{\text{multiples of 3 from 1 to 25}\}$

  a. List the elements in *E*, *O*, *P*, and *T*.
  b. Determine $n(E)$, $n(O)$, $n(P)$, and $n(T)$.
  c. List the elements in $E \cap O$.
  d. List the elements in $E \cap T$.
  e. Determine $n(P \cup T)$.
  f. Which two of the given sets are disjoint?
  g. List the elements in the complement of $O \cap P$.
- The following table shows the four hockey teams that made it to the semi-finals of the NHL playoffs from 2009 to 2011. Draw a Venn diagram that shows the relationship between the playoff years and the participating teams, where each circle represents a playoff year.

2009	2010	2011
Carolina Hurricanes	Montreal Canadiens	Tampa Bay Lightning
Pittsburgh Penguins	Philadelphia Flyers	Boston Bruins
Detroit Red Wings	San Jose Sharks	Vancouver Canucks
Chicago Blackhawks	Chicago Blackhawks	San Jose Sharks

Section 3.4 – Applications of Set Theory (pp. 179-194)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **LR2 (F G H)** **After this lesson, students will be expected to:** • use sets to model and solve problems **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 191-194:** #1-14	**Possible Instructional Strategies:** • Encourage students to use Venn diagrams when solving problems involving sets. Remind them that they should always start from the intersection of all sets and work outward. **Possible Assessment Strategies:** • One semester in a university math class, ➢ 14 students failed due to poor attendance, ➢ 23 students failed due to not studying, ➢ 15 students failed due to not turning in assignments, ➢ 9 students failed due to poor attendance and not studying, ➢ 8 students failed due to not studying and not turning in assignments. ➢ 5 students failed due to poor attendance and not turning in assignments, ➢ 2 students failed due to all three reasons. a. How many students failed for exactly two of the three reasons? b. How many students failed because of poor attendance and not studying, but not because of not turning in assignments? c. How many students failed because of exactly one of the three reasons? d. How many students failed because of poor attendance and not turning in assignments, but not because of not studying? e. How many students failed the course? • There are 900 employees at Canto Crafts Inc. Of these, ➢ 615 are female, ➢ 345 are under 35 years old, ➢ 482 are single, ➢ 295 are single females, ➢ 187 are singles under 35 years old, ➢ 190 are females under 35 years old, ➢ 120 are single females under 35 years old. Use a Venn diagram to determine how many employees are married males who are at least 35 years old.

Section 3.5 – Conditional Statements and Their Converse (pp. 195-207)

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<tr><th>ELABORATIONS &
SUGGESTED PROBLEMS</th><th>POSSIBLE INSTRUCTIONAL &
ASSESSMENT STRATEGIES</th></tr>
<tr><td>

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **LR3 (A B C D F G)**

After this lesson, students will be expected to:

- understand and interpret conditional statements

After this lesson, students should understand the following concepts:

- **conditional statement –** an “if-then” statement; for example, “If it is Monday, then it is a school day”
- **hypothesis –** an assumption; for example, in the statement, “If it is Monday, then it is a school day,” the hypothesis is “it is Monday”
- **conclusion –** the result of a hypothesis; for example, in the statement, “If it is Monday, then it is a school day,” the conclusion is “it is a school day”
- **counterexample –** an example that disproves a statement; for example, “If it is Monday, then it is a school day” is disproved by the counterexample that there is no school on Thanksgiving Monday; only one counterexample is needed to disprove a statement
- **converse –** a conditional statement in which the hypothesis and the conclusion are switched; for example, the converse of “If it is Monday, then it is a school day” is “If it is a school day, then it is Monday”
- **biconditional –** a conditional statement whose converse is also true; in logic notation, a biconditional statement is written as “*p* if and only if *q*”; for example, the statement “If a number is even, then it is divisible by 2” is true; the converse “if a number is divisible by 2, then it is even”, is also true; the biconditional statement is “A number is even if and only if it is divisible by 2”

Suggested Problems in *Foundations of Mathematics 12*:

- **pp. 203-206:** #1-15

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Possible Instructional Strategies:

- Ensure that students are able to convert statements to “if-then” form.
- Ensure that students understand the relationship between a conditional statement and its converse.

Possible Assessment Strategies:

- Determine the truth value of each conditional statement. If **true**, explain your reasoning. If **false**, give a counterexample.
  a. If two opposite sides of a quadrilateral are parallel, and one interior angle is 90^0, then the quadrilateral is a rectangle.
  b. If you determine the reciprocal of a real number, then the result is a real number.
  c. If a point lies in the third quadrant, then its *y*-coordinate is negative.
  d. If a triangle is a right triangle, then it does not contain an obtuse angle.
- Write the converse of each of the following true conditional statements. Then determine whether each related conditional is **true** or **false**. If a statement is false, find a counterexample.
  a. If it is raining, then it is cloudy.
  b. If a real number is positive, then its absolute value is positive.
  c. If $b > 1$, then $b^2 > 1$.
  d. If a right triangle is isosceles, then it has two angles measuring 45^0.
- Write each statement in “if *p*, then *q*” form. If the statement is biconditional, rewrite it in biconditional form. If the statement is not biconditional, provide a counterexample.
  a. A right angle has a measure of 90^0.
  b. A terminating decimal can be written as a fraction.
  c. A square has four sides of equal length.

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Section 3.6 – The Inverse and the Contrapositive of Conditional Statements
(pp. 208-216)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **LR3 (C D E G)** **After this lesson, students will be expected to:** • understand and interpret the contrapositive and inverse of a conditional statement **After this lesson, students should understand the following concepts:** • **inverse –** a statement that is formed by negating both the hypothesis and the conclusion of a conditional statement; for example, for the statement "If a number is even, then it is divisible by 2," the inverse is "If a number is not even, then it is not divisible by 2" • **contrapositive –** a statement that is formed by negating both the hypothesis and the conclusion of the converse of a conditional statement; for example, for the statement "If a number is even, then it is divisible by 2," the contrapositive is "If a number is not divisible by 2, then it is not even" **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 214-216:** #1-13	**Possible Instructional Strategies:** • Ensure that students understand the relationships between a conditional statement, and its inverse and contrapositive. **Possible Assessment Strategies:** • Write the inverse and contrapositive of each of the following true conditional statements. Then determine whether each related conditional is **true** or **false**. If a statement is false, find a counterexample. a. If it is raining, then it is cloudy. b. If a real number is positive, then its absolute value is positive. c. If $b > 1$, then $b^2 > 1$. d. If a right triangle is isosceles, then it has two angles measuring 45^0. • John claims that if $x = -1$, then $x^2 = 1$ is true. a. Is John correct? Explain. b. Is the converse true? Explain. c. Is the inverse true? Explain. d. Is the contrapositive true? Explain. • State whether each of the following statements is **true** or **false**. a. When a conditional statement is false, then its contrapositive will be false. b. When the converse of a conditional statement is false, then its inverse will be true.

CHAPTER 4
COUNTING METHODS

SUGGESTED TIME

10 classes

Section 4.1 – Counting Principles (pp. 228-237)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **P4 (A B C D)** **After this lesson, students will be expected to:** • determine the Fundamental Counting Principle and use it to solve problems **After this lesson, students should understand the following concept:** • **Fundamental Counting Principle –** If there are *a* ways to perform one task and *b* ways to perform another, then there are $a \bullet b$ ways of performing both **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 235-237:** #1-17	**Possible Instructional Strategies:** • Ensure that students understand the Fundamental Counting Principle, as it is the basis for the solution of all counting problems. **Possible Assessment Strategies:** • a. In 1985, a Prince Edward Island license plate consisted of two letters followed by three digits, where the first letter was a P (for Prince County), Q (for Queens County), or K (for Kings county). How many possible license plates were possible in 1985? b. In 1995, the rules governing license plates were changed so that the first two characters could be any letter. How many more possible license plates did this rule change create? • How many positive four-digit even numbers are there? • There are four blood types: A, B, AB, and O. Blood is also either Rh^+ or Rh^-. If a local blood bank labels donations according to its type, Rh factor, and gender of the donor, how many different ways can a blood sample be labelled? • a. The letters A, B, C, D, and E are to be used in a four-letter identification card. How many different cards are possible if letters are allowed to be repeated? b. How many cards are possible if each letter can only be used once? • A store manager has considered four possible applicants for two different positions at a department store. In how many ways can the manager fill the positions? • In how many ways can a teacher seat four girls and three boys in a row of seven seats if a boy must be seated at each end of the row? • How many three-digit numbers are there using only the digits 1, 2, 3, 4, and 5 with no repetitions allowed? • a. How many six-letter arrangements can be made using all of the letters A, B, C, D, E, and F, with no repetitions? b. How many of those arrangements begin and end with a consonant?

Section 4.2 – Introducing Permutations and Factorial Notation (pp. 238-245)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **P5 (A B C D)**

After this lesson, students will be expected to:

- use factorial notation to solve simple permutation problems

After this lesson, students should understand the following concepts:

- **permutation –** an arrangement of distinguishable objects in a definite order; for example, the objects *a* and *b* have two permutations, *ab* and *ba*
- **factorial notation –** a concise representation of the product of consecutive descending natural numbers:

  $$n! = n(n-1)(n-2)\cdots(3)(2)(1)$$

  for example, $4! = 4 \bullet 3 \bullet 2 \bullet 1$

Suggested Problems in *Foundations of Mathematics 12*:

- **pp. 243-245:** #1-16

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Ensure that students become comfortable with factorial and permutation notation.

Possible Assessment Strategies:

- Simplify each of the following expressions.

  a. $\dfrac{8!}{4!3!}$

  b. $(2!)(5!)$
- A family of five is taking a family picture. In how many different ways can the members of the family line up for the family picture?
- In how many different ways can seven types of laser printer be displayed on a shelf in a computer store?
- In how many ways can the letters of the word TRIANGLE be arranged?
- In how many ways can 12 members of a basketball team be seated on a bench if the captain sits at the left end of the bench?
- Six children are to line up for a photograph.

  a. How many different arrangements are possible?

  b. How many arrangements are possible if Brenda is in the third position?

  c. How many arrangements are possible if Ahmed is on the far left and Yen is on the far right?

  d. How many arrangement are possible if Hans and Brian must be together?
- A 12-volume encyclopedia is to be placed on a shelf. How many incorrect arrangements are there?
- Solve each of the following equations for *n*, where *n* is a whole number.

  a. $\dfrac{(n+2)!}{(n+1)!} = 12$

  b. $\dfrac{(n+2)!}{n!} = 56$

Section 4.3 – Permutations When All Objects Are Distinguishable (pp. 246-257)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **P5 (E H)** **After this lesson, students will be expected to:** • determine the number of permutations of *n* objects taken *r* at a time, where $0 \le r \le n$ **PERMUTATIONS** The number of permutations from a set of *n* different objects, where *r* of them are used in each arrangement, can be calculated using the formula $${}_nP_r = \frac{n!}{(n-r)!},\ 0 \le r \le n$$ **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 255-257:** #1-18	**Possible Instructional Strategies:** • Remind students that order is important when solving a problem involving permutations. **Possible Assessment Strategies:** • How many different four-letter permutations can be formed from the letters in the word DECAGON? • Out of a group of 8 students serving on the student council, in how many ways can a president, a vice president, a secretary, and a treasurer be selected? • How many different ways can 5 raffle tickets be selected from 100 tickets if each ticket wins a different prize? • How many ways can Laura colour 4 adjacent regions on a map if she has a set of 12 coloured pencils? • If you have a standard deck of 52 cards, in how many different ways can you deal out a. 5 cards? b. 3 cards? c. 5 red cards? d. 5 face cards? • Suppose you are designing a coding system for data relayed by a satellite. To make transmission errors easier to detect, each code must have no repeated digits. If you need at least 60,000 different codes, how many digits long should each code be? • How many different 3-digit even numbers greater than 300 can be made with the digits 1, 2, 3, 4, and 5, if no digits are repeated?

Section 4.4 – Permutations When Objects Are Identical (pp. 260-270)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **P5 (F G)** **After this lesson, students will be expected to:** • determine the number of permutations when some of the objects are identical **PERMUTATIONS** The number of permutations of *n* objects, where *a* are identical, another *b* are identical, another *c* are identical, and so on is $P = \frac{n!}{a!b!c!\cdots}$ **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 266-269:** #1-18	**Possible Instructional Strategies:** • Ensure that students understand the difference between permutation problems when objects are distinguishable and when some objects are identical. **Possible Assessment Strategies:** • A builder has three models of homes from which customers can choose, A, B, and C. On one side of a street, the builder sold three model A homes, four model B homes, and two model C homes. in how many ways can the homes be arranged along the street? • Rita lives 5 blocks east and 3 blocks north of her friend Marion. How many different routes can Marion take by walking only eastward or northward when going to visit Rita? • The word BOOKKEEPER is unusual in that it has three consecutive double letters. How many permutations are there of the letters in BOOKKEEPER? • A coin is tossed 8 times. In how many different orders could 2 heads and 6 tails occur? • Kathryn's soccer team played a good season, finishing with 16 wins, 3 losses, and 1 tie. In how many orders could these results have happened? • Six friends shared a bag of assorted doughnuts. In how many ways can the friends share the doughnuts if the bag contains a. 6 different doughnuts? b. 3 plain doughnuts and 3 Boston cream doughnuts? c. 2 plain doughnuts, 2 Boston cream doughnuts, and 2 jelly doughnuts? • How many 7-digit even numbers less than 3,000,000 can be formed using all the digits 1, 2, 2, 3, 5, 5, and 6?

Section 4.5 – Exploring Combinations (pp. 271-272)

ELABORATIONS & SUGGESTED PROBLEMS
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **P6 (A)** **After this lesson, students will be expected to:** • explore how counting combinations differs from counting permutations **After this lesson, students should understand the following concept:** • **combination –** a grouping of objects where order does not matter; for example, the two objects *a* and *b* have one combination because *ab* is the same as *ba* **Suggested Problems in *Foundations of Mathematics 12*:** • **p. 272:** #1-4

Section 4.6 – Combinations (pp. 273-282)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **P6 (B C)** **After this lesson, students will be expected to:** • solve problems involving combinations **COMBINATIONS** The number of combinations from a set of *n* different objects, where only *r* of them are used in each combination is $${}_nC_r = \frac{n!}{r!(n-r)!},\ 0 \le r \le n$$ **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 280-282:** #1-19	**Possible Instructional Strategies:** • Ensure that students become comfortable with combination notation. • Ensure that students are able to explain the differences between permutations and combinations, and know when to use each. **Possible Assessment Strategies:** • a. How many three-card permutations can be made from the 10, jack, queen, king, and ace of spades? b. How many three-card combinations can be made from the 10, jack, queen, king, and ace of spades? c. How are the answers in (a) and (b) related? • Determine the number of possible lottery tickets that can be created in a 6/49 lottery, where each ticket has 6 different numbers, in no particular order, chosen from the numbers 1 to 49, inclusive. • How many different sampler dishes with 3 different flavours could you get at an ice cream shop with 31 different flavours? • Each player in a bridge game receives a hand of 13 cards dealt from a standard deck of 52 cards. How many different bridge hands are possible? Express the answer in scientific notation with three significant digits. • A group of 5 students is to be selected from a class of 35 students. a. How many different groups can be selected? b. Lisa, Gwen, and Al are students in the class. How many of the possible groups include all three of these students? c. How many groups do not include all three of these students? • Solve the equation ${}_{n+1}C_2 = 10$. • The student council of a high school consists of 7 girls and 3 boys. From this group, 3 girls and 2 boys will be chosen to go to a leadership conference. In how many ways can this group of 5 students be chosen? • How many 5-card hands are there that contain exactly 3 aces?

Section 4.7 – Solving Counting Problems (pp. 283-290)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **P4 (D)** • **P5 (H)** • **P6 (C)** **After this lesson, students will be expected to:** • solve counting problems that involve permutations and combinations **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 288-290:** #1-17	**Possible Instructional Strategies:** • Ensure that students are able to distinguish among the different types of counting problems. **Possible Assessment Strategies:** • From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done? • In how many different ways can the letters of the word LEADING be arranged in such a way that the vowels always come together? • Out of 7 consonants and 4 vowels, how many groups of 3 consonants and 2 vowels can be formed? • How many 3-digit numbers can be formed from the digits 2, 3, 5, 6, 7 and 9, which are divisible by 5 and none of the digits is repeated? • In how many ways a committee, consisting of 5 men and 6 women can be formed from a group of 8 men and 10 women? • How many 4-letter words, with or without meaning, can be formed out of the letters of the word LOGARITHMS, if repetition of letters is not allowed? • A teacher is making a multiple choice quiz. She wants to give each student the same questions, but have each student's questions appear in a different order. If there are twenty-seven students in the class, what is the least number of questions the quiz must contain? • A basketball coach must choose five starters from a team of 12 players. How many different ways can the coach choose the starters? • There are fourteen juniors and twenty-three seniors in the Service Club. The club is to send four representatives to the national conference. a. How many different ways are there to select a group of four students to attend the conference? b. If the members of the club decide to send two juniors and two seniors, how many different groupings are possible?

CHAPTER 5
PROBABILITY

SUGGESTED TIME
10 classes

Section 5.1 – Exploring Probability (pp. 302-303)

ELABORATIONS & SUGGESTED PROBLEMS
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **P1 (A)** **After this lesson, students will be expected to:** • use probability to make predictions **After this lesson, students should understand the following concept:** • **fair game –** a game in which all players are equally likely to win; for example, tossing a coin to get heads or tails is a fair game **Suggested Problems in *Foundations of Mathematics 12*:** • **p. 303:** #1-4

Section 5.2 – Probability and Odds (pp. 304-312)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **P1 (B C D E F)** **After this lesson, students will be expected to:** • understand and interpret odds, and relate them to probability **After this lesson, students should understand the following concepts:** • **odds in favour –** the ratio of the probability that an event will occur to the probability that the event will not occur, or the ratio of the number of favourable outcomes to the number of unfavourable outcomes; if the odds in favour of event *A* occurring are $m:n$, then $$P(A)=\frac{m}{m+n}$$ • **odds against –** the ratio of the probability that an event will not occur to the probability that the event will occur, or the ratio of the number of unfavourable outcomes to the number of favourable outcomes **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 310-312:** #1-19	**Possible Instructional Strategies:** • Ensure that students clearly understand the difference between odds and probability. **Possible Assessment Strategies:** • Calculate the odds in favour and the odds against each event. a. Christmas falling on a Monday b. tossing exactly three heads with three coins c. randomly drawing a face card from a standard deck of 52 cards d. a random digit being odd e. rolling two dice and getting a sum of 7 f. winning Lotto 6/49 if there are 13,983,816 different six-number combinations • The odds that the Toronto Blue Jays will beat the Boston Red Sox are $3:4$. What is the probability that Toronto will beat Boston in their next game? • Boomer Gallant gives a 30% probability of precipitation for tomorrow. a. What are the odds in favour of precipitation for tomorrow? b. What are the odds against precipitation for tomorrow?

Section 5.3 – Probabilities Using Counting Methods (pp. 313-324)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **P5 (I)** • **P6 (D)** **After this lesson, students will be expected to:** • solve probability problems that involve counting techniques **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 321-324:** #1-18	**Possible Instructional Strategies:** • Review the difference between permutations and combinations with the students. **Possible Assessment Strategies:** • What is the probability that at least two out of a group of eight friends will have the same birthday? Express the answer as a decimal, rounded off to four decimal places. • An athletic committee with three members is to be randomly selected from a group of 6 gymnasts, 4 weightlifters, and 8 long-distance runners. a. Determine the probability that the committee is comprised entirely of runners. b. Determine the probability that the committee is represented by each of the three types of athletes. • A messy drawer contains 3 black socks, 5 blue socks, and 8 white socks, none of which are paired up. If the owner grabs two socks without looking, what is the probability that both will be white? • Laura, Dave, Monique, Marcus, and Sarah are going to a party. What is the probability that two of the girls will arrive first, if they all arrive individually? • A hockey team has two goalies, six defenders, eight wingers, and four centres. If the team randomly selects four players to attend a charity function, what is the probability that a. they are all wingers? b. no goalies or centres are selected? • Four friends, two females and two males, are playing bridge. Partners are randomly assigned for each game. What is the probability that the two females will be partners for the first game?

Section 5.4 – Mutually Exclusive Events (pp. 328-343)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **P2 (A B C D E F)** **After this lesson, students will be expected to:** • understand and solve problems that involve mutually exclusive and non-mutually exclusive events **MUTUALLY EXCLUSIVE EVENTS** Given two mutually exclusive events, *A* and *B*, the probability that either *A* or *B* will occur is $P(A \cup B) = P(A) + P(B)$ **NON-MUTUALLY EXCLUSIVE EVENTS** Given two non-mutually exclusive events, *A* and *B*, the probability that either *A* or *B* will occur is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 338-342:** #1-18	**Possible Instructional Strategies:** • Ensure that students understand the difference between mutually exclusive and non-mutually exclusive events. **Possible Assessment Strategies:** • Determine which events are mutually exclusive and which are not, when a single die is rolled. a. Rolling an odd number or rolling an even number. b. Rolling a 3 or rolling an odd number. c. Rolling an odd number or rolling an number less than 4. d. Rolling a number greater than 4 or rolling a number less than 4. • A box contains 3 glazed doughnuts, 4 jelly doughnuts, and 5 chocolate doughnuts. If a person selects a doughnut at random, find the probability that it is either a glazed doughnut or a chocolate doughnut. • At a political rally, there are 20 Liberals, 13 Conservatives, and 6 New Democrats. If a person is selected at random, find the probability that he or she is either a Liberal or a New democrat. • A single card is drawn from an ordinary deck of cards. Find the probability that it is either an ace or a black card. • In a hospital unit, there are 8 nurses and 5 physicians. Of the staff in the unit, 7 nurses and 3 physicians are female. If a staff person is selected at random, find the probability that the person is a nurse or a male. • On New Year's Eve, the probability of a person driving while intoxicated is 0.15, the probability of a person having a driving accident is 0.03, and the probability of a person having a driving accident while intoxicated is 0.02. What is the probability of a person driving while intoxicated or having a driving accident?

Section 5.5 – Conditional Probability (pp. 344-353)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **P3 (A B C D)**

After this lesson, students will be expected to:

- understand and solve problems that involve dependent events

After this lesson, students should understand the following concepts:

- **dependent events –** events whose outcomes are affected by each other; for example, if two cards are drawn from a deck without replacement, the outcome of the second event depends on the outcome of the first event (the first card drawn)
- **conditional probability –** the probability of an event occurring given that another event has already occurred; $P(B \mid A)$ is the notation for conditional probability; it is read, "the probability that event *B* will occur, given that event *A* has already occurred"; the formula for $P(B \mid A)$ is

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

DEPENDENT EVENTS
If event *B* depends on event *A* occurring, then the probability that both events will occur can be represented as follows: $P(A \cap B) = P(A) \bullet P(B \mid A)$

Suggested Problems in *Foundations of Mathematics 12*:

- **pp. 350-353:** #1-21

Possible Instructional Strategies:

- Ensure that students understand the difference between dependent and independent events.

Possible Assessment Strategies:

- Classify each of the following as independent or dependent events.

	FIRST EVENT	SECOND EVENT
a.	Attending a rock concert on Tuesday night	Passing a final exam the following Wednesday morning
b.	Eating chocolate	Winning at checkers
c.	Graduating from university	Running a marathon
d.	Going to a mall	Purchasing a new shirt

- Two cards are drawn from a standard deck, without replacement. What is the probability that two face cards will be drawn?
- A recent survey asked 100 people if they thought women in the armed forces should be permitted to participate in combat. The results of the survey are shown.

GENDER	YES	NO	TOTAL
MALE	32	18	50
FEMALE	8	42	50
TOTAL	40	60	100

  a. What is the probability that the respondent answered yes, given that the respondent was a female?
  b. What is the probability that the respondent was a male, given that the respondent answered no?

- A math teacher gave her class two tests. If 25% of the class passed both tests and 40% of the class passed the first test, what percent of those who passed the first test also passed the second test?

Section 5.6 – Independent Events (pp. 354-363)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **P3 (B C D)** **After this lesson, students will be expected to:** • understand and solve problems that involve independent events **INDEPENDENT EVENTS** The probability that two independent events, *A* and *B*, will both occur is the product of their individual probabilities: $P(A \cap B) = P(A) \cdot P(B)$ **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 360-363:** #1-16	**Possible Instructional Strategies:** • Ensure that students understand the difference between dependent and independent events. **Possible Assessment Strategies:** • A coin is flipped and a die is rolled. What is the probability of flipping tails and rolling an even number in a single trial? • At work, John determines that there is a 0.8 probability that he will talk to his friend and a 0.4 chance that he will have a meeting. What is the probability that John will talk to his friend and not have a meeting? • Jane travels the same route to work every day. She has determined that there is a 0.7 probability that she will wait for at least one red light and that there is a 0.4 probability that she will hear her favourite new song on her way to work. a. What is the probability that she will not have to wait at a red light and will hear her favourite song? b. What is the probability that she will have to wait at a red light and not hear her favourite song? • There are two tests for a particular antibody. Test A gives a correct result 95% of the time. Test B gives a correct result 89% of the time. If a patient is given both tests, find the probability that a. both tests give the correct result b. neither test gives the correct result c. at least one of the tests gives the correct result

CHAPTER 6
POLYNOMIAL FUNCTIONS

SUGGESTED TIME

8 classes

Section 6.1 – Exploring the Graphs of Polynomial Functions (pp. 380-383)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **RF1 (A)**

After this lesson, students will be expected to:

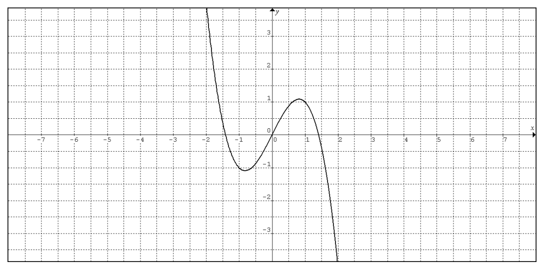
- identify characteristics of the graphs of polynomial functions

After this lesson, students should understand the following concepts:

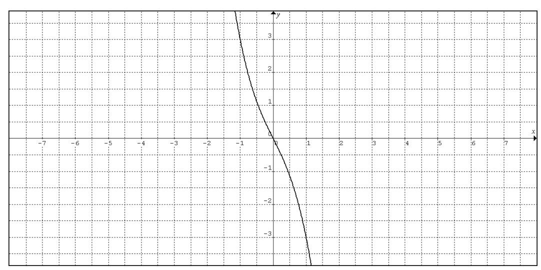
- **polynomial function –** a function that contains only the operations of multiplication and addition with real number coefficients, whole number exponents, and two variables; for example, $f(x) = 5x^3 + 6x^2 - 3x + 7$
- **end behaviour –** the description of the shape of a graph, from left to right, on the coordinate plane
- **cubic function –** a polynomial function of the third degree, whose greatest exponent is 3; for example, $f(x) = 5x^3 + x^2 - 4x + 1$

After this lesson, students should understand the following concepts:

- **turning point –** any point where the graph of a function changes from increasing to decreasing, or from decreasing to increasing; for example,
  - this curve has two turning points, since the *y*-values change from decreasing to increasing to decreasing



  - this curve does not have any turning points, since the *y*-values are always decreasing



Suggested Problems in *Foundations of Mathematics 12*:

- **p. 383:** #1-4

Section 6.2 – Characteristics of the Equations of Polynomial Functions

(pp. 384-398)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **RF1 (B C)**

After this lesson, students will be expected to:

- make connections between the coefficients and constant in the equation of a function and the characteristics of the graph of the function

After this lesson, students should understand the following concepts:

- **standard form**
  - the standard form for a linear function is $f(x) = ax + b$, where $a \neq 0$
  - the standard form for a quadratic function is $f(x) = ax^2 + bx + c$, where $a \neq 0$
  - the standard form for a cubic function is $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$
- **constant term –** the constant term in a polynomial function in standard form is the term that does not have a variable; for example, in the function $f(x) = 2x^2 + 7x - 5$, the constant term is –5
- **leading coefficient –** the coefficient of the term with the greatest degree in a polynomial function in standard form; for example, the leading coefficient in the function $f(x) = 2x^3 + 7x$ is 2

Suggested Problems in *Foundations of Mathematics 12*:

- **pp. 393-397:** #1-16

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Review the concept of a polynomial with the class. Classify and give examples of polynomials that are constant, linear and quadratic.

Possible Assessment Strategies:

- Identify whether each of the following is a polynomial function.

  a. $f(x) = 3 - 2x + \frac{1}{2}x^2$

  b. $f(x) = \frac{1}{x^2 - 4x + 4}$

  c. $f(x) = (x-1)^3$

  d. $f(x) = 6 - \sqrt{x}$

  e. $f(x) = x^{-2} + 5x^{-1} + 6$

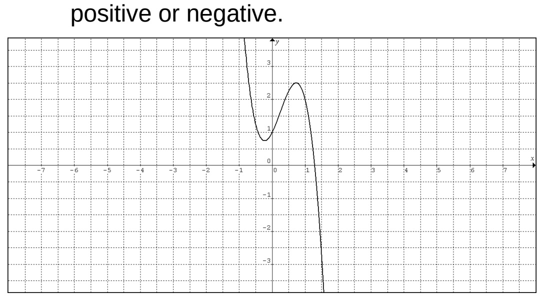
- Determine the degree, leading coefficient, and the constant term of each polynomial function.

  a. $f(x) = x^3 + 2x^2 - 3x - 4$

  b. $f(x) = 2x^4 - 5x^2 + 2x + 2$

  c. $f(x) = 3 - 2x - x^2$

- For the graph of the given function,
  - determine whether the graph represents an odd-degree or an even-degree function;
  - determine whether the leading coefficient is positive or negative.



- Describe the end behaviour of the corresponding graphs of each of the following functions. State the possible number of x-intercepts and y-intercepts.

  a. $f(x) = x^2 - 3x + 2$

  b. $f(x) = 4 - 3x^2 - x^3$

Section 6.3 – Modelling Data with a Line of Best Fit (pp. 401-412)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **RF1 (D E F)**

After this lesson, students will be expected to:

- determine the linear function that best fits a set of data, and use the function to solve a problem

After this lesson, students should understand the following concepts:

- **scatter plot –** a set of points on a grid, used to visualize a relationship or possible trend in the data
- **line of best fit –** a straight line that best approximates the trend in a scatter plot
- **regression function –** a line or curve of best fit, developed through a statistical analysis of data
- **interpolation –** the process used to estimate a value within the domain of a set of data, based on a trend
- **extrapolation –** the process used to estimate a value outside the domain of a set of data, based on a trend

Suggested Problems in *Foundations of Mathematics 12*:

- **pp. 407-412:** #1-13

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Use graphing technology to demonstrate how to find the line of best fit.

Possible Assessment Strategies:

- The following table lists the heights and masses for a group of fire-department trainees.

HEIGHT (cm)	MASS (kg)
177	91
185	88
173	82
169	79
188	87
182	85
175	79

  a. Determine the equation of the line of best fit. Round off all values to three significant digits.
  b. Predict the mass of a trainee whose height is 165 cm. Round off the answer to the nearest whole number.
  c. Predict the height of a 79-kg trainee. Round off the answer to the nearest whole number.

- A random survey of a small group of high school students collected information on the students' ages and the number of books they had read in the past year. Determine the equation of the line of best fit. Round off all values to three significant digits.

AGE (years)	BOOKS READ
16	5
15	3
18	8
17	6
16	4
15	4
14	5

Section 6.4 – Modelling Data with a Curve of Best Fit (pp. 413-423)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **RF1 (D E F)**

After this lesson, students will be expected to:

- determine the quadratic or cubic function that best fits a set of data, and use the function to solve a problem

After this lesson, students should understand the following concept:

- **curve of best fit –** a curve that best approximates the trend on a scatter plot

Suggested Problems in *Foundations of Mathematics 12*:

- **pp. 419-422:** #1-10

Possible Instructional Strategies:

- Use graphing technology to demonstrate how to find the curve of best fit.

Possible Assessment Strategies:

- The heights of a stand of pine trees were measured along with the area under the cone formed by their branches.

HEIGHT (m)	AREA (m^2)
2.0	5.9
1.5	3.4
1.8	4.8
2.4	8.6
2.2	7.3
1.2	2.1
1.8	4.9
3.1	14.4

a. Determine the equation of the quadratic regression function for the data. Round off all values to three significant digits.
b. Predict the area under a tree whose height is 2.7 m. Round off the answer to one decimal place.
c. Predict the height of a tree whose area covered is 30.0 m^2. Round off the answer to one decimal place.

CHAPTER 7
EXPONENTIAL AND LOGARITHMIC FUNCTIONS

SUGGESTED TIME

11 classes

Section 7.1 – Exploring the Characteristics of Exponential Functions (pp. 436-439)

ELABORATIONS & SUGGESTED PROBLEMS
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **RF2 (A)** **After this lesson, students will be expected to:** • investigate the characteristics of the graphs of exponential functions **After this lesson, students should understand the following concept:** • **exponential function –** a function of the form $y = a(b)^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$ **Suggested Problems in *Foundations of Mathematics 12*:** • **p. 439:** #1-3

Section 7.2 – Relating the Characteristics of an Exponential Function to Its Equation (pp. 440-453)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **RF2 (A B C)** **After this lesson, students will be expected to:** • predict the characteristics of an exponential function by examining its equation **After this lesson, students should understand the following concept:** • ***e* –** a constant known as Euler's number; it is an irrational number that equals 2.718...; this number occurs naturally in some situations where a quantity increases continuously, such as in increasing populations **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 448-453:** #1-17	**Possible Instructional Strategies:** • Ensure that students understand the general characteristics of the graph of an exponential function. **Possible Assessment Strategies:** • Determine whether each of the following functions is exponential. a. $y = x^4$ b. $y = \left(\frac{1}{2}\right)^x$ c. $y = (1.03)^x$ • For each of the following exponential functions, state the intercepts, the end behaviour, the domain, the range, and determine whether the function is increasing or decreasing. Then, sketch the graph of each function. a. $y = 5^x$ b. $y = \left(\frac{1}{3}\right)^x$ c. $y = \frac{1}{2}e^x$ d. $y = -3(0.1)^x$ • The doubling period of a bacterium is 20 min. If there are 300 bacteria initially in a culture, how many bacteria will there be after a. 40 min? b. 2 h? • The half-life of a certain isotope is 2 days. How much will be left from a mass of 500 g after a. 6 days? b. 2 weeks?

Section 7.3 – Modelling Data Using Exponential Equations (pp. 454-468)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **RF2 (D E F)**

After this lesson, students will be expected to:

- represent data using an exponential function, and interpret the graph to solve a problem

After this lesson, students should understand the following concepts:

- **exponential growth function –** an exponential function whose *y*-values increase as you move from left to right along the *x*-axis; for an exponential function of the form $y = a(b)^x$, exponential growth occurs when $a > 0$ and $b > 1$
- **exponential decay function –** an exponential function whose *y*-values decrease as you move from left to right along the *x*-axis; for an exponential function of the form $y = a(b)^x$, exponential decay occurs when $a > 0$ and $0 < b < 1$

Suggested Problems in *Foundations of Mathematics 12*:

- **pp. 461-468:** #1-17

Possible Instructional Strategies:

- Use graphing technology to demonstrate how to find the exponential curve of best fit.

Possible Assessment Strategies:

- The following table shows the population of Canada from 1901 to 2011, in 10-year intervals.

YEAR	POPULATION (millions)
1901	5.4
1911	7.2
1921	8.8
1931	10.4
1941	11.5
1951	14.0
1961	18.2
1971	22.0
1981	24.8
1991	28.0
2001	31.0
2011	33.5

a. Determine the equation of the exponential regression function for the data. Use the number of years after 1901 as the independent variable. Round off all values to four significant digits.

b. Using this regression function, predict Canada’s population in the year 2051. Round off the answer to one decimal place.

c. Is the answer in (b) a reasonable answer?

Section 7.4 – Characteristics of Logarithmic Functions with Base 10 and Base *e*

(pp. 474-487)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **RF2 (A B C)** **After this lesson, students will be expected to:** • investigate the characteristics of logarithmic functions with base 10 and base *e* from graphs and equations **After this lesson, students should understand the following concepts:** • **logarithmic function –** a function of the form $y = a\ \log_b x$, where $b > 0$, $b \neq 1$, and $a \neq 0$, where *a* and *b* are real numbers • **common logarithm –** a logarithm with a base of 10; the common logarithm of a number, *x*, is written as $\log x$ • **natural logarithm –** a logarithm with a base of *e*; the natural logarithm of a number, *x*, is written as $\ln x$ **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 482-487:** #1-13	**Possible Instructional Strategies:** • Ensure that students understand that logarithms and exponents are inverses of each other. **Possible Assessment Strategies:** • For each of the following logarithmic functions, state the intercepts, the end behaviour, the domain, the range, and determine whether the function is increasing or decreasing. Then, sketch the graph of each function. a. $y = 2\log x$ b. $y = -\frac{1}{3}\log x$ c. $y = \frac{2}{5}\ln x$ d. $y = -2\ln x$ • The equivalent amount of energy, *E*, in kilowatt-hours, released for an earthquake with a Richter magnitude of *R* is determined by the function. $R = 0.67\ \log\ 0.36E + 1.46$. If an earthquake released 100,000 kWh of energy, what would be its measure on the Richter scale. Round off the answer to one decimal place.

Section 7.5 – Modelling Data Using Logarithmic Functions (pp. 488-500)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **RF2 (D E F)**

After this lesson, students will be expected to:

- represent data using a logarithmic function and interpret the graph to solve a problem

Suggested Problems in *Foundations of Mathematics 12*:

- **pp. 494-499:** #1-10

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- Use graphing technology to demonstrate how to find the logarithmic curve of best fit.

Possible Assessment Strategies:

- A new car bought for $18,000 depreciates in value every year, according to the following table.

YEARS OWNED	VALUE ($)
1	13,940
2	11,431
3	9,373
4	7,686
5	6,303
6	5,168

Determine the equation of the logarithmic regression function for the data. Round off all values to the nearest whole number.

CHAPTER 8
SINUSOIDAL FUNCTIONS

SUGGESTED TIME

11 classes

Section 8.1 – Understanding Angles (pp. 514-520)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

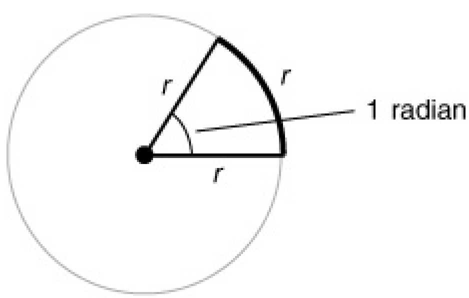
- **RF3 (A)**

After this lesson, students will be expected to:

- estimate and determine benchmarks for angle measure

After this lesson, students should understand the following concept:

- **radian –** the measure of the central angle of a circle subtended by an arc that is the same length as the radius of the circle



Suggested Problems in *Foundations of Mathematics 12*:

- **pp. 519-520:** #1-10

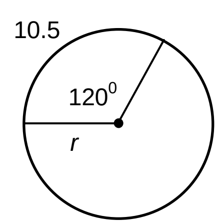
POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- When an angle measure is given without units, remind students that it is assumed to be in radian measure.
- When solving trigonometric problems, ensure that students always verify that their calculators are in the correct mode.
- When calculating arc length, remind students that the angle subtended by the arc must be measured in radians.

Possible Assessment Strategies:

- Convert each of the following degree measures to radians. Express the answer both as a multiple of π, and as an approximate measure, correct to two decimal places.
  a. 225^0
  b. -120^0
- Convert each of the following radian measures to degrees. Where necessary, round off the answer to one decimal place.
  a. $\frac{\pi}{4}$
  b. -2
- Determine the value of *r* in the diagram below. Round off the answer to one decimal place.



Section 8.2 – Exploring Graphs of Periodic Functions (pp. 521-526)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

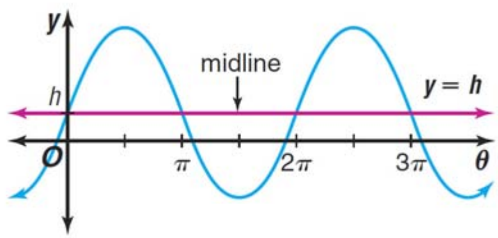
- **RF3 (B)**

After this lesson, students will be expected to:

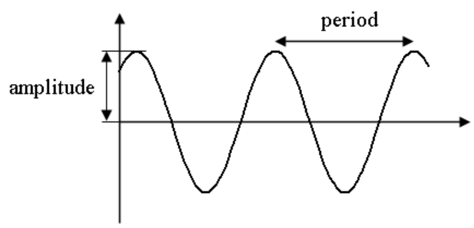
- investigate the characteristics of the graphs of sine and cosine functions

After this lesson, students should understand the following concepts:

- **periodic function –** a function whose graph repeats in regular intervals or cycles
- **midline –** the horizontal line halfway between the maximum and minimum values of a periodic function



- **amplitude –** the distance from the midline to either the maximum or minimum value of a periodic function; the amplitude is always expressed as a positive number
- **period –** the length of the interval of the domain to complete one cycle



Suggested Problems in *Foundations of Mathematics 12*:

- **pp. 524-525:** #1-9

Section 8.3 – The Graphs of Sinusoidal Functions (pp. 527-542)

ELABORATIONS & SUGGESTED PROBLEMS

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **RF3 (B)**

After this lesson, students will be expected to:

- identify characteristics of the graphs of sinusoidal functions

After this lesson, students should understand the following concepts:

- **sinusoidal function –** any periodic function whose graph has the same shape as that of $y = \sin x$
- **frequency –** the number of times that a cycle occurs in a given time period; for example, the fourth A note on a piano has a frequency of 440 Hz, or 440 cycles per second

SINUSOIDAL GRAPH CHARACTERISTICS
The **period** is the horizontal distance between two successive maximum or minimum values: $\text{period} = \dfrac{360^0}{\text{coefficient of } x}$, in degrees or $\text{period} = \dfrac{2\pi}{\text{coefficient of } x}$, in radians
The **equation of the midline** is the average of the maximum and minimum values: $y = \dfrac{\text{maximum value} + \text{minimum value}}{2}$
The **amplitude** is the positive vertical distance between the midline and either a maximum or a minimum value. It is also half of the vertical distance between a maximum and a minimum value: $\text{Amplitude} = \dfrac{\text{maximum value} - \text{minimum value}}{2}$

Suggested Problems in *Foundations of Mathematics 12*:

- **pp. 536-542:** #1-15

POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Possible Instructional Strategies:

- When graphing trigonometric functions, use eight evenly-spaced points within a period in order to get an accurate sketch of the graph. For example, if the period is 360^0, consider using 0^0, 45^0, 90^0, 135^0, 180^0, 225^0, 270^0, and 315^0 as the values for x. Then, finish off the graph by using 360^0 as the final value of x.
- Demonstrate to the class that the graph of $y = \cos x$ is a horizontal translation of the graph of $y = \sin x$.

Possible Assessment Strategies:

- State the amplitude and period, in degrees and radians, of each of the following sinusoidal functions. Then, sketch the graph of each function over two periods.

  a. $y = 0.5 \cos \dfrac{3x}{5}$

  b. $y = -3 \sin \dfrac{2x}{3}$

- Write an equation of a cosine function with the given characteristics.

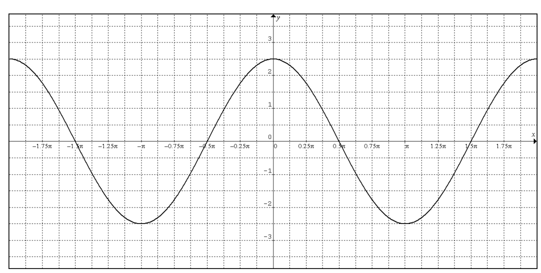
  a. amplitude 3, period 2π

  b. amplitude 7, period 150^0

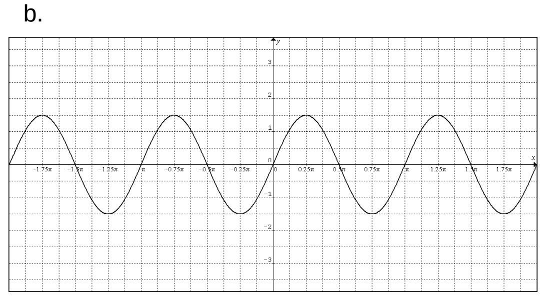
  c. amplitude 0.5, period 720^0

- Determine an equation for each of the following sinusoidal functions.

  a.



  b.



Section 8.4 – The Equations of Sinusoidal Functions (pp. 546-562)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES
Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed: • **RF3 (C D)** **After this lesson, students will be expected to:** • identify characteristics of the equations of sinusoidal functions **SINUSOIDAL FUNCTION CHARACTERISTICS** A sinusoidal function of the form $y = a \sin b(x-c)+d$ or $y = a \cos b(x-c)+d$ has the following characteristics: Amplitude: $\lvert a \rvert$ Period: $\frac{360^0}{b}$ or $\frac{2\pi}{b}$ Midline: $y = d$ Maximum Value: $d+a$ Minimum Value: $d-a$ **Suggested Problems in *Foundations of Mathematics 12*:** • **pp. 558-561:** #1-21	**Possible Instructional Strategies:** • Review with the class the different types of transformations. Make connections between translations, and phase shifts and vertical displacements; and between stretches and amplitudes. **Possible Assessment Strategies:** • Determine the amplitude, period, equation of the midline, horizontal translation, maximum value, and minimum value for each sinusoidal function. Then, sketch the graph of each function over two periods. a. $y = \sin x + 2$ b. $y = 0.5 \sin 2x - 1$ c. $y = -2 \cos \left(x + 30^0\right)$ d $y = \sin 6\left(x - 20^0\right)$ e. $y = \sin \left(x - \frac{\pi}{3}\right) - 1$ f. $y = -3 \cos 4\left(x + \frac{\pi}{4}\right) + 5$ • At a seaport, the depth of the water, *h* metres, at time *t* hours during a certain day is given by the function $h = 2.5 \sin 2\pi\left(\frac{t-1.5}{12.4}\right) + 4.3$ a. What is the minimum depth of the water? When does it occur? b. Estimate the approximate depth of the water at 9:30 A.M. Round off the answer to one decimal place. c. Determine when the water is 4 m deep. Round off the answers to the nearest minute. • In a given region, the number of daylight hours varies, depending on the time of year. This variation can be approximated with a sinusoidal function. The model for a certain region is given by the function $d(t) = 5 \sin \frac{2\pi}{365}(t-95) + 13$ where $d(t)$ is in hours and *t* represents the the day of the year. Find two days when the approximate number of daylight hours is 16 h. Assume that the year is not a leap year.

Section 8.5 – Modelling Data with Sinusoidal functions (pp. 563-578)

ELABORATIONS & SUGGESTED PROBLEMS	POSSIBLE INSTRUCTIONAL & ASSESSMENT STRATEGIES

Specific Curriculum Outcome(s) and Achievement Indicator(s) addressed:

- **RF3 (E F G)**

After this lesson, students will be expected to:

- determine the sinusoidal function that best models a set of data, and use the model to solve a problem

Suggested Problems in *Foundations of Mathematics 12*:

- **pp. 571-577:** #1-12

Possible Instructional Strategies:

- Use graphing technology to demonstrate how to find the sinusoidal curve of best fit.

Possible Assessment Strategies:

- The following table shows the average monthly temperature, in degrees Celsius, for Charlottetown.

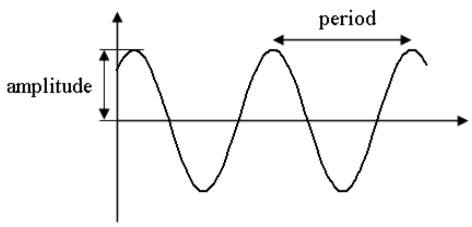
MONTH (*m*)	TEMPERATURE (^{0}C)
1 – January	–7.5
2 – February	–7.8
3 – March	–3.1
4 – April	2.5
5 – May	8.8
6 – June	14.4
7 – July	18.6
8 – August	18.2
9 – September	14.0
10 – October	8.5
11 – November	2.6
12 - December	–3.9

Determine the equation of the sinusoidal regression function for the data. Round off all values to three significant digits.

GLOSSARY OF MATHEMATICAL TERMS

A

- **amortization table –** a table that lists regular payments of a loan and shows how much of each payment goes toward the interest charged and the principal borrowed, as the balance of the loan is reduced to zero
- **amplitude –** the distance from the midline to either the maximum or minimum value of a periodic function; the amplitude is always expressed as a positive number



- **appreciation –** increase in the value of an asset over time
- **asset –** an item or a portion of an item owned; also known as property; assets include such items as real estate, investment portfolios, vehicles, art, and gems

B

- **Bank of Canada prime rate –** a value set by Canada's central bank, which other financial institutions use to set their interest rates
- **biconditional –** a conditional statement whose converse is also true; in logic notation, a biconditional statement is written as "*p* if and only if *q*"; for example, the statement "If a number is even, then it is divisible by 2" is true; the converse "if a number is divisible by 2, then it is even", is also true; the biconditional statement is "A number is even if and only if it is divisible by 2"

C

- **collateral –** an asset that is held as security against the repayment of a loan
- **combination –** a grouping of objects where order does not matter; for example, the two objects *a* and *b* have one combination because *ab* is the same as *ba*
- **common logarithm –** a logarithm with a base of 10; the common logarithm of a number, *x*, is written as $\log x$
- **complement –** all the elements of a universal set that do not belong to a subset of it; for example, $O' = \{0, 2, 4, 6, 8\}$ is the complement of $O = \{1, 3, 5, 7, 9\}$, a subset of the universal set of digits, *D*; the complement is denoted with a prime sign, O'
- **compound interest –** the interest that is earned or paid on both the principal and the accumulated interest
- **compounded annually –** when compound interest is determined or paid yearly
- **compounding period –** the time over which interest is determined; interest can be compounded annually, semi-annually (every 6 months), quarterly (every 3 months), monthly, weekly, or daily
- **conclusion –** the result of a hypothesis; for example, in the statement, "If it is Monday, then it is a school day," the conclusion is "it is a school day"
- **conditional probability –** the probability of an event occurring given that another event has already occurred; $P(B \mid A)$ is the notation for conditional probability; it is read, "the probability that event *B* will occur, given that event *A* has already occurred"; the formula for $P(B \mid A)$ is

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

- **conditional statement –** an "if-then" statement; for example, "If it is Monday, then it is a school day"
- **constant term –** the constant term in a polynomial function in standard form is the term that does not have a variable; for example, in the function $f(x) = 2x^2 + 7x - 5$, the constant term is –5
- **contrapositive –** a statement that is formed by negating both the hypothesis and the conclusion of the converse of a conditional statement; for example, for the statement "If a number is even, then it is divisible by 2," the contrapositive is "If a number is not divisible by 2, then it is not even"

- **converse –** a conditional statement in which the hypothesis and the conclusion are switched; for example, the converse of "If it is Monday, then it is a school day" is "If it is a school day, then it is Monday"
- **counterexample –** an example that disproves a statement; for example, "If it is Monday, then it is a school day" is disproved by the counterexample that there is no school on Thanksgiving Monday; only one counterexample is needed to disprove a statement
- **cubic function –** a polynomial function of the third degree, whose greatest exponent is 3; for example, $f(x) = 5x^3 + x^2 - 4x + 1$
- **curve of best fit –** a curve that best approximates the trend on a scatter plot

D

- **dependent events –** events whose outcomes are affected by each other; for example, if two cards are drawn from a deck without replacement, the outcome of the second event depends on the outcome of the first event (the first card drawn)
- **depreciation –** decrease in the value of an asset over time
- **disjoint –** two or more sets having no elements in common; for example, the set of even numbers and the set of odd numbers are disjoint
- **disposable income –** the amount of income that someone has available to spend after all regular expenses and taxes have been deducted

E

- ***e* –** a constant known as Euler's number; it is an irrational number that equals 2.718...; this number occurs naturally in some situations where a quantity increases continuously, such as in increasing populations
- **element –** an object in a set; for example, 3 is an element of *D*, the set of digits
- **empty set –** a set with no elements; for example, the set of odd numbers divisible by 2 is the empty set; the empty set is denoted by $\{\ \}$ or $\varnothing$
- **end behaviour –** the description of the shape of a graph, from left to right, on the coordinate plane
- **equity –** the difference between the value of an item and the amount still owing on it; can be thought of as the portion owned; for example, if a $25,000 down payment is made on a $230,000 home, $205,000 is still owing and $25,000 is the equity or portion owned
- **exponential decay function –** an exponential function whose *y*-values decrease as you move from left to right along the *x*-axis; for an exponential function of the form $y = a(b)^x$, exponential decay occurs when $a > 0$ and $0 < b < 1$
- **exponential function –** a function of the form $y = a(b)^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$
- **exponential growth function –** an exponential function whose *y*-values increase as you move from left to right along the *x*-axis; for an exponential function of the form $y = a(b)^x$, exponential growth occurs when $a > 0$ and $b > 1$
- **extrapolation –** the process used to estimate a value outside the domain of a set of data, based on a trend

F

- **factorial notation –** a concise representation of the product of consecutive descending natural numbers:

  $$n! = n(n-1)(n-2)\cdots(3)(2)(1)$$

  for example, $4! = 4 \bullet 3 \bullet 2 \bullet 1$
- **fair game –** a game in which all players are equally likely to win; for example, tossing a coin to get heads or tails is a fair game
- **finite set –** a set with a countable number of elements; for example, the set of positive even numbers less than 10, $E = \{2, 4, 6, 8\}$ is finite
- **fixed interest rate –** an interest rate that is guaranteed not to change during the term of an investment or loan
- **frequency –** the number of times that a cycle occurs in a given time period; for example, the fourth A note on a piano has a frequency of 440 Hz, or 440 cycles per second
- **Fundamental Counting Principle –** If there are *a* ways to perform one task and *b* ways to perform another, then there are $a \bullet b$ ways of performing both

- **future value –** the amount, *A*, that an investment will be worth after a specified period of time; the simple interest formula for future value is

$$A = P(1 + rt)$$

where *P* is the principal, *r* is the annual interest rate, and *t* is the time, in years

H

- **hypothesis –** an assumption; for example, in the statement, "If it is Monday, then it is a school day," the hypothesis is "it is Monday"

I

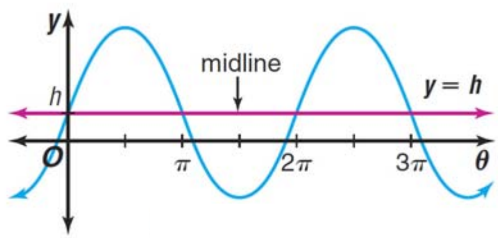
- **infinite set –** a set with an infinite number of elements; for example, the set of natural numbers $N = \{1, 2, 3, ...\}$, is infinite
- **interest –** the amount of money earned on an investment or paid on a loan
- **interpolation –** the process used to estimate a value within the domain of a set of data, based on a trend
- **intersection –** the set of elements that are common to two or more sets; in set notation, $A \cap B$ denotes the intersection of sets *A* and *B*; for example, if $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $A \cap B = \{3\}$
- **inverse –** a statement that is formed by negating both the hypothesis and the conclusion of a conditional statement; for example, for the statement "If a number is even, then it is divisible by 2," the inverse is "If a number is not even, then it is not divisible by 2"

L

- **leading coefficient –** the coefficient of the term with the greatest degree in a polynomial function in standard form; for example, the leading coefficient in the function $f(x) = 2x^3 + 7x$ is 2
- **lease –** a contract for purchasing the use of property, such as a building or vehicle, from another, the lessor, for a specified period
- **line of best fit –** a straight line that best approximates the trend in a scatter plot
- **line of credit –** a pre-approved loan that offers immediate access to funds, up to a pre-defined limit, with a minimum monthly payment based on accumulated interest; a secure line of credit has a lower interest rate because it is guaranteed against the client's assets, usually property
- **logarithmic function –** a function of the form $y = a \log_b x$, where $b > 0$, $b \neq 1$, and $a \neq 0$, where *a* and *b* are real numbers

M

- **maturity –** the contracted end date of an investment or loan, at the end of the term
- **midline –** the horizontal line halfway between the maximum and minimum values of a periodic function



- **mortgage –** a loan, usually for the purchase of real estate, with the real estate purchased used as collateral to secure the loan
- **mutually exclusive –** two or more events that cannot occur at the same time; for example, the sun rising and the sun setting are mutually exclusive events

N

- **natural logarithm –** a logarithm with a base of *e*; the natural logarithm of a number, *x*, is written as $\ln x$

O

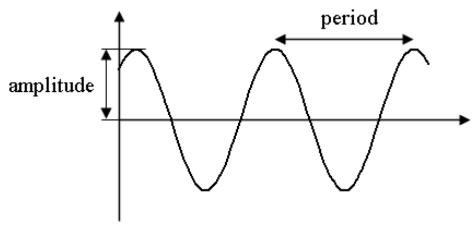
- **odds against –** the ratio of the probability that an event will not occur to the probability that the event will occur, or the ratio of the number of unfavourable outcomes to the number of favourable outcomes

- **odds in favour –** the ratio of the probability that an event will occur to the probability that the event will not occur, or the ratio of the number of favourable outcomes to the number of unfavourable outcomes; if the odds in favour of event *A* occurring are $m : n$, then

$$P(A) = \frac{m}{m+n}$$

P

- **period –** the length of the interval of the domain to complete one cycle



- **periodic function –** a function whose graph repeats in regular intervals or cycles
- **permutation –** an arrangement of distinguishable objects in a definite order; for example, the objects *a* and *b* have two permutations, *ab* and *ba*
- **polynomial function –** a function that contains only the operations of multiplication and addition with real number coefficients, whole number exponents, and two variables; for example, $f(x) = 5x^3 + 6x^2 - 3x + 7$
- **portfolio –** one or more investments held by an individual investor or by a financial organization
- **present value –** the amount that must be invested now to result in a specific future value in a certain time at a given interest rate; the formula for present value is

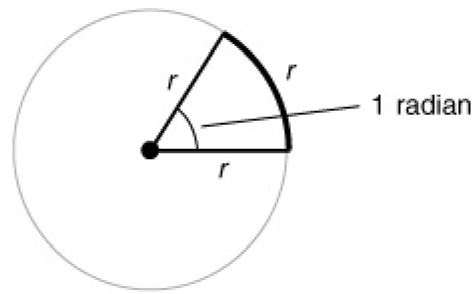
$$P = \frac{A}{(1+i)^n}$$

  where *A* is the future value, *i* is the interest rate per compounding period, and *n* is the number of compounding periods
- **principal –** the original amount of money invested or loaned
- **Principle of Inclusion and Exclusion –** the number of elements in the union of two sets is equal to the sum of the number of elements in each set, less the number of elements in both sets; using set notation, this is written as

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

R

- **radian –** the measure of the central angle of a circle subtended by an arc that is the same length as the radius of the circle



- **rate of return –** the ratio of money earned (or lost) on an investment relative to the amount of money invested, usually expressed as a decimal or a percent
- **regression function –** a line or curve of best fit, developed through a statistical analysis of data
- **Rule of 72 –** a simple formula for estimating the doubling time of an investment; 72 is divided by the annual interest rate as a percent to estimate the doubling time of an investment in years; the Rule of 72 is most accurate when the interest is compounded annually

S

- **scatter plot –** a set of points on a grid, used to visualize a relationship or possible trend in the data
- **set –** a collection of distinguishable objects; for example, the set of whole numbers is $W = \{0, 1, 2, 3, ...\}$
- **simple interest –** the amount of interest earned on an investment or paid on a loan based on the original amount (the principal) and the simple interest rate; the formula for simple interest is

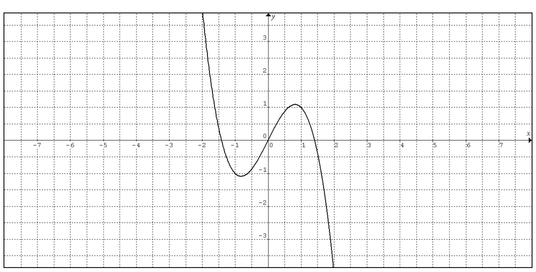
$$I = Prt$$

  where *P* is the principal, *r* is the annual interest rate, and *t* is the time, in years

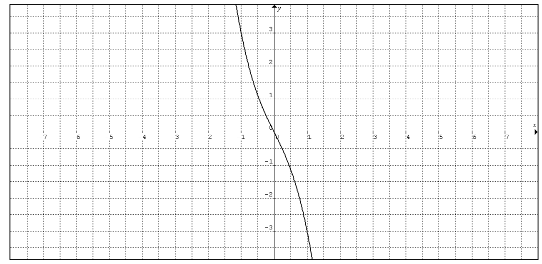
- **sinusoidal function –** any periodic function whose graph has the same shape as that of $y = \sin x$
- **standard form**
  - the standard form for a linear function is $f(x) = ax + b$, where $a \neq 0$
  - the standard form for a quadratic function is $f(x) = ax^2 + bx + c$, where $a \neq 0$
  - the standard form for a cubic function is $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$
- **subset –** a set whose elements all belong to another set; for example, the set of odd digits, $O = \{1, 3, 5, 7, 9\}$, is a subset of *D*, the set of digits; in set notation, the relationship is written as $O \subset D$

T

- **term –** the contracted duration of an investment or loan
- **turning point –** any point where the graph of a function changes from increasing to decreasing, or from decreasing to increasing; for example,
  - this curve has two turning points, since the *y*-values change from decreasing to increasing to decreasing



  - this curve does not have any turning points, since the *y*-values are always decreasing



U

- **union –** the set of all the elements in two or more sets; in set notation, $A \cup B$ denotes the union of sets *A* and *B*; for example, if $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $A \cup B = \{1, 2, 3, 4, 5\}$
- **universal set –** a set of all the elements under consideration for a particular context (also called the sample space); for example, the universal set of digits is $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

SOLUTIONS TO POSSIBLE ASSESSMENT STRATEGIES

SECTION 1.1

- a. $960
  b. $1303.13
  c. $3856.22
- 8.43%
- $141,509.43
- 3 years

SECTION 1.3

-

	Annual Interest Rate	Frequency	*t*	*i*	*n*
a.	4%	annually	10 years	0.04	10
b.	2%	quarterly	9 months	0.005	3
c.	3%	monthly	18 months	0.0025	18

- a. $A = \$448.22,\ I = \$248.22$
  b. $A = \$1209.17,\ I = \$459.17$
- approximately 12 years
- The first option pays 1.045 times the principal after one year, and the second option pays approximately 1.04525 times the principal after one year, so the second option is better.
- $12,175.94
- $88,776.27
- $340,089.36
- $14,532.94

SECTION 1.4

- $23,137.74
- 5.16%

SECTION 1.5

- $38,886.29
- $69,870.71
- His investment at age 65 will be worth $1,307,962.28, so he will have exceeded his goal by $307,962.28.

SECTION 1.6

- $3713.83

SECTION 2.1

- a. $26,878.33
  b. $223.99
- $112.40
- a. $835.08
  b. $131,523.85
  c. $271,523.85

SECTION 2.3

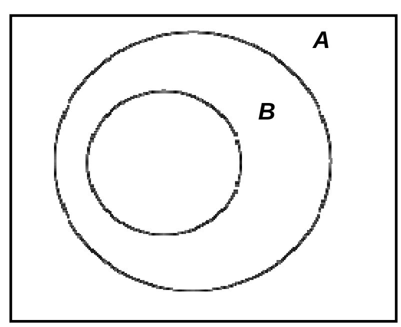
- $30
- a. 39 months
  b. $1974.53
  c. $474.53

SECTION 2.4

- The first option will cost her $9000 in total, and the second option will cost her $9750 in total, so the first option is better.
- $7910.16

SECTION 3.1

- a. $A = \{3,\ 6,\ 9,\ 12,\ 15,\ 18,\ 21,\ 24,\ 27,\ 30,$
  $33,\ 36,\ 39,\ 42,\ 45,\ 48,\ 51,\ 54,\ 57,\ 60\}$
  $B = \{6,\ 12,\ 18,\ 24,\ 30,\ 36,\ 42,\ 48,\ 54,\ 60\}$
  b. $B \subset A$

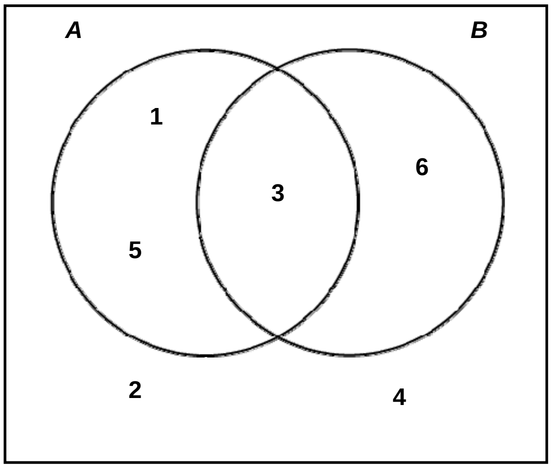


- a. $t_n = n^2$

  b. ➢ 10
  ➢ 10
  ➢ 380

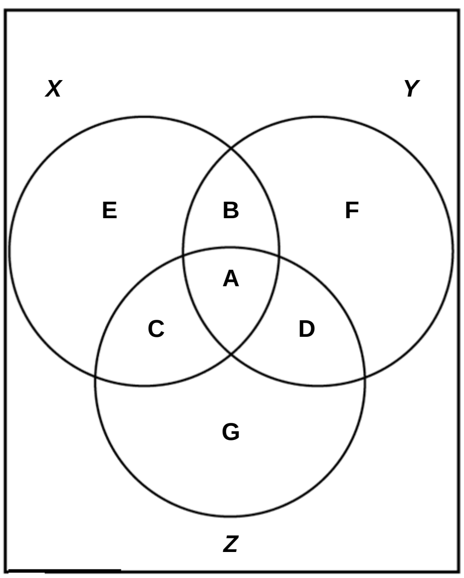
  c. 20

- a.



  b. 2 rolls

-



SECTION 3.3

- a. $E = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24\}$

  $O = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25\}$

  $P = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

  $T = \{3, 6, 9, 12, 15, 18, 21, 24\}$

  b. $n(E) = 12,\ n(O) = 13,\ n(P) = 9,\ n(T) = 8$

  c. $\varnothing$

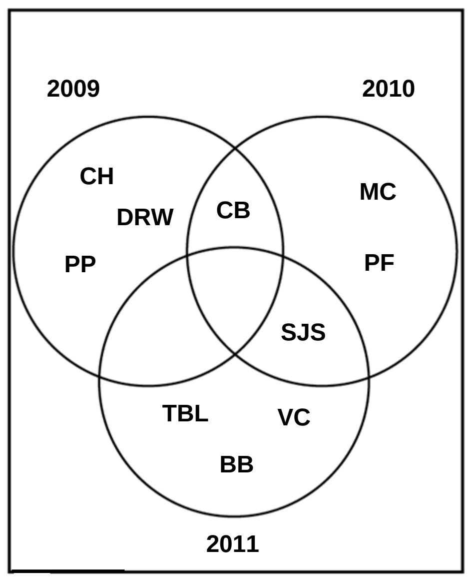
  d. $\{6, 12, 18, 24\}$

  e. 16

  f. E and O

  g. $\{1, 2, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25\}$

-



SECTION 3.4

- a. 16 students

  b. 7 students

  c. 14 students

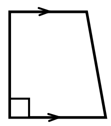
  d. 3 students

  e. 32 students

- 10 employees

SECTION 3.5

- a. false; The quadrilateral could be a trapezoid, such as the one pictured below.



  b. false; The number zero has no reciprocal.

  c. true; Any point in the third quadrant is of the form $(\text{negative, negative})$.

  d. true; If a triangle is right, then one of its angles measures 90^0. Therefore, the sum of the measures of the other two angles is 90^0, which means that each of the other two angles must have a measure less than 90^0, which means they are both acute. Therefore, there cannot be any obtuse angle in a right triangle.

- a. If it is cloudy, then it is raining. False; If it is cloudy, other forms of precipitation could be falling, or there may not be any precipitation falling.

  b. If the absolute value of a real number is positive, then the real number is positive. False; This statement is false for any negative real number.

  c. If $b^2 > 1$, then $b > 1$. False; If $b < -1$, then the hypothesis is true, but the conclusion is false.

  d. If a right triangle has two angles measuring 45^0, then it is isosceles. True; Since this triangle has two congruent angles, it would have two equal sides, making it isosceles.

- a. If an angle is right, then it has a measure of 90^0. Biconditional; An angle is right if and only if it has a measure of 90^0.

  b. If a decimal is terminating, then it can be written as a fraction. Not biconditional, since it is not true for fractions that whose decimal equivalents are repeating decimals, such as $\frac{1}{3}$.

  c. If a figure is a square, then it has four sides of equal length. Not biconditional, since a rhombus also is a figure with four sides of equal length.

SECTION 3.6

- a. **Inverse:** If it is not raining, then it is not cloudy. False; If it is not raining, it can still be cloudy.

  Contrapositive: If it is not cloudy, then it is not raining. True; If it is not cloudy, no rain can fall.

  b. **Inverse:** If a real number is not positive, then its absolute value is not positive. False; This statement is false for negative numbers.

  Contrapositive: If the absolute value of a real number is not positive, then the real number is not positive. True; If the absolute value of a real number is not positive, then the number must be zero. The absolute value of zero is zero, which is not positive.

  c. **Inverse:** If $b \le 1$, then $b^2 \le 1$. False; If $b < -1$, then the hypothesis is true, but the conclusion is false.

  Contrapositive: If $b^2 \le 1$, then $b \le 1$. True; If $b^2 \le 1$, then $-1 \le b \le 1$, which is within the solution set of $b \le 1$.

  d. **Inverse:** If a right triangle is not isosceles, then it does not have two angles measuring 45^0. True; If a right triangle is not isosceles, then it must be scalene, since it cannot be equilateral. Scalene triangles have three angles which are all different, therefore the conclusion is true.

  Contrapositive: If a right triangle does not have two angles measuring 45^0, then it is not isosceles. True; The sum of the measures of the two non-right angles in a right triangle is 90^0. Since these angles cannot measure 45^0, these two acute angles cannot be congruent. Therefore, the measures of the three angles of the triangle must all be different, which means that the triangle cannot be isosceles.

- a. Yes; If $x = -1$ is substituted into the equation $x^2 = 1$, the resulting equation is true.

  b. No; If $x^2 = 1$, then $x = \pm 1$.

  c. No; If $x \ne -1$, then x^2 could still equal 1 if $x = 1$.

  d. Yes; If $x^2 \ne 1$, then $x \ne \pm 1$.

- a. true

b. false

SECTION 4.1

- a. 78,000 license plates
  b. 598,000 license plates
- 4500 four-digit even numbers
- 16 ways
- a. 625 cards
  b. 120 cards
- 12 ways
- 720 ways
- 60 three-digit numbers
- a. 720 arrangements
  b. 288 arrangements

SECTION 4.2

- a. 280
  b. 240
- 120 ways
- 5040 ways
- 40,320 ways
- 39,916,800 ways
- a. 720 arrangements
  b. 120 arrangements
  c. 24 arrangements
  d. 240 arrangements
- 479,001,599 incorrect arrangements
- a. $n = 10$
  b. $n = 6$

SECTION 4.3

- 840 permutations
- 1680 ways
- 9,034,502,400 ways
- 11,880 ways
- a. 311,875,200 ways
  b. 132,600 ways
  c. 7,893,600 ways
  d. 95,040 ways
- 6 digits
- 15 numbers

SECTION 4.4

- 1260 ways
- 56 routes
- 151,200 permutations
- 28 orders
- 19,380 orders
- a. 720 ways
  b. 20 ways
  c. 90 ways
- 210 even numbers

SECTION 4.6

- a. 60 permutations
  b. 10 combinations
  c. The answer in (b) is equal to the answer in (a) divided by $3!$.
- 13,983,816 lottery tickets
- 4495 sampler dishes
- 6.35×10^{11} hands
- a. 324,632 groups
  b. 496 groups
  c. 324,136 groups
- $n = 4$
- 105 ways
- 4512 hands

SECTION 4.7

- 756 ways
- 720 ways
- 210 groups
- 20 three-digit numbers
- 11,760 ways
- 5040 words
- 5 questions
- 792 ways
- a. 66,045 ways
  b. 23,023 groupings

SECTION 5.2

- a. odds in favour: $1:6$; odds against: $6:1$
  b. odds in favour: $1:7$; odds against: $7:1$
  c. odds in favour: $3:10$; odds against: $10:3$

d. odds in favour: $1:1$; odds against: $1:1$
e. odds in favour: $1:5$; odds against: $5:1$
f. odds in favour: $1:13,983,815$; odds against: $13,983,815:1$

- $\frac{3}{7}$
- a. $3:7$
  b. $7:3$

SECTION 5.3

- 0.0743
- a. $\frac{7}{102}$
  b. $\frac{4}{17}$
- $\frac{7}{30}$
- $\frac{3}{10}$
- a. $\frac{14}{969}$
  b. $\frac{1001}{4845}$
- $\frac{1}{3}$

SECTION 5.4

- a. mutually exclusive
  b. not mutually exclusive
  c. not mutually exclusive
  d. mutually exclusive
- $\frac{2}{3}$
- $\frac{2}{3}$
- $\frac{7}{13}$
- $\frac{10}{13}$
- 0.16

SECTION 5.5

- a. dependent events
  b. independent events
  c. independent events
  d. dependent events
- $\frac{11}{221}$
- a. $\frac{4}{25}$
  b. $\frac{3}{10}$
- 62.5%

SECTION 5.6

- $\frac{1}{4}$
- 0.48
- a. 0.12
  b. 0.42
- a. 84.55%
  b. 0.55%
  c. 99.45%

SECTION 6.2

- a. polynomial
  b. not a polynomial
  c. polynomial
  d. not a polynomial
  e. not a polynomial
- a. Degree: 3; Leading Coefficient: 1; Constant Term: –4
  b. Degree: 4; Leading Coefficient: 2; Constant Term: 2
  c. Degree: 2; Leading Coefficient: –1; Constant Term: 3
- odd-degree function; negative
- a. quadratic function with positive leading coefficient; extends up into Quadrant II and up into Quadrant I
  b. cubic function with negative leading coefficient; extends from Quadrant II to Quadrant IV

SECTION 6.3

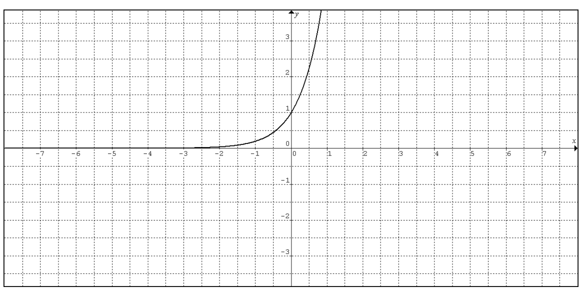
- a. $y = 0.442x + 5.51$
  - b. 78 kg
  - c. 166 cm
- $y = 0.921x - 9.61$

SECTION 6.4

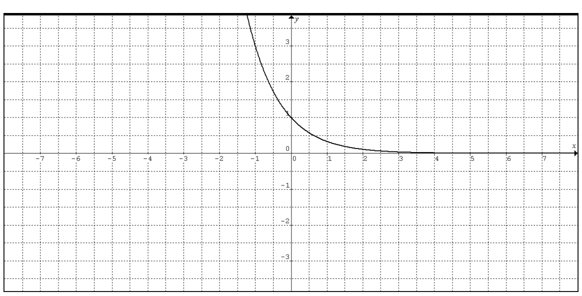
- a. $y = 1.49x^2 + 0.0487x - 0.0765$
  - b. 10.9 m^2
  - c. 4.5 m

SECTION 7.2

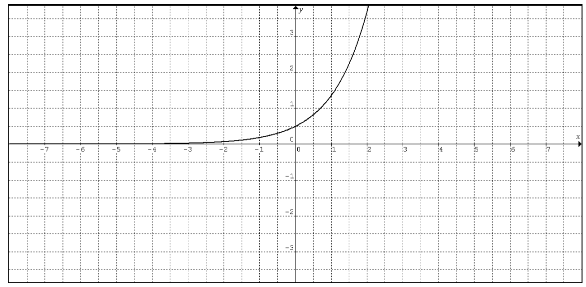
- a. not exponential
  - b. exponential
  - c. exponential
- a. y-intercept: 1; Curve extends from Quadrant II to Quadrant I; $D = \{x \mid x \in R\}$; $R = \{y \mid y > 0\}$; Increasing



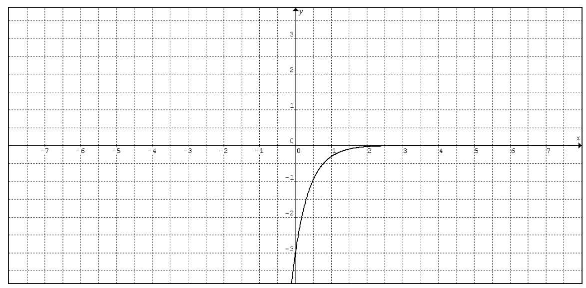
  - b. y-intercept: 1; Curve extends from Quadrant II to Quadrant I; $D = \{x \mid x \in R\}$; $R = \{y \mid y > 0\}$; Decreasing



  - c. y-intercept: $\frac{1}{2}$; Curve extends from Quadrant II to Quadrant I; $D = \{x \mid x \in R\}$; $R = \{y \mid y > 0\}$; Increasing



  - d. y-intercept: –3; Curve extends from Quadrant III to Quadrant IV; $D = \{x \mid x \in R\}$; $R = \{y \mid y < 0\}$; Increasing



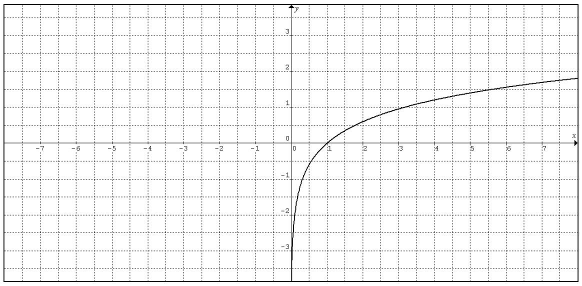
- a. 1200 bacteria
  - b. 19,200 bacteria
- a. 62.5 g
  - b. 3.90625 g

SECTION 7.3

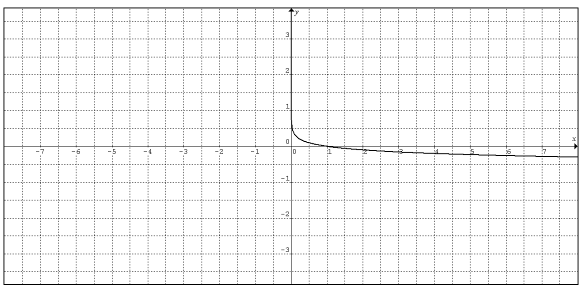
- a. $y = (6.096)(1.017)^x$
  - b. 76.4 million
  - c. No, because the natural rate of growth to Canada's population has decreased over time, and much of Canada's population growth is due to immigration, which does not increase at a naturally consistent rate. As well, the supply of resources in Canada is finite, and at some point would not be able to support an ever-increasing population.

SECTION 7.4

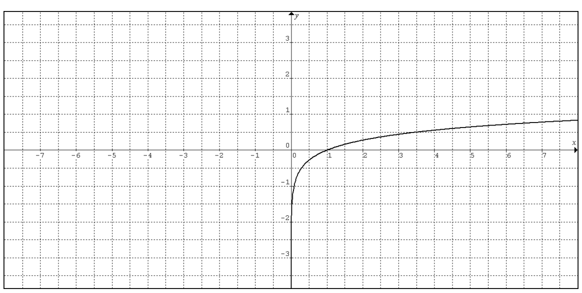
- a. x-intercept: 1; Curve extends from Quadrant IV to Quadrant I; $D = \{x \mid x > 0\}$;
  $R = \{y \mid y \in R\}$; Increasing



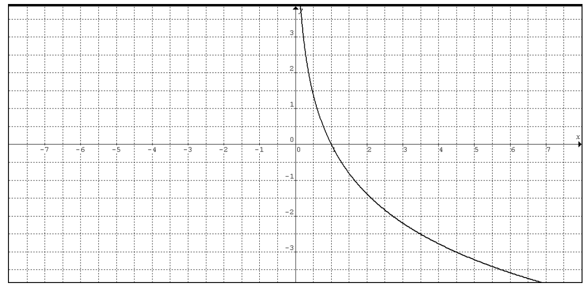
  b. x-intercept: 1; Curve extends from Quadrant I to Quadrant IV; $D = \{x \mid x > 0\}$;
  $R = \{y \mid y \in R\}$; Decreasing



  c. x-intercept: 1; Curve extends from Quadrant IV to Quadrant I; $D = \{x \mid x > 0\}$;
  $R = \{y \mid y \in R\}$; Increasing



  d. x-intercept: 1; Curve extends from Quadrant I to Quadrant IV; $D = \{x \mid x > 0\}$;
  $R = \{y \mid y \in R\}$; Decreasing



- 4.5

SECTION 7.5

- $y = 14,390 - 4391 \ln x$

SECTION 8.1

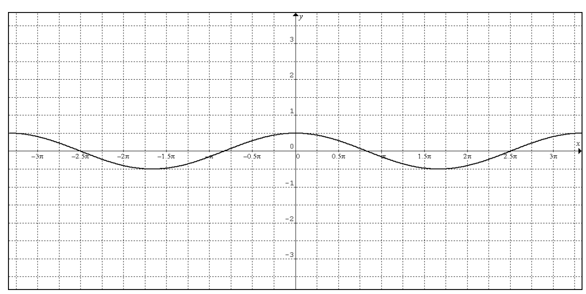
- $\frac{5\pi}{4}$, 3.93

  b. $-\frac{2\pi}{3}$, –2.09
- a. 45^0

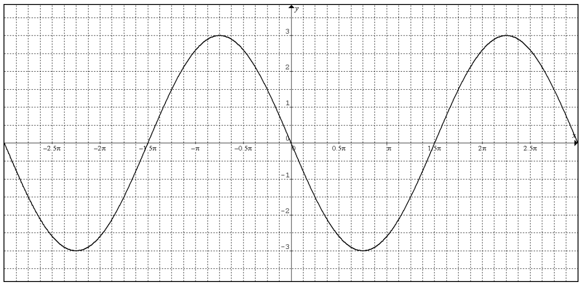
  b. -114.6^0
- 5.0

SECTION 8.3

- a. Amplitude: 0.5; Period: 600^0 or $\frac{10\pi}{3}$



b. Amplitude: 3; Period: 540^0 or 3π



- a. $y = 3\cos x$

  b. $y = 7\cos\dfrac{12x}{5}$

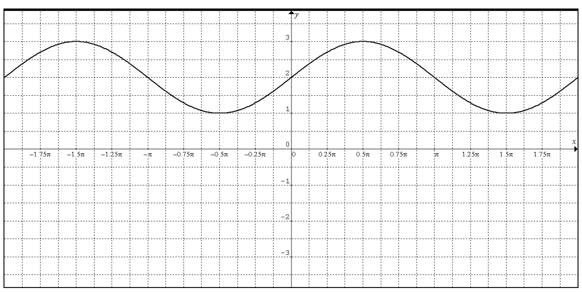
  c. $y = 0.5\cos\dfrac{x}{2}$

- a. $y = 2.5\cos x$

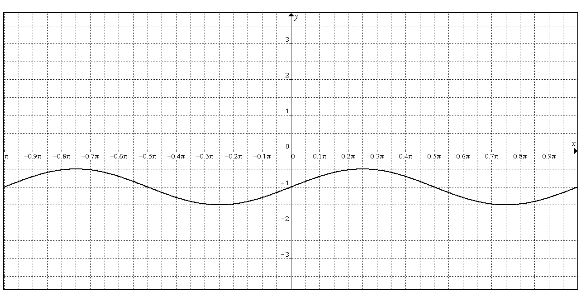
  b. $y = 1.5\sin 2x$

SECTION 8.4

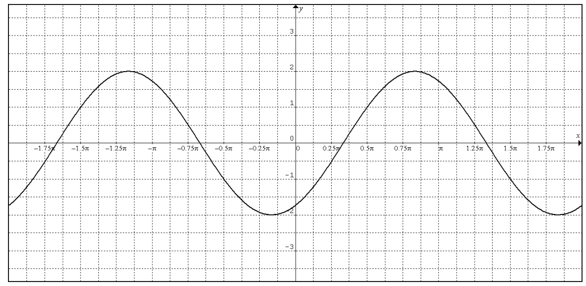
- a. Amplitude: 1; Period: 2π; Midline: $y = 2$; Horizontal translation: none; Vertical translation: upward 2; Maximum value: 3; Minimum value: 1



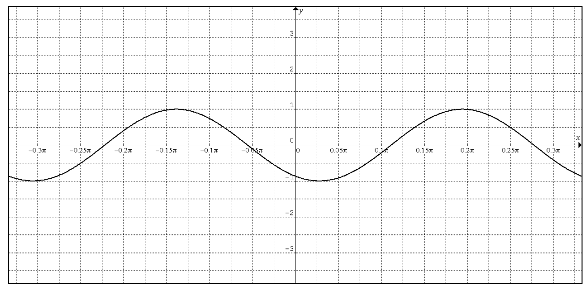
b. Amplitude: 0.5; Period: π; Midline: $y = -1$; Horizontal translation: none; Vertical translation: downward 1; Maximum value: –0.5; Minimum value: –1.5



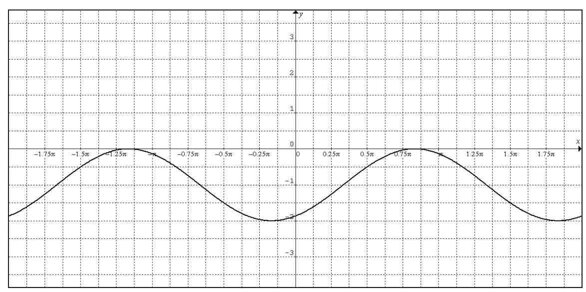
c. Amplitude: 2; Period: 360^0; Midline: $y = 0$; Horizontal translation: left 30^0; Vertical translation: none; Maximum value: 2; Minimum value: –2



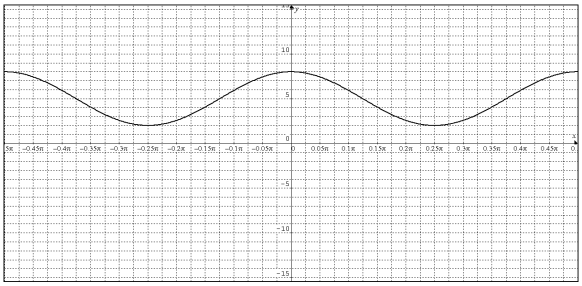
d. Amplitude: 1; Period: 60^0; Midline: $y = 0$; Horizontal translation: right 20^0; Vertical translation: none; Maximum value: 1; Minimum value: –1



e. Amplitude: 1; Period: 2π; Midline: $y = -1$; Horizontal translation: right $\dfrac{\pi}{3}$; Vertical translation: downward 1; Maximum value: 0; Minimum value: –2



f. Amplitude: 3; Period: $\frac{\pi}{2}$; Midline: $y = 5$; Horizontal translation: left $\frac{\pi}{4}$; Vertical translation: upward 5; Maximum value: 8; Minimum value: 2



- a. 1.8 m; 10:48 A.M. and 11:12 P.M.
  b. 2.3 m
  c. 1:16 A.M., 7:56 A.M., 1:40 P.M., and 8:20 P.M.
- May 12, August 28

SECTION 8.5

- $y = 13.2 \sin 0.515(x - 4.42) + 5.24$

APPENDIX

MATHEMATICS RESEARCH PROJECT

➢ Introduction

A research project can be a very important part of a mathematics education. Besides the greatly increased learning intensity that comes from personal involvement with a project, and the chance to show universities, colleges, and potential employers the ability to initiate and carry out a complex task, it gives the student an introduction to mathematics as it is – a living and developing intellectual discipline where progress is achieved by the interplay of individual creativity and collective knowledge. A major research project must successfully pass through several stages. Over the next few pages, these stages are highlighted, together with some ideas that students may find useful when working through such a project.

➢ Creating an Action Plan

As previously mentioned, a major research project must successfully pass though several stages. The following is an outline for an action plan, with a list of these stages, a suggested time, and space for students to include a probable time to complete each stage.

STAGE	SUGGESTED TIME	PROBABLE TIME
Select the topic to explore.	1 – 3 days	
Create the research question to be answered.	1 – 3 days	
Collect the data.	5 – 10 days	
Analyse the data.	5 – 10 days	
Create an outline for the presentation.	2 – 4 days	
Prepare a first draft.	3 – 10 days	
Revise, edit and proofread.	3 – 5 days	
Prepare and practise the presentation.	3 – 5 days	

Completing this action plan will help students organize their time and give them goals and deadlines that they can manage. The times that are suggested for each stage are only a guide. Students can adjust the time that they will spend on each stage to match the scope of their projects. For example, a project based on primary data (data that they collect) will usually require more time than a project based on secondary data (data that other people have collected and published). A student will also need to consider his or her personal situation – the issues that each student deals with that may interfere with the completion of his or her project. Examples of these issues may include:

- a part-time job;
- after-school sports and activities;
- regular homework;
- assignments for other courses;
- tests in other courses;
- time they spend with friends;
- family commitments;
- access to research sources and technology.

➢ Selecting the Research Topic

To decide what to research, a student can start by thinking about a subject and then consider specific topics. Some examples of subjects and topics may be:

SUBJECT	TOPIC
Entertainment	• effects of new electronic devices • file sharing
Health care	• doctor and/or nurse shortages • funding
Post-secondary education	• entry requirements • graduate success
History of Western and Northern Canada	• relations among First Nations • immigration

It is important to take the time to consider several topics carefully before selecting a topic for a research project. The following questions will help a student determine if a topic that is being considered is suitable.

- **Does the topic interest the student?**

  Students will be more successful if they choose a topic that interests them. They will be more motivated to do research, and they will be more attentive while doing the research. As well, they will care more about the conclusions they make.

- **Is the topic practical to research?**

  If a student decides to use first-hand data, can the data be generated in the time available, with the resources available? If a student decides to use second-hand data, are there multiple sources of data? Are the sources reliable, and can they be accessed in a timely manner?

- **Is there an important issue related to the topic?**

  A student should think about the issues related to the topic that he or she has chosen. If there are many viewpoints on an issue, they may be able to gather data that support some viewpoints but not others. The data they collect from all viewpoints should enable them to come to a reasoned conclusion.

- **Will the audience appreciate the presentation?**

  The topic should be interesting to the intended audience. Students should avoid topics that may offend some members of their audience.

➢ Creating the Research Question or Statement

A well-written research question or statement clarifies exactly what the project is designed to do. It should have the following characteristics:

- The research topic is easily identifiable.

- The purpose of the research is clear.
- The question or statement is focused. The people who are listening to or reading the question or statement will know what the student is going to be researching.

A good question or statement requires thought and planning. Below are three examples of initial questions or statements and how they were improved.

UNACCEPTABLE QUESTION OR STATEMENT	WHY?	ACCEPTABLE QUESTION OR STATEMENT
Is mathematics used in computer technology?	Too general	What role has mathematics played in the development of computer animation?
Water is a shared resource.	Too general	Homes, farms, ranches, and businesses east of the Rockies all use runoff water. When there is a shortage, that water must be shared.
Do driver's education programs help teenagers parallel park?	Too specific, unless the student is generating his or her own data	Do driver's education programs reduce the incidence of parking accidents?

The following checklist can be used to determine if the research question or statement is effective.

- Does the question or statement clearly identify the main objective of the research? After the question or statement is read to someone, can they tell what the student will be researching?
- Is the student confident that the question or statement will lead him or her to sufficient data to reach a conclusion?
- Is the question or statement interesting? Does it make the student want to learn more?
- Is the topic that the student chose purely factual, or is that student likely to encounter an issue, with different points of view?

➢ Carrying Out the Research

As students continue with their projects, they will need to conduct research and collect data. The strategies that follow will help them in their data collection.

There are two types of data that students will need to consider – primary and secondary. Primary data is data that the student collects himself or herself using surveys, interviews and direct observations. Secondary data is data that the student obtains through other sources, such as online publications, journals, magazines, and newspapers.

Both primary and secondary data have their advantages and disadvantages. Primary data provide specific information about the research question or statement, but may take time to collect and process. Secondary data is usually easier to obtain and can be analysed in less time. However, because the data was originally gathered for other purposes, a student may need to sift through it to find exactly what he or she is looking for.

The type of data chosen can depend on many factors, including the research question, the skills of the student, and available time and resources. Based on these and other factors, the student may chose to use primary data, secondary data, or both.

When collecting primary data, the student must ensure the following:

- For surveys, the sample size must be reasonably large and the random sampling technique must be well designed.
- For surveys and interviews, the questionnaires must be designed to avoid bias.
- For experiments and studies, the data must be free from measurement bias.
- The data must be compiled accurately.

When collecting secondary data, the student should explore a variety of resources, such as:

- textbooks, and other reference books;
- scientific and historical journals, and other expert publications;
- the Internet;
- library databases.

After collecting the secondary data, the student must ensure that the source of the data is reliable:

- If the data is from a report, determine what the author's credentials are, how recent the data is, and whether other researchers have cited the same data.
- Be aware that data collection is often funded by an organization with an interest in the outcome or with an agenda that it is trying to further. Knowing which organization has funded the data collection may help the student decide how reliable the data is, or what type of bias may have influenced the collection or presentation of the data.
- If the data is from the Internet, check it against the following criteria:
    - authority – the credentials of the author should be provided;
    - accuracy – the domain of the web address may help the student determine the accuracy;
    - currency – the information is probably being accurately managed if pages on a site are updated regularly and links are valid.

➢ Analysing the Data

Statistical tools can help a student analyse and interpret the data that is collected. Students need to think carefully about which statistical tools to use and when to use them, because other people will be scrutinizing the data. A summary of relevant tools follows.

Measures of central tendency will give information about which values are representative of the entire set of data. Selecting which measure of central tendency (mean, median, or mode) to use depends on the distribution of the data. As the researcher, the student must decide which measure most accurately describes the tendencies of the population. The following criteria should be considered when deciding upon which measure of central tendency best describes a set of data.

- Outliers affect the mean the most. If the data includes outliers, the student should use the median to avoid misrepresenting the data. If the student chooses to use the mean, the outliers should be removed before calculating the mean.

- If the distribution of the data is not symmetrical, but instead strongly skewed, the median may best represent the set of data.
- If the distribution of the data is roughly symmetrical, the mean and median will be close, so either may be appropriate to use.
- If the data is not numeric (for example, colour), or if the frequency of the data is more important than the values, use the mode.

Both the range and the standard deviation will give the student information about the distribution of data in a set. The range of a set of data changes considerably because of outliers. The disadvantage of using the range is that it does not show where most of the data in a set lies – it only shows the spread between the highest and the lowest values. The range is an informative tool that can be used to supplement other measures, such as standard deviation, but it is rarely used as the only measure of dispersion.

Standard deviation is the measure of dispersion that is most commonly used in statistical analysis when the mean is used to calculate central tendency. It measures the spread relative to the mean for most of the data in the set. Outliers can affect standard deviation significantly. Standard deviation is a very useful measure of dispersion for symmetrical distributions with no outliers. Standard deviation helps with comparing the spread of two sets of data that have approximately the same mean. For example, the set of data with the smaller standard deviation has a narrower spread of measurement around the mean, and therefore has comparatively fewer high or low scores, than a set of data with a higher standard deviation.

When working with several sets of data that approximate normal distributions, you can use *z*-scores to compare the data values. A *z*-score table enables a student to find the area under a normal distribution curve with a mean of zero and a standard deviation of one. To determine the *z*-score for any data value in a set that is normally distributed, the formula $z = \frac{x - \bar{x}}{s}$ can be used where x is any observed data value in the set, $\bar{x}$ is the mean of the set, and is s is the standard deviation of the set.

When analysing the results of a survey, a student may need to interpret and explain the significance of some additional statistics. Most surveys and polls draw their conclusions from a sample of a larger group. The margin of error and the confidence level indicate how well a sample represents a larger group. For example, a survey may have a margin of error of plus or minus 3% at a 95% level of confidence. This means that if the survey were conducted 100 times, the data would be within 3 percent points above or below the reported results in 95 of the 100 surveys.

The size of the sample that is used for a poll affects the margin of error. If a student is collecting data, he or she must consider the size of the sample that is needed for a desired margin of error.

➢ Identifying Controversial Issues

While working on a research project, a student may uncover some issues on which people disagree. To decide on how to present an issue fairly, he or she should consider some questions to ask as the research proceeds.

- **What is the issue about?**

  The student should identify which type of controversy has been uncovered. Almost all controversy revolves around one or more of the following:
  - Values – What should be? What is best?
  - Information – What is the truth? What is a reasonable interpretation?

  - Concepts – What does this mean? What are the implications?

- **What positions are being taken on the issue?**

  The student should determine what is being said and whether there is reasonable support for the claims being made. Some questions to ask are:
  - Would you like that done to you?
  - Is the claim based on a value that is generally shared?
  - Is there adequate information?
  - Are the claims in the information accurate?
  - Are those taking various positions on the issue all using the same term definitions?

- **What is being assumed?**

  Faulty assumptions reduce legitimacy. The student can ask:
  - What are the assumptions behind an argument?
  - Is the position based on prejudice or an attitude contrary to universally held human values, such as those set out in the United Nations Declaration of Human Rights?
  - Is the person who is presenting a position or an opinion an insider or an outsider?

- **What are the interests of those taking positions?**

  The student should try to determine the motivations of those taking positions on the issue. What are their reasons for taking their positions? The degree to which the parties involved are acting in self-interest could affect the legitimacy of their opinions.

➢ The Final Product and Presentation

The final presentation should be more than just a factual written report of the information gathered from the research. To make the most of the student's hard work, he or she should select a format for the final presentation that suits his or her strengths, as well as the topic.

To make the presentation interesting, a format should be chosen that suits the student's style. Some examples are:

- a report on an experiment or an investigation;
- a short story, musical performance, or play;
- a web page;
- a slide show, multimedia presentation, or video;
- a debate;
- an advertising campaign or pamphlet;
- a demonstration or the teaching of a lesson.

Sometimes, it is also effective to give the audience an executive summary of the presentation. This is a one-page summary of the presentation that includes the research question and the conclusions that were made.

Before giving the presentation, the student can use these questions to decide if the presentation will be effective.

- Did I define my topic well? What is the best way to define my topic?
- Is my presentation focused? Will my classmates find it focused?
- Did I organize my information effectively? Is it obvious that I am following a plan in my presentation?
- Am I satisfied with my presentation? What might make it more effective?
- What unanswered questions might my audience have?

➢ Peer Critiquing of Research Projects

After the student has completed his or her research for the question or statement being studied, and the report and presentation have been delivered, it is time to see and hear the research projects developed by other students. However, rather than being a passive observer, the student should have an important role – to provide feedback to his or her peers about their projects and presentations.

Critiquing a project does not involve commenting on what might have been or should have been. It involves reacting to what is seen and heard. In particular, the student should pay attention to:

- strengths and weaknesses of the presentation;
- problems or concerns with the presentation.

While observing each presentation, students should consider the content, the organization, and the delivery. They should take notes during the presentation, using the following rating scales as a guide. These rating scales can also be used to assess the presentation.

Content

Shows a clear sense of audience and purpose.	1	2	3	4	5
Demonstrates a thorough understanding of the topic.	1	2	3	4	5
Clearly and concisely explains ideas.	1	2	3	4	5
Applies knowledge and skills developed in this course.	1	2	3	4	5
Justifies conclusions with sound reasoning.	1	2	3	4	5
Uses vocabulary, symbols and diagrams correctly.	1	2	3	4	5

Organization

Presentation is clearly focused.	1	2	3	4	5
Engaging introduction includes the research question, clearly stated.	1	2	3	4	5
Key ideas and information are logically presented.	1	2	3	4	5
There are effective transitions between ideas and information.	1	2	3	4	5
Conclusion follows logically from the analysis and relates to the question.	1	2	3	4	5

Delivery

Speaking voice is clear, relaxed, and audible.	1	2	3	4	5
Pacing is appropriate and effective for the allotted time.	1	2	3	4	5
Technology is used effectively.	1	2	3	4	5
Visuals and handouts are easily understood.	1	2	3	4	5
Responses to audience's questions show a thorough understanding of the topic.	1	2	3	4	5

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